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NOISE ANALYSIS OF A ZERO POWER CRITICAL LINE REACTOR USING LANGEVIN'S TECHNIQUE

ŞARMAN GENÇAY[1] and ZIYA AKCASU[2]

[1]Institute for Nuclear Energy, Technical University of Istanbul, Emirgân, Istanbul, Turkey
[2]Nuclear Engineering Department, University of Michigan, Ann Arbor, MI 48105, U.S.A.

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Abstract—Noise analysis of a critical, infinite-length, zero-power line reactor is given using the Langevin technique. A one-speed model is used and delayed neutrons are ignored. Stochastic analysis is carried out starting from the Boltzman equation with the assumption that neutrons move only in both directions in the line reactor. The power spectrum of neutron fluctuations is obtained and compared with that computed from diffusion approximation. Exact expressions for auto and cross-power spectrums of non-fission neutron detectors' outputs are also obtained.

1. INTRODUCTION

In the present study, neutron and detectron fluctuation in an infinite, one-dimensional, one-speed prompt critical line reactor is analysed using Langevin's technique. This model was chosen because stochastic analysis can be carried out rigorously starting from the transport equation.

It is assumed that the neutrons move only in two different directions on the line. As a result of this assumption, fission neutrons are born in one of these two directions with a probability of 1/2. Scattering events are limited to that of backscatter. Delayed neutrons are ignored in this study.

This work is an application of the space-dependent reactor noise analysis using Langevin's technique (Akcasu and Osborn, 1966) to the above mentioned model (Gençay, 1977). One may find Langevin's technique in general in (Lax, 1966) and fluctuation analysis in (Lax, 1960). The noise equivalent source concept which is used in this work is an elaboration of the method used by Cohn (1960). Extensive literature on the other applications of the same technique are cited in (Satio, 1974). The fundamental aspects of the technique are also given in (Williams, 1974) besides the related references.

2. POWER SPECTRAL DENSITY

$N(x, \mathbf{\Omega}, t)$ represents the instantaneous neutron density at phase point $(x, \mathbf{\Omega})$ at instant t. The average of rapidly fluctuating neutron density $N(x, t)$ which is given by $\int N(x, \mathbf{\Omega}, t)\,\mathrm{d}\mathbf{\Omega}$ is constant for the critical reactor under consideration and will be represented by $\langle N \rangle$. $\langle N \rangle$ satisfies the following equation:

$$\mathscr{B}\langle N \rangle = 0 \tag{2.1}$$

where $\mathscr{B}$ is the Boltzman operator. $N(x, \mathbf{\Omega}, t)$ satisfies the stochastic equation (Akcasu and Osborn, 1966);

$$[\partial/\partial t + \mathscr{B}]N(x, \mathbf{\Omega}, t) = s(x, \mathbf{\Omega}, t) \tag{2.2}$$

$s(x, \mathbf{\Omega}, t)$ is called 'noise equivalent source' (NES). Defining

$$n(x, \mathbf{\Omega}, t) = N(x, \mathbf{\Omega}, t) - \langle N \rangle$$

and subtracting (2.1) from (2.2)

$$[\partial/\partial t + \mathscr{B}]n(x, \mathbf{\Omega}, t) = s(x, \mathbf{\Omega}, t) \tag{2.3}$$

Operator $\mathscr{B}$ for one speed reactor model is given by

$$\mathscr{B} = v(\mathbf{\Omega}\cdot\nabla) + r_t - \langle v \rangle r_f F(\mathbf{\Omega}) - r_s \int F(\mathbf{\Omega}'' \rightarrow \mathbf{\Omega})\,\mathrm{d}\mathbf{\Omega}'' \tag{2.4}$$

where v is neutron speed and $\langle v \rangle$ represents the average number of neutrons liberated for each neutron absorbed in a fission reaction, $r_i = v\Sigma_i$, Σ_i is macroscopic cross section for reaction i and i refers to r, f, s, a, t which represents fission, scattering, absorption and absorption plus scattering events respectively. $F(\mathbf{\Omega}'' \rightarrow \mathbf{\Omega})$ is scattering frequency and $F(\mathbf{\Omega})\,\mathrm{d}\mathbf{\Omega}$ is the probability that a neutron born of fission will emerge in $\mathrm{d}\Omega$ about $\mathbf{\Omega}$.

Neutron population for the model under consideration may be given with the aid of Dirac delta function as

$$N(x, \mathbf{\Omega}, t) = N_1(x, t)\delta(\mathbf{\Omega} - \mathbf{\Omega}_0) + N_2(x, t)\delta(\mathbf{\Omega} + \mathbf{\Omega}_0) \tag{2.5}$$

where the indices 1 and 2 represents the neutrons' move in the direction of $\mathbf{\Omega}_0$, $-\mathbf{\Omega}_0$ respectively. $\mathbf{\Omega}_0$ is a unit vector which lies on the line indicating positive direction. Thus, for the fluctuations in neutron population and NES, the following expressions may be written:

$$\left.\begin{aligned} n_1(x,t) &= n(x,\mathbf{\Omega}_0,t) = N_1(x,t) - \langle N_1\rangle \\ n_2(x,t) &= n(x,-\mathbf{\Omega}_0,t) = N_2(x,t) - \langle N_2\rangle \end{aligned}\right\} \quad (2.6)$$

$$\left.\begin{aligned} s_1(x,t) &= s(x,\mathbf{\Omega}_0,t) \\ s_2(x,t) &= s(x,-\mathbf{\Omega}_0,t) \end{aligned}\right\} \quad (2.7)$$

Since the reactor is assumed to be in a steady state condition, the average density of neutrons moving in directions 1 and 2 is equal to ($\langle N_1\rangle = \langle N_2\rangle$) and $\langle N_1\rangle + \langle N_2\rangle = \langle N\rangle$. As mentioned above, neutrons born of fission have equal chances of having the direction $\mathbf{\Omega}_0$ or $-\mathbf{\Omega}_0$ in the reactor, therefore

$$F(\mathbf{\Omega}) = \tfrac{1}{2}[\delta(\mathbf{\Omega}-\mathbf{\Omega}_0) + \delta(\mathbf{\Omega}+\mathbf{\Omega}_0)] \quad (2.8)$$

Since scattering events are limited to that of backscatter,

$$F(\mathbf{\Omega}'' \to \mathbf{\Omega}) = \delta(\mathbf{\Omega}'' + \mathbf{\Omega}) \quad (2.9)$$

In a critical reactor $r_d = \langle v\rangle r_f$, then defining

$$2A = 2r_t - \langle v\rangle r_f = \langle v\rangle r_f + 2r_s \quad (2.10)$$

and integrating both sides of (2.3) over $d\mathbf{\Omega}$, using equations (2.4)–(2.10), the following sets of equations are obtained:

$$\left.\begin{aligned} &\frac{\partial n_1(x,t)}{\partial t} + v\frac{\partial n_1(x,t)}{\partial x} \\ &\qquad + A[n_1(x,t) - n_2(x,t)] = s_1(x,t) \\ &\frac{\partial n_2(x,t)}{\partial t} - v\frac{\partial n_2(x,t)}{\partial x} \\ &\qquad - A[n_1(x,t) - n_2(x,t)] = s_2(x,t) \end{aligned}\right\} \quad (2.11)$$

Operators $\mathscr{D}_1$, $\mathscr{D}_2$, and $\mathscr{L}$ are defined as

$$\left.\begin{aligned} \mathscr{D}_1 &= \frac{\partial}{\partial t} + v\frac{\partial}{\partial x} + A \\ \mathscr{D}_2 &= \frac{\partial}{\partial t} - v\frac{\partial}{\partial x} + A \\ \mathscr{L} &= \mathscr{D}_1\mathscr{D}_2 - A^2 \end{aligned}\right\} \quad (2.12)$$

A prime will be used to indicate the same operators of independent variables of t' and x'. Then (2.11) gives

$$\left.\begin{aligned} \mathscr{L}n_1(x,t) &= \mathscr{D}_2 s_1(x,t) + As_2(x,t) \\ \mathscr{L}n_2(x,t) &= \mathscr{D}_1 s_2(x,t) + As_1(x,t) \end{aligned}\right\} \quad (2.13)$$

