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STRENGTH OF THE $Q_\pi \cdot Q_\nu$ INTERACTION AND THE STRONG-COUPLED PSEUDO-SU(3) LIMIT†

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Abstract: The suitability of pseudo SU(3) strong coupled wave functions in representing the normal parity part of yrast eigenstates of systems with protons and neutrons filling *different* major shells and interacting with one another via a $Q_\pi \cdot Q_\nu$ interaction is explored. In addition, we find that the overlaps of yrast eigenstates of H with the leading SU(3) symmetry follow the correlation of $Q \cdot Q$ with H very closely and are large even when $Q_\pi \cdot Q_\nu$ contributes less than half to the full width of the eigenvalue spectrum of H. The results suggest that the pseudo-SU(3) coupling scheme will prove useful in microscopic shell-model studies of high-spin phenomena in rare earth and actinide nuclei.

1. Introduction

A theoretical interpretation of high-spin and related phenomena in heavy deformed nuclei, where the number of spectroscopically active nucleons is large, has come about mainly through phenomenological quasiparticle-plus-rotor studies or through more microscopic Hartree–Fock–Bogoliubov mean field theories. The advantage of a fully microscopic approach has recently led us to propose a shell-model theory for high-spin phenomena. The necessarily severe basis truncation is effected by recognizing and utilizing special group symmetries. The model is based on the weak coupling of normal (N) parity orbitals comprised of all but the largest j-member of an oscillator shell and the unique or abnormal (A) parity level of largest j from the next higher oscillator shell; e.g. the (N) orbitals $g_{7/2}d_{5/2}d_{3/2}s_{1/2}$ plus the (A) $h_{11/2}$ orbital of the Ba region. Quasispin symmetry is used to organize the abnormal parity configurations into states of good seniority. The normal-parity configurations are classified according to representations of pseudo SU(3) symmetry [1]). This symmetry is based on a correspondence such as $(g_{7/2}d_{5/2}d_{3/2}s_{1/2}) \rightarrow (\tilde{f}_{7/2}\tilde{f}_{5/2}\tilde{p}_{3/2}\tilde{p}_{1/2})$ which maps the (N) states onto states of a complete major pseudo oscillator shell. This mapping is

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motivated by the fact that realistic interactions for deformed nuclei seem to be strongly correlated [2]) with the second-order Casimir invariant, $\tilde{C}_2$, of the pseudo-SU(3) symmetry group. The deformation inducing quadrupole–quadrupole interaction is simply related to C_2, the Casimir invariant of real SU(3) symmetry, and $\tilde{C}_2$ and C_2 are themselves very strongly correlated.

Results of model calculations have led to a microscopic interpretation of backbending [2]), the so-called prealignment $B(E2)$ anomaly [3]) in ^{126}Ba, and the branching from a single 6^+ state to three 8^+ states [4]), a phenomenon called forking, in ^{68}Ge. For simplicity, these initial examples have concentrated on regions in which the spectroscopically active neutrons and protons are filling the same oscillator levels. In such nuclei the goodness of the pseudo-SU(3) symmetry may be open to some question. Calculations by Strottman to test the validity of pseudo-SU(3) symmetry [5]) using the simple identical nucleon configurations $(\widetilde{fp})^3$ and $(\widetilde{fp})^4$ and a surface-delta interaction, seem to cast some doubt on the validity of pseudo-SU(3) symmetry truncations. [These calculations have been extended by McGrory to identical nucleon configurations $(\widetilde{fp})^6$ and $(\widetilde{fp})^8$ with similar results [6]).] However, these calculations are based on configurations of identical nucleons (neutrons only, for example) and use an interaction in which, perforce, the deformation inducing quadrupole–quadrupole component of the proton–neutron (π–ν) interaction can play no role. It is the underlying feature of the Bohr–Mottelson picture of nuclear deformation that the sphericity favoring short-range residual interactions are overcome by the tendency of nucleons to align themselves with the non-spherical field produced by the valence nucleons themselves as more valence nucleons are added to the system. In this picture stable strong equilibrium deformations depend on the presence of a sufficient number of both valence protons and neutrons (particles and/or holes), and the importance of the quadrupole–quadrupole component of the proton–neutron (π–ν) interaction in stabilizing the deformation is indicated by many alternative theoretical approaches.

It is the purpose of this article to investigate the probable "goodness" of strong-coupled pseudo-SU(3) wave functions for the normal-parity configurations of nuclei in which protons and neutrons are filling different major shells. If this strong coupling approximation is good so the normal-parity proton and neutron configurations couple to resultant SU(3) representations with the largest possible Casimir eigenvalues, then, by weak coupling the leading SU(3) symmetries to quasispin truncated abnormal-parity configurations, as done in the Ge and Ba region, we can anticipate the possibility of making realistic shell-model calculations of high-spin phenomena in rare earth and actinide nuclei. The most direct and detailed measure of the "goodness" of the pseudo-SU(3) symmetry would come from a comparison between the exact yrast eigenstates and the states of good SU(3) symmetry. The calculation of the needed overlaps for such a comparison for configurations with many spectroscopically active protons and neutrons would lead to matrix diagonalization problems in spaces of prohibitive dimensions. In this work we focus instead

on $\pi^2\nu^2$ systems for which the diagonalizations can be carried out in the full model space. Such systems might be considered unfavorable to the point we wish to make since strong intrinsic deformations are not expected in the separate two-particle spaces. However, our results are encouraging even for this case. If the π–ν interaction is of the quadrupole–quadrupole type with a magnitude comparable to that of the π–π and ν–ν interactions in their separate spaces, even though these are far from $Q \cdot Q$ type, the SU(3) strong-coupled wave functions are good approximations for the yrast states. Since these results are valid for the simple $\pi^2\nu^2$ configurations, the pseudo-SU(3) strong coupling approximation can be expected to be even more appropriate for systems that are much richer in the number of π and ν valence nucleons.

