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Neutrino Mass in the SO(10) Grand Unified Gauge Model

YUKIO TOMOZAWA

Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48109

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The problem of neutrino mass generation by a large Majorana mass term in the $SO(10)$ grand unified gauge model is discussed. In particular, it is pointed out that the left-hand Majorana mass induced by the radiative corrections does not exceed that obtained by diagonalization of the mass matrix.

I. INTRODUCTION

The question of neutrino mass has been with us for a long time. This is because there is no principle which prevents neutrinos from acquiring mass on the one hand and all the existing experiments indicate that the masses must be small on the other [1]. In recent articles [2, 3] several authors have suggested that the Higgs mechanism in the $SO(10)$ model [4] can give a large Majorana mass for the right-handed (RH) neutrino and then a small mass for the left-handed (LH) neutrino results from the diagonalization of the mass matrix.

In this article, we look into the problem of the LH neutrino mass induced by radiative corrections and see whether the above mechanism for the explanation of the observed small neutrino mass is spoiled or not. In Section II, a general description of the Majorana mass and the Dirac mass is given. The LH Majorana mass generated by the radiative corrections is discussed in Section III.

II. THE MAJORANA MASS AND THE DIRAC MASS

A general expression for the mass term for neutrinos is given by

$$-\mathscr{L} = \tfrac{1}{2}M_{\mathrm{R}}(\bar{\nu}_{\mathrm{R}}{}^{c}\nu_{\mathrm{R}} + \bar{\nu}_{\mathrm{R}}\nu_{\mathrm{R}}{}^{c}) + \tfrac{1}{2}m_{\mathrm{L}}(\bar{\nu}_{\mathrm{L}}{}^{c}\nu_{\mathrm{L}} + \bar{\nu}_{\mathrm{L}}\nu_{\mathrm{L}}{}^{c}) + m(\bar{\nu}_{\mathrm{L}}\nu_{\mathrm{R}} + \bar{\nu}_{\mathrm{R}}\nu_{\mathrm{L}}), \quad (2.1)$$

where $M_{\mathrm{R}}(m_{\mathrm{L}})$ and m are the Majorana masses for the RH (LH) neutrino ν_{R} (ν_{L}) and the Dirac mass, respectively. The suffix c stands for the charge conjugated field and is defined by

$$\psi^{c} = C\bar{\psi}^{T}, \qquad \bar{\psi}^{c} = -\psi^{T}C^{-1}, \quad (2.2)$$

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where C is the 4×4 charge conjugation matrix. Hence, it is easy to see the identity

$$\bar{\psi}_1{}^c\psi_2 = \bar{\psi}_2{}^c\psi_1 \tag{2.3}$$

for anti-commuting fields ψ_i.

Defining the Majorana fields

$$\begin{aligned} \xi &\equiv \nu_L + \nu_L{}^c, \\ \eta &\equiv \nu_R + \nu_r{}^c \end{aligned} \tag{2.4}$$

and using the identities

$$\begin{aligned} \bar{\xi}\xi &= \bar{\nu}_L{}^c\nu_L + \bar{\nu}_L\nu_L{}^c, \\ \bar{\eta}\eta &= \bar{\nu}_R{}^c\nu_R + \bar{\nu}_R\nu_R{}^c, \\ \bar{\xi}\eta &= \bar{\nu}_L\nu_R + \bar{\nu}_L{}^c\nu_R{}^c = \bar{\nu}_L\nu_R + \bar{\nu}_R\nu_L \\ = \bar{\eta}\xi &= \bar{\nu}_R\nu_L + \bar{\nu}_R{}^c\nu_L{}^c, \end{aligned} \tag{2.5}$$

we can reexpress Eq. (2.1) as

$$\begin{aligned} -\mathscr{L} &= \frac{1}{2} m_L\bar{\xi}\xi + \frac{1}{2} M_R\bar{\eta}\eta + \frac{m}{2}(\bar{\xi}\eta + \bar{\eta}\xi) \\ &= \frac{1}{2}(\bar{\xi}, \bar{\eta})\begin{pmatrix} m_L & m \\ m & M_R \end{pmatrix}\begin{pmatrix} \xi \\ \eta \end{pmatrix}. \end{aligned} \tag{2.6}$$