For auto- and cross-correlation functions of $n_1(x,t)$ and $n_2(x,t)$, which are given by

$$\langle n_i(x,t)n_i(x',t')\rangle = \Phi_{n_i n_i}(x,t;x',t')$$

and

$$\langle n_i(x,t)n_j(x',t')\rangle = \Phi_{n_i n_j}(x,t;x',t')$$

for $i,j = 1,2$, $i \neq j$, the following relations can be obtained:

$$\begin{aligned} \mathscr{L}\mathscr{L}'\Phi_{n_i n_i}&(x,t;x',t') \\ &= \langle[\mathscr{D}_j s_i(x,t) + As_j(x,t)][\mathscr{D}'_j s_i(x',t') \\ &\quad + As_j(x',t')]\rangle \\ &= \mathscr{D}_j\mathscr{D}'_j\langle s_i(x,t)s_i(x',t')\rangle + A\mathscr{D}_j\langle s_i(x,t)s_j(x',t')\rangle \\ &\quad + A\mathscr{D}'_j\langle s_j(x,t)s_i(x',t')\rangle \\ &\quad + A^2\langle s_j(x,t)s_j(x',t')\rangle \end{aligned} \quad (2.14)$$

$$\begin{aligned} \mathscr{L}\mathscr{L}'\Phi_{n_i n_j}&(x,t;x',t') \\ &= \langle[\mathscr{D}_j s_i(x,t) + As_j(x,t)][\mathscr{D}'_i s_j(x',t') \\ &\quad + As_i(x',t')]\rangle \\ &= \mathscr{D}_i\mathscr{D}'_i\langle s_i(x,t)s_j(x',t')\rangle + A\mathscr{D}_j\langle s_i(x,t)s_i(x',t')\rangle \\ &\quad + A\mathscr{D}'_i\langle s_j(x',t')s_j(x,t)\rangle \\ &\quad + A^2\langle s_j(x,t)s_j(x',t')\rangle \end{aligned} \quad (2.15)$$

where $i,j = 1,2$, $i \neq j$ in both equations. The correlation functions of NES on the right hand side of (2.14) and (2.15) will be evaluated in Section 3. Let operator $\mathscr{M}$ be defined

$$\mathscr{M} = \mathscr{L}\mathscr{L}' \quad (2.16)$$

Since the reactor is assumed to be in a stationary state and have infinite length,

$$\mathscr{M}\Phi_{n_i n_j}(x,t;x',t') = \mathscr{M}\Phi_{n_i n_j}(y,\tau) \quad (i,j = 1,2)$$

where $y = x - x'$, $\tau = t - t'$. As a result of this transformation

$$\left.\begin{aligned} \mathscr{D}_1 &= \frac{\partial}{\partial\tau} + v\frac{\partial}{\partial y} + A \\ \mathscr{D}'_1 &= -\frac{\partial}{\partial\tau} - v\frac{\partial}{\partial y} + A \\ \mathscr{D}_2 &= \frac{\partial}{\partial\tau} - v\frac{\partial}{\partial y} + A \\ \mathscr{D}'_2 &= -\frac{\partial}{\partial\tau} + v\frac{\partial}{\partial y} + A \end{aligned}\right\} \quad (2.17)$$

Equation (2.15) gives

$$\begin{aligned} \mathscr{M}\Phi_{n_i n_j}(y,\tau) &= \left(\frac{\partial^2}{\partial\tau^2} + 2A\frac{\partial}{\partial\tau} - v^2\frac{\partial^2}{\partial y^2}\right) \\ &\quad \times \left(\frac{\partial^2}{\partial\tau^2} - 2A\frac{\partial}{\partial\tau} - v^2\frac{\partial^2}{\partial y^2}\right)\Phi_{n_i n_j}(y,\tau) \end{aligned} \quad (2.18)$$

and Fourier transform with respect to τ

$$\mathscr{F}\{\mathscr{M}\Phi_{n_i n_j}(y,\tau)\} = \left(v^2 \frac{\mathrm{d}^2}{\mathrm{d}y^2} + \eta^2\right) \times \left(v^2 \frac{\mathrm{d}^2}{\mathrm{d}y^2} + \eta^{*2}\right)\mathscr{F}\{\Phi_{n_i n_j}(y,\tau)\} \quad (2.19)$$

where

$$\eta^2 = \omega(\omega - 2Ai) \quad (2.20)$$

On the other hand, equations (2.14) and (2.15) can be written as follows:

$$\mathscr{M}\phi_{n_i n_j}(y,\tau) = H_{ij}(y,\tau) \quad (i,j = 1,2) \quad (2.21)$$

The Fourier transform of the functions on the right hand side of (2.21), which are given in Appendix A(A-2), are obtained using the relations found in Section 3. The Dirac delta functions in the expressions of $\tilde{H}_{ij}$ suggest the application of Green's function. We then seek to obtain Green's function which satisfies the equation

$$\mathscr{M}G(y,\tau) = \delta(y)\delta(\tau)$$

Taking Fourier transform with respect to τ

$$\mathscr{F}\{\mathscr{M}G(y,\tau)\} = \delta(y) \quad (2.22)$$

First, we should find that the Green's function satisfies $\mathscr{F}\{\mathscr{M}G(y,\tau)\} = 0$ which can be written as $\tilde{\mathscr{M}}\tilde{G}(y,\omega) = 0$ where (see 2.19)

$$\tilde{\mathscr{M}} = \left(v^2 \frac{\mathrm{d}^2}{\mathrm{d}y^2} + \eta^2\right)\left(v^2 \frac{\mathrm{d}^2}{\mathrm{d}y^2} + \eta^{*2}\right) \quad (2.23)$$

Since $\tilde{G}(y,\omega)$ tends to zero, as y tends to $\pm\infty$ and symmetric about $y = 0$ axis,

$$\tilde{G}(y,\omega) = c_2 \exp\{-(\beta - i\alpha)|y|/v\} + c_4 \exp\{-(\beta + i\alpha)|y|/v\} \quad (2.24)$$

where

$$\eta^2 = (\alpha - i\beta)^2 \quad (2.25)$$

$$\left.\begin{aligned} \alpha &= \frac{\omega}{\sqrt{2}}[1 + \sqrt{1 + 4(A/\omega)^2}]^{1/2} \\ \beta &= \frac{A\omega}{\alpha} \\ \alpha^2 + \beta^2 &= \omega^2\sqrt{1 + 4(A/\omega)^2} \\ \alpha^2 - \beta^2 &= \omega^2 \end{aligned}\right\} \quad (2.26)$$

Continuity of the first derivative at $y = 0$ gives $c_4 = -c_2(\beta - i\alpha)/(\beta + i\alpha)$.

In Green's function, the first and second derivatives are continuous at every y; the third derivative is also continuous at every y, except $y = 0$. Thus, discontinuity at $y = 0$ in (2.22) can be investigated easily, integrating both sides on the interval $(+\epsilon, -\epsilon)$. Then, $c_2 = 1/(8i\beta\alpha v(\beta - i\alpha)$ is obtained and the solution for $\tilde{G}(y,\omega)$ is written as,

$$\tilde{G}(y,\omega) = \frac{\exp\{-\beta|y|/v\}}{4v\beta(\beta^2 + \alpha^2)} \times \left[\frac{\beta}{\alpha}\sin\left(\frac{\alpha}{v}|y|\right) + \cos\left(\frac{\alpha}{v}|y|\right)\right] \quad (2.27)$$

Fourier transform of (2.21) with respect to τ is $\tilde{\mathscr{M}}\tilde{\Phi}_{n_i n_j}(y,\omega) = \tilde{H}_{ij}(y,\omega)$. Then $\tilde{\Phi}(y,\omega)$ can be evaluated by the following integral (see Appendix A(A-2) for $\tilde{H}_{ij}$):

$$\tilde{\Phi}_{n_i n_j}(y,\omega) = \int_{-\infty}^{+\infty} \tilde{G}(y - y',\omega)\tilde{H}_{ij}(y',\omega)\,\mathrm{d}y' \quad (2.28)$$

It is obvious from (2.5) and (2.6) that

$$n(x,t) = n_1(x,t) + n_2(x,t),$$

then

$$\Phi_{nn}(y,\tau) = \langle n(x,t)n(x',t')\rangle$$

can be written as

$$\begin{aligned} &\langle n(x,t)n(x',t')\rangle \\ &= \langle [n_1(x,t) + n_2(x,t)][n_1(x',t') + n_2(x',t')]\rangle \\ &= \langle n_1(x,t)n_1(x',t')\rangle + \langle n_1(x,t)n_2(x',t')\rangle \\ &\quad + \langle n_2(x,t)n_1(x',t')\rangle + \langle n_2(x,t)n_2(x',t')\rangle \end{aligned}$$

or

$$\begin{aligned} &\tilde{\Phi}_{nn}(y,\omega) \\ &= \tilde{\Phi}_{n_1 n_1}(y,\omega) + \tilde{\Phi}_{n_1 n_2}(y,\omega) + \tilde{\Phi}_{n_2 n_1}(y,\omega) \\ &\quad + \tilde{\Phi}_{n_2 n_2}(y,\omega) \end{aligned} \quad (2.29)$$

Substituting (2.27) and the expressions for $\tilde{H}_{ij}(y,\omega)$ [Appendix A(A-2)] in (2.28), and integrating the right hand side with the aid of formulas given in Appendix B, then using (2.29), the power spectral density function of neutron fluctuations for the model under consideration is obtained as

$$\begin{aligned} \tilde{\Phi}_{nn}(y,\omega) = \frac{\langle N\rangle \exp\{-\beta|y|/v\}}{4v\beta} &\times \left\{\frac{\beta}{\alpha}[(\mu - 2A)\sqrt{1 + 4(A/\omega)^2} - (\mu + 2A)]\sin\left(\frac{\alpha}{v}|y|\right)\right. \\ &\left. + [(\mu - 2A)\sqrt{1 + 4(A/\omega)^2} + (\mu + 2A)]\cos\left(\frac{\alpha}{v}|y|\right)\right\} \end{aligned} \quad (2.30)$$