In deformed nuclei with large numbers of valence nucleons there may be no certain simple quantitative measures for the "goodness" of a symmetry. However, recent work in statistical spectroscopy suggests that the symmetry is good if there is a large correlation between the system hamiltonian and the second-order Casimir invariant of the symmetry group [7]). In the present study where precise overlaps can be calculated it is thus instructive to compare the yrast state overlap results with the correlation measure in the simple $\pi^2\nu^2$ systems. The results suggest that the correlation measure can be used with confidence to gauge the goodness of pseudo-SU(3) symmetry in real deformed nuclei where the overlaps are beyond the scope of possible calculations.

2. Eigenstate overlaps and operator correlations

We are interested in determining the extent to which various π–ν interactions yield yrast eigenstates that belong predominantly to a single irreducible representation of pseudo-SU(3) symmetry, in particular, the so-called leading representation with largest possible eigenvalue of the quadratic Casimir invariant, $\tilde{C}_2$, and hence large real intrinsic quadrupole deformation. [The real quadrupole operator, Q, is effectively identical with $\tilde{Q}$, the quadrupole operator of the pseudo space [8]). Likewise the symmetry preserving operator $\tilde{Q} \cdot \tilde{Q}$ is effectively equivalent to the real quadrupole–quadrupole interaction.] A direct unambiguous measure of the validity of the symmetry comes from the overlap between eigenstates of the model hamiltonian, H, and the eigenstates of the Casimir operator or any similar symmetry preserving operator such as $\tilde{Q} \cdot \tilde{Q}$. Therefore, if $|\phi_\alpha\rangle$ and $|\Psi_\beta\rangle$, respectively, are normalized eigenstates of $\tilde{Q} \cdot \tilde{Q}$ and H, then the overlaps

$$N_{\alpha\beta} = \langle \phi_\alpha | \Psi_\beta \rangle \tag{1}$$

provide a detailed measure of the goodness of the pseudo-SU(3) symmetry. The full overlaps can of course be calculated only for those systems for which a complete solution of the matrix problem is feasible for both H and $\tilde{Q} \cdot \tilde{Q}$. In most cases of real interest the model space dimensionality is simply much too large to be amenable

to matrix methods. (If it were not, there would be no need for a severe basis truncation.) In those cases we need an alternative simple measure for the goodness of the symmetry and in particular an understanding of the meaning of that measure for the yrast states.

The trace of the product of two operators, which measures how the expectation values of one vary as a function of the eigenvalues of the other, defines the inner product of an operator geometry. In particular,

$$\mathrm{Tr}\,(\mathcal{O}^+\mathcal{O}) = \sum_{ij} \langle i|\mathcal{O}^+|j\rangle\langle j|\mathcal{O}|i\rangle \equiv \|\mathcal{O}\|^2\,, \tag{2a}$$

$$\mathrm{Tr}\,(\mathcal{O}_\alpha^+\mathcal{O}_\beta)/\|\mathcal{O}_\alpha\|\|\mathcal{O}_\beta\| \equiv \zeta_{\alpha\beta} = \cos\theta_{\alpha\beta}\,, \tag{2b}$$

are, respectively, the *norm* of $\mathcal{O}$ and the *correlation* between or inner product of $\mathcal{O}_\alpha$ and $\mathcal{O}_\beta$. The norm is a proper measure of the size or strength of a traceless operator. It is the width of its eigenvalue distribution. The correlation, which can also be interpreted as the cosine of an angle between the two operators, has a value that can range between plus and minus one. It is a measure of the similarity of the two operators in the model space.

Statistical spectroscopy studies have shown that all "reasonable" interactions have similar, near gaussian, eigenvalue distributions. The underlying dynamics that leads to this behavior is the operation in the model space of a central limit theorem. A consequence of this that is of importance here is that for the correlation coefficient between two interactions, H and $\tilde{Q}\cdot\tilde{Q}$, to be large requires that their eigenvalue distributions must *on the average* be very similar. A large correlation certainly does not necessarily imply large overlaps. In statistical terms, when the correlation is small the system is "free to fluctuate" but when the correlation is large the system is "constrained". In the present study, where overlaps can be calculated, it is instructive to explore the complementary nature of these two measures. We will see that the correlation measure, which can be calculated with equal ease in large and small model spaces, anticipates the yrast overlap results. The results suggest that it can be used with confidence in applications where overlaps cannot be calculated.

