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AN EXAMPLE FOR FRICTIONAL SLIP PROGRESSING INTO A CONTACT ZONE OF A CRACK

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Abstract—We consider a crack in a linearly varying field of normal stress that is kept constant, and apply shearing tractions that increase with time. This leads eventually to slip progressing into the closed part of the crack. If the crack lies entirely in the compressive part of the normal stress field, the problem can be solved in closed form, and it is easy to get results also for shearing tractions that start to decrease and eventually lead to backslip.

INTRODUCTION

It is fair to say that most elasticity solutions for cracks are aimed at understanding fracture under generally tensile conditions. Even if a specific boundary value problem leads to contradictory results, because the crack faces are seen to overlap after the solution is constructed, the dilemma is more often than not dismissed by saying that some other loads can always be superposed to keep the crack open. However, in some applications and perhaps most notably in geophysics, cracks need be considered in an essentially compressive environment. This means that the cracks are partially, if not fully closed, and that friction between the crack faces is liable to play an important role in the ensuing phenomena.

There is to date only a handful of elasticity solutions considering one aspect or another of closed cracks involving contact between their faces [1–8]. The most fundamental among them appears to be that by Bowie and Freese [8] who considered a crack which extends into the compression part of a pure bending field, and assumed that the crack faces stick in the contact zone. The assumption of stick is obviously justified only for relatively high values of the friction coefficient. We considered recently the case when this assumption no longer holds and slip takes place, so that the problem must be reformulated accordingly [9]. It was found that due to some special features of this problem, conditions of either stick or slip prevail over the entire contact zone at any given time.

It will be instructive to study an example where progressing slip takes place in the contact zone of a crack as the loading changes. We consider for this purpose a crack in a linearly varying field of normal stress that is kept constant, and suppose that the solid is subsequently also loaded in shear. The shear load is assumed to increase gradually with time, so that slip eventually starts and progresses into the contact zone.

PARTIALLY CLOSED CRACK

Consider a crack of length $L_1 + L_2$ in a field of linearly varying normal stress that is kept constant, and apply a homogeneous shearing stress that may change with time. The geometry and the placement of the coordinate axes are shown in Fig. 1. The linearly varying normal stress may be viewed as earth stress in a geophysical context, if it is purely compressive, or a bending field in engineering applications. We assume at the beginning that the crack extends into the tension part of the normal stress field so that it is partially open. The open part of the crack corresponds to the interval $-L_1 < x < b$, and the closed part or contact zone to $b < x < L_2$. Moreover, the contact zone consists of the slip zone $b < x < c$ and the stick zone $c < x < L_2$.

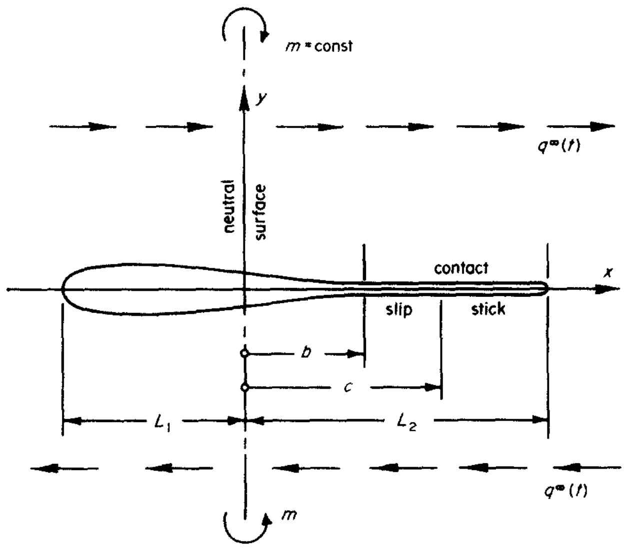

Fig. 1. Geometry of the partially closed crack.

The tractions on $y = 0$ corresponding to the linearly varying field of normal stresses and the applied shear are

$$\sigma_{xy}(x, 0) = q^{\infty}, \quad \sigma_{yy}(x, 0) = -mx \tag{1, 2}$$

where m is a constant, and $q^{\infty} = q^{\infty}(t)$ is at the beginning restricted to be a monotonically increasing function of time with $q^{\infty}(0) = 0$. Consequently, the extent of the slip zone is also expected to depend on time, or that $c = c(t)$.

