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THE GENERATION OF DIGITAL RANDOM TIME HISTORIES

H.T. CUONG,* A.W. TROESCH† and T.G. BIRDSALL†

Department of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, MI 48109, U.S.A.

Abstract—"A method to generate digital random time histories is described. A random number sequence is shaped to give the desired spectral density curve. This finite set of numbers is then Inverse Fast Fourier Transformed (IFFT). The result is a pseudo random time history which has given spectral characteristics. An application of this technique is described."

INTRODUCTION

THE GENERATION of digital random time histories which have given stochastic properties is of interest to those working in the field of marine dynamics. Projects involving computer simulation or model towing tank experiments find these random records quite useful and in many cases essential.

Various techniques have been devised to create these random records. Johnson (1981) catalogs the ones currently used by different towing tanks throughout the world. He classifies them as either non-deterministic or deterministic generation. Non-deterministic generation implies a non-repeating record while deterministic generation implies a record that contains a finite number of discrete components.

The generation method described in this paper contains aspects of both the non-deterministic and deterministic methods. A random number generator is used to generate digital white noise. This non-deterministic source is then operated on by a shaping filter and the result is inverse fast Fourier transformed (IFFT) to give the desired time history. The use of the IFFT in fact means that the record is represented by the sum of a finite number of sine waves. However, this number can be made as large as the IFFT algorithm allows. The details are given in the following sections. An example where the method is used to generate random waves is also described.

PROBLEM STATEMENT

The objective of this work is to describe the means by which a "random" time history may be generated such that the record would have a specified spectral density function. In the application described in this work, the "random" record is used to drive a wave maker that produces irregular seas. The block diagram shown in Fig. 1 helps to define this problem in a more concrete manner.

*University of Michigan, Ann Arbor, Michigan; currently Westinghouse Electric Co., Marine Division, Sunnyvale, California, U.S.A.

†University of Michigan, Ann Arbor, Michigan, U.S.A.

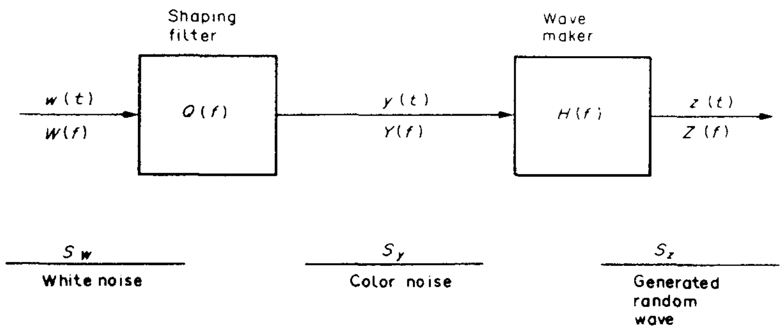

Fig. 1. Procedural block diagram.

In Figure 1 and throughout the rest of this work, the following comments apply:

— The lower case letters $w(t)$, $y(t)$ and $z(t)$ denote time functions.
— The upper case letters $W(f)$, $Y(f)$ and $Z(f)$ denote their corresponding Fourier Transforms.
— S_w, S_y and S_z are the corresponding spectral densities.
— $w(t)$ is white noise with spectral density: $S_w = 1$.
— $y(t)$ is the control time history to be used to drive the wave generator.
— $z(t)$ is the wave height at a particular point.

The problem can be restated precisely as follows:
Given $H(f)$, the system function of the wave generator, and S_z, the desired spectral density function of the generated waves, the required control function $y(t)$ has to be found so that the wave generator produces S_z.
In the above scheme, $y(t)$ can be regarded as a colored noise. Consequently, the problem may be viewed as that of finding a shaping filter $Q(f)$ for whitening the colored noise. After obtaining $Q(f)$ the process can be reversed by passing white noise through this shaping filter. The desired control function $y(t)$ will appear at the output. It should be noted that white noise can be generated fairly easily by a number of standard procedures. For the sake of simplicity and for the ease of implementation, the additional constraint that $Q(f)$ is a real function will be made. In other words, the shaping filter will not distort the phase angle of the white noise input.

DERIVATION

In this section, the relationship between $Q(f)$ and the given quantities, $S_z(f)$ and $H(f)$, will be illustrated. It will be shown that the problem of generating a simulated time history from a given spectral density function is nothing more than a corrollary of the wave generator problem.