3. Schematic hamiltonian

With valence protons and neutrons in different major shells, we take the hamiltonian to have the form

$$H(\kappa) = H_\pi + H_\nu + 2\kappa Q_\pi \cdot Q_\nu\,, \tag{3}$$

that is, the π–ν interaction is approximated by a $Q\cdot Q$ interaction. The π–π and ν–ν interactions in their separate spaces must include sizable pairing effects. For this reason H_π and H_ν have been chosen to have the form of a surface–delta

interaction. Such a form for H has been used in numerous calculations, including for example quadrupole-plus-pairing model studies. Since the real quadrupole operators Q are effectively identical with $\tilde{Q}$, the Q-operators of the pseudo-space, the $Q_\pi \cdot Q_\nu$ term will for simplicity be replaced by $\tilde{Q}_\pi \cdot \tilde{Q}_\nu$. [This difference will affect final results by at most 2–5%; see, e.g. the pseudo-SU(3) expansions of the real Q-operator in ref. [8]).] In addition, with total pseudo-spin, $\tilde{S} = 0$, we have $\tilde{L} = J$, where J is the (real) total angular momentum quantum number. For economy of notation it will therefore be useful to omit the tildes, and *henceforth* it will be understood that operators $\tilde{Q}$ and quantum numbers such as $\tilde{L}$, $\tilde{S}$, $\tilde{\lambda}$, $\tilde{\mu}$ are simply abbreviated by Q and L, S, λ, μ.

It is our aim to investigate the goodness of the SU(3) symmetry using the form (3) for H. Our strength measure will be the norm of $\kappa(2Q_\pi \cdot Q_\nu)$ measured relative to the norm of H_π and H_ν. Clearly, if we were to choose $H_\pi = Q_\pi \cdot Q_\pi$, $H_\nu = Q_\nu \cdot Q_\nu$ and $\kappa = 1$, then $H = Q \cdot Q$, and SU(3) would be an exact symmetry for this H.

Matrix elements of the various parts of H can be calculated either in an SU(3) weak- or strong-coupled basis:

$$|[(\lambda_\pi\mu_\pi)\kappa_\pi L_\pi S_\pi J_\pi T_\pi; (\lambda_\nu\mu_\nu)\kappa_\nu L_\nu S_\nu J_\nu T_\nu]J\rangle \leftarrow \text{weak coupling},$$

$$|[(\lambda_\pi\mu_\pi)S_\pi T_\pi; (\lambda_\nu\mu_\nu)S_\nu T_\nu]\rho(\lambda\mu)\kappa LSJ\rangle \leftarrow \text{strong coupling},$$

The procedure we found most convenient for this study was to work with SU(3) weak-coupled basis states. The overlaps of the yrast eigenstates of $H(\kappa)$ with leading symmetry SU(3) strong coupled wave functions was then determined by taking for the latter the yrast eigenstates of $Q \cdot Q$ itself. Note that the $2Q_\pi \cdot Q_\nu$ part of H can be written as

$$\begin{aligned} 2Q_\pi \cdot Q_\nu &= Q \cdot Q - (Q_\pi \cdot Q_\pi + Q_\nu \cdot Q_\nu) \\ &= (4C_2 - 3L^2) - (4C_2^\pi - 3L_\pi^2 + 4C_2^\nu - 3L_\nu^2), \end{aligned} \tag{5}$$

where the general result, $Q \cdot Q = 4C_2 - 3L^2$, has been used to convert each homogeneous $Q \cdot Q$ operator into an SU(3) second-order invariant, C_2 with eigenvalue $(\lambda+\mu+3)(\lambda+\mu)-\lambda\mu$, and the corresponding second-order invariant of R(3), L^2 with eigenvalue $L(L+1)$. The second part of this form for $2Q_\pi \cdot Q_\nu$ is diagonal in the SU(3) weak-coupled basis. The first part, $4C_2 - 3L^2$, is diagonal in the SU(3) strong-coupled basis. The transformation from one to the other

$$\begin{aligned} &|[(\lambda_\pi\mu_\pi)\kappa_\pi L_\pi S_\pi J_\pi T_\pi; (\lambda_\nu\mu_\nu)\kappa_\nu L_\nu S_\nu J_\nu T_\nu]J\rangle \\ &= \sum_{LS} \begin{Bmatrix} L_\pi & S_\pi & J_\pi \\ L_\nu & S_\nu & J_\nu \\ L & S & J \end{Bmatrix} \sum_{\rho(\lambda\mu)\kappa} \langle (\lambda_\pi\mu_\pi)\kappa_\pi L_\pi; (\lambda_\nu\mu_\nu)\kappa_\nu L_\nu \| (\lambda\mu)\kappa L\rangle_\rho \\ &\quad \times |[(\lambda_\pi\mu_\pi)S_\pi T_\pi (\lambda_\nu\mu_\nu)S_\nu T_\nu]\rho(\lambda\mu)\kappa LSJ\rangle \end{aligned} \tag{6}$$

was used to determine the matrix elements of the $4C_2 - 3L^2$ part of $Q_\pi \cdot Q_\nu$. The

