SOFT BREAKING OF TWO-LOOP FINITE $N = 1$ SUPERSYMMETRIC GAUGE THEORIES

D.R.T. JONES
Department of Physics, University of Colorado, Boulder, CO 80309, USA

L. MEZINCESCU
Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA

and

York-Peng YAO
Randall Laboratory, University of Michigan, Ann Arbor, MI 48109, USA [1]
and Research Institute for Fundamental Physics, Kyoto University, Kyoto, Japan

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The effect of soft supersymmetry-breaking terms on one-loop finite $N = 1$ supersymmetric gauge theories is investigated, and the general conditions that finiteness be preserved given. Particular attention is paid to the kinds of breakings which arise in low energy supergravity models, and it is shown that in this case the susy-breaking gaugino and scalar masses and cubic scalar interactions are related.

The search for finite supersymmetric theories in four dimensions began in 1974 when it was pointed out [1] that a $N = 1$ supersymmetric gauge theory with three matter multiplets in the adjoint representation had vanishing one-loop gauge β-function ($\beta_g^{(1)} = 0$). Subsequently it was shown that, for a $N = 1$ theory with no superpotential, there exists no choice of group and representation such that both $\beta_g^{(1)}$ and the two-loop gauge β-function ($\beta_g^{(2)}$) vanish [2]. Then in 1977 it was shown that, for the $N = 4$ supersymmetric gauge theory, $\beta_g^{(1)} = \beta_g^{(2)} = 0$ [3]. This theory corresponds in $N = 1$ language to a theory with three matter multiplets in the adjoint representation and a superpotential

$$W = \sqrt{2} g f^{abc} \phi_1^a \phi_2^b \phi_3^c \,. \tag{1}$$

This result was later extended to the three-loop level [4] and eventually to all orders [5].

More finite theories were found, when, using the susy no-renormalisation theorems [6], it was shown that $\beta_g^{(n)}$ vanished for $n > 1$ in $N = 2$ theories [7] [‡1]. $N = 2$ theories are defined by the superpotential [‡2]

$$W = \sqrt{2} g \psi_v (R^a)_u^v \xi^u \phi^a \,, \tag{2}$$

where ψ, ξ, ϕ transform according to the R^*, R and adjoint representation respectively. By choosing R so that $\beta_g^{(1)} = 0$ a class of finite theories is obtained; the $N = 4$ case (eq. (1)) being a special case. $N = 1$ supersymmetric mass terms, and certain soft susy-breaking interactions are consistent with finiteness [10] and attempts (largely unsuccessful) have been made to construct a realistic model with $N = 2$ [11].

[1] Permanent address.
[‡1] For a review and references see, e.g., West [8].
[‡2] Eq. (13) of ref. [9] is incorrect by a factor 2.

Recently the complete two-loop divergence structure for a $N = 1$ susy gauge theory with arbitrary superpotential has been given, both from the interplay of the Adler–Bardeen theorem with the supersymmetry anomaly [9, 12] and by direct calculation [13,14]. It was found that any $N = 1$ susy theory that is one-loop finite is also two-loop finite. (By finiteness we mean vanishing of all field anomalous dimensions when calculated in a fully supersymmetric and gauge invariant way. Thus the theories without a superpotential examined in ref. [2] were not one-loop finite by this definition).

For the superpotential

$$W = a_i\phi^i + \tfrac{1}{2}(m^s)_{ij}\phi^i\phi^j + \tfrac{1}{6}c_{ijk}\phi^i\phi^j\phi^k \tag{3}$$

the finiteness conditions are

$$T(\mathrm{R}) = 3C_2(\mathrm{G}) \tag{4}$$

and

$$c^{ikl}c_{jkl} = 4C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^i_j g^2 , \tag{5}$$

where $c^{ijk} = c^*_{ijk}$, ϕ transforms according to the representation R of the gauge group, and

$$T(\mathrm{R})\delta^{ab} = \mathrm{Tr}[\mathrm{R}^a\mathrm{R}^b], \quad (\mathrm{R}^a\mathrm{R}^a)^i_j = C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^i_j, \quad C_2(\mathrm{G})\delta^{ab} = f^{acd}f^{bcd} , \tag{6}$$

where f^{abc} are the structure constants, and $(\mathrm{E}_\beta)^i_j$ is the projector onto the irreducible representation R_β. Note that, as a trivial consequence of the non-renormalisation theorem, finiteness places no constraint on either m_s [13] or a_i; note however that eq. (5) debars gauge singlets from the cubic part of W so that linear terms in W exist only for free fields.

