

**PRECONTRACTUAL INFORMATION ASYMMETRY  
BETWEEN PRINCIPAL AND AGENT**  
**The Continuous Case**

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In the presence of precontractual information asymmetry between principal and risk-neutral agent, the optimal strategy for the principal will be to deliberately induce outcomes which are ex post Pareto inefficient, except under rare circumstances.

**1. Introduction**

A standard result in the principal–agent literature holds that the observed contract between a principal and risk-neutral agent will always be socially efficient, ex post [see, for example, Harris and Raviv (1979), Holmstrom (1979), and Shavell (1979)]. This result, though, hinges crucially on the assumption that the principal and agent share symmetric precontractual beliefs about the distribution of a random state of nature,  $\theta$ , which ultimately affects the agent's productivity.

It is the purpose of this research to demonstrate with a particular class of examples that absent such symmetry of beliefs, the observed contract will, except in rare cases, not be socially efficient.

The type of information asymmetry considered here, which is similar in nature to that explored by Green (1979) and Green and Stokey (1980a,b), is made explicit in section 2, as is the entire model to be analyzed. Then in section 3, the major finding of this research is restated more precisely and proved. Some concluding observations are offered in the final section.

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## 2. Statement of the problem

In the model considered here, the principal owns a productive technology which requires as an input the (unobservable) effort,  $a$ , of a risk-neutral agent. The technology is given by  $x = X(a, \theta)$ , where  $x$  is the (observable) value of output produced and  $\theta$  is a random state of nature which has a continuous distribution with strictly positive support on the interval  $[\theta_1, \theta_n]$ . Only the agent is able to observe the actual realization of  $\theta$ , and although this observation is made before he chooses the level of effort he wishes to put forth, it is not made until after the agent has agreed to the terms of a binding contract governing his future relations with the principal. Note that this contract,  $S(x)$ , must specify dollar payments,  $S$ , to the agent as a function only of the value of the output actually produced and transferred to the principal, since the principal cannot observe any of the other relevant parameters in the model (i.e.,  $a$  or  $\theta$ ).

Letting subscripts denote partial derivatives, it is assumed that the productive technology is such that  $X(0, \theta) = 0$ , and  $X_a > 0 \forall \theta$ , and that  $X_\theta > 0 \forall a > 0$ . The risk-neutral agent's state-dependent utility function over effort and payments is defined indirectly in terms of  $x$ ,  $S$  and  $\theta$  as  $U^A(x, S|\theta) = S - W(x, \theta)$ , where  $W(x, \theta)$  is the dollar value of the disutility of producing  $x$  in state  $\theta$ . Consistent with the assumptions on the productive technology, it is assumed that  $W_x \geq 0$ ,  $W_{xx} \geq 0$ ,  $W_{x\theta} \leq 0$ , and  $W_\theta \leq 0 \forall \theta$ , with strict inequality for all  $x > 0$ , and also that  $W_x(0, \theta) < 1 \forall \theta$ . In addition,  $W(x, \theta)$  is assumed to have continuous partial derivatives.

In keeping with most of the principal-agent literature, it is assumed that the principal knows both the agent's utility function and the minimum level of expected utility (normalized here at zero) that any contract must promise the agent in order for him to accept it. In contrast to most studies, though, it is assumed here that the principal does not know precisely the true distribution,  $f^t(\theta)$ , of the random state of nature at the time when a contract must be signed. Instead, he is assumed to know only the probability ( $p^i > 0$ ) with which the true distribution will take on each of two possible known distributions,  $f^i(\theta)$ ,  $i = 1, 2$ ,  $p^1 + p^2 = 1$ .

The principal though, is aware that the agent knows whether  $f^t(\theta) = f^1(\theta)$  or  $f^t(\theta) = f^2(\theta)$  before any contract is signed. Thus, the precontractual information asymmetry analyzed here stems from the fact that the agent's information about the distribution of  $\theta$  is better [in the sense of Blackwell (1951)] than that of the principal.

In an informational environment of this class, the principal's optimal strategy, i.e., the one which maximizes his expected utility,<sup>1</sup> is to design a set of at most two distinct contracts from which the agent, before observing the realization of  $\theta$ , can

<sup>1</sup> The principal's utility function is assumed throughout to be given by  $U^P(x, S) = x - S$ .

either choose one or decide not to contract at all.<sup>2</sup> The agent's choice is final and binding upon both parties.

Any non-trivial (set of) contract(s) designed by the principal can, without loss of generality, be classified as one of the following: (i) a single *non-revealing* contract, i.e., one that will be accepted by the agent whatever the true distribution of  $\theta$ , (ii) a single *revealing* contract, i.e., one that will be accepted by the agent under one distribution of  $\theta$ , but not under the other, or (iii) a pair of revealing contracts, i.e., two contracts,  $S^1(x)$  and  $S^2(x)$ , the former of which will always be selected by the agent when  $f^t(\theta) = f^1(\theta)$ , and the latter when  $f^t(\theta) = f^2(\theta)$ . These classifications are instrumental in the statement of the major findings in section 3.

Before proceeding to these findings, one final definition is in order. An *efficient* contract is defined to be one that induces an outcome which is *ex post* Pareto efficient, i.e., efficient in the particular state of nature that is realized, whatever that state might be.

### 3. Findings of the model

The major findings of this research are stated here as a series of propositions leading to the concluding theorem.

*Proposition 1.* Two (or more) non-trivial revealing contracts cannot both (all) be efficient.

The proof of Proposition 1 follows directly from the fact that all efficient contracts must be of the form  $S(x) = x - k$  ( $k$  a constant) when  $\theta$  has a connected support on a closed interval,<sup>3</sup> and from the fact that the smaller the magnitude of  $k$ , the greater the agent's expected utility under an efficient contract, whatever the true distribution of  $\theta$ .