symbol in brackets is a *unitary* R(3) $9-j$ symbol and the SU(3) Wigner coefficient effects the coupling of $(\lambda_\pi\mu_\pi)\times(\lambda_\nu\mu_\nu)$ to $(\lambda\mu)$. The index ρ is introduced to distinguish multiple occurrences of $(\lambda\mu)$ in the direct product. For the $H_\pi^{\mathrm{SDI}}+H_\nu^{\mathrm{SDI}}$ parts of H the two-particle $jj\to LS\to(\lambda\mu)LS$ transformation,

$$|(\lambda\mu)\kappa LSJT\rangle=\sum_{l_a l_b}\langle(\eta_a 0)l_a;(\eta_b 0)l_b\|(\lambda\mu)\kappa L\rangle \times\sum_{j_a j_b}\sqrt{1+\delta_{j_a j_b}}\begin{Bmatrix} l_a & l_b & L\\ \frac{1}{2} & \frac{1}{2} & S\\ a & b & J\end{Bmatrix}|(j_a j_b)JT\rangle \tag{7}$$

together with the *jj*-coupled two-body matrix elements of the surface-delta interaction was used to determine the SU(3) weak-coupled matrix elements. The last step is the only part of the procedure we used that is not general. Tools for determining matrix elements of H_π and H_ν in spaces with larger particle numbers are available; but, as we have elected to restrict ourselves in this work to $\pi^2\nu^2$ systems, they are not needed here.

The general result for matrix elements of $H(\kappa)$ in the SU(3) weak-coupled basis is

$$\begin{aligned}
&\langle[[(\lambda'_\pi\mu'_\pi)\kappa'_\pi L'_\pi S'_\pi J'_\pi T'_\pi;(\lambda'_\nu\mu'_\nu)\kappa'_\nu L'_\nu S'_\nu J'_\nu T'_\nu]J|H|[(\lambda_\pi\mu_\pi)\kappa_\pi L_\pi S_\pi J_\pi T_\pi;\\
&\quad\times(\lambda_\nu\mu_\nu)\kappa_\nu L_\nu S_\nu J_\nu T_\nu]J\rangle\\
&=\delta_{\nu'\nu}\langle(\lambda'_\pi\mu'_\pi)\kappa'_\pi L'_\pi S'_\pi J'_\pi T'_\pi|H_\pi|(\lambda_\pi\mu_\pi)\kappa_\pi L_\pi S_\pi J_\pi T_\pi\rangle\\
&\quad+\delta_{\pi'\pi}\langle(\lambda'_\nu\mu'_\nu)\kappa'_\nu L'_\nu S'_\nu J'_\nu T'_\nu|H_\nu|(\lambda_\nu\mu_\nu)\kappa_\nu L_\nu S_\nu J_\nu T_\nu\rangle\\
&\quad+\delta_{\nu'\nu}\delta_{\pi'\pi}\kappa\{3[L_\pi(L_\pi+1)+L_\nu(L_\nu+1)]\\
&\quad-4[(\lambda_\pi+\mu_\pi+3)(\lambda_\pi+\mu_\pi)-\lambda_\pi\mu_\pi+(\lambda_\nu+\mu_\nu+3)(\lambda_\nu+\mu_\nu)-\lambda_\nu\mu_\nu]\}\\
&\quad+\kappa\sum_{\substack{\rho(\lambda\mu)\kappa\\ LS}}\begin{Bmatrix} L'_\pi & S_\pi & J_\pi\\ L'_\nu & S_\nu & J_\nu\\ L & S & J\end{Bmatrix}\begin{Bmatrix} L_\pi & S_\pi & J_\pi\\ L_\nu & S_\nu & J_\nu\\ L & S & J\end{Bmatrix}\\
&\quad\times[\langle(\lambda_\pi\mu_\pi)\kappa'_\pi L'_\pi;(\lambda_\nu\mu_\nu)\kappa'_\nu L'_\nu\|(\lambda\mu)\kappa L\rangle_\rho\langle(\lambda_\pi\mu_\pi)\kappa_\pi L_\pi;(\lambda_\nu\mu_\nu)\kappa_\nu L_\nu\|(\lambda\mu)\kappa L\rangle_\rho]\\
&\quad\times\{4[(\lambda+\mu+3)(\lambda+\mu)-\lambda\mu]-3L(L+1)\}\\
&\quad\times\delta_{S'_\pi S_\pi}\delta_{T'_\pi T_\pi}\delta_{S'_\nu S_\nu}\delta_{T'_\nu T_\nu}\delta_{\lambda'_\pi\lambda_\pi}\delta_{\mu'_\pi\mu_\pi}\delta_{\lambda'_\nu\lambda_\nu}\delta_{\mu'_\nu\mu_\nu}\,.
\end{aligned}\tag{8}$$

Off-diagonal couplings in $(\lambda_\pi\mu_\pi)$ and $(\lambda_\nu\mu_\nu)$ come only from the H_π and H_ν parts of H. When $\kappa=1$ and $H_\pi=Q_\pi\cdot Q_\pi$ and $H_\nu=Q_\nu\cdot Q_\nu$ this result yields the matrix elements of $Q\cdot Q$ in the weak-coupled basis. That choice can therefore be used to generate eigenstates that are the SU(3) strong-coupled basis states. Even more

simply, the same result can be achieved by diagonalizing the last term in (8), i.e. by setting H_π and H_ν to zero as well as the part of $Q \cdot Q$ that is diagonal in the weak-coupled basis. This procedure was employed in determining eigenstates of $Q \cdot Q$. Of course the inverse of (6) also yields the relationship between the strong- and weak-coupled basis states.