Applying a fundamental result of Linear System theory (see, for example, Crandall and Mark (1973)) to the block diagram, the following relationship can be written:

$$Z(f) = H(f) \cdot Q(f) \cdot W(f) \;, \tag{1}$$

and

$$Y(f) = Q(f) \cdot W(f) \;. \tag{2}$$

In a similar manner, the spectral densities are related by:

$$S_z(f) = |H(f)|^2 \cdot |Q(f)|^2 \cdot S_w(f) \;.$$

Note that the white noise spectral density S_w was chosen to be unity. Hence,

$$S_z(f) = |H(f)|^2 \cdot |Q(f)|^2 \;.$$

Therefore:

$$|Q(f)|^2 = \frac{S_z(f)}{|H(f)|^2} \;.$$

Recalling that we have placed a constraint by selecting $Q(f)$ to be a real function the above relationship becomes:

$$Q(f) = \frac{\sqrt{S_z(f)}}{|H(f)|} \;. \tag{3}$$

Substituting the above transfer function for the shaping filter back into Equation (2) we arrive at:

$$Y(f) = \frac{\sqrt{S_z(f)}}{|H(f)|} \cdot W(f) \;. \tag{4}$$

Finally, note that $Y(f)$ is the Fourier transform of the plunger control time history. Thus, the desired record may be produced by taking the inverse Fourier transform of $Y(f)$.

Implementation

Referring back to the block diagram shown in Fig. 1, if we set:

$$H(f) = 1 \;,$$

then, $Y(f)$ and $Z(f)$ are identical. They are the Fourier transform of the desired time record, and

$$Y(f) = \sqrt{S(f)} \cdot W(f) \ . \tag{5}$$

For the purpose of convenience, this simplifying assumption will be used in this paper. Having a non-unity transfer function does not change the basic technique described here.

Recalling that $W(f)$ is the Fourier transform of the white noise, it follows that $W(f)$ is a complex random function. In digital computer simulation, $W(f)$ is represented by a sequence:

$$\{W_k \ , \ k = 1 \ , \ 2^n\} \ ,$$

where:

$$W_1 = W(0) \ ,$$

and

$$W_k = W[(k - 1)\Delta f] \ .$$

The number 2^n frequencies is selected to enable efficient use of fast Fourier transforms (FFT) algorithms. The frequency increment Δf is related to the length of the data record T by:

$$\Delta f = \frac{1}{T} \ .$$

Let f_{max} be the frequency range of $W(f)$ and, hence, $Y(f)$, then

$$\begin{aligned} f_{max} &= (2^n - 1)\Delta f \\ &= (2^n - 1) \frac{1}{T} \end{aligned}$$

or

$$(2^n - 1) = f_{max} \cdot T \ . \tag{6}$$

This equation gives us the relationship between the number of terms of the sequence $\{W_k\}$ to be used and the range of frequency and data length.

The complex random function $W(f)$ can be represented by 2^n complex random numbers, W_k, where

$$W_k = R_R e^{j\theta 2\pi} \tag{7}$$

and where

R_R is a Rayleigh random variable with variance of unity,

and

θ is a random variable, uniformly distributed between [0,1].

Using Equations (4) or (5), 2^n points of $Y(f)$ can be generated as a sequence of $\{Y_k , k = 1 , 2^n\}$. Taking the IFFT of $\{Y_k\}$, will produce a time sequence $\{y_j , j = 1 , 2^n\}$. However, the result will show a great deal of scattering. Smoothing can be achieved by the following scheme: recall that the spectral density function is already assumed nonzero in the frequency range $[0, f_{\max}]$. Here $f_{\max}$ is the highest frequency of interest. However, the sampling time must be carried out at the Nyquist rate,

$$f_{\text{sampling}} = 2 f_{\max} \ .$$

Therefore, the data must be padded by another 2^n of zeros adding to the high end of the frequency range. The zeros will ensure that the time record will not have any component of frequency higher than the maximum frequency of the information contained in the spectral density function. A further degree of smoothing can be achieved by padding the data with 2^{n+1}, 2^{n+2}, etc. of zeros. However, this might increase the cost of the IFFT process.

