

## TECHNICAL NOTE

### A METHOD FOR MEASURING SHAPES OF ANATOMICAL SURFACES\*†

In the course of a study of biomechanical aspects of carpal tunnel syndrome, a three dimensional point encoder was built to record surface geometries of hand-wrist structures. The point encoder shown in Fig. 1 consists of a pointer that slides axially and rotates in perpendicular planes. The encoder is instrumented with a LVDT (linear voltage differential transformer) and with two potentiometers such that voltages proportional to the axial and two angular positions of the pointer are produced. These voltages of course correspond to the spherical coordinates of the pointer. The encoder voltages were measured via an A-D (Analogue-Digital) Converter and an HP 2100 computer.

Each dimension of the system was calibrated by measuring the voltage differences between known positions of the pointer. Scale factors for each channel were expressed as:

$$S_A = \Delta A / \Delta V_A \quad (1a)$$

$$S_B = \Delta B / \Delta V_B \quad (1b)$$

$$S_R = \Delta R / \Delta V_R, \quad (1c)$$

where  $S_A$ ,  $S_B$  and  $S_R$  are scale factors for each of three channels with corresponding spherical coordinates  $A$ ,  $B$  and  $R$ ;  $\Delta A$ ,  $\Delta B$  and  $\Delta R$  are known differences introduced in spherical coordinates for calibration; and  $\Delta V_A$ ,  $\Delta V_B$  and  $\Delta V_R$  are measured voltage differences associated with known differences in spherical coordinates of the pointer.

The computer was then programmed to calculate spherical coordinates from A-D converter voltages of each point,  $P_i$ , and then to convert the results to Cartesian coordinates.

Spherical coordinates of point  $P_i(A_i, B_i, R_i)$  were calculated as:

$$A_i = V_{A_i} S_A \quad (2a)$$

$$B_i = V_{B_i} S_B \quad (2b)$$

$$R_i = R_0 + V_{R_i} S_R, \quad (2c)$$

where  $A_i$ ,  $B_i$  and  $R_i$  are the spherical coordinates of point  $P_i$ ;  $V_{A_i}$ ,  $V_{B_i}$  and  $V_{R_i}$  are voltages corresponding to the spherical coordinates  $A_i$ ,  $B_i$  and  $R_i$  of point  $P_i$ ; and  $R_0$  is the distance between the axes of rotation and the end of the pointer when the LVDT voltage is zero.

The spherical coordinates of each point  $P_i$  (see Fig. 2) were then converted to Cartesian coordinates,  $P_i(X_i, Y_i, Z_i)$  by the following calculations:

$$X_i = R_i \cos A_i \sin B_i \quad (3a)$$

$$Y_i = R_i \cos A_i \cos B_i \quad (3b)$$

$$Z_i = R_i \sin A_i. \quad (3c)$$

The coordinates  $X_i$ ,  $Y_i$  and  $Z_i$  are located with respect to the point encoder. It was desirable that the coordinates be located with respect to selected points on the anatomical surfaces; therefore, a program was written that transformed the coordinates of point  $P_i(X_i, Y_i, Z_i)$  into a new Cartesian coordinate system  $P_i(X'_i, Y'_i, Z'_i)$  that was defined by points  $P_{ref_1}(X_{ref_1}, Y_{ref_1}, Z_{ref_1})$ ,  $P_{ref_2}(X_{ref_2}, Y_{ref_2}, Z_{ref_2})$  and  $P_{ref_3}(X_{ref_3}, Y_{ref_3}, Z_{ref_3})$ . A convention in which  $P_{ref_2}(X_{ref_2}, Y_{ref_2}, Z_{ref_2})$  was selected as the origin of this new reference system was used.

Calculation of unit vectors  $\bar{V}_{ref_x}$ ,  $\bar{V}_{ref_y}$  and  $\bar{V}_{ref_z}$  that correspond to each axis of the new coordinate systems was the first step in the translation process (see Fig. 2).

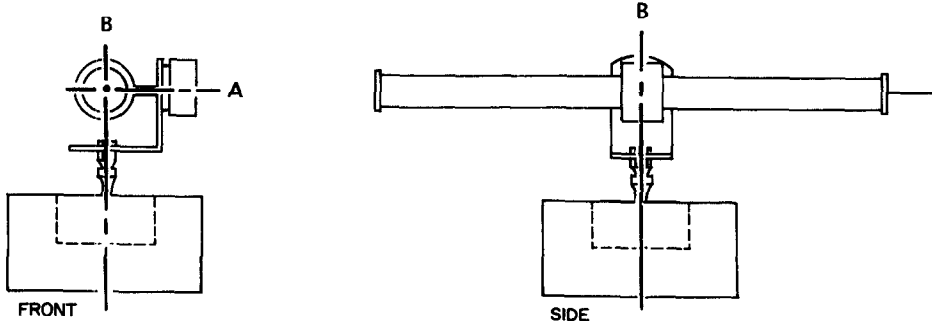


Fig. 1. The encoder is constructed from two servo potentiometers and a linear voltage differential transformer so as to produce voltages,  $V_A$ ,  $V_B$  and  $V_R$ , that correspond to spherical coordinates,  $A$ ,  $B$  and  $R$ , of the pointer.

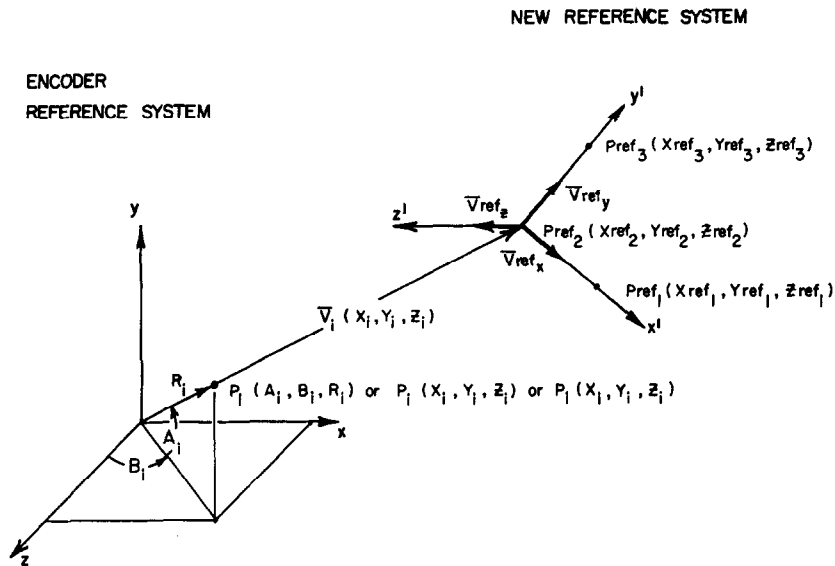


Fig. 2. Points are encoded in spherical coordinates,  $P_i(A_i, B_i, R_i)$  and converted to Cartesian coordinates,  $P_i(X_i, Y_i, Z_i)$ , with respect to the encoder reference system. The Cartesian coordinates can then be transformed to a new reference system,  $P_i(X'_i, Y'_i, Z'_i)$ , that is determined from points  $P_{ref_1}$ ,  $P_{ref_2}$  and  $P_{ref_3}$ .

$$\bar{V}_{ref_x} = (|X_{ref_1} - X_{ref_2}| \hat{i} + |Y_{ref_1} - Y_{ref_2}| \hat{j} + |Z_{ref_1} - Z_{ref_2}| \hat{k}) / |\bar{V}_{ref_x}| \quad (4a)$$

$$\bar{V}_{ref_y} = (|X_{ref_3} - X_{ref_2}| \hat{i} + |Y_{ref_3} - Y_{ref_2}| \hat{j} + |Z_{ref_3} - Z_{ref_2}| \hat{k}) / |\bar{V}_{ref_y}| \quad (4b)$$

$$\bar{V}_{ref_z} = \bar{V}_{ref_x} \times \bar{V}_{ref_y} \quad (4c)$$

Since  $\bar{V}_{ref_x}$  and  $\bar{V}_{ref_y}$  may not be orthogonal, a new  $\bar{V}'_{ref_y}$  was calculated.

$$\bar{V}'_{ref_y} = \bar{V}_{ref_x} \times \bar{V}_{ref_z} \quad (4d)$$

Calculation of a vector  $\bar{V}_i(X_i, Y_i, Z_i)$  between each point  $P_i(X_i, Y_i, Z_i)$  and the origin of the reference system,  $P_{ref_2}(X_{ref_2}, Y_{ref_2}, Z_{ref_2})$  was the next step in the translation process (see Fig. 2).

$$\bar{V}_i(X_i, Y_i, Z_i) = |X_i - X_{ref_2}| \hat{i} + |Y_i - Y_{ref_2}| \hat{j} + |Z_i - Z_{ref_2}| \hat{k} \quad (5)$$

Finally, the transformed coordinates,  $X'_i, Y'_i$  and  $Z'_i$  were calculated as scalar products between each axis of the new reference system and the  $\bar{V}_i(X_i, Y_i, Z_i)$ .

$$X'_i = \bar{V}_i(X_i, Y_i, Z_i) \cdot \bar{V}_{ref_x} \quad (6a)$$

$$Y'_i = \bar{V}_i(X_i, Y_i, Z_i) \cdot \bar{V}'_{ref_y} \quad (6b)$$

$$Z'_i = \bar{V}_i(X_i, Y_i, Z_i) \cdot \bar{V}_{ref_z} \quad (6c)$$

The resulting data points were then stored in a file in disc memory for later plotting and analysis.

The area of coverage is a function of the mechanical range of the LVDT, the scale factors of the encoder, and the maximum input voltage of the A-D converter. Using scale factors,  $S_A, S_B$  and  $S_R$ , of  $80^\circ/V, 80^\circ/V$ , and  $90 \text{ mm}/V$ , and an

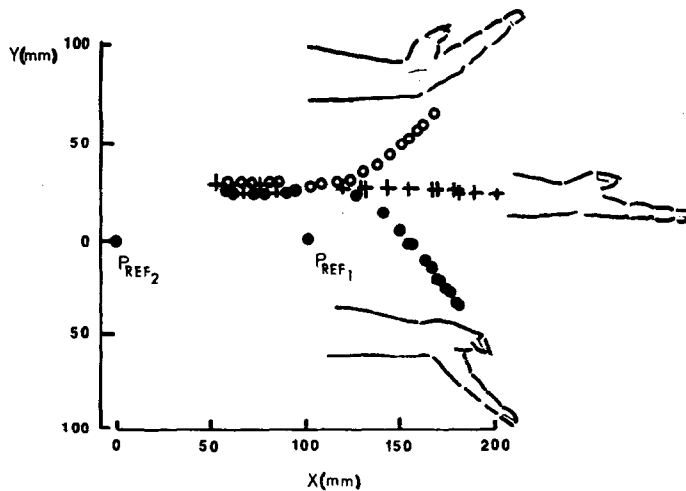


Fig. 3. Encoded points along the flexor digitorum profundus tendon of the second digit in flexed, neutral and extended positions of a partially dissected wrist.

A-D converter with a range of  $\pm 1$  V, the encoder built in this lab covered a volume of  $160^\circ$  about the *A* and *B* axes and from 100 to 280 mm along the *R* axis. A ten bit A-D converter resulted in a resolution greater than  $\pm 1$  mV, so that the system is likely to be affected by stray electrical noise. Random noise problems can be controlled by averaging multiple samples from the A-D converter.

As indicated in the introduction to this paper, the development of this system was motivated by an interest in surface shapes of hand and wrist structures. Sample point locations along the flexor digitorum profundus tendon of the second digit in three positions of a partially dissected wrist are shown in Fig. 3. Methods including cubic splines are currently under study for analysis of surface data. We believe that the type of

data encoder described is of interest to other persons concerned with recording shapes and positions of anatomical structures. This type of data encoder should also be useful for determining plans or coordinates of points in photographs.

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