The diagonalization of the mass matrix in Eq. (2.6) leads to

$$-\mathscr{L} = \tfrac{1}{2}m_1\bar{\psi}_1\psi_1 + \tfrac{1}{2}m_2\bar{\psi}_2\psi_2, \tag{2.7}$$

where the eigenvalues m_i and the eigenfunctions ψ_i are, for m, $m_L \ll M_R$, given by

$$m_1 = m_L - \frac{m^2}{M_R}, \qquad \psi_1 = \xi\cos\theta - \eta\sin\theta \tag{2.8}$$

and

$$m_2 = M_R + \frac{m^2}{M_R}, \qquad \psi_2 = \xi\sin\theta + \eta\cos\theta$$

with

$$\tan\theta = \frac{m}{M_R}. \tag{2.9}$$

The idea of Ref. [2] is to endow M_R with the unification mass of $SO(10)$, $M(\approx 10^{15}$–10^{19} GeV) and for $m_L = 0$ and $m \approx m_q$ (quark mass) we have

$$m_1 = -\frac{m^2}{M}, \tag{2.10}$$

which is quite small ($10^{-12} \sim 10^{-16}$ eV for $m = 1$ MeV). The Majorana field ψ_1 (ψ_2) is predominantly the LH (RH) neutrino and the inverse of Eq. (2.8) is

$$\begin{aligned} \xi &= \nu_L + \nu_L{}^c = \psi_1 \cos\theta + \psi_2 \sin\theta, \\ \eta &= \nu_R + \nu_R{}^c = -\psi_1 \sin\theta + \psi_2 \cos\theta \end{aligned} \tag{2.11}$$

and therefore

$$\begin{aligned} \nu_L &= (\psi_1)_L \cos\theta + (\psi_2)_L \sin\theta, \\ \nu_R &= -(\psi_1)_R \sin\theta + (\psi_2)_R \cos\theta. \end{aligned} \tag{2.12}$$

III. Neutrino Mass in the SO(10) Model

(a) *The Minimum Higgs System*

The fundamental representation, ψ_{16}, of the $SO(10)$ gauge group is given by [4]

$$\psi_{16} = (\nu, u_1, u_2, u_3, e, d_1, d_2, d_3, -d_3{}^c, d_2{}^c, d_1{}^c, -e^c, u_3{}^c, -u_2{}^c, -u_1{}^c, \nu^c)_L . \tag{3.1}$$

The RH neutrino, ν_R, belongs to a $SU(5)$ singlet in $\psi_{\overline{16}} \approx \psi_{16}^c$ (denoted by $(\overline{16}, 1)$ or $(\overline{16}, 1)_R$). Then the Majorana mass for the *RH* neutrino, $\bar{\nu}_R{}^c \nu_R$, has the transformation property of $(\overline{16}, 1) \times (\overline{16}, 1) = (\overline{126}, 1)$. Therefore, we need either a 126 representation of Higgs fields or a combination of Higgs fields that transforms as a 126 representation, in order to have the RH Majorana mass generated by spontaneous symmetry breaking.

Witten [3] considered the $SO(10)$ model with the minimum Higgs system, i.e., a ϕ_{16} to break the $SO(10)$ symmetry to $SU(5)$, a ϕ_{45} to further reduce the symmetry [6] to $SU_c(3) \times SU(2) \times U(1)$ and several ϕ_{10}'s to give a final residual symmetry $U(1)$. These 10's are also necessary to give masses to leptons and quarks including the Dirac mass for neutrinos. In this model, therefore, the RH Majorana mass term $(\overline{126}, 1)$ must be generated by a multi-Higgs diagram. Witten showed that this can be done by the two-loop diagram shown in Fig. 1. This diagram introduces the relevant vacuum expectation values that behave like $(\overline{16}, 1) \times (\overline{16}, 1) = (\overline{126}, 1)$.

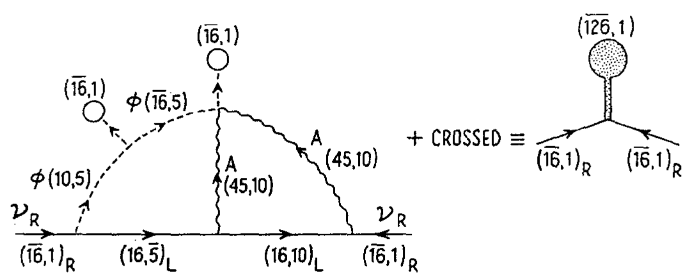

FIG. 1. Two loop diagram generating the Majorana mass for the right-handed neutrino (Witten's diagram).