Following the approach of our previous article[9], we represent the crack as an array of distributed edge dislocations with the densities $B_x(x)$ and $B_y(x)$, where

$$B_x(x) = -\frac{dh(x)}{dx}, \quad B_y = -\frac{dg(x)}{dx}, \tag{3, 4}$$

$$h(x) = u_x(x, 0^+) - u_x(x, 0^-) \tag{5}$$

is the tangential shift of the upper crack face with respect to the lower, and

$$g(x) = u_y(x, 0^+) - u_y(x, 0^-) \tag{6}$$

is the gap between the crack faces.

The boundary condition requiring that the normal tractions vanish in the open part of the crack leads to the same result as in our previous article by setting $\alpha = \pi/2$. Thus, in the new variables

$$x = \delta\zeta + \sigma \tag{7}$$

$$\delta = \frac{1}{2}(b + L_1), \quad \sigma = \frac{1}{2}(b - L_1) \tag{8}$$

the density of the climb dislocations is as before

$$B_y(\zeta) = \frac{m}{C}(\delta\zeta + b)(1 - \zeta)^{1/2}(1 + \zeta)^{-1/2}, \quad |\zeta| < 1. \tag{9}$$

Consequently, the normal tractions outside the separation zone are

$$N(\zeta) = -m\{\sigma|\zeta - 1|^{1/2}|\zeta + 1|^{-1/2} + \delta sgn\zeta(\zeta^2 - 1)^{1/2}\}, \quad |\zeta| > 1 \tag{10}$$

and

$$b = L_1/3. \tag{11}$$

The last result is again a consequence of the condition of single valued displacements as employed in [9].

In view of the geometry and the applied tractions, we anticipate that slip will be in the positive direction, or that $\mathrm{d}h/\mathrm{d}t > 0$ for monotonically increasing $q^\infty(t)$. Therefore

$$S(x) = -fN(x), \quad b < x < c \tag{12}$$

where $N(x)$ and $S(x)$ denote normal and shearing tractions as before [9]. Furthermore, the shearing tractions must vanish in the separation zone of the crack. This condition and (12) can be combined into the single integral equation valid in both the separation and slip zones:

$$\frac{C}{\pi}\int_{-L_1}^{c} \frac{B_x(\xi)}{\xi - x}\,\mathrm{d}\xi = q^\infty - f\left\{mx + \frac{C}{\pi}\int_{-L_1}^{b} \frac{B_y(\xi)}{\xi - x}\,\mathrm{d}\xi\right\}[H(x-b) - H(x-c)], \quad -L_1 < x < c. \tag{13}$$

Normalizing the integration interval by the change of variables

$$\begin{gathered} x = \hat{\delta}s + \hat{\sigma}, \quad \xi = \hat{\delta}r + \hat{\sigma} \\ \hat{\delta} = \frac{1}{2}(c + L_1), \quad \hat{\sigma} = \frac{1}{2}(c - L_1) \end{gathered} \tag{14}$$

and using (9), eqn (13) becomes

$$\begin{aligned} \frac{C}{\pi}\int_{-1}^{1} \frac{B_x(r)}{r - s}\,\mathrm{d}r = q^\infty - fm\{(\hat{\delta}s + \hat{\delta})^{1/2}(\hat{\delta}s + \hat{\sigma} - b)^{1/2} \\ + \sigma(\hat{\delta}s + \hat{\sigma} - b)^{1/2}(\hat{\delta}s + \hat{\delta})^{-1/2}\}[H(\hat{\delta}s + \hat{\sigma} - b) - H(s - 1)], \quad -1 < s < 1. \end{aligned} \tag{15}$$