In summary, the following four steps describe the procedure to generate a random time history:

1. Generate 2^n of random complex numbers W_k, using Equation (7)

   $$W_k = R_R e^{j2\pi\theta} \ ;$$

2. Multiply W_k by the transfer function $Q(f)$ of the shaping filter; Equations (4) or (5); getting a sequence of the output,

   $$\{Y_k , k = 1 , 2^n\} \ ;$$

3. Pad the sequence of the output $\{Y_k\}$ with 2^n zeros.
4. Taking the IFFT of $\{Y_k\}$ will produce the time sequence $\{Y_j\}$. The smoothing is automatically achieved by data padding of step 3.

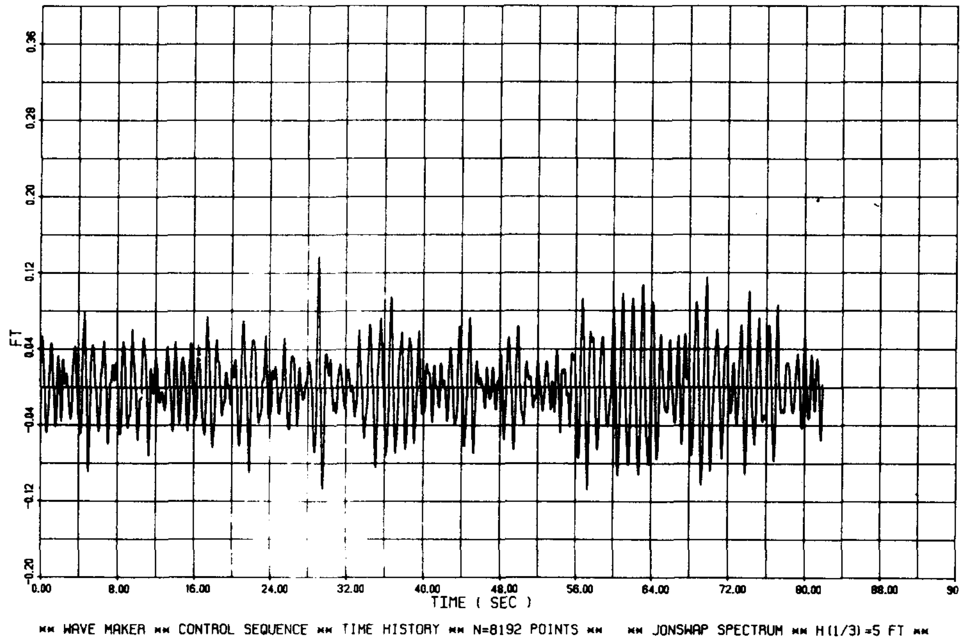

Fig. 2. Time history for a JONSWAP spectrum. Scale ratio 1:66.

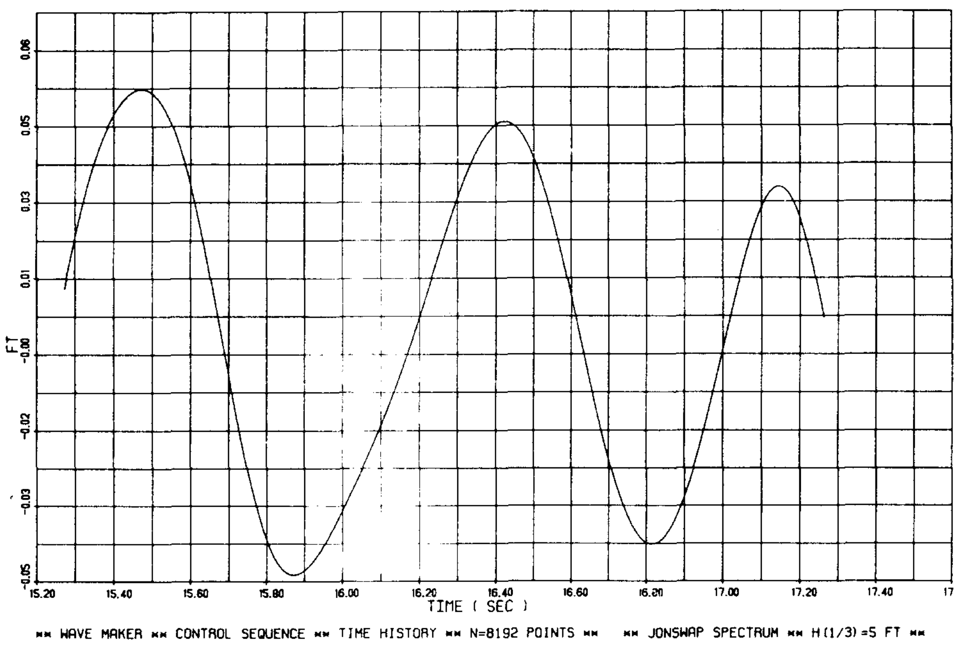

Fig. 3. Time history for a JONSWAP spectrum. Scale ratio 1:66.