TABLE I
$SO(10)$ Multiplication Table

$10 \times 10 = 1 + 45 + 54$
$10 \times 16 = \overline{16} + 144$
$16 \times \overline{16} = 1 + 45 + 210$
$16 \times 16 = 10 + 120 + 126$
$10 \times 45 = 10 + 120 + 320$
$16 \times 45 = 16 + \overline{144} + 560$
$45 \times 45 = 1 + 45 + 54 + 210 + 770 + 945$
$10 \times 120 = 45 + 210 + 945$

TABLE II
$SU(5)$ Contents and Index of $SO(10)$ Irreducible Representation[a]

Representation, ϕ	Dimension, $N(\phi)$	$SU(5)$ contents	Index, $l(\phi)$
(00000)	1	1	0
(10000)	10	$5, \bar{5}$	2
(00001)	16	$10, \bar{5}, 1$	4
(00010)	$\overline{16}$	$\overline{10}, 5, 1$	4
(01000)	45	$24, 10, \overline{10}, 1$	16
(20000)	54	$24, 15, \overline{15}$	24
(00100)	120	$45, \overline{45}, 10, \overline{10}, 5, \bar{5}$	56
(00002)	126	$50, \overline{45}, \overline{15}, 10, \bar{5}, 1$	70
(00020)	$\overline{126}$	$\overline{50}, 45, 15, \overline{10}, 5, 1$	70
(10001)	144	$\overline{45}, 40, 24, \overline{15}, \overline{10}, \bar{5}, 5$	68
(10010)	$\overline{144}$	$45, \overline{40}, 24, 15, 10, 5, \bar{5}$	68
(00011)	210	$75, 40, \overline{40}, 24, 10, \overline{10}, 5, \bar{5}, 1$	112
(11000)	320	$70, \overline{70}, 45, \overline{45}, 40, \overline{40}, 5, \bar{5}$	192
(01001)	560	$175, 50, \overline{70}, 75, 45, \overline{45}, \overline{40}, 24,$ $10, 10, \overline{10}, \bar{5}, 1$	364
(01010)	$\overline{560}$	conjugate of above	364
(02000)	770	$200, 175, \overline{175}, 50, \overline{50}, 75, 24,$ $10, \overline{10}, 1$	616
(10100)	945	$126, \overline{126}, 175, \overline{175}, 75, 45,$ $\overline{45}, 40, \overline{40}, 15, \overline{15}, 24, 24, 10, \overline{10}$	672

[a] Taken from Ref. 7.

In order to understand possible vertices allowed by the gauge group, we list the multiplication table and the $SU(5)$ contents of some of the irreducible representations of $SO(10)$ group in Tables I and II. Table II is a direct transcription of Ref. [7]. The multiplication Table I is obtained by comparing the $SU(5)$ contents of both sides and/or by using the index rule explained in Ref. [7, p. 14]. [In the reduction of a product of the irreducible representations ϕ and ϕ' into the sum of ϕ_i, $\phi \times \phi' = \sum_i \phi_i$, we have the relationship $N(\phi) + N(\phi') = \sum N(\phi_i)$ and $l(\phi)N(\phi') + l(\phi')N(\phi) = \sum l(\phi_i)$, where $N(\phi)$ and $l(\phi)$ are the dimension and the index of the representation ϕ, respectively].

The estimated Majorana mass for the RH neutrino for the diagram of Fig. 1 is shown to be [3] $g^4\epsilon(m/M_W)M$, or including an appropriate factor of π,

$$M_{\mathrm{R}} = \left(\frac{\alpha}{\pi}\right)^2 \epsilon \frac{m}{M_W} M, \tag{3.2}$$

where M and M_W stand for the $SO(10)$ unification mass ($10^{15} \sim 10^{19}$ GeV) and the $SU(2) \times U(1)$ unification mass ($\sim$100 GeV). ϵ is the mixing angle of $\phi_{(10,\bar{5})}$ and $\phi_{(16,\bar{5})}$, which originates from the trilinear Higgs coupling term $\phi_{10}\phi_{16}\phi_{16}$. (The symmetry breaking by the vacuum expectation value of (16, 1) causes a mixing of (10, $\bar{5}$) and (16, $\bar{5}$). For the value of $\epsilon \approx 1/10$ and $M \approx 10^{15}$ GeV, Eq. (3.2) leads to a considerable amount of reduction of M_{R} and consequently an enjancement of m^2/M_{R}. Namely,

$$M_{\mathrm{R}} \approx 10^{-11}M \qquad \text{for} \quad m = 1 \text{ MeV}$$

and

$$\frac{m^2}{M_{\mathrm{R}}} = \frac{mM_W}{(\alpha/\pi)^2 \epsilon M} \simeq 10^{-7}\, m. \tag{3.3}$$

We will show now that a diagram similar to that of Fig. 1 can give rise to a Majorana mass for the LH neutrino, provided that all the symmetry breakings are taken into account. Such an example is shown in Fig. 2, where the symmetry breakups are caused by two of the (16.$\bar{5}$)'s obtaining vacuum expectation values. Here we have

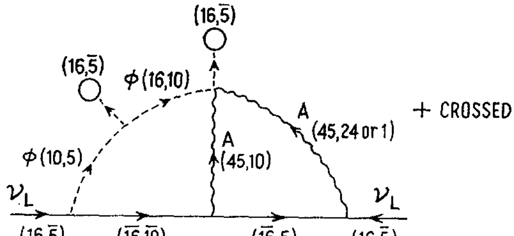

FIG. 2. Two loop diagram generating the Majorana mass for the left-handed neutrino.

used the fact that the trilinear Higgs interaction term $\phi_{10}\phi_{16}\phi_{16}$ + h.c., which was described above, can cause a mixing among $(10, \bar{5})$ and $(16, 10)$ when the $(16, \bar{5})$ has a nonzero vacuum expectation value. (As noticed earlier, $\phi_{(10,\bar{5})}$ and $\phi_{(16,\bar{5})}$ are mixed by the $SO(10)$ symmetry breaking).

An estimate of Fig. 2 can be done in a manner similar to Witten's analysis and is given by

$$m_{\mathrm{L}} = \left(\frac{\alpha}{\pi}\right)^2 \frac{m}{M_w} \epsilon' M_w . \tag{3.4}$$

The difference between Eqs. (3.4) and (3.2) is twofold. Firstly, the mass scale of the tadpole $(16, \bar{5})$ in Fig. 2 is given by M_W instead of M for $(\overline{16}, 1)$ in Fig. 1. Secondly, the mixing angle ϵ' (from the tadpole on the left-hand side of Fig. 2) is dictated by the ratio of the mass scale of the two symmetry breakings, i.e.,

$$\epsilon' = \frac{M_W}{M} . \tag{3.5}$$

Hence, we obtain

$$m_{\mathrm{L}} = \left(\frac{\alpha}{\pi}\right)^2 \frac{m M_W}{M} \approx (10^{-18} \sim 10^{-22})\, m \ll \frac{m^2}{M_{\mathrm{R}}} . \tag{3.6}$$

For $m = 1$ MeV, however, this value is comparable to that of Eq. (2.10), m^2/M, since $(\alpha/\pi)^2 M_W \approx 1$ MeV.

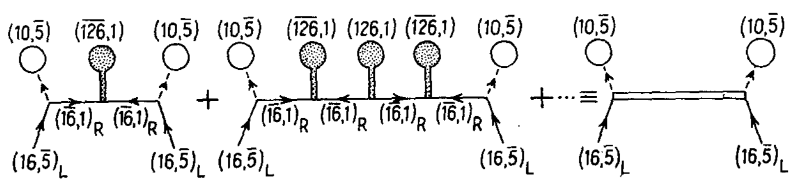

FIG. 3. Propagator for the Majorana neutrino with mass.

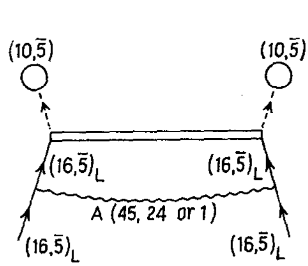

FIG. 4. One loop diagram generating the Majorana mass for the left-handed neutrino.