3. NOISE EQUIVALENT SOURCE EVALUATION

Fluctuations in $s_1(x,t)$ and $s_2(x,t)$, which are the NES for the neutrons move in directions $\mathbf{\Omega}_0$ and $-\mathbf{\Omega}_0$ respectively (see 2.7), are due to the random fluctuations in the capture, scattering events and fluctuations in the number of neutrons produced per fission. We can thus write

$$\left.\begin{aligned} s_1(x,t) &= -s_1^c(x,t) + s_1^f(x,t) - s_1^s(x,t) \\ s_2(x,t) &= -s_2^c(x,t) + s_2^f(x,t) - s_2^s(x,t) \end{aligned}\right\} \quad (3.1)$$

where c, f, s represent capture, fission and scattering events, respectively. Scattering and capture cause loss of neutrons for their NES. This is represented by minus signs. Since there is no correlation between capture fission and scattering events, the correlation function, which is given generally by

$$\Phi_{ss}(x,t;x',t') = \langle s(x,t)s(x',t')\rangle,$$

can be written as

$$\left.\begin{aligned} \Phi_{s_1s_1} &= \Phi^c_{s_1s_1} + \Phi^f_{s_1s_1} + \Phi^s_{s_1s_1} \\ \Phi_{s_2s_2} &= \Phi^c_{s_2s_2} + \Phi^f_{s_2s_2} + \Phi^s_{s_2s_2} \\ \Phi_{s_1s_2} &= \Phi^c_{s_1s_2} + \Phi^f_{s_1s_2} + \Phi^s_{s_1s_2} \\ \Phi_{s_2s_1} &= \Phi^c_{s_2s_1} + \Phi^f_{s_2s_1} + \Phi^s_{s_2s_1} \end{aligned}\right\} \quad (3.2)$$

For the sake of simplicity, $\phi_{ss}(x,t;x',t')$ is denoted by Φ_{ss} in (3.2). The reactor is assumed in a stationary state and having infinite length; therefore,

$$\Phi_{ss}(x,t;x',t') = \Phi_{ss}(y,\tau)$$

where $y = x - x'$, $\tau = t - t'$. In the following calculations, $\Phi_{ss}(y,\tau)$ will be denoted by Φ_{ss}.

The terms on the right hand side of equations (2.14) and (2.15) will be evaluated according to the expressions developed by Akcasu and Osborn (1966). The expression of the correlation function of NES for capture is given as

$$\begin{aligned} &\Phi^c_{ss}(x,\mathbf{\Omega},t;x',\mathbf{\Omega}',t') \\ &\quad = \delta(t-t')\delta(x-x')\delta(\mathbf{\Omega},-\mathbf{\Omega}')r_c\langle N(x,\mathbf{\Omega})\rangle \quad (3.3) \end{aligned}$$

where $\langle N(x,\mathbf{\Omega})\rangle$ is the average neutron density at phase point $(x,\mathbf{\Omega})$. Hence, for the model under consideration

$$\left.\begin{aligned} \Phi^c_{s_1s_1} &= \langle s_1^c(x,t)s_1^c(x',t')\rangle = \delta(y)\delta(\tau)r_c\langle N_1\rangle \\ \Phi^c_{s_2s_2} &= \langle s_2^c(x,t)s_2^c(x',t')\rangle = \delta(y)\delta(\tau)r_c\langle N_2\rangle \\ \Phi^c_{s_1s_2} &= \Phi^c_{s_2s_1} = \langle s_1^c(x,t)s_2^c(x',t')\rangle \\ &= \langle s_2^c(x,t)s_1^c(x',t')\rangle = 0 \end{aligned}\right\} \quad (3.4)$$

The last equation shows that there is no correlation between the capture of the neutrons' movement in the $\mathbf{\Omega}_0$ and $-\mathbf{\Omega}_0$ directions.

The fission event causes both gain and loss of neutrons; it is therefore found more convenient to consider that fission comprises these two events. The gain and loss events in fission are two correlated different random processes and their NES will be denoted by $s^g(x,\mathbf{\Omega},t)$, $s^l(x,\mathbf{\Omega},t)$ respectively. Thus, using (2.7)

$$\begin{aligned} \Phi^f_{s_is_j} &= \langle [s_i^g(x,t) - s_i^l(x,t)][s_j^g(x',t') - s_j^l(x',t')]\rangle \\ \Phi^f_{s_is_j} &= \langle s_i^g(x,t)s_j^g(x',t')\rangle - \langle s_i^g(x,t)s_j^l(x',t')\rangle \\ &\quad - \langle s_i^l(x,t)s_j^g(x',t')\rangle + \langle s_i^l(x,t)s_j^l(x',t')\rangle \quad (3.5) \end{aligned}$$

where $i,j = 1,2$. As given in Akcasu and Osborn (1966):

$$\begin{aligned} &\langle s^l(x,\mathbf{\Omega},t)s^g(x',\mathbf{\Omega}',t')\rangle \\ &\quad = \delta(x-x')\delta(t-t')r_f\langle N(x,\mathbf{\Omega})\rangle \\ &\quad\quad \times \sum_{\nu,\alpha} \alpha B_\alpha^\nu(\mathbf{\Omega},\mathbf{\Omega}') \quad (3.6) \end{aligned}$$

$$\begin{aligned} &\langle s^g(x,\mathbf{\Omega},t)s^g(x',\mathbf{\Omega}',t')\rangle \\ &\quad = \delta(x-x')\delta(t-t')r_f \int\Big\{\langle N(x,\mathbf{\Omega}'')\rangle \\ &\quad\quad \times \sum_{\substack{\nu,\alpha\\ \beta}} \alpha\beta B^\nu_{\alpha\beta}(\mathbf{\Omega}''|\mathbf{\Omega},\mathbf{\Omega}')\Big\}\,d\mathbf{\Omega}'' \quad (3.7) \end{aligned}$$

where $B^\nu_{\alpha\beta}(\mathbf{\Omega}''|\mathbf{\Omega},\mathbf{\Omega}')\,d\Omega\,d\Omega'$ is the probability that when a neutron in the direction $\mathbf{\Omega}''$ induces a fission event, ν prompt neutrons are emitted of which α neutrons have the direction $\mathbf{\Omega} \in d\Omega$ and β neutrons have the direction $\mathbf{\Omega}' \in d\Omega'$. Then, $B^\nu_\alpha(\mathbf{\Omega}'',\mathbf{\Omega})\,d\Omega$ will be defined as the probability, that, when a neutron in direction $\mathbf{\Omega}''$ induces a fission event, ν prompt neutrons are emitted of which α neutrons have the direction $\mathbf{\Omega} \in d\Omega$, and $B^\nu(\mathbf{\Omega}'')$ will be defined as the probability that, when a neutron in direction $\mathbf{\Omega}''$ induces a fission event, ν neutrons are emitted. And the following relations are self-evident if $\bar{B}^\nu_{\alpha\beta}(\mathbf{\Omega}''/\mathbf{\Omega},\mathbf{\Omega}')$ is defined only for the $\mathbf{\Omega} \neq \mathbf{\Omega}'$.

$$\left.\begin{aligned} B^\nu_{\alpha\beta}(\mathbf{\Omega}''|\mathbf{\Omega},\mathbf{\Omega}') &= \bar{B}^\nu(\mathbf{\Omega}''|\mathbf{\Omega},\mathbf{\Omega}') \\ &\quad + \delta(\mathbf{\Omega}-\mathbf{\Omega}')\delta_{\alpha\beta}B^\nu_\alpha(\mathbf{\Omega}'',\mathbf{\Omega}) \\ \int \bar{B}^\nu_{\alpha\beta}(\mathbf{\Omega}''|\mathbf{\Omega},\mathbf{\Omega}')\,d\Omega &= \delta_{\beta(\nu-\alpha)}B^\nu_\alpha(\mathbf{\Omega}'',\mathbf{\Omega}) \\ \int B^\nu_\alpha(\mathbf{\Omega}'',\mathbf{\Omega})\,d\Omega &= \delta_{\alpha\nu}B^\nu(\mathbf{\Omega}'') \end{aligned}\right\} \quad (3.8)$$

where $B^\nu_\alpha(\mathbf{\Omega}'',\mathbf{\Omega}_0) = B^\nu_\alpha(\mathbf{\Omega}'',-\mathbf{\Omega}_0)$ because of the assumption made about the direction of the neutrons born in fission. Then, for the model under consideration (3.8) yields

$$\left.\begin{aligned} B^{\nu}_{\alpha\beta}(\mathbf{\Omega}''|\mathbf{\Omega}_0, \mathbf{\Omega}) &= \delta^{\nu}_{\alpha\beta} B^{\nu}_{\alpha}(\mathbf{\Omega}'', \mathbf{\Omega}_0) \\ B^{\nu}_{\alpha\beta}(\mathbf{\Omega}''|\mathbf{\Omega}_0, -\mathbf{\Omega}_0) &= \delta_{\beta(\nu-\alpha)} B^{\nu}_{\alpha}(\mathbf{\Omega}'', \mathbf{\Omega}_0) \\ B^{\nu}_{\alpha}(\mathbf{\Omega}'', \mathbf{\Omega}_0) + B^{\nu}_{\alpha}(\mathbf{\Omega}'', -\mathbf{\Omega}_0) &= \delta_{\alpha\nu} B^{\nu}(\mathbf{\Omega}'') \\ B^{\nu}_{\alpha}(\mathbf{\Omega}'', \mathbf{\Omega}_0) = B^{\nu}_{\alpha}(\mathbf{\Omega}'', -\mathbf{\Omega}_0) &= \tfrac{1}{2}\delta_{\nu\alpha} B^{\nu}(\mathbf{\Omega}'') \end{aligned}\right\} \quad (3.9)$$

Multiplying both sides of (3.7) by $\delta(\mathbf{\Omega} - \mathbf{\Omega}')$ and integrating over the $\mathbf{\Omega}'\langle s^g(x, \mathbf{\Omega}t)s^g(x', \mathbf{\Omega}, t')\rangle$ is obtained. Substituting $\mathbf{\Omega}_0$ for $\mathbf{\Omega}$, then $-\mathbf{\Omega}_0$ for $\mathbf{\Omega}$, and using (2.7), (3.9) one may write

$$\left.\begin{aligned} &\langle s^g_1(x, t)s^g_1(x', t')\rangle \\ &\quad = \tfrac{1}{2}\delta(x - x')\delta(t - t')r_f\langle N\rangle\langle \nu^2\rangle \\ &\langle s^g_2(x, t)s^g_2(x', t')\rangle \\ &\quad = \tfrac{1}{2}\delta(x - x')\delta(t - t')r_f\langle N\rangle\langle \nu^2\rangle \end{aligned}\right\} \quad (3.10)$$

Following the same steps, but using $\delta(\mathbf{\Omega} + \mathbf{\Omega}')$ instead of $\delta(\mathbf{\Omega} - \mathbf{\Omega}')$

$$\langle s^g_1(x, t)s^g_2(x', t')\rangle = \langle s^g_2(x, t)s^g_1(x', t')\rangle = 0 \quad (3.11)$$

is obtained.

Since the fission event considered consists of two different events, called gain and loss events, the loss of neutrons in fission is no different than in capture. Thus, substituting r_f for r_c in (3.4)

$$\left.\begin{aligned} \langle s^l_1(x, t)s^l_1(x', t')\rangle &= \delta(y)\delta(\tau)r_f\langle N_1\rangle \\ \langle s^l_2(x, t)s^l_2(x', t')\rangle &= \delta(y)\delta(\tau)r_f\langle N_2\rangle \\ \langle s^l_1(x, t)s^l_2(x', t')\rangle &= \langle s^l_2(x, t)s^l_1(x', t')\rangle = 0 \end{aligned}\right\} \quad (3.12)$$

are obtained. Starting from (3.6) and following the similar steps used in obtaining (3.10) and (3.11)

$$\left.\begin{aligned} \langle s^g_1(x, t)s^l_1(x', t')\rangle &= \langle s^l_1(x, t)s^g_1(x', t')\rangle \\ &= \delta(y)\delta(\tau)r_f\frac{\langle N_1\rangle}{2}\langle \nu\rangle \\ \langle s^g_2(x, t)s^l_2(x', t')\rangle &= \langle s^l_2(x, t)s^g_2(x', t')\rangle \\ &= \delta(y)\delta(\tau)r_f\frac{\langle N_2\rangle}{2}\langle \nu\rangle \\ \langle s^l_1(x, t)s^g_2(x', t')\rangle &= \langle s^g_2(x, t)s^l_1(x', t')\rangle \\ &= \delta(y)\delta(\tau)r_f\frac{\langle N_1\rangle}{2}\langle \nu\rangle \\ \langle s^l_2(x, t)s^g_1(x', t')\rangle &= \langle s^g_1(x, t)s^l_2(x', t')\rangle \\ &= \delta(y)\delta(\tau)r_f\frac{\langle N_2\rangle}{2}\langle \nu\rangle \end{aligned}\right\} \quad (3.13)$$

Substituting (3.10), (3.11), (3.12), (3.13) in (3.5) for $i, j = 1, 2$, the following expressions are obtained for the correlation function of the NES for fission:

$$\left.\begin{aligned} \Phi^f_{s_1s_1} &= \Phi^f_{s_2s_2} \\ &= \delta(y)\delta(\tau)r_f\frac{\langle N\rangle}{2}(\langle \nu^2\rangle - \langle \nu\rangle + 1) \\ \Phi^f_{s_1s_2} &= \Phi^f_{s_2s_1} = -\delta(y)\delta(\tau)r_f\frac{\langle N\rangle}{2}\langle \nu\rangle \end{aligned}\right\} \quad (3.14)$$

Scattering events will be considered as fission events with $\langle \nu\rangle = 1$. But the assumption which was made about scattering events in the introduction leads us to the following model. The neutron born as a result of fission will be in the opposite direction of the neutron which induces a fission event. Therefore,

$$B^{\nu}_{\alpha}(\mathbf{\Omega}_0, \mathbf{\Omega}_0) = B^{\nu}_{\alpha}(-\mathbf{\Omega}_0, -\mathbf{\Omega}_0) = 0$$

for $\alpha = 1$ and equal to one for $\alpha = 0$. Replacing r_f by r_s and following similar steps as for fission events, (3.6), (3.7) and (3.5) yield

$$\left.\begin{aligned} \Phi^s_{s_1s_1} &= \Phi^s_{s_2s_2} = \delta(y)\delta(\tau)r_s\langle N\rangle \\ \Phi^s_{s_1s_2} &= \Phi^s_{s_2s_1} = -\delta(y)\delta(\tau)r_s\langle N\rangle \end{aligned}\right\} \quad (3.15)$$

Substituting (3.4), (3.14), (3.15) in (3.2), the following results are obtained:

$$\left.\begin{aligned} \Phi_{s_1s_1} &= \Phi_{s_2s_2} = \delta(y)\delta(\tau)\frac{\langle N\rangle}{2} \\ &\quad \times [r_t + r_f(\langle \nu^2\rangle - \langle \nu\rangle) + r_s] \\ \Phi_{s_1s_2} &= \Phi_{s_2s_1} \\ &= -\delta(y)\delta(\tau)\frac{\langle N\rangle}{2}(2r_s + r_f\langle \nu\rangle) \end{aligned}\right\} \quad (3.16)$$

The correlation function of NES may be written as $\Phi_{ss} = \Phi_{s_1s_1} + \Phi_{s_1s_2} + \Phi_{s_2s_1} + \Phi_{s_2s_2}$. Thus (3.16) yields

$$\phi_{ss} = (\mu - 2A)\langle N\rangle\delta(y)\delta(\tau) \quad (3.17)$$

where

$$\mu = r_t + r_f(\langle \nu^2\rangle - \langle \nu\rangle) + r_s \quad (3.18)$$

$2A$ is given in (2.10).

In the case of having a detector in the reactor $s^c_i(x, t)$ $(i = 1, 2)$ in (3.1) should be written as

$$s^c_i(x, t) = s^d_i(x, t) + s^{c'}_i(x, t)$$

where $s^d_i(x, t)s^{c'}_i(x, t)$ are NES for detection and capture events respectively. There is no correlation between these two events and those of fission and scattering. Therefore,

$$\langle s_i(x, t)s^d_j(x', t')\rangle = \langle s^d_i(x, t)s^d_j(x', t')\rangle = \Phi_{d_id_j} \quad (3.19)$$

where $i, j = 1, 2$. Since a detection event is the same as that of capture, (3.4) yields

$$\left.\begin{aligned} \Phi_{d_id_j} &= r_d\langle N_i\rangle\delta(y)\delta(\tau), & i &= j \\ \Phi_{d_id_j} &= 0. & i &\neq j \end{aligned}\right\} \quad (3.20)$$

The correlation function of NES for detector counts may be written $\Phi_{dd} = \phi_{d_1d_1} + \phi_{d_2d_2}$ then

$$\phi_{dd} = r_d\langle N\rangle\delta(y)\delta(\tau) \tag{3.21}$$

In Section 5, we will need $\langle\tilde{s}_i(x,\omega)\tilde{s}_j^d(x',\omega')\rangle$ which is average of the products of the Fourier transform with respect to t of the NES for the fluctuation of the neutrons' move in direction i and NES for the detection of the neutrons' move in direction j. Then, one can write

$$\langle\tilde{s}_i(x,\omega)\tilde{s}_j^d(x',\omega')\rangle = \iint_{-\infty}^{+\infty}\langle s_i(x,t)s_j^d(x',t')\rangle\exp\{-i(\omega t + \omega' t')\}\,dt\,dt'$$

where $i, j = 1, 2$. Using (3.19) and (3.20) for $i = j$

$$\langle\tilde{s}_i(x,\omega)\tilde{s}_i^d(x',\omega')\rangle = r_d\langle N_i\rangle\delta(x - x')\int_{-\infty}^{+\infty}\exp\{-i(\omega + \omega')t\}\,dt$$

$$= 2\pi r_d\langle N_i\rangle\delta(x - x')\delta(\omega + \omega') \tag{3.22}$$

and for $i \neq j$

$$\langle\tilde{s}_i(x,\omega)\tilde{s}_j^d(x',\omega')\rangle = 0 \tag{3.23}$$

are obtained.

4. DIFFUSION APPROXIMATION

In the case of diffusion approximation, direction of the neutrons will not be considered. Then,

$$n_1(x,t) + n_2(x,t) = n(x,t)$$

and

$$s_1(x,t) + s_2(x,t) = s(x,t)$$

is written. Adding both sides of equation (2.11)

$$\frac{\partial n(x,t)}{\partial t} + v\frac{\partial}{\partial x}[n_1(x,t) - n_2(x,t)] = s(x,t) \tag{4.1}$$

Defining $J(x,t)$ as the net current on direction $\mathbf{\Omega}_0$ for neutron fluctuations

$$J(x,t)\mathbf{\Omega}_0 = v[n_1(x,t) - n_2(x,t)]\mathbf{\Omega}_0$$

and using the expression of Fick's Law,† which is given as

$$J(x,t) = -Dv\frac{\partial}{\partial x}n(x,t)$$

one can write,

$$v\frac{\partial}{\partial x}[n_1(x,t) - n_2(x,t)] = -Dv\frac{\partial^2}{\partial x^2}n(x,t)$$

where D is the diffusion coefficient. Substituting the last equation in (4.1)

$$\frac{\partial n(x,t)}{\partial t} - Dv\frac{\partial^2 n(x,t)}{\partial x^2} = s(x,t) \tag{4.2}$$

the diffusion equation is obtained. Let $\mathscr{L}_0$ be defined as

$$\mathscr{L}_0 = \frac{\partial}{\partial t} - Dv\frac{\partial^2}{\partial x^2} \tag{4.3}$$

Then, with the aid of (4.2)

$$\mathscr{L}_0\mathscr{L}_0'\langle n(x,t)n(x',t')\rangle = \langle s(x,t)s(x',t')\rangle$$

is obtained. Using (4.3), (3.17) and representing the correlation function of the neutron fluctuation in the case of diffusion approximation by $\Phi_{nn}^D(x,t;x',t')$,

$$\left(\frac{\partial}{\partial t} - Dv\frac{\partial^2}{\partial x^2}\right)\left(\frac{\partial}{\partial t'} - Dv\frac{\partial^2}{\partial x'^2}\right)\Phi_{nn}^D(x,t;x',t') = \langle N\rangle(\mu - 2A)\delta(x - x')\delta(t - t')$$

is obtained. Since the reactor is assumed to be in a stationary state and have infinite length, one may write,

$$-\left(\frac{\partial}{\partial\tau} - Dv\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial}{\partial\tau} + Dv\frac{\partial^2}{\partial y^2}\right)\Phi_{nn}^D(y,\tau) = \langle N\rangle(\mu - 2A)\delta(\tau)\delta(y) \tag{4.4}$$

where $y = x - x'$ and $\tau = t - t'$ as defined before. Fourier transform of the last equation with respect to τ gives

$$\left[\frac{\partial^4}{\partial y^4} + \frac{\omega^2}{(Dv)^2}\right]\Phi_{nn}^D(y,\omega) = \frac{\langle N\rangle(\mu - 2A)}{(Dv)^2}\delta(y)$$

Following similar steps as those followed in obtaining (2.27), one can obtain the Green's function,

$$\tilde{G}_D(y,\omega) = \frac{\exp\{-(|\omega|/2Dv)^{1/2}|y|\}}{8(|\omega|/2Dv)^{3/2}} \times \{\cos[(|\omega|/2Dv)^{1/2}|y|] + \sin[(|\omega|/2Dv)^{1/2}|y|]\}$$

which satisfies the equation

$$\left[\frac{\partial^4}{\partial y^4} + \frac{\omega^2}{(Dv)^2}\right]\tilde{G}_D(y,\omega) = \delta(y)$$

† Stochastic equation $J(x,t) = -D\nabla\phi(x,t) + s_j(x,t)$ might be written. But, Fick's approximation was used. According to this approximation $\partial J/\partial t$ and $s_j(x,t)$ were neglected. $s_j(x,t)$ is NES for current and gives rise to the term $\nabla s_j(x,t)$ which may be considered as a random process uncorrelated in space and time.

Then

$$\tilde{\Phi}^D_{nn}(y,\omega) = \langle N\rangle \int_{-\infty}^{+\infty} \tilde{G}_D(y-y',\omega) \times \frac{(\mu-2A)}{(Dv)^2}\,\delta(y')\,\mathrm{d}y'$$

gives the power spectral density in the case of diffusion approximation of the reactor model under consideration, as follows:

$$\tilde{\Phi}^D_{nn}(y,\omega) = \frac{\langle N\rangle(\mu-2A)}{2|\omega|(2|\omega|Dv)^{1/2}}\exp\{-(|\omega|/2Dv)^{1/2}|y|\} \times \{\cos[(|\omega|/2Dv)^{1/2}|y|] + \sin[(|\omega|/2Dv)^{1/2}|y|]\} \qquad (4.5)$$

Comparison of the exact and diffusion approximation solutions at the low frequency range

The frequency range where $|\omega|/A \ll 1$ will be considered. Here, $A = \frac{1}{2}\langle\nu\rangle r_f + r_s$. Under this restriction (2.26) gives that $\alpha \simeq \beta \simeq (A\omega)^{1/2}$ for $\omega > 0$ and $\alpha \simeq -(A|\omega|)^{1/2}$, $\beta \simeq (A|\omega|)^{1/2}$ for $\omega < 0$. Using Fick's law, one can easily show that $D = v/2A$. Thus

$$\beta/v \simeq (|\omega|/2Dv)^{1/2}$$

$$\alpha/v \simeq \begin{cases} (|\omega|/2Dv)^{1/2} & \text{if} \quad \omega > 0 \\ -(|\omega|/2Dv)^{1/2} & \text{if} \quad \omega < 0 \end{cases}$$

Substituting the last relations in (2.30) and assuming

$$\frac{|\omega|}{2A}\,\frac{\mu+2A}{\mu-2A} \ll 1 \qquad (4.6)$$

(2.30) is reduced to (4.5). From (2.10) and (3.18)

$$\mu + 2A = r_t + 3r_s + r_f\langle\nu^2\rangle$$

$$\mu - 2A = r_f\langle\nu(\nu-1)\rangle$$

Since $r_a = \langle\nu\rangle r_f$ for critical reactor and $r_t = r_a + r_s$, (4.6) can be written as

$$\frac{|\omega|}{A} \ll \frac{\langle\nu(\nu-1)\rangle}{(2r_s/r_f) + \frac{1}{2}\langle\nu(\nu+1)\rangle} \qquad (4.7)$$

It is understood that at a low frequency range, the difference between exact solution and diffusion approximation for power spectral density function of neutron fluctuations is negligible if $|\omega|/2A \ll 1$ and (4.6) (or 4.7) are satisfied.

For comparison of both solutions, the following model is chosen:

$v = 2.2\;10^5$ cm/s, $\quad \langle\nu\rangle = 2.42$,
$r_t = 1.1582\;10^5\;\mathrm{s}^{-1}$, $\quad r_f = 2.1593\;10^4\;\mathrm{s}^{-1}$,
$r_s = 9.0374\;10^5\;\mathrm{s}^{-1}$.

Then,

$A = 1.650\;10^5\;\mathrm{s}^{-1}$, $\quad \mu + 2A = 5.3973\;10^5\;\mathrm{s}^{-1}$, $\mu - 2A = 1.0053\;10^5\;\mathrm{s}^{-1}$

are obtained. It is assumed that $\langle N\rangle = 1000$ and $\langle\nu^2\rangle - \langle\nu\rangle$ is evaluated using Diven parameter as 4.6558. The right-hand side of (4.7) is found to be 0.052. Results are given in Figs 1–6.

In Figs 1a, 2a and 3 both solutions for power spectral density versus y are given in three different frequencies, which shows that the difference between two solutions increases as frequency increases. ψ is defined as $(\tilde{\phi}_{nn} - \tilde{\phi}^D_{nn})/\tilde{\phi}_{nn}$ to give a better idea of the difference between two solutions. It is seen in Figs 1b and 2b that $|\psi|$ is greater near $y = 0$ and decreases as y increases. After a certain value of y depending on f, $|\psi|$ starts to increase.

Power spectral densities at four different y ($y = 0$, 3, 5 and 10 cm) are given in Figs 4 and 5, and Fig. 6 gives the variation on $|\psi|$ versus frequency at six different y. On this last figure, it is seen that $|\psi|$ increases as the frequency itself increases, and this increase is sharp after certain frequencies depending on

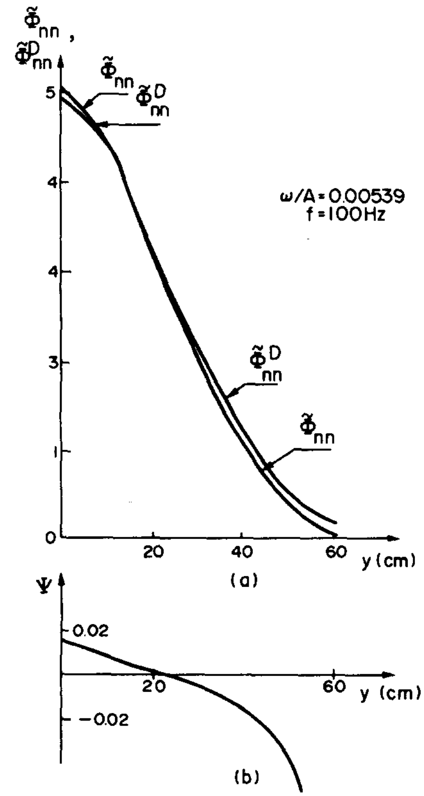

Fig. 1.