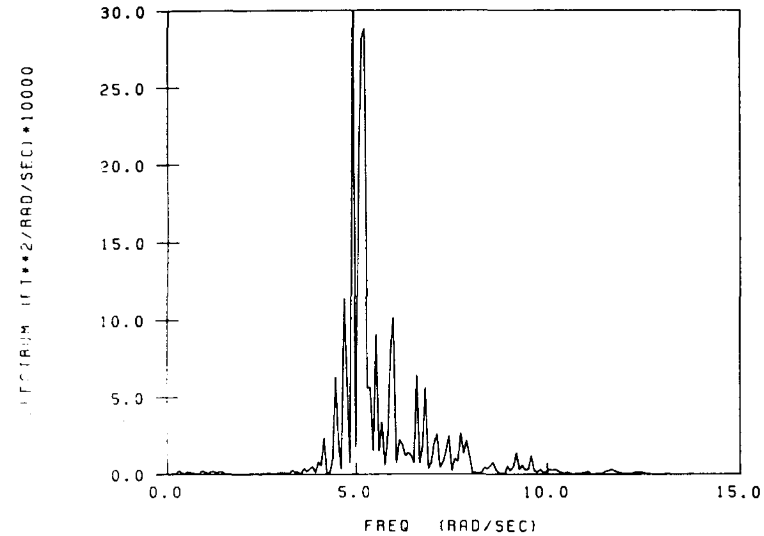

FIG. 4. Periodogram of run No. 3. JONSWAP spectrum. $H_{1/3}$ = 12 ft (3.66 m). Scale ratio 1:66.

Example

As an example of the technique, a series of time histories were generated to drive the wave maker of the Ship Hydrodynamics Laboratory, University of Michigan. The desired output was a JONSWAP spectrum that had either a 5-ft (1.52m) or a 12-ft (3.66m) significant wave height. The model scale was 1:66.

Figure 2 shows a typical 80 sec, model scale, time history of $\{y_j\}$, the input to the wave maker.

Figure 3 shows a short segment of another run to demonstrate the effects of zero padding. As can be seen, the record is sufficiently smooth.

Figure 4 shows the periodogram of a single run. The ordinates of the periodogram are given by

$$\{S_k\} = \{|Z|_k\}^2/\Delta f \ .$$

Figure 5 shows the averaged periodogram for the five runs. The technique used to average was simply summing the values of S_k for a given k and dividing by five. This and other methods of spectral smoothing are described by Otnes and Enochson (1978). The desired JONSWAP spectrum is also shown in Fig. 5. The actual curve and desired curve compare well up to the frequency of maximum S_k. However, for higher frequencies, the

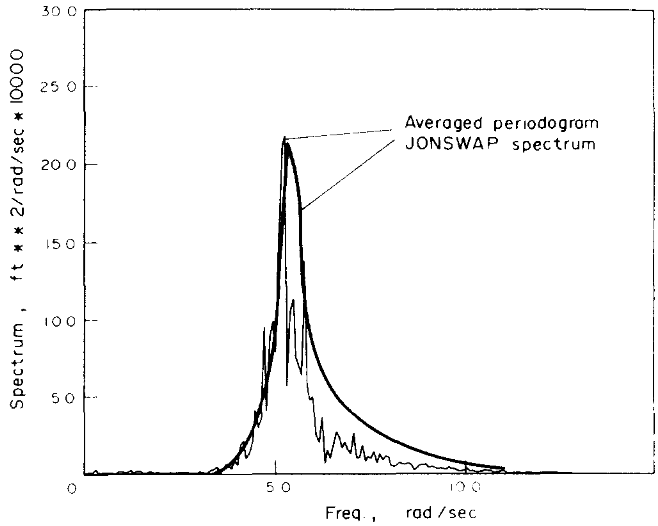

Fig. 5. Averaged periodogram of runs 1–5. JONSWAP spectrum. $H_{1/3}$ = 12 ft (3.66 m). Scale ratio 1:66.

comparison is poor. This is a result of inadequately estimating the transfer function of the wave maker for short waves. The discrepancy, though, does ot detract from the usefulness of the method outlined in this work for generating random time histories.

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