*Proposition 2.* A single revealing contract will lead to an inefficient outcome in every state of nature under one (of the two possible) distribution(s) of  $\theta$ .

The inefficient outcome referred to in Proposition 2 is the 'no production' outcome, which is inefficient because  $W_x(0, \theta) < 1 \forall \theta$ , i.e., in every state of nature, the marginal disutility to the agent of increasing output slightly above zero is less than the value of that additional output to the principal. By definition, this 'no production' outcome will always ensue under one of the two distributions of  $\theta$  whenever the principal chooses to offer the agent a single revealing contract.

<sup>2</sup> This result follows from the work of Harris and Townsend (1979).

<sup>3</sup> This result is proved formally in Sappington (1980a).

*Proposition 3.* The non-revealing contract most preferred by the principal will be efficient if and only if the agent's expected utility is identically zero under this contract whatever the true distribution of  $\theta$ .

*Proof.* When he has decided to offer a single non-revealing contract to the agent, the principal's problem can be stated as

$$\max_{S(x)} \phi \int [x(\theta) - S(x(\theta))] f^1(\theta) d\theta + (1 - \phi) \int [x(\theta) - S(x(\theta))] f^2(\theta) d\theta ,$$

subject to

$$\int [S(x(\theta)) - W(x, \theta)] f^i(\theta) d\theta \geq 0 , \quad i = 1, 2 , \tag{1}$$

$$x(\theta) = \operatorname{argmax}_{x'} S(x') - W(x', \theta) , \tag{2}$$

$$x(\theta) \geq 0 \quad \forall \theta . \tag{3}$$

The first constraint ensures that the contract will be accepted by the agent whatever the true distribution of  $\theta$ . The second constraint reflects the fact that the agent will choose his level of effort to maximize his own utility after observing the realization of  $\theta$  and the terms of the contract. All integrals are defined over the interval  $[\theta_1, \theta_n]$ .

Treating  $S(\cdot)$  and  $x(\cdot)$  as state variables and  $x_\theta$  as the control variable, the Hamiltonian associated with this problem can be formulated, and the necessary conditions for a maximum derived. Then, manipulation of these necessary conditions reveals that if constraint (1) is satisfied as a strict inequality at the optimum for either  $f^1(\theta)$  or  $f^2(\theta)$  or both, it cannot be the case that the optimal contract has  $x_\theta > 0 \forall \theta$  and  $S_x = 1 \forall x$ , i.e., the optimal contract is not efficient.

The proof of the converse follows directly from the fact that an efficient contract maximizes the total expected surplus. Q.E.D.

The results stated as Propositions 1 through 3 constitute the proof to the major conclusion of this research.

*Theorem.* Whenever

$$\int [x^*(\theta) - W(x^*(\theta), \theta)] f^1(\theta) d\theta \neq \int [x^*(\theta) - W(x^*(\theta), \theta)] f^2(\theta) d\theta ,$$

[where  $x^*(\theta)$  is the efficient value of output in state  $\theta$ ] the strategy which is optimal for the principal will induce the risk-neutral agent to realize an inefficient outcome in some states of nature.

The theorem states that it is only if the total possible surplus under both distributions is identical <sup>4</sup> that the principal will find it in his own interest to induce the risk-neutral agent to realize an efficient outcome in every state of nature. In all other cases, even though the total surplus is reduced when inefficient contracts are employed, the principal's total expected utility is increased through the induced inefficiency.

#### 4. Concluding observations

The conclusions of this research were drawn under highly simplified assumptions concerning the nature of the precontractual information asymmetry between principal and agent. Preliminary research, though, suggests that the conclusions remain essentially unchanged under far more general specifications of the informational environment as long as the distribution of  $\theta$  is continuous. When the distribution is discrete, however, the conditions under which efficient contracts between principal and risk-neutral agent will be observed differ qualitatively. <sup>5</sup>

#### References

- Blackwell, David, 1951, Comparison of experiments, in: Jerzy Neyman, ed., *Proceedings of the second Berkeley symposium on mathematical statistics and probability* (University of California Press, Berkeley, CA) 93–102.
- Green, Jerry R., 1979, Statistical decision theory requiring incentives for information transfer, Discussion Paper no. 708 (Harvard Institute of Economic Research, Cambridge, MA).
- Green, Jerry R. and Nancy L. Stokey, 1980a, A two-person game of information transmission, Discussion Paper no. 751 (Harvard Institute of Economic Research, Cambridge, MA).
- Green, Jerry R. and Nancy L. Stokey, 1980b, The value of information in the delegation problem, Mimeo. (Harvard University, Cambridge, MA).
- Harris, Milton and Arthur Raviv, 1979, Optimal incentive contracts with imperfect information, *The Journal of Economic Theory* 20, no. 2, 231–259.
- Harris, Milton and Robert M. Townsend, 1979, Resource allocation under asymmetric information, Working Paper no. 43-77-78 (Carnegie-Mellon University, Pittsburgh, PA).
- Holmstrom, Bengt, 1979, Moral hazard and observability, *The Bell Journal of Economics* 10, no. 1, 74–91.
- Sappington, David, 1980a, The inefficiency of indirect control under information asymmetry, Unpublished Doctoral dissertation at Princeton University.
- Sappington, David, 1980b, Precontractual information asymmetry and optimal inefficiency: The discrete case, Unpublished Working Paper.
- Shavell, Steven, 1979, Risk sharing and incentives in the principal and agent relationship, *The Bell Journal of Economics* 10, no. 1, 55–73.

<sup>4</sup> If and only if this condition is satisfied can the principal design a non-revealing, efficient contract which leaves the agent with zero expected utility under both distributions of  $\theta$ .

<sup>5</sup> See Sappington (1980b).