While, unlike in the special cases $N = 2, 4$, the couplings c_{ijk} are not obviously related by an identifiable *symmetry* to g, the fact that these theories are two-loop finite leads us to speculate optimistically that we are here confronted with one of the mysterious miracles of supersymmetry. A classification of what groups and representations permit solutions of eqs. (4),(5) has been given [15], and an SU_5 candidate for a realistic theory has been presented [16]. (The theory developed in ref. [16] was independently mentioned as being of possible phenomenological interest in ref. [15].)

In this paper we explore what classes of soft susy breakings preserve finiteness at the one-loop level ‡3. We will give the most general conditions to be satisfied; the most interesting result, however, is that soft breakings of the type which emerge in low energy supergravity models [17] are allowed, but the finiteness condition relates previously arbitrary parameters.

As in ref. [18], which treated the $N = 2$ case, we find it convenient to use the effective potential formalism [19], with component fields, in the Landau gauge. While this formalism is not manifestly supersymmetric (so that even in a finite theory the anomalous dimension of the scalar fields is non-zero) it permits a compact treatment of the general case.

The effective potential V obeys a renormalisation group equation

$$[\mu\partial/\partial_\mu + \beta_p\partial/\partial\lambda_p - (\gamma^i_j\phi_i\partial/\partial_j + \mathrm{c.c.})]\, V = 0 , \tag{7}$$

where $\lambda_p = \{g, c^{ijk}, c_{ijk}, m^{ij}, m_{ij}, a^i, a_i\}$ and γ^i_j is the scalar field anomalous dimension in the Landau gauge. Now including one-loop effects

$$V(\phi) = V_0(\phi) + (1/64\pi^2)\, \mathrm{STr}\, M^4 \ln(M^2/\mu^2) , \tag{8}$$

where V_0 is the tree potential, the mass matrices M are functions of the classical fields ϕ, and STr is short for the usual spin-weighted trace.

In a finite theory, for which $\beta_p = 0$ (all p) we obtain

‡3 The existence of such classes is discussed briefly in ref. [13].

$$\Delta = \tfrac{1}{2}\mathrm{STr}\,M^4 + 16\pi^2(\gamma^i_j\phi_i\partial/\partial\phi_j + \text{c.c.})\,V_0 = 0\,. \tag{9}$$

Now in general $N = 1$ theory [20,21]

$$16\pi^2\gamma^i_j = \tfrac{1}{2}c^{ikl}c_{jkl} - g^2C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^i_j\,. \tag{10}$$

So in the one-loop finite class [‡4]

$$16\pi^2\gamma^i_j = g^2C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^i_j\,. \tag{11}$$

Using eq. (4), (5) and (11) and the expression STr M^4 in a general susy theory [20–22] it is straightforward to verify that eq. (9) is satisfied for a one-loop finite theory without soft breaking terms.

We now introduce soft breaking terms as follows. For V_0 we take

$$V_0 = W^iW_i + \tfrac{1}{2}g^2\mathrm{D}_a^2 + \phi_i(M^2)^i_j\phi^j + \tfrac{1}{2}(m^2_{ij}\phi^i\phi^j + \text{c.c.})$$
$$+ \tfrac{1}{6}(f^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}) + \tfrac{1}{2}(g_i{}^{jk}\phi^i\phi_j\phi_k + \text{c.c.})\,, \tag{12}$$

where $\mathrm{D}_a = \phi_i(\mathrm{R}^a)^i_j\phi^j$ and we also add a gaugino mass term $\tfrac{1}{2}\bar{\lambda}\bar{M}\lambda$ and fermion mixing terms $\bar{\lambda}^a m_{ai}\psi^i$ where λ, ψ are gaugino and matter fermions respectively. We need not consider matter–fermion mass terms since we can regard these as (susy minus scalar) mass terms and any susy mass term preserves finiteness by the no-renormalisation theorem [6].

Our strategy now is to compute STr M^4 with the revised lagrangian and impose on the parameters that eq. (9) be still satisfied. This will ensure finiteness at the one-loop level. (Note that we do not need to consider STr M^2 because the finiteness constraint eq. (5) forbids gauge singlets from the cubic part of W.) As already stated, $\Delta = 0$ is an identity [given eqs. (4), (5)] in the absence of soft breaking terms, so we retain only contributions dependent on the soft breakings below. Writing $\Delta = \Delta_\mathrm{S} + \Delta_\mathrm{F}$ for the scalar and fermion contributions to Δ respectively we obtain:

$$\Delta_\mathrm{S} = (M^2)^i_jW_{ki}W^{kj} + f_{ijl}\phi^lW^{kij}W_k + g^i{}_{jl}(2\phi^lW_{ki}W^{kj} + \phi_iW^{kjl}W_k)$$
$$+ \phi^k\phi_l(\tfrac{1}{2}f^{ijl}f_{ijk} + \tfrac{1}{2}g_k{}^{ij}g^l{}_{ij} + g^i{}_{jk}g_i^{jl}) + \phi^k\phi^l(f_{ijk}g_l^{ij} + g^j{}_{ik}g^i{}_{jl})$$
$$+ 2g^2C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^j_i[\phi^i(M^2)^k_j\phi_k + g^i{}_{kl}\phi_j\phi^k\phi^l] + 2g^2g^i{}_{jl}(\mathrm{R}^a)^j_i\phi^l\mathrm{D}^a\ (+\text{c.c.})\,, \tag{13}$$

$$\Delta_\mathrm{F} = -2W^{jk}W_{ik}m^{ia}m_{aj} + g(4\sqrt{2})\,W^{jm}W_i(\mathrm{R}^a)^i_m m_{aj} - g^3 8\sqrt{2}(\mathrm{R}^a\mathrm{R}^b)^j_l(\mathrm{R}^a)^i_k\phi^k\phi^l\phi_j\mathrm{m}_{bi}$$
$$+ 4g^2\bar{M}\,C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^l_j(W^j\phi_l - \phi^j\phi_l\bar{M}) - 4g^2m_{bi}m^{jb}(\mathrm{R}^a)^i_k(\mathrm{R}^a)^l_j\phi^k\phi_l$$
$$- 4g^2m_{aj}m^{jb}(\mathrm{R}^a)^i_k(\mathrm{R}^b)^l_i\phi^k\phi_l - 4g^2m_{aj}m_{bi}(\mathrm{R}^a)^i_k(\mathrm{R}^b)^j_l\phi^k\phi^l\ (+\text{c.c.})\,. \tag{14}$$

Now for a finite theory we require $\Delta = 0$ for arbitrary fields ϕ^i, so we obtain four identities: for the coefficients of ϕ^3, $\phi^2\phi^*$, ϕ^2 and $\phi\phi^*$ respectively. These are:

$$c_{kmn}\phi^l\phi^m\phi^n\,[f_{ijl}c^{ijk} + 4\bar{M}g^2C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^k_l] = 0\,, \tag{15}$$

$$\phi_l\phi^m\phi^n\,[\tfrac{1}{2}g^l{}_{ij}c^{ijk}c_{mnk} + 2g^i{}_{jm}c^{jkl}c_{ikn} + 2g^2g^i{}_{jn}(\mathrm{R}^a)^l_m(\mathrm{R}^a)^j_i + 2g^2g^i{}_{mn}C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^l_i$$
$$- (4\sqrt{2})g^3C_2(\mathrm{G})m_{an}(\mathrm{R}^a)^l_m - (4\sqrt{2})gc_{ikm}c^{jkl}(\mathrm{R}^a)^i_n m_{aj}] = 0\,, \tag{16}$$

‡4 We once again emphasize that this γ^i_j differs from the γ^i_j of refs. [9,13,14] because we have adopted the Wess–Zumino–Landau gauge.

$$\phi^l\phi^m\,[f_{ijl}c^{ijk}(m^s)_{km} + 2g^i{}_{jl}c_{ikm}(m^s)^{kj} + f_{ijm}g_l{}^{ij}\,3\,g^i{}_{jm}g^j{}_{il} + 4g^2\bar{M}(m^s)_{jm}C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^j_l$$

$$- 4g^2(\mathrm{R}^a)^i_m(\mathrm{R}^b)^j_l m_{aj}m_{bi}] = 0\,, \tag{17}$$

$$\phi_l\phi^m\,[c^{kil}c_{kjm}(M^2)^j_i + \tfrac{1}{2}f^{ijl}f_{ijm} + \tfrac{1}{2}g_m{}^{ij}g^l{}_{ij} + g^i{}_{jm}g_i{}^{jl} + g^l{}_{ij}c^{kij}(m_s)_{km}$$

$$+ 2g^i{}_{jm}c^{kjl}(m_s)_{ki} + 2g^2(M^2)^l_k C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^k_m - 4g^2\bar{M}2C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^l_m - 4g^2(\mathrm{R}^a)^i_m(\mathrm{R}^a)^l_j m_{bi}m^{jb}$$

$$- 4g^2(\mathrm{R}^a\mathrm{R}^b)^l_m m_{bj}m^{ja} - 2c^{jkl}c_{ikm}m_{aj}m^{ia} - (4\sqrt{2})g(\mathrm{R}^a)^i_m c^{jlk}m^{\mathrm{s}}_{ki}m_{aj}](+\ \mathrm{c.c.}) = 0\,. \tag{18}$$

Note that none of the equations depend on m_{ij} so finiteness places no restriction on ϕ^2 type mass terms. (This fact – and that it holds also at two loops – was pointed out in ref. [13].)