4. Overlap and correlation measures

Results for overlaps of the exact $\pi^2\nu^2$ yrast states of $H(\kappa) = H_\pi^{\mathrm{SDI}} + H_\nu^{\mathrm{SDI}} + 2\kappa Q_\pi \cdot Q_\nu$ with yrast eigenstates of $Q \cdot Q$ are plotted in fig. 1 (left) as a function of the magnitude of κ for the model spaces ds–fp and fp–gds. As the surface–delta interaction acts only between pairs of particles that are spatially symmetrically coupled, antisymmetric two-particle $(\lambda\mu)$ values in the product $(\lambda\mu)_\pi \times (\lambda\mu)_\nu$ will not contribute to low-lying eigenstates and have therefore not been included in the bases. For example, for states of the ds–fp case we have in the weak-coupled basis retained all possible states arising out of the $(40)\times(60)$, $(40)\times(22)$, $(02\times(60)$, and $(02)\times(22)$ couplings only. The dimensions for each J are given as part of fig. 1. The surface–delta interactions have been normalized to yield a binding of the $J=0$ two-particle state equal to $\sum_j (j+\frac{1}{2})$ where the j sum runs over all j's of the shell, e.g. 6 for $(\mathrm{ds})^2$. Our normalization for $Q \cdot Q$ was determined by the equality of $Q \cdot Q$ and $4C_2 - 3L^2$ where the eigenvalue of C_2 is $(\lambda+\mu+3)(\lambda+\mu)-\lambda\mu$. The values of $\kappa_{Q\cdot Q}$ which makes the norm of $2\kappa Q_\pi \cdot Q_\nu$ equal to the norm of $H_\pi^{\mathrm{SDI}} + H_\nu^{\mathrm{SDI}}$ are given in the legends of the overlap plots of fig. 1. Note how increasing the strength of the $Q_\pi \cdot Q_\nu$ term rapidly drives the yrast states towards the SU(3) strong-coupled limit. For $\kappa = 0.1$ the 2^+ yrast state of the $(\mathrm{ds})^2$–$(\mathrm{fp})^2$ configuration has a 90% overlap with the SU(3) strong-coupled state with $(\lambda\mu) = (10, 0)$. For the $(\mathrm{fp})^2$–$(\mathrm{gds})^2$ configuration the yrast states are dominated by the SU(3) strong-coupled component with $(\lambda\mu) = (14, 0)$, as $\kappa \to 0.1$. Note also that the overlaps are large even for κ-values (~ 0.05) for which the $Q_\pi \cdot Q_\nu$ interaction contributes less than half to the full width of the eigenvalue spectrum of H.

Correlations between $H_\pi^{\mathrm{SDI}} + H_\nu^{\mathrm{SDI}} + 2\kappa Q_\pi \cdot Q_\nu$ and $Q \cdot Q$ as a function of κ, calculated according to (2b) for the various fixed-J subspaces of the ds–fp and fp–dgs model spaces, are also shown in fig. 1 (right). Note that the overlap and correlation results show a remarkable similarity. Remember that the correlation is an averaged feature of all eigenstates whereas the overlap is a detailed comparison of eigenstates. That they are so similar for the yrast states is a positive feature of the statistical theory.

The overlaps of non-yrast states are also interesting. On the left side of tables 1 and 2 we compare, for the three lowest energy states of each J, the square of the overlaps of the eigenstates of $Q \cdot Q$ with those for both $H(\kappa = 0)$ and $H(\kappa = 0.1)$. The $|\langle\phi_\beta|\psi_\beta\rangle|^2$ overlaps, for $\beta = 1$, repeat information already given in fig. 1 and are included for comparison purposes only. (In the tables it is the square of overlaps