The solution of (16) is [10]

$$\begin{aligned} B_x(s) = -\frac{q^\infty}{C}(1+s)^{-1/2}(1-s)^{1/2} + \frac{fm}{\pi C}(1+s)^{-1/2}(1-s)^{1/2} \\ \times \int_{(b-\hat{\sigma})/\hat{\delta}}^{1} \frac{[(\hat{\delta}r + \hat{\delta})^{1/2}(\hat{\delta}r + \hat{\sigma} - b)^{1/2} + \sigma(\hat{\delta}r + \hat{\sigma} - b)^{1/2}(\hat{\delta}r + \hat{\delta})^{-1/2}](1+r)^{1/2}}{(r-s)(1-r)^{1/2}}\,\mathrm{d}r. \end{aligned} \tag{16}$$

The requirement of single-valued tangential displacements

$$\int_{c}^{L_1} B_x(x)\,\mathrm{d}x = 0 \tag{17}$$

applied on (16) in the new variables leads to

$$\frac{q^\infty}{fmL_1} = \frac{1+\lambda}{2\pi}\int_{\hat{\lambda}}^{1} F(r)(1+r)^{1/2}(1-r)^{-1/2}\,\mathrm{d}r \tag{18}$$

where

$$F(r) = (r - \hat{\lambda})^{1/2}\left[(r+1)^{1/2} - \frac{2}{3(1+\lambda)}(r+1)^{-1/2}\right] \tag{19}$$

and
$$\lambda = c/L_1, \quad \hat{\lambda} = \frac{5-3\lambda}{3(1+\lambda)}.$$

Thus by specifying λ we obtain from (18) q^∞/fmL_1. The result is shown in Fig. 2. It is noted that the numerical computation predicts linear dependence of c/L_1 on q^∞/fmL_1. We have also plotted the normalized value of the shear stress at the end of the slip zone $S(c)/fmL_1$ in the same figure for comparison.

The shear stress intensity factor at the open end of the crack can be obtained from $B_x(x)$ as

$$K_2(-L_1) = -C \lim_{x\to -L_1} [(L_1+x)^{1/2}\sqrt{2}B_x(x)] \tag{20}$$

and the normalized quantity is plotted vs q^∞/fmL_1 in Fig. 3.

The shear stress in the stick zone is

$$S(x) = q^\infty - \frac{C}{\pi}\int_{-L_1}^{c} \frac{B_x(\xi)}{\xi - x}\,d\xi, \quad x > c. \tag{21}$$

Using the normalized variables, (16) and (18) in (21) we obtain after some elementary integrations

$$\frac{S(s)}{fmL_1} = \frac{1+\lambda}{2\pi}|s-1|^{1/2}|s+1|^{1/2}\int_{\hat{\lambda}}^{1} F(r)(1+r)^{1/2}(1-r)^{-1/2}\left(1-\frac{1}{r-s}\right)dr, \quad s>1. \tag{22}$$

A plot of $S(x)/fmL_1$ vs x is shown in Fig. 4 for $c/L_1 = 1.4$.

FORMULATION AND SOLUTION: FULLY CLOSED CRACK

If the crack is fully embedded in the compression zone it will remain completely closed. The new geometry is shown in Fig. 5. The normal tractions are simply

