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ERRATUM

CORRECTION TO "NEWTON POLYHEDRA AND FACTORIAL RINGS"

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The Grothendieck theorem cited in Section 2 of my paper [3] needs an additional assumption: $Y = V(\mathcal{J})$ is an ample divisor in X. Consequently, the main theorem of the paper and its corollary require an additional assumption on the Newton polyhedron Δ_F of a Laurent polynomial F: Δ_F is nonsingular in every one-dimensional face. The latter means the following. For every face Γ of Δ set

$$\sigma_\Gamma = \bigcup_{\lambda \geq 0} \lambda(\Delta - m),$$

where m lies strictly inside of the face Γ (see [1, 5.8]). Then Δ is said to be nonsingular in Γ, if the dual cone $\check{\sigma}_\Gamma$ is spanned by a part of a basis of $Z^n \subset \mathbb{R}^n$. For example, let $\{v_1, \ldots, v_m\}$ be the set of vertices of Δ and $\Gamma = \langle v_i, v_j \rangle$ is a one-dimensional face of Δ. Then Δ is nonsingular in Γ if $\det(v_{j_1} - v_i, \ldots, v_{j_k} - v_i) = \pm 1$, where $\langle v_i, v_{j_1} \rangle, \ldots, \langle v_i, v_{j_k} \rangle$ is the set of all 1-faces passing through the vertex v_i and $k = n$.

The proof goes in a similar way. However, instead of taking a fan Σ for which the torus variety X_Σ is nonsingular and the supporting function f of Δ_F is linear on every $\sigma \in \Sigma$, we have to take the uniquely defined fan Σ formed by the maximal cones σ on which the function f is linear. In this case the function f is strictly convex with respect to Σ and the corresponding invertible sheaf $\mathcal{L}_\Sigma$ is ample on $X = X_\Sigma$ (see [1, 6.9]). The condition of nonsingularity of Δ_F implies that the variety X is nonsingular outside of a finite set of points, which are fixed under the torus action. This implies that every nondegenerate Laurent polynomial supported in Δ_F defines a nonsingular hypersurface Y in X with $\mathcal{O}_X(Y) \simeq \mathcal{L}_\Sigma$. Applying the Grothendieck theorem we get an open set U containing Y such that its complement $X - U$ consists of a finite set of torus invariant points and the canonical map $\mathrm{Pic}(U) \to \mathrm{Pic}(Y)$ is bijective. The rest of the proof remains unchanged.

For general Δ_F satisfying other conditions of the theorem, one can compute the ideal class group $C(A_F)$ via Δ_F. For example, if Δ_F is simplicial in every vertex [2, Section 2], then $C(A_F)$ is a finite abelian group for any nondegenerate Laurent polynomial supported in Δ_F.

References

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[3] I. Dolgachev, Newton polyhedra and factorial rings, J. Pure Appl. Algebra 18 (1980) 253–258.