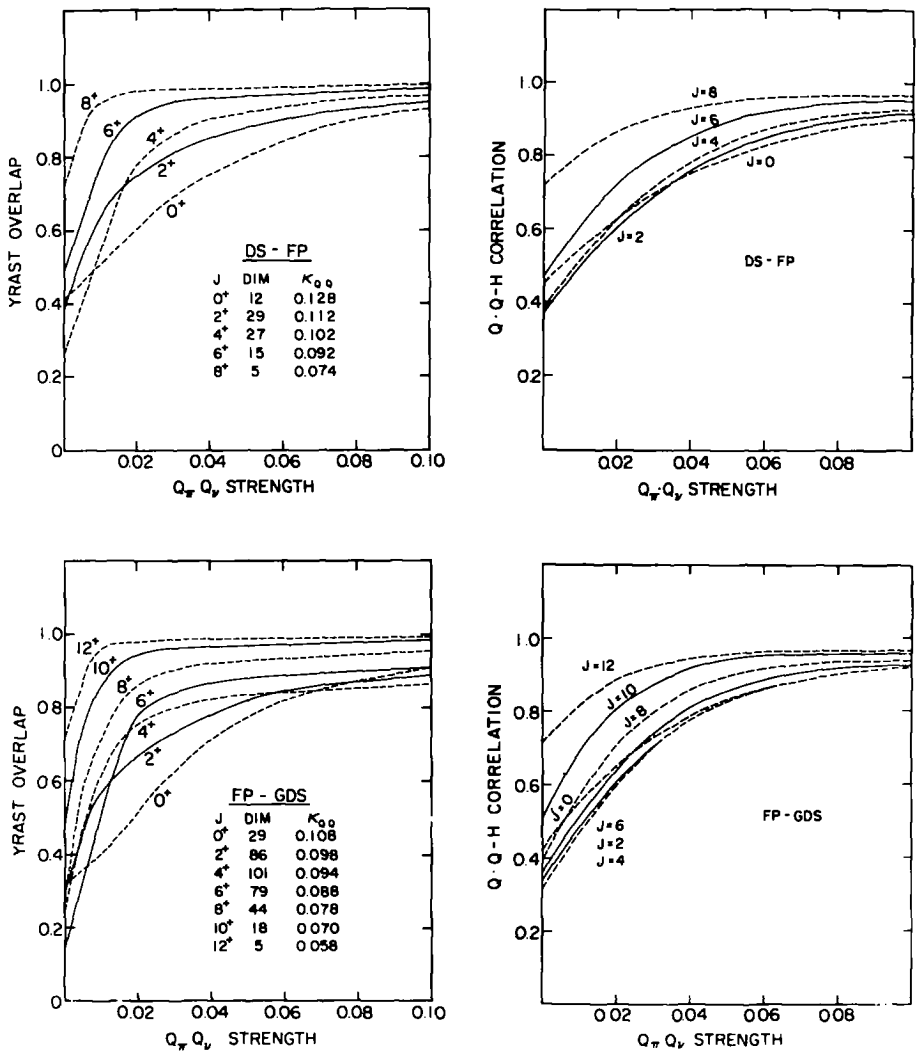

Fig. 1. Variation of the yrast overlaps and fixed-J correlations as a function of the strength (κ) of the $2Q_\pi \cdot Q_\nu$ part of our model hamiltonian. $\kappa_{Q\cdot Q}$ denotes the strength of κ which makes the norm of the π–ν interaction equal to that of the π–$\pi + \nu$–ν interactions.

that are given whereas in the figure the absolute values of the overlaps are plotted.) Note that, except for the 0_2^+ state, switching on the π–ν interaction drives the states towards good SU(3) symmetry. That the 0_2^+ state is not noticeably affected is possibly indicative of a pairing vibrational nature for this state. The non-yrast states, nevertheless, are not as strongly driven towards good SU(3) symmetry as are the yrast states. This serves to re-emphasize the fact that a strong correlation between $Q \cdot Q$ and $H(\kappa = 0.1)$ does not necessarily imply that all overlaps are large. What

TABLE 1

The projection of the first three eigenstates (ψ_β) of $H(\kappa) = H_{\pi\pi}^{\mathrm{SDI}} + H_\nu^{\mathrm{SDI}} + 2\kappa Q_\pi \cdot Q_\nu$ for $\kappa = 0.0$ and $\kappa = 0.1$ into a subspace of the $\pi^2\nu^2$ ds–fp system spanned by eigenstates (ϕ_α) of $Q \cdot Q$

J_β	$\lvert\langle\phi_\beta\lvert\psi_\beta\rangle\rvert^2$		$\sum_{\alpha=1}^{\beta+1} \lvert\langle\phi_\alpha\lvert\psi_\beta\rangle\rvert^2$		$\sum_{\alpha=1}^{6} \lvert\langle\phi_\alpha\lvert\psi_\beta\rangle\rvert^2$	
	$\kappa = 0.0$	$\kappa = 0.1$	$\kappa = 0.0$	$\kappa = 0.1$	$\kappa = 0.0$	$\kappa = 0.1$
0_1	0.18	0.86	0.39	0.96	0.82	1.0
0_2	0.08	0.11	0.57	0.57	0.90	0.96
0_3	0.44	0.35	0.53	0.94	0.77	0.99
2_1	0.16	0.90	0.28	0.90	0.63	1.0
2_2	0.07	0.89	0.29	0.93	0.64	0.96
2_3	0.03	0.21	0.22	0.56	0.23	0.89
4_1	0.07	0.94	0.27	0.94	0.37	1.0
4_2	0.03	0.86	0.47	0.94	0.84	0.97
4_3	0.0	0.25	0.44	0.65	0.74	0.92
6_1	0.24	0.97	0.64	0.97	0.77	1.0
6_2	0.25	0.88	0.32	0.94	0.55	0.98
6_3	0.02	0.74	0.51	0.95	0.80	0.96
8_1	0.54	0.99	0.92	0.99		
8_2	0.55	0.94	0.90	0.98		
8_3	0.0	0.93	0.07	0.97		

then does the strong correlation measure imply about the eigenstates of the two operators? The answer is demonstrated by results given in the middle and right-hand columns of tables 1 and 2. A large correlation means that the overlap of an eigenstate ψ_α of $H(\kappa = 0.1)$ with a subspace spanned by several neighboring (in energy) eigenstates of $-Q \cdot Q$ ($\phi_{\alpha-2}$, $\phi_{\alpha-1}$, ϕ_α, $\phi_{\alpha+1}$, $\phi_{\alpha+2}$, for example) is large. Thus, as the tabulated results show, the three lowest energy eigenstates of $H(\kappa = 0.1)$ are almost wholly contained in the subspace spanned by the six lowest energy eigenstates of $-Q \cdot Q$. This behavior must hold for the rest of the eigenstates of $H(\kappa = 0.1)$ as well because the correlation is a measure of how the expectation value of one operator varies as a function of the eigenvalues of the other. Thus a strong correlation implies that eigenstates of H for which the energy E is large and negative span a space within which the result of diagonalizing $Q \cdot Q$ would yield eigenvalues which are also large and negative, etc. An operator measure comparable to an eigenstate overlap would be the expectation value of the operator in say the nth eigenstate of H compared to the nth eigenvalue of the operator in the same space. The correlation integrates all such measures into a single number.

Once we know that the correlation is large the fact that the eigenstate overlaps are large for yrast states can be understood by noting the special character of these

TABLE 2

The projection of the first three eigenstates (ψ_β) of $H(\kappa) = H_\pi^{\mathrm{SDI}} + H_\nu^{\mathrm{SDI}} + 2\kappa Q_\pi \cdot Q_\nu$ for $\kappa = 0.0$ and $\kappa = 0.1$ into a subspace of the $\pi^2\nu^2$ fp–sdg system spanned by eigenstates (ϕ_α) of $Q \cdot Q$

J_β	$\lvert\langle\phi_\beta\lvert\psi_\beta\rangle\rvert^2$		$\sum_{\alpha=1}^{\beta+1} \lvert\langle\phi_\alpha\lvert\psi_\beta\rangle\rvert^2$		$\sum_{\alpha=1}^{6} \lvert\langle\phi_\alpha\lvert\psi_\beta\rangle\rvert^2$	
	$\kappa = 0.0$	$\kappa = 0.1$	$\kappa = 0.0$	$\kappa = 0.1$	$\kappa = 0.0$	$\kappa = 0.1$
0_1	0.10	0.83	0.17	0.94	0.47	0.99
0_2	0.13	0.09	0.55	0.55	0.63	0.91
0_3	0.15	0.40	0.25	0.96	0.39	0.99
2_1	0.10	0.79	0.10	0.79	0.34	0.98
2_2	0.03	0.80	0.16	0.88	0.37	0.90
2_3	0.03	0.74	0.18	0.94	0.20	0.95
4_1	0.06	0.75	0.07	0.96	0.28	0.99
4_2	0.06	0.33	0.25	0.91	0.45	0.98
4_3	0.09	0.34	0.28	0.84	0.32	0.96
6_1	0.02	0.84	0.11	0.84	0.24	0.99
6_2	0.14	0.63	0.37	0.90	0.44	0.98
6_3	0.0	0.60	0.24	0.90	0.25	0.97
8_1	0.09	0.92	0.35	0.92	0.44	1.0
8_2	0.10	0.70	0.12	0.92	0.20	0.97
8_3	0.09	0.69	0.46	0.95	0.80	0.96
10_1	0.23	0.97	0.63	0.97	0.76	1.0
10_2	0.35	0.88	0.47	0.95	0.61	0.99
10_3	0.02	0.82	0.49	0.96	0.72	0.97
12_1	0.51	0.99	0.91	0.99		
12_2	0.53	0.93	0.90	0.98		
12_3	0.0	0.93	0.04	0.98		

states. As pointed out earlier, statistical spectroscopy studies teach us that the eigenvalue spectra of H and $-Q \cdot Q$ are both nearly gaussian. This means that for each interaction and spin there is a single unique and energetically well-separated state of minimum energy. A strong correlation between H and $-Q \cdot Q$ implies that both their yrast eigenvalues are large and negative simultaneously. This is only possible when the yrast states of H are dominated by the single leading irreducible representation of SU(3). For non-yrast states representation mixing can occur at no cost to the expectation value of $-Q \cdot Q$; the system is not constrained and representation mixing can and normally does occur, reducing the overlaps even though the correlation is large.

5. Discussion

Previous efforts towards a microscopic interpretation of the structure of heavy and deformed nuclei have taken a $Q_\pi \cdot Q_\nu$ term in H to simulate the proton–neutron

interaction. Calculations using the random phase approximation [9]), the Kumar–Baranger method [10]) and the boson expansion technique [11]) have found that data are fit by a pure $Q \cdot Q$ interaction added to a pairing force just as well as by interactions for which the strengths of $Q_\pi \cdot Q_\pi + Q_\nu \cdot Q_\nu$ and $2Q_\pi \cdot Q_\nu$ vary freely. Within the framework of the RPA, it is known that the strength of $Q_\pi \cdot Q_\nu$ must be appreciable or else a doublet of low-lying 2^+ states is predicted, each state representing a state of collective proton or neutron character, respectively. Since the experimental data for numerous nuclei denies this possibility it is clear that a relatively large $Q_\pi \cdot Q_\nu$ term is required.

