MECHANICS RESEARCH COMMUNICATIONS Vol. 7(5),309-315,1980. Printed in the USA.
0093-6413/80/050309-07$02.00/0

THE INTERFACE CRACK LOADED IN BENDING

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(Received 14 May 1980; accepted for print 5 June 1980)

Introduction

The crack in a bending field was recently considered by Bowie and Freese [1], who assumed conditions of stick over the closed segment of the crack. Their solution is valid for relatively large coefficients of friction f. Comninou and Dundurs [2] solved the same problem but for smaller values of friction, for which slip occurs.

In this paper we consider the interface crack in a pure bending field. Since the interface crack must necessarily have closed tips [3,4], there is a contact zone on both sides of its open part, and the contact zone that runs into the compressive field is expected to be much larger. In the following analysis we neglect friction ($f = 0$) and deal only with that aspect of the problem which is influenced by the material interface. The effect of friction was studied in [5] for the interface crack loaded in shear. It was found that the main consequence of friction, besides adding considerable computational complexity to the problem, is to slightly increase the extent of the contact zones and the maximum value of the gap.

Formulation

The geometry of the problem is shown in Fig. 1. The interface crack of length 2L is open in the interval (-a,b) and its faces are in frictionless contact over the intervals (-L,-a) and (b,L). Although the left tip of the crack is in the tension part of the bending field, it must remain closed because a

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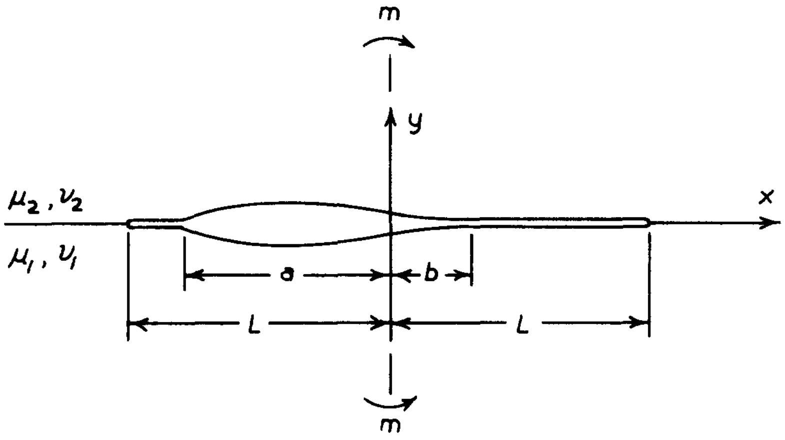

FIG. 1
Geometry of the interface crack.

direct transition from interface bond to separation leads to oscillatory singularities and material overlapping [3,4]. For the same reason the right crack tip is also closed. The effect of the compressive field due to bending is merely to enlarge the right contact zone.

The bending field in the absence of the crack is

$$\sigma_{yy} = - mx, \quad \sigma_{xy} = 0 \tag{1}$$

where m is a constant.

Since the analysis follows closely that of [3,6] we omit some of the details. We represent the crack as an array of distributed glide and climb dislocations with densities $B_x(x)$ and $B_y(x)$. Denoting the gap between the crack faces by

$$g(x) = u_y^{(2)}(x,0) - u_y^{(1)}(x,0) \tag{2}$$

and the tangential shift of the upper crack face with respect to the lower by

$$h(x) = u_x^{(2)}(x,0) - u_x^{(1)}(x,0), \tag{3}$$

the dislocation densities are

$$B_x(x) = -\frac{dh}{dx},\ B_y(x) = -\frac{dg}{dx} \tag{4}$$

Thus $B_x(x)$ and $B_y(x)$ vanish outside the intervals (-L,L) and (-a,b) respectively. The boundary conditions of vanishing normal tractions in (-a,b) and vanishing shear tractions in (-L,L) yield [3]

$$S(x) = C\left[\beta B_y(x) - \frac{1}{\pi}\int_{-L}^{L} \frac{B_x(\xi)}{\xi - x}\,d\xi\right] = 0, \quad -L < x < L \tag{5}$$

$$N(x) = -mx - C\left[\beta B_x(x) + \frac{1}{\pi}\int_{-a}^{b} \frac{B_y(\xi)}{\xi - x}\,d\xi\right] = 0, \ -a < x < b \tag{6}$$

where

$$C = \frac{2\mu_1(1 + \alpha)}{(\kappa_1 + 1)(1 - \beta^2)} \tag{7}$$

$$\alpha = \frac{\mu_2(\kappa_1 + 1) - \mu_1(\kappa_2 + 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)}, \quad \beta = \frac{\mu_2(\kappa_1 - 1) - \mu_1(\kappa_2 - 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)}, \tag{8}$$

and $\kappa = 3 - 4\nu$ for plane strain. In addition we must require single-valued displacements or

$$\int_{-L}^{L} B_x(\xi)d\xi = 0, \quad \int_{-a}^{b} B_y(\xi)d\xi = 0 \tag{9,10}$$