An alternative diagram which leads to the m_L term is given in Fig. 4, where the double line represents the repeated insertions of an effective ($\overline{126}$, 1) tadpole, an example of which is Witten's diagram shown in Fig. 1 (see Fig. 3). We notice that the double line in Figs. 3 and 4 amounts to a propagator

$$(-i)^3 \frac{-i}{i\gamma p} M_R \frac{-i}{i\gamma p} \sum_{n=0}^{\infty} \left(M_R \frac{-i}{i\gamma p} M_R \frac{-i}{i\gamma p} \right)^n = \frac{-iM_R}{p^2 + M_R^2} . \tag{3.7}$$

This is precisely the propagator for the Majorana particle with mass M_R sandwiched between the two projection operators $(1 + \gamma_5)/2$,

$$\overline{\nu_L{}^c} \frac{-i}{i\gamma p + M_R} \nu_L = \overline{\nu_L{}^c} \frac{-iM_R}{p^2 + M_R{}^2} \nu_L \tag{3.8}$$

An estimate of the order of magnitude of the diagram in Fig. 4 is given by $g^2(m^2/M_R)$: m^2 comes from the two tadpoles and the one loop integration is dictated by the mass M_R of the double lined propagator. Thus we have

$$m_L = \left(\frac{\alpha}{\pi} \right) \frac{m^2}{M_R} . \tag{3.9}$$

In conclusion, the LH Majorana mass induced by radiative corrections is smaller than that obtained by diagonalization of the mass matrix, m^2/M_R .

(b) *Addition of* ϕ_{126}

The empirical mass relations among quarks and leptons in various generations seem to indicate that a more complicated Higgs structure is needed in the $SO(10)$ model [4, 8]. For simplicity, however, let us assume that there exists in addition only Higgs particles of the 126 representation, ϕ_{126} . Then the RH Majorana mass term can be generated directly by ϕ_{126} , since it contains a $SU(5)$ singlet, (126, 1).

The magnitude of M_R is dictated by the mass scale M of the $SO(10)$ symmetry breaking, but it could be somewhat smaller than M. Then empirical relationship between the mass of the quarks and the electroweak gauge vector mesons

$$m_q \approx (10^{-5} - 1)\, M_W \tag{3.10}$$

may suggest the similar relationship

$$M_R \approx (10^{-5} - 1) M. \tag{3.11}$$

The diagonalized mass for the LH Majorana neutrino is given by

$$m_1 = -m^2/m_R , \tag{3.12}$$

while the estimate of the LH Majorana mass induced by radiative correction which is obtained in the previous subsection, is unchanged. The value of m_L obtained for

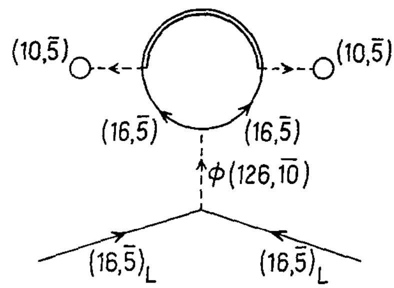

FIG. 5. One loop diagram generating the Majorana mass for the left-handed neutrino, in the presence of the Higgs particles, ϕ_{112}.

Fig. 2, Eq. (3.6), can be comparable to that of the diagonalized mass m_1, Eq. (3.12), only if $M \approx M_R$ and $m \approx 1$ MeV, while the value of m_L obtained for Fig. 4, Eq. (3.9) is obviously smaller than Eq. (3.12) by a factor of (α/π).

The presence of ϕ_{126} makes another type of one-loop diagram contribute to the LH Majorana mass term, such as that given in Fig. 5. This is similar to a diagram considered in Ref. [9], in which it is claimed that the LH Majorana mass induced by Fig. 5 is of the order of m^2/M. The dimensional coupling suggests that the contribution of Fig. 5 to m_L is

$$g^2 \left(\frac{M_R}{M}\right)^2 \frac{m^2}{M_R}. \tag{3.13}$$

The factor gM_R/M is the coupling constant of the $\phi_{126}\psi_{16}\psi_{16}$ interaction term. Again Eq. (3.13) is smaller than Eq. (3.12) by a factor $(gM_R/M))^2$.

In summary, we have shown that the LH Majorana mass generated by radiative corrections is of the order of $g^4 m M_W/M$, $g^2 m^2/M_R$ or $g^2(M_R/M)^2(m^2/M_R)$ and does not exceed the LH Majorana mass obtained by diagonalization of the mass matrix which is $-m^2/M_R$.

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