The importance of the $Q_\pi \cdot Q_\nu$ interaction is also indicated by the work of Federman and Pittel [12]). They argue that deformation arises in the nuclear system when the isoscalar component of the π–ν interaction dominates over the $T = 1$ pairing interaction. In our analyses the $Q_\pi \cdot Q_\nu$ term is the source of the isoscalar interaction. Also, Hartree–Fock–Bogoliubov studies by Banerjee *et al.* [13]) in the rare earth region included a $2Q_\pi \cdot Q_\nu$ strength slightly larger than $Q_\pi \cdot Q_\pi + Q_\nu \cdot Q_\nu$ in order to fit data. It also appears that the $Q_\pi \cdot Q_\nu$ interaction will play an important role in any future microscopic versions of the interacting boson approximation [14]). The net result of these studies is to indicate that a relatively strong $Q_\pi \cdot Q_\nu$ term is a reasonable and necessary part of the residual interaction.

In the present work we have used the very simple $\pi^2\nu^2$ system to make a detailed quantitative study of the validity of pseudo-SU(3) symmetry truncations as a function of the strength of the $Q_\pi \cdot Q_\nu$ interaction with a simple hamiltonian for which the π–π and ν–ν interactions in their separate spaces are far from $Q \cdot Q$ type. The specific cases $\tilde{d}\tilde{s}$–$\tilde{f}\tilde{p}$ and $\tilde{f}\tilde{p}$–$\tilde{g}\tilde{d}\tilde{s}$, involving the normal parity (N) spaces relevant for rare earth and actinide nuclei, have been discussed in detail. We have carried out similar calculations for the cases $\tilde{d}\tilde{s}$–$\tilde{d}\tilde{s}$ (Ge region) and $\tilde{f}\tilde{p}$–$\tilde{f}\tilde{p}$ (Ba region) and exactly the same trends as for $\tilde{d}\tilde{s}$–$\tilde{f}\tilde{p}$ and $\tilde{f}\tilde{p}$–$\tilde{g}\tilde{d}\tilde{s}$ were obtained. However, for neutrons and protons filling the same major shell, an isospin invariant interaction should be chosen. Despite the lack of this in $H(\kappa)$ the yrast states were nevertheless dominantly $T = 0$. Even so, we place no emphasis on our quantitative results in these cases. An earlier study of the $(\tilde{s}\tilde{d})^4$ configuration by Strottman [15]), using Kuo–Brown interactions, found that pseudo-SU(3) symmetry is poor in the $\nu^4(^{60}\mathrm{Ni})$ case, but fairly good in the $\pi^2\nu^2(^{60}\mathrm{Zn})$ nucleus. In terms of the present analysis, we note that the deformation-inducing $Q_\pi \cdot Q_\nu$ part of the Kuo–Brown interaction is only effective in a $\pi^2\nu^2$ system; albeit an even stronger $Q_\pi \cdot Q_\nu$ component appears to be needed to attain the high degree of symmetry found in π–ν systems by our study.

For nuclei with a large number of valence particles outside the closed shells, overlap comparisons like those given in the present work cannot be generated. Indeed we intend to use the pseudo-SU(3) symmetry to truncate the basis so as to make calculations tractable. It is an underlying feature of the Bohr–Mottelson picture of nuclear deformation that as we proceed into the shell, the sphericity

favoring short-ranged residual interactions are overcome by the tendency of nucleons to align themselves with the non-spherical core, which is itself generated by the field produced by the valence nucleons. Since the $Q \cdot Q$ interaction gives a contribution to the binding which goes as N^2 whereas the pair binding contribution is proportional to N, the system tends to deform as more particles are added. Since this picture has enjoyed overwhelming success in understanding properties of many nuclei, the encouraging result of the present study indicates that pseudo-SU(3) symmetry truncations might be even more valid in nuclei with a large number of valence protons and neutrons. If the major effects of the unnatural parity orbitals can still be treated within the seniority scheme, as in our previous studies, then fully microscopic shell-model calculation may be possible to explore high-spin phenomena in rare earth and actinide nuclei.

In summary, we have been able to show in the present study that even a small admixture of a $Q_\pi \cdot Q_\nu$ interaction, whether H_π and H_ν are $Q \cdot Q$-like or not, induces a strong correlation between H and $Q \cdot Q$ and forces each yrast eigenstate of H to have a large overlap with the corresponding single leading SU(3) strong-coupled basis state. A significant feature is that the correlation can apparently be used to anticipate the overlap result. We know this to be the case from ds and fp shell-model studies and have demonstrated it here for various $\pi^2\nu^2$ model spaces. As the correlation measure is an average property of operators it can be calculated with relative ease even for larger numbers of valence particles. Thus we anticipate the correlation measure being a useful tool in basis truncation considerations. Finally we hope to initiate shell-model studies of high-spin phenomena in rare earth and actinide nuclei similar to those carried out for Ge and Ba but with the π–ν direct product restricted to the leading strong-coupled irreducible representation of pseudo SU(3).

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