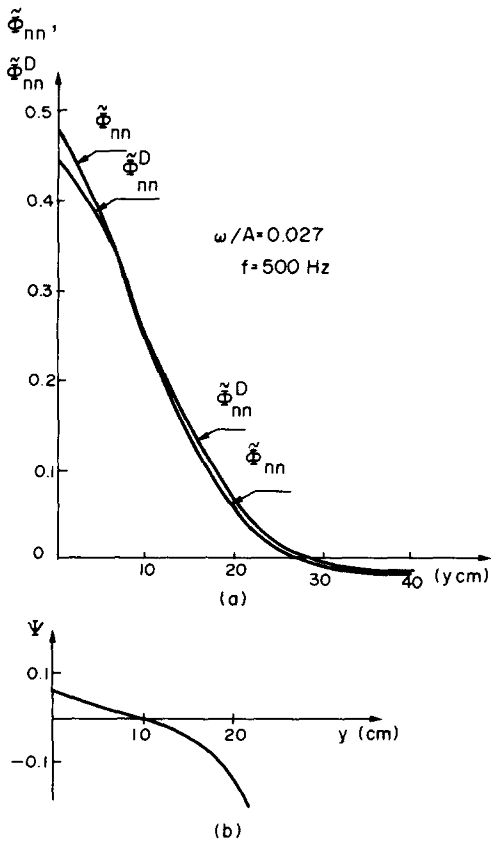

Fig. 2.

y. At a fixed frequency, starting from $y = 0$ there is a decrease, and then an increase on $|\psi|$ as y increases, as shown in Figs 1, 2 and 3. We can also see from those figures that the correlation function, which is obtained by diffusion approximation, reached the zero later than the exact correlation function thus showing artificial correlation.

5. POWER SPECTRAL DENSITY FUNCTION OF DETECTOR OUTPUT

Fluctuations of the output of a neutron detector of length L placed on the line reactor may be given as

$$c(t) = \int_{-L/2}^{+L/2} z(x, t)\,dx$$

Here

$$c(t) = C(t) - \langle C \rangle$$

$$z(x, t) = Z(x, t) - \langle Z \rangle$$

and $C(t)$ is the instantaneous value, $\langle C \rangle$ is the average value of the detector output, $Z(x, t)$ is the instantaneous detection rate per unit length about x at time t, $\langle Z(x) \rangle$ is the average detection rate per unit length, $z(x, t)$ is the fluctuation about $\langle Z(x) \rangle$. It is assumed that the mid-point of the detector is placed at the origin. The auto-correlation function of the detector, and cross-correlation of two detectors are given as follows:

$$\langle c(t)c(t') \rangle = \iint_{-L/2}^{+L/2} \langle z(x, t)z(x', t') \rangle \,dx\,dx' \quad (5.1)$$

$$\langle c_1(t)c_2(t') \rangle = \int_{x_0-L/2}^{x_0+L/2} \int_{-L/2}^{+L/2} \times \langle z(x, t)z(x', t') \rangle \,dx\,dx' \quad (5.2)$$

In the second case, the distance between mid-points of the detectors is x_0, and $x_0 > L$ is assumed.

The Fourier transform of (5.1) and (5.2) with respect to τ gives the power and cross-power spectral density functions of one and two detectors respectively.

NES in the detection process will be denoted by $s^d(x, t)$. Then

$$z(x, t) = r_d n(x, t) + s^d(x, t)$$

stochastic relation may be given. Here $r_d = \Sigma_d v$ and Σ_d is the cross section for neutron detection. For the model under consideration

$$z(x, t) = r[n_1(x, t) + n_2(x, t)] + s_1^d(x, t) + s_2^d(x, t)$$

is written. Thus, the correlation function

$$\Phi_z(x, t; x', t') = \langle z(x, t)z(x', t') \rangle$$

is given as

$$\Phi_z(x, t; x', t') = r_d^2 \Phi_{nn}(x, t; x', t') + \Phi_{dd}(x, t; x', t') + r_d \left\langle \sum_{i,j=1}^{2} n_i(x, t)s_j^d(x', t') + \sum_{i,j=1}^{2} s_j^d(x, t)n_i(x', t') \right\rangle \quad (5.3)$$

The first and second terms in this relation are given in (2.30), (3.21). The last term can be obtained as follows: the Fourier transform of (2.13) with respect to t, using (2.12) and (2.20) gives

$$\left.\begin{aligned} \left(v^2 \frac{\partial^2}{\partial x^2} + \eta^2\right)\tilde{n}_1(x, \omega) &= -F_1(x, \omega) \\ \left(v^2 \frac{\partial^2}{\partial x^2} + \eta^2\right)\tilde{n}_2(x, \omega) &= -F_2(x, \omega) \end{aligned}\right\} \quad (5.4)$$

where

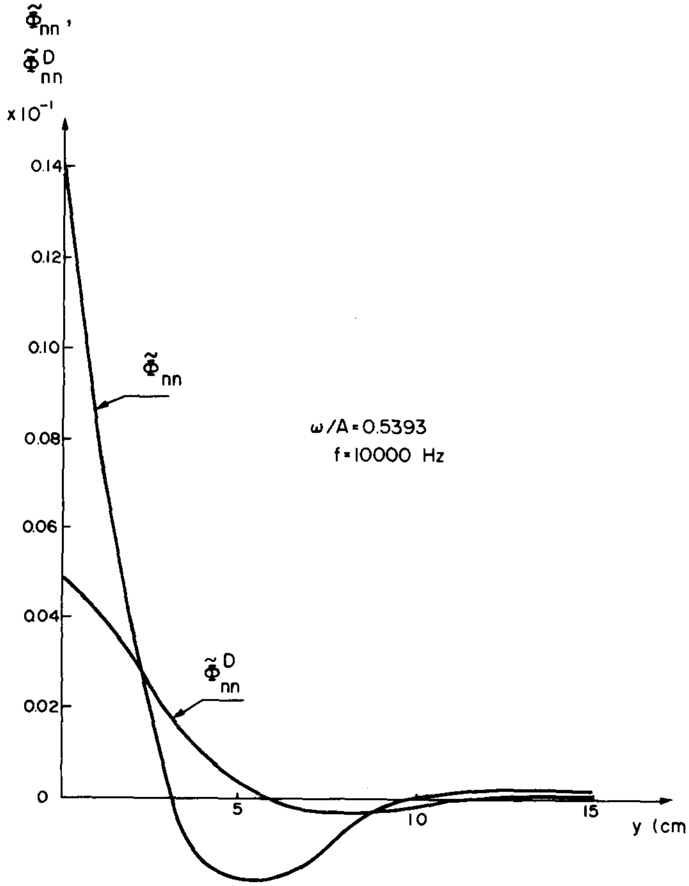
$\tilde{\Phi}_{nn}$, $\tilde{\Phi}^D_{nn}$
x 10⁻¹
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
5
10
15
y (cm)
$\tilde{\Phi}_{nn}$
$\tilde{\Phi}^D_{nn}$
ω/A = 0.5393
f = 10000 Hz

Fig. 3.

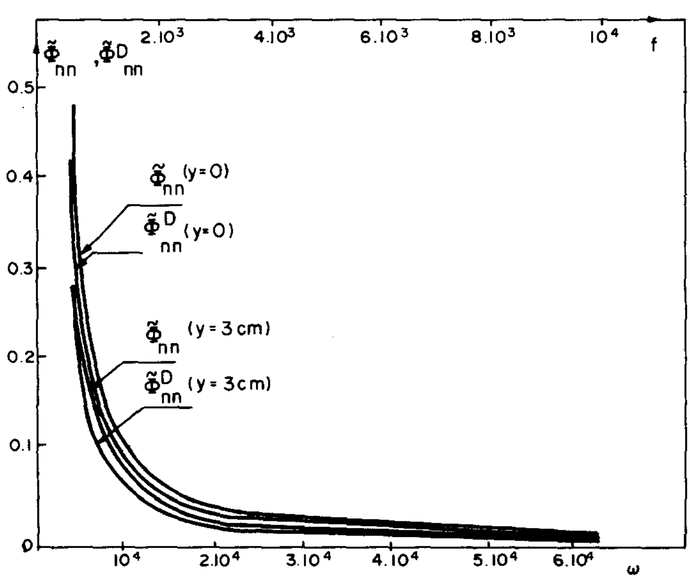
$\tilde{\Phi}_{nn}$, $\tilde{\Phi}^D_{nn}$
0.5
0.4
0.3
0.2
0.1
0
2.10³
4.10³
6.10³
8.10³
10⁴
f
$\tilde{\Phi}_{nn}$ (y = 0)
$\tilde{\Phi}^D_{nn}$ (y = 0)
$\tilde{\Phi}_{nn}$ (y = 3 cm)
$\tilde{\Phi}^D_{nn}$ (y = 3 cm)
10⁴
2.10⁴
3.10⁴
4.10⁴
5.10⁴
6.10⁴
ω

Fig. 4.

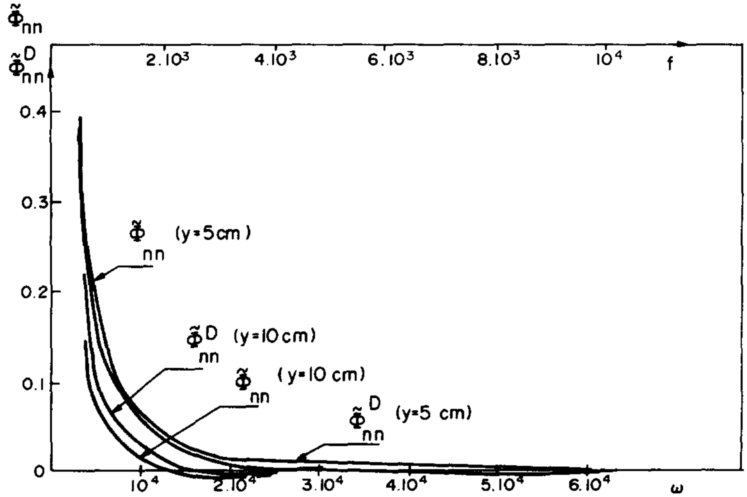

Fig. 5.

$$
\left.\begin{aligned}
F_1(x, \omega) &= i\omega\tilde{s}_1(x, \omega) - v\frac{\partial}{\partial x}\tilde{s}_1(x, \omega) \\
&\quad + A[\tilde{s}_1(x, \omega) + \tilde{s}_2(x, \omega)] \\
F_2(x, \omega) &= i\omega\tilde{s}_2(x, \omega) + v\frac{\partial}{\partial x}\tilde{s}_2(x, \omega) \\
&\quad + A[\tilde{s}_1(x, \omega) + \tilde{s}_2(x, \omega)]
\end{aligned}\right\} \quad (5.5)
$$

One obtains the solution of equation (5.4) by the method of the Green's function. Green's function, which satisfies the equation

$$
v^2\frac{d^2}{dx^2}\tilde{G}(x - x'', \omega) + \eta^2\tilde{G}(x - x'', \omega) = \delta(x - x'')
$$

is obtained by following similar steps as those mentioned in Section 2. Thus

$$
\tilde{G}(x - x'', \omega) = -\frac{1}{2v(\beta + i\alpha)} \times \exp\left\{-\frac{1}{v}(\beta + i\alpha)|x - x''|\right\} \quad (5.6)
$$

is found where α, β are given by (2.25). Then, $\tilde{n}_1(x, \omega)$ and $\tilde{n}_2(x, \omega)$ is obtained as

$$
\left.\begin{aligned}
\tilde{n}_1(x, \omega) &= -\int_{-\infty}^{+\infty}\tilde{G}(x - x'', \omega)F_1(x'', \omega)\,dx'' \\
\tilde{n}_2(x, \omega) &= -\int_{-\infty}^{+\infty}\tilde{G}(x - x'', \omega)F_2(x'', \omega)\,dx''
\end{aligned}\right\} \quad (5.7)
$$

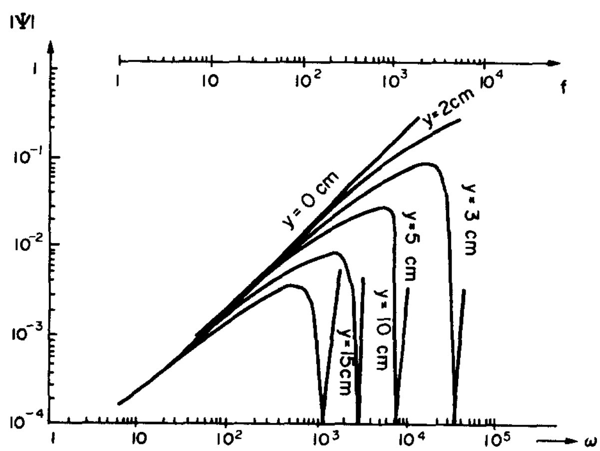

Fig. 6.

An inverse Fourier transform gives

$$n_1(x,t)s_1^d(x',t') = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\tilde{n}_1(x,\omega)\exp\{i\omega t\}\,d\omega \times \int_{-\infty}^{+\infty}\tilde{s}_1^d(x',\omega')\exp\{i\omega' t\}\,d\omega' \quad (5.8)$$

Substituting the first relation of (5.7) into (5.8), and using (5.5), (5.6) and (3.22), and the formulae in Appendix B, the correlation function

$$\Phi_{n_1d_1}(x,t;x',t') = \langle n_1(x,t)s_1^d(x',t')\rangle$$

is obtained (Gençay, 1977). Thus

$$\left.\begin{aligned}\Phi_{n_1d_1}(y,\tau) &= -\frac{r_d\langle N_1\rangle}{4\pi v}\int_{-\infty}^{+\infty}\left[\frac{A+i\omega}{\beta+i\alpha}+1\right] \times \exp\left\{-\frac{1}{v}(\beta+i\alpha)|y|\right\} \times \exp\{i\omega\tau\}\,d\omega\\ \tilde{\Phi}_{n_1d_1}(y,\omega) &= -\frac{r_d\langle N_1\rangle}{2v}\left[\frac{A+i\omega}{\beta+i\alpha}+1\right] \times \exp\left\{-\frac{1}{v}(\beta+i\alpha)|y|\right\}\end{aligned}\right\} \quad (5.9)$$

where $y = x - x'$, $\tau = t - t'$. Following similar steps

$$\tilde{\Phi}_{n_2d_2}(y,\omega) = -\frac{r_d\langle N_2\rangle}{2v}\left[\frac{A+i\omega}{\beta+i\alpha}-1\right] \times \exp\left\{-\frac{1}{v}(\beta+i\alpha)|y|\right\} \quad (5.10)$$

$$\tilde{\Phi}_{d_1n_1}(y,\omega) = -\frac{r_d\langle N_1\rangle}{2v}\left[\frac{A-i\omega}{\beta-i\alpha}+1\right] \times \exp\left\{-\frac{1}{v}(\beta-i\alpha)|y|\right\} \quad (5.11)$$

$$\tilde{\Phi}_{d_2n_2}(y,\omega) = -\frac{r_d\langle N_2\rangle}{2v}\left[\frac{A-i\omega}{\beta-i\alpha}-1\right] \times \exp\left\{-\frac{1}{v}(\beta-i\alpha)|y|\right\} \quad (5.12)$$

$$\tilde{\Phi}^*_{d_2n_1}(y,\omega) = \tilde{\Phi}_{n_1d_2}(y,\omega) = -\frac{r_d\langle N_2\rangle A}{2v(\beta+i\alpha)}\exp\left\{-\frac{1}{v}(\beta+i\alpha)|y|\right\} \quad (5.13)$$

$$\tilde{\Phi}^*_{d_1n_2}(y,\omega) = \tilde{\Phi}_{n_2d_1}(y,\omega) = -\frac{r_d\langle N_1\rangle A}{2v(\beta+i\alpha)}\exp\left\{-\frac{1}{v}(\beta+i\alpha)|y|\right\} \quad (5.14)$$

are obtained. Substituting (2.30) and (3.21), also relations (5.9) through (5.14) into the Fourier transform with respect to τ of (5.3),

$$\tilde{\Phi}_z(y,\omega) = \langle N\rangle r_d^2\exp\{-\beta|y|/v\}[K_1^0(\omega)\sin(\alpha|y|/v) + K_2^0(\omega)\cos(\alpha|y|/v)] + r_d^2\langle N\rangle\delta(y) \quad (5.15)$$

where $\langle N\rangle/2 = \langle N_1\rangle = \langle N_2\rangle$ and

$$\left.\begin{aligned}K_1^0(\omega) &= \frac{1}{4v\alpha}[(\mu-2A)\sqrt{1+4(A/\omega)^2} - (\mu+2A)]\\ K_2^0(\omega) &= -\frac{2A\beta+\omega\alpha}{v(\alpha^2+\beta^2)}+\frac{1}{4v\beta} \times [(\mu-2A)\sqrt{1+4(A/\omega)^2} + (\mu+2A)]\end{aligned}\right\} \quad (5.16)$$

(5.1) and (5.2) may be written in the following form:

$$\Phi_c(\tau) = \langle c(t)c(t')\rangle = \int_{-L/2}^{+L/2}\Phi_z(x,x',\tau)\,dx\,dx'$$

$$\Phi_{c_1c_2}(\tau) = \langle c_1(t)c_2(t')\rangle = \int_{-L/2}^{+L/2}\int_{x_0-L/2}^{x_0+L/2}\Phi_z(x,x',\tau)\,dx'\,dx$$

Taking the Fourier transform with respect to τ and substituting (5.15) into the last equations with $|y| = |x - x'|$, one obtains the following results after the integration (Gençay, 1977):

$$\tilde{\Phi}_c(\omega) = \frac{2\langle N\rangle r_d^2v^2}{\omega^4[1+4(a/\omega)^2]}\exp\{-\beta L/v\} \times [K_1(\omega)\cos(\alpha L/v) - K_2(\omega)\sin(\alpha L/v)] - K_1(\omega) + r_d^2\langle N\rangle L \quad (5.17)$$

$$\begin{aligned}\tilde{\Phi}_{c_1c_2}(\omega) = \frac{\langle N\rangle r_d^2v^2}{\omega^4[1+4(A/\omega)^2]}\exp\{-\beta x_0/v\} &\times [\![K_1(\omega)\{\exp\{\beta L/v\}\cos[(x_0-L)\alpha/v]\\ &+ \exp\{-\beta L/v\}\cos[(x_0+L)\alpha/v]\\ &- 2\cos(x_0\alpha/v)\}\\ &- K_2(\omega)\{\exp\{\beta L/v\}\sin[(x_0-L)\alpha/v]\\ &+ \exp\{-\beta L/v\}\sin[(x_0+L)\alpha/v]\\ &- 2\sin(x_0\alpha/v)\}]\!]\end{aligned} \quad (5.18)$$

Here

$$K_1(\omega) = 2A\omega K_1^0(\omega) - \omega^2K_2^0(\omega)$$
$$K_2(\omega) = 2A\omega K_2^0(\omega) + \omega^2K_1^0(\omega)$$

(5.17) and (5.18) give the auto- and cross-power spectral density functions of one and two non-overlapping neutron detectors respectively, on the reactor model under consideration.