One can verify that the $N = 2$ soft breakings obtained in ref. [10] are a solution of the above equations. The $N = 2$ content is realised by the decomposition $\phi_i = (\phi^a, \xi^u, \psi^v)$ with self-interactions given in eq. (2). In terms of our notation here the $N = 2$ soft breakings are given by

$$f^{auv} = \sqrt{2}g\bar{M}(\mathrm{R}^a)^{uv}, \quad g^a{}_{uv} = \sqrt{2}g(m_{11})^{ab}(\mathrm{R}^b)_{uv}\,, \quad g_u{}^{au} = \sqrt{2}gm^{ab}(\mathrm{R}^b)^u_u\,, \quad g_v{}^{av} = \sqrt{2}gm^{ab}(\mathrm{R}^b)^v_v\,, \tag{19}$$

with other components of f^{ijk} and $g^i{}_{jk}$ equal to zero. $(m_{11})^{ab}$ is the $N = 1$ supersymmetric mass term for ϕ^a; terms quadratic in m_{11} obey the finiteness condition by the no-renormalisation theorem. Eq. (19) satisfies eqs. (15), (16) automatically and in eqs. (17), (18) leads to the $N = 2$ mass formula eq. (2.16) of ref. [18].

We do not here pursue the interesting problem of the construction of general solutions to eqs. (15)–(18), but instead consider a much simplified special case, with the following motivation: In models of low energy supersymmetry in which the soft susy breakings are induced by supergravity [17] it has been shown that the most general soft breakings thereby obtainable correspond in our language to the case $m^{ia} = g^i{}_{jk} = 0$. We therefore adopt this ansatz, in order to show that our finiteness conditions are compatible with supergravity-induced susy breaking.

Eqs. (15)–(18) become as follows: eq. (15) is unchanged, eq. (16) is trivially satisfied and eqs. (17) and (18) become respectively

$$(m_s)_{km}\phi^l\phi^m\,[f_{ijl}c^{ijk} + 4g^2\bar{M}C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^k_l] = 0 \tag{17a}$$

and

$$\phi_l\phi^m\,\{c^{kil}c_{kjm}(M^2)^j_i + \tfrac{1}{2}f^{ijl}f_{ijm} + 2g^2C_2(\mathrm{R}_\beta)(\mathrm{E}_\beta)^k_m\,[(M^2)^l_k - 2\bar{M}^2\delta^l_k]\} = 0\,. \tag{18a}$$

The constraint (18a) in the special case $\bar{M} = f_{ijk} = 0$ appears in ref. [13], where the possibility of the more general case considered here is also mentioned.

A solution to eqs. (16), (17a), and (18a) immediately presents itself:

$$f_{ijk} = -\bar{M}c_{ijk}\,, \quad (M^2)^j_i = \tfrac{1}{3}\bar{M}^2\delta^j_i\,. \tag{20,21}$$

Eqs. (20), (21) form a very interesting solution from the point of view of low energy supergravity models, and thus our central result. With general assumptions about the nature of the gravitational interactions, arbitrary f_{ijk} and $(M^2)^i_j$ are allowed; however the "minimal" Kahler potential assumption leads to [17]

$$f_{ijk} = Am_{3/2}c_{ijk}\,, \quad (M^2)^i_j = m^2_{3/2}\delta^i_j\,, \tag{22}$$

where $m_{3/2}$ is the gravitino mass, and A is an arbitrary parameter. Our solution is consistent with eq. (22), but $m_{3/2}$ is related to the gaugino mass $[m_{3/2} = (1/\sqrt{3})\bar{M}]$ and the parameter $A = -\sqrt{3}$. Note that $|A| < 3$, which is important in realistic models to avoid the appearance of undesirable charge-violating vacua [23].

Recently an SU_5 model which satisfies the finiteness conditions eqs. (4), (5) has been constructed [16]. In order to have susy breaking but preserve finiteness in this model the only available mechanism is soft breaking terms satisfying our constraints. We will explore elsewhere whether the restriction of finiteness at the unification mass leads to $SU_2 \times U_1$ breaking in a natural way. As discussed in ref. [16], we may hope to obtain interesting predictions for and relations among quark and lepton (and their susy partners') masses, Cabibbo angles, and nucleon decay.

After this manuscript was prepared we received a paper [24] which contains the startling assertion that supersymmetric gauge theories cannot be regularized so as to simultaneously preserve both unitarity and supