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LETTER TO THE EDITOR

ON THE PLANE EXTERNAL CRACK

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The problem of the plane external crack in an unbounded elastic solid was recently reconsidered by Stallybrass [1]. In this paper, Stallybrass showed that existing solutions imply the presence of loads remote from the crack tips and he attempted to reduce these loads to zero by superposition of classical solutions. In other words, the total force R^∞ and the total moment M^∞ on the crack plane (including the applied loads) were required to vanish. It is the purpose of this letter to show that it is not always possible to prescribe arbitrarily R^∞ and M^∞ and still maintain that the crack remains completely open. Consider, for instance, the example given in [1]: a pair of compressive forces P are applied at distance b-a from the right crack tip on the crack faces. If R^∞ and M^∞ are not required to vanish the normal tractions along the bond and the derivative of the gap along the cracks are easily obtained as

$$\sigma_{yy}(x,0) = \frac{P}{\pi}\frac{1}{(a^2 - x^2)^{1/2}}F(x), \qquad |x| < a \qquad (1)$$

$$\frac{dg(x)}{dx} = \frac{\kappa + 1}{2\mu\pi}\frac{P\,\mathrm{sgn}x}{(x^2 - a^2)^{1/2}}F(x), \qquad |x| > a \qquad (2)$$

where

$$F(x) = (b^2 - a^2)^{1/2}\left(\frac{1}{b - x} + \frac{2x}{a^2}\right) + \frac{2M^\infty}{Pa^2}x + \frac{R^\infty}{P} \qquad (3)$$

This solution is obtained on the assumption that the cracks remain completely open. It is, therefore, physically meaningful only when

$$g \geq 0, \quad |x| > a \qquad (4)$$

This condition is not satisfied if $R^\infty = M^\infty = 0$ as in [1]. Equation (1) then reduces to

$$\sigma_{yy}(x,0) = \frac{P}{\pi}\left(\frac{b^2 - a^2}{a^2 - x^2}\right)^{1/2}\left(\frac{1}{b - x} + \frac{2x}{a^2}\right), \quad |x| < a \tag{5}$$

which is identical to equation (17) of [1]. It is immediately clear that the stress intensity factor is negative at the left tip $(x = -a)$ and it follows from asymptotic analysis [2] that (4) is violated near $x = -a$.

Alternatively, we can demonstrate that a negative gap is developed in this case by setting $R^\infty = M^\infty = 0$ in equation (2). We then obtain

$$\frac{dg}{dx} = \frac{(\kappa + 1)P}{2\pi\mu} \operatorname{sgn} x \left(\frac{b^2 - a^2}{x^2 - a^2}\right)^{1/2}\left(\frac{1}{b - x} + \frac{2x}{a^2}\right), \quad |x| > a \tag{6}$$

from which we see that $dg/dx > 0$ near $x = -a$.

Furthermore, if we examine the expression for dg/dx in the more general case where R^∞ and M^∞ are not required to vanish (1 - 3) we see that as $|x| \to \infty$, dg/dx tends to the constant value $(\kappa + 1)[M^\infty + P(b^2 - a^2)^{1/2}]/\mu\pi a^2$. Hence, (4) will be violated at large positive or negative x for all values of M^∞ except

$$M^\infty = -P(b^2 - a^2)^{1/2} \tag{7}$$

In addition, the requirement that no interpenetration occurs near the tips imposes the condition

$$R^\infty \geq -P\left(\frac{b - a}{b + a}\right), \tag{8}$$

as seen from equation (2).

In view of these considerations, we suggest that the author should extend his investigation of the problem to allow the possibility of regions of contact between the crack faces.

REFERENCES

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