6. CONCLUSIONS

Starting from the Boltzman equation and using Langevin's technique, exact expressions for the power spectral density of neutron fluctuations and fluctuations of detector counts are given for one and two detector cases. In the latter case, detectors are assumed non-overlapping. An expression for the neutron fluctuation, in the case of diffusion approximation, is also obtained and compared with the exact solution. It is understood that at the low frequency region, difference between both solutions is negligible. The term which resulted from detector noise on the auto-power spectral density function disappeared on the cross-power spectral density function of the two detectors, as expected.

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APPENDIX A

One may obtain the Fourier transform of $H_{ij}(y, \tau)$ (see 2.21) with respect to τ as follows: substituting (3.16) in (2.14) and (2.15) then using (2.21),

$$\left.\begin{aligned}
\tilde{H}_{11}(y,\omega) &= \mathscr{F}\left\{[\mu\mathscr{D}_2\mathscr{D}_2' - 2A^2\mathscr{D}_2 - 2A^2\mathscr{D}_2' + \mu A^2]\frac{\langle N\rangle}{2}\delta(y)\delta(\tau)\right\} \\
\tilde{H}_{22}(y,\omega) &= \mathscr{F}\left\{[\mu\mathscr{D}_1\mathscr{D}_1' - 2A^2\mathscr{D}_1 - 2A^2\mathscr{D}_1' + \mu A^2]\frac{\langle N\rangle}{2}\delta(y)\delta(\tau)\right\} \\
\tilde{H}_{12}(y,\omega) &= \mathscr{F}\left\{[-2A\mathscr{D}_2\mathscr{D}_1' + \mu A\mathscr{D}_2 + \mu A\mathscr{D}_1' - 2A^3]\frac{\rangle N\rangle}{2}\delta(y)\delta(\tau)\right\} \\
\tilde{H}_{21}(y,\omega) &= \mathscr{F}\left\{[-2A\mathscr{D}_1\mathscr{D}_2' + \mu A\mathscr{D}_1 + \mu A\mathscr{D}_2' - 2A^3]\frac{\langle N\rangle}{2}\delta(y)\delta(\tau)\right\}
\end{aligned}\right\} \quad (A.1)$$

is obtained. Here, μ and A are given by (2.10) and (3.18) respectively. On the other hand, with the aid of (2.17) we may write

$$\mathscr{F}\{\mathscr{D}_1\mathscr{D}_1'[\delta(y)\delta(\tau)]\} = \left(\omega^2 - 2iv\omega\frac{d}{dy} - v^2\frac{d^2}{dy^2} + A^2\right)\delta(y)$$

$$\mathscr{F}\{\mathscr{D}_2\mathscr{D}_2'[\delta(y)\delta(\tau)]\} = \left(\omega^2 + 2iv\omega\frac{d}{dy} - v^2\frac{d^2}{dy^2} + A^2\right)\delta(y)$$

$$\mathscr{F}\{\mathscr{D}_1\mathscr{D}_2'[\delta(y)\delta(\tau)]\} = \left(\omega^2 + 2vA\frac{d}{dy} + v^2\frac{d^2}{dy^2} + A^2\right)\delta(y)$$

$$\mathscr{F}\{\mathscr{D}_2\mathscr{D}_1'[\delta(y)\delta(\tau)]\} = \left(\omega^2 - 2vA\frac{d}{dy} + v^2\frac{d^2}{dy^2} + A^2\right)\delta(y)$$

and

$$\mathscr{F}\{\mathscr{D}_1[\delta(y)\delta(\tau)]\} = \left(i\omega + v\frac{d}{dy} + A\right)\delta(y)$$

$$\mathscr{F}\{\mathscr{D}_1'[\delta(y)\delta(\tau)]\} = \left(-i\omega - v\frac{d}{dy} + A\right)\delta(y)$$

$$\mathscr{F}\{\mathscr{D}_2[\delta(y)\delta(\tau)]\} = \left(i\omega - v\frac{d}{dy} + A\right)\delta(y)$$

$$\mathscr{F}\{\mathscr{D}_2'[\delta(y)\delta(\tau)]\} = \left(-i\omega + v\frac{d}{dy} + A\right)\delta(y)$$

Substituting the last equations in (A.1) $\tilde{H}_{ij}(y, \omega)$ is obtained as

$$\left.\begin{aligned}
\tilde{H}_{11}(y,\omega) &= \frac{\langle N\rangle}{2}\{[\mu\omega^2 + 2A(\mu - 2A)]\delta(y) + 2\mu v\omega i\delta'(y) - \mu v^2\delta''(y)\} \\
\tilde{H}_{22}(y,\omega) &= \frac{\langle N\rangle}{2}\{[\mu\omega^2 + 2A(\mu - 2A)]\delta(y) - 2\mu v\omega i\delta'(y) - \mu v^2\delta''(y)\} \\
\tilde{H}_{12}(y,\omega) &= \frac{\langle N\rangle}{2}\{[-2A\omega^2 + 2A(\mu - 2A)]\delta(y) - 2Av(\mu - 2A)\delta'(y) - 2Av^2\delta''(y)\} \\
\tilde{H}_{21}(y,\omega) &= \frac{\langle N\rangle}{2}\{[-2A\omega^2 + 2A(\mu - 2A)]\delta(y) + 2Av(\mu - 2A)\delta'(y) - 2Av^2\delta''(y)\}
\end{aligned}\right\} \quad (A.2)$$

APPENDIX B

$$\int_{-\infty}^{\infty} dy' \exp\left\{-\frac{\beta}{v}|y - y'|\right\}\sin\left(\frac{\alpha}{v}|y - y'|\right)\delta(y') = \exp\left\{-\frac{\beta}{v}|y|\right\}\sin\left(\frac{\alpha}{v}|y|\right) \quad (B.1)$$

$$\int_{-\infty}^{\infty} dy' \exp\left\{-\frac{\beta}{v}|y - y'|\right\}\cos\left(\frac{\alpha}{v}|y - y'|\right)\delta(y') = \exp\left\{-\frac{\beta}{v}|y|\right\}\cos\left(\frac{\alpha}{v}|y|\right) \quad (B.2)$$

$$\int_{-\infty}^{\infty} dy' \exp\left\{-\frac{\beta}{v}|y - y'|\right\}\sin\left(\frac{\alpha}{v}|y - y'|\right)\delta'(y') = \exp\left\{-\frac{\beta}{v}|y|\right\}\left[\left(\frac{\beta}{v}\right)\sin\left(\frac{\alpha}{v}|y|\right) - \frac{\alpha}{v}\cos\left(\frac{\alpha}{v}|y|\right)\right] \quad (B.3)$$

$$\int_{-\infty}^{\infty} dy' \exp\left\{-\frac{\beta}{v}|y-y'|\right\}\cos\left(\frac{\alpha}{v}|y-y'|\right)\delta'(y')$$

$$= \exp\left\{-\frac{\beta}{v}|y|\right\}\left[\left(\frac{\beta}{v}\right)\cos\left(\frac{\alpha}{v}|y|\right)+\frac{\alpha}{v}\sin\left(\frac{\alpha}{v}|y|\right)\right] \quad \text{(B.4)}$$

$$\int_{-\infty}^{\infty} dy' \exp\left\{-\frac{\beta}{v}|y-y'|\right\}\sin\left(\frac{\alpha}{v}|y-y'|\right)\delta''(y')$$

$$= \exp\left\{-\frac{\beta}{v}|y|\right\}\left[\left(\frac{\beta}{v}\right)^2\sin\left(\frac{\alpha}{v}|y|\right)\right.$$

$$\left.-\frac{2\alpha\beta}{v^2}\cos\left(\frac{\alpha}{v}|y|\right)-\left(\frac{\alpha}{v}\right)^2\sin\left(\frac{\alpha}{v}|y|\right)\right] \quad \text{(B.5)}$$

$$\int_{-\infty}^{\infty} dy' \exp\left\{-\frac{\beta}{v}|y-y'|\right\}\cos\left(\frac{\alpha}{v}|y-y'|\right)\delta''(y')$$

$$= \exp\left\{-\frac{\beta}{v}|y|\right\}\left[\left(\frac{\beta}{v}\right)^2\cos\left(\frac{\alpha}{v}|y|\right)\right.$$

$$\left.+\frac{2\alpha\beta}{v^2}\sin\left(\frac{\alpha}{v}|y|\right)-\left(\frac{\alpha}{v}\right)^2\cos\left(\frac{\alpha}{v}|y|\right)\right] \quad \text{(B.6)}$$