$$N(x) = -mx, \quad a < x < L_2. \tag{23}$$

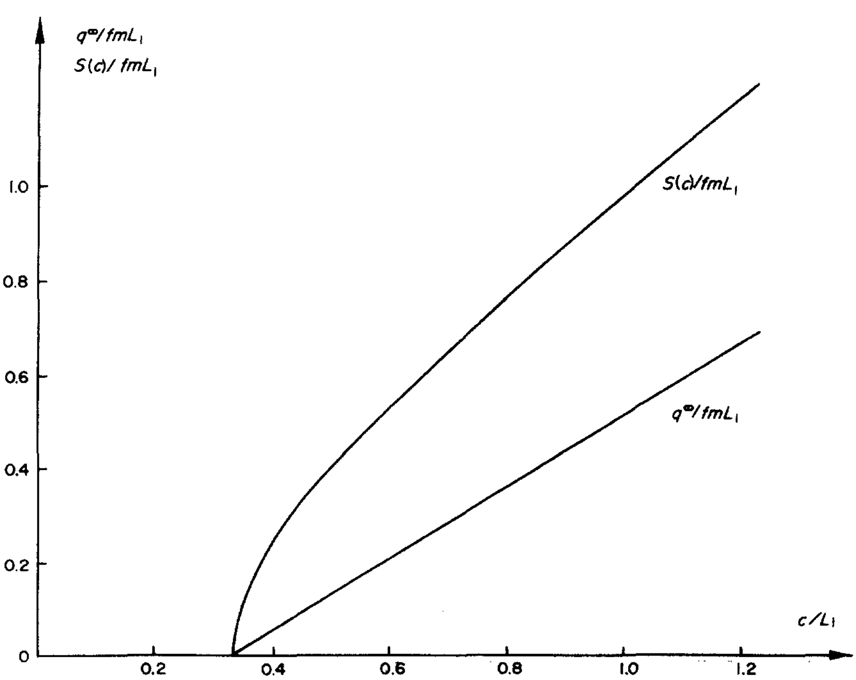

Fig. 2. Variation of the applied shear stress and shear traction at c.

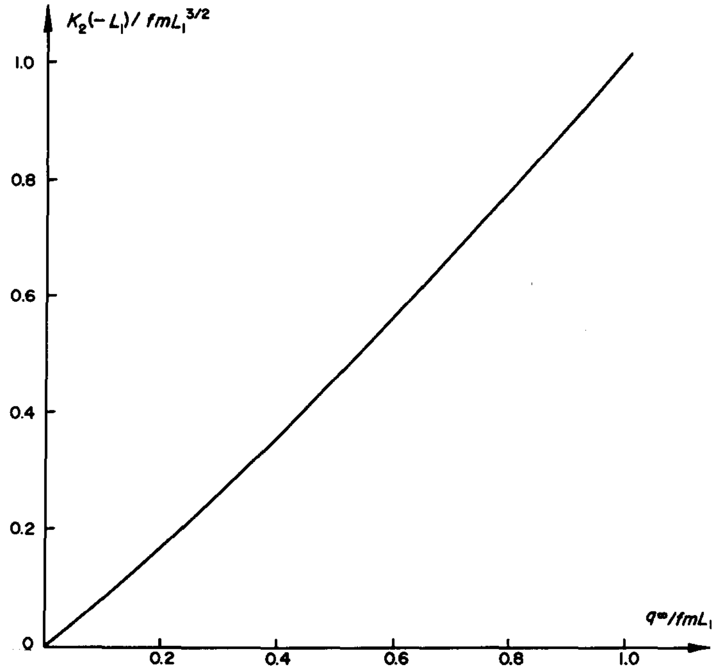

Fig. 3. Shear stress intensity factor at open end of crack.

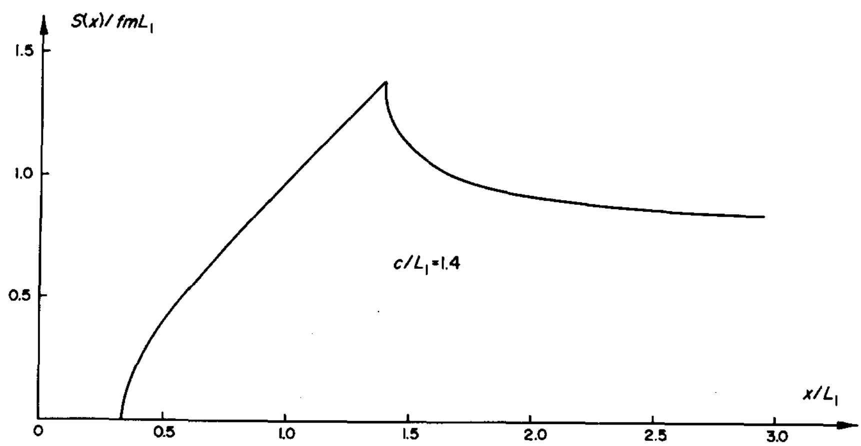

Fig. 4. Shear stress in the slip and stick zones for $c/L = 1.4$.