Solving formally (5) for $B_x(x)$ and using (9) we obtain

$$B_x(s) = -\frac{\beta}{\pi}(1 - s^2)^{-\frac{1}{2}} \int_{\gamma_1}^{\gamma_2} \frac{B_y(r)(1 - r^2)^{\frac{1}{2}}}{r - s}\,dr, \quad |s| < 1 \tag{11}$$

where

$$x = Ls, \quad \xi = Lr, \quad \gamma_1 = -\frac{a}{L}, \quad \gamma_2 = \frac{b}{L} \tag{12}$$

Substituting (11) into (6) yields

$$(1 - \beta^2) \int_{\gamma_1}^{\gamma_2} \frac{B_y(r)}{r - s} dr - \beta^2 \int_{-1}^{1} \frac{B_y(r)}{r - s} \left[\left(\frac{1 - r^2}{1 - s^2}\right)^{\frac{1}{2}} - 1\right] dr$$

$$= -\frac{mL\pi s}{C}, \quad |s| < 1 \tag{13}$$

Of interest are the normal tractions in the contact zones and the stress intensity factors:

$$N(s) = -mLs + \frac{C}{\pi}\left[\int_{\gamma_1}^{\gamma_2} \frac{B_y(r)}{r - s} \left[\beta^2\left(\frac{1 - r^2}{1 - s^2}\right)^{\frac{1}{2}} - 1\right] dr\right],$$

$$-1 < s < \gamma_1, \; \gamma_2 < s < 1 \tag{14}$$

$$K_2(\pm L) = \lim_{x \to \pm L} [2(L \mp x)]^{\frac{1}{2}} \sigma_{xy}(x,0)$$

$$= \mp \frac{\beta C L^{\frac{1}{2}}}{\pi} \int_{\gamma_1}^{\gamma_2} \frac{B_y(r)(1 - r^2)^{\frac{1}{2}}}{r \mp 1} dr \tag{15}$$

Numerical results

The numerical solution employs the quadrature developed by Erdogan and Gupta [7]. The details of the iteration procedure which solves (13) and (10) for $B_y(x)$ and at the same time determines the unknown parameters a/L and b/L can be found in [6] and are omitted here. The iteration procedure is started by taking b/L = 1/3, which is the exact value for identical materials ($\beta = 0$) [1,2]. A guess for γ_1 is obtained from the bilateral solution which admits oscillatory singularities

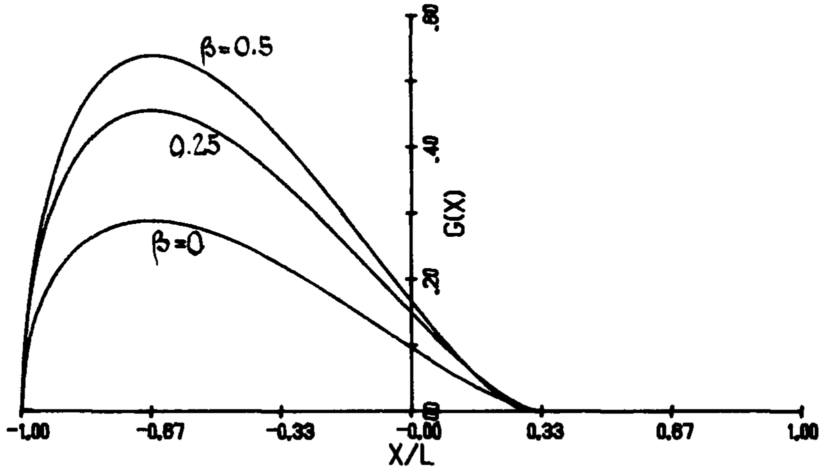

Fig. 2
Normalized gap between the crack faces,

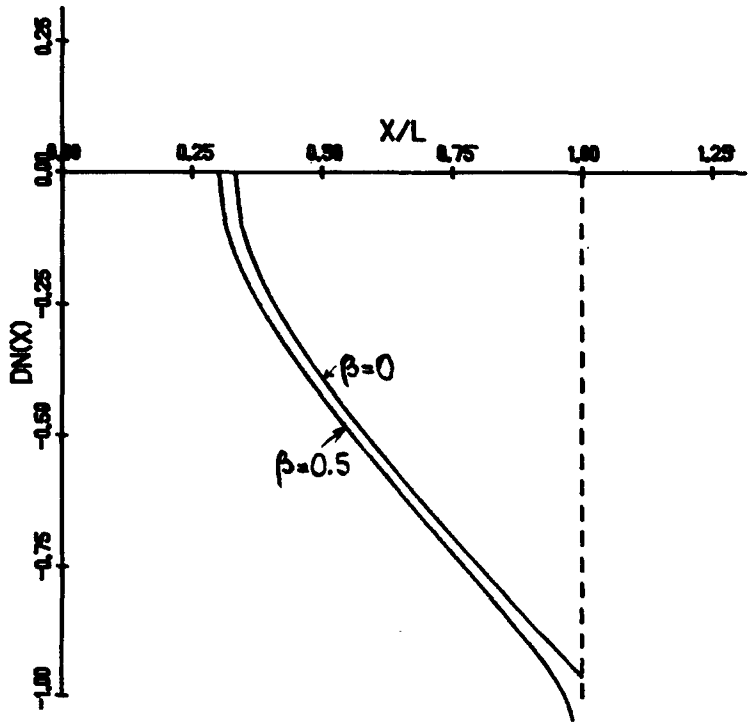

Fig. 3
Normalized contact pressure in the large contact zone.

$$g(x) = -\frac{m}{2C(1-\beta^2)^{\frac{1}{2}}}(L^2 - x^2)^{\frac{1}{2}}\left\{x\cos\left[\omega\log\left(\frac{L-x}{L+x}\right)\right]\right.$$

$$\left. - 2L\omega\sin\left[\omega\log\left(\frac{L-x}{L+x}\right)\right]\right\} \quad (16)$$

where

$$\omega = \frac{1}{2\pi}\log\left(\frac{1+\beta}{1-\beta}\right) \quad (17)$$

For $\beta = 0.5$ the first zero of $g(x)$ occurs in $0.99996 < |\frac{x}{L}| < 0.99997$ for $x < 0$ and gives an idea about the order of magnitude of the extent of the left contact zone which is not due to the applied compressive field. The results of the numerical computations are for $\beta = 0.5$

$\frac{a}{L} = 1 - 4 \times 10^{-5}$, $\frac{b}{L} = 0.301$, $K_2(L)/mL^{3/2} = 0.040$,

$K_2(-L)/mL^{3/2} = -0.600$

and for $\beta = 0.25$

$\frac{a}{L} = 1 - 8 \times 10^{-7}$, $\frac{b}{L} = 0.326$, $K_2(L)/mL^{3/2} = 0.018$,

$K_2(-L)/mL^{3/2} = -0.400$

The shapes of the gaps for various values of β are plotted using the dimensionless variable $G(x) = Cg(x)/mL^2$ in Fig. 2. The case $\beta = 0$ cannot be obtained as a limiting case since the numerical procedure incorporates the nature of the singularities for $\beta \neq 0$, which is distinctly different. For comparison purposes the gap for identical materials ($\beta = 0$) is obtained from [2] as

$$g(x) = \frac{m}{2C}\left(\frac{L}{3} - x\right)^{3/2}(x + L)^{1/2}, \; -L < x < \frac{L}{3} \quad (18)$$

It is noted that the mismatch in the elastic properties decreases slightly the extent of the gap, but increases its magnitude. The normal stress using the dimensionless variable $DN(x) = N(x)/mL$ is shown in Fig. 3 for the right contact zone.

Observe that for $\beta = 0$, the normal stress is bounded, while for $\beta \neq 0$ the normal stress is square root singular behind the crack tip [3]. In fact the corresponding stress intensity factors are given by

$$K_1(\pm L^-) = \mp \beta K_2(\pm L) \tag{19}$$

If the material properties of the solids are interchanged (β taken of opposite sign), $K_2(L)$ and $K_2(-L)$ exchange algebraic signs but not magnitudes. All other results remain unaffected. In particular the gap remains positive and the normal contact tractions compressive as they should.

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