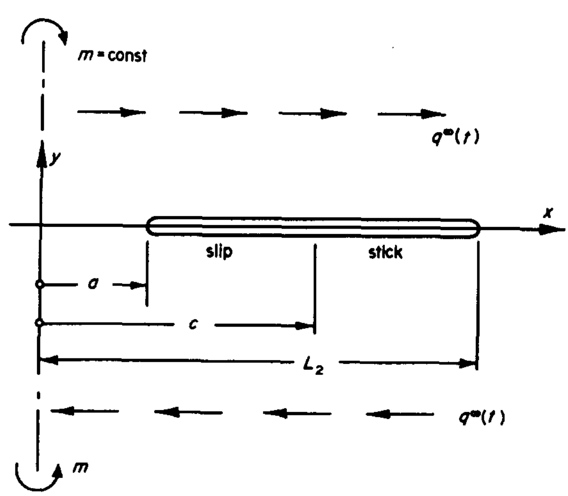

Fig. 5. Geometry of the fully closed crack.

Coulomb law in the slip zone gives

$$q^{\infty}-\frac{C}{\pi}\int_a^c \frac{B_x(\xi)}{\xi-x}\,\mathrm{d}\xi = fmx,\quad a<x<c. \tag{24}$$

Using the new variables

$$x=\delta' s+\sigma',\quad \xi=\delta' r+\sigma'$$
$$\delta'=\frac{1}{2}(c-a),\quad \sigma=\frac{1}{2}(c+a) \tag{25}$$

and solving (24) as in the previous section we obtain

$$B_x(s)=\frac{1}{C}(1+s)^{-1/2}(1-s)^{1/2}[fm(\delta' s+c)-q^{\infty}]. \tag{26}$$

Applying (17) on (26) yields

$$\frac{c}{a}=\frac{1}{3}\left(\frac{4q^{\infty}}{fma}-1\right). \tag{27}$$

Since $c/a\geq 1$, slip is possible only for $q^{\infty}/fma \geq 1$ in which case the extent of the slip zone is determined by (27). We note that, as in the previous section, c depends linearly on q^{∞}. The shear stress intensity factor is computed to be

$$K_2(a)=fm\left(\frac{c-a}{2}\right)^{3/2}. \tag{28}$$

In the limiting case of no slip zone or $c=a$ the shear stress intensity factor vanishes as it should.

The shear tractions in the stick zone are

$$S(x)=fm\left\{x-\left[x-\frac{1}{4}(c+3a)\right]|x-c|^{1/2}|x-a|^{-1/2}\right\},\quad x>c. \tag{29}$$

It can be verified from (29) that

$$\lim_{x\to\infty} S(x)=q^{\infty} \tag{30}$$

and

$$S(x)<-fN(x),\quad x>c. \tag{31}$$

In the special case where the slip zone extends over the entire crack, care must be taken to admit a square root singularity at L_2. However, the solution does not present any particular difficulties and it is not recorded here.

The simplicity of the solution for the fully closed crack leads itself to the study of backslip. For this purpose we assume that the applied shear $q^{\infty}(t)$ increases monotonically, reaches a maximum q_1 at $t=t_1$ and then decreases monotonically as $q^{\infty}(t)=q_1-q_{II}(t)$, where $q_{II}(t)$ is a monotonically increasing function of time.

The condition

$$sgn\, S(x)=sgn\, \dot{h}(x),\quad a<x<c \tag{32}$$

which must hold under conditions of slip, is satisfied up to t_1. For $t > t_1$ (32) is violated and stick must set in everywhere in the contact zone. The total tractions, viz. residual tractions due to locking of the crack and the additional applied tractions become

$$N(x) = -mx, \quad a < x < c \tag{33}$$

$$S(x) = fmx - q_{II}(t), \quad a < x < c, \ t > t_1. \tag{34}$$

Stick prevails as long as

$$|S(x)| < f|N(x)|, \quad a < x < c \tag{35}$$

holds. Substituting (33) and (34) into (35), it is seen that stick lasts until the instant $t = t_2$ determined by

$$q_{II}(t_2) = 2\,fma. \tag{36}$$

For $t > t_2$ backslip starts. Moreover, since (35) is violated first at $x = a$, backslip progresses from a. During backslip, the shear tractions in the slip zone are

$$S(x) = -fmx, \quad a < x < c \tag{37}$$

and the extent of the slip zone is

$$\frac{c}{a} = \frac{1}{3}\left(\frac{4q_{II}}{fma} - 1\right). \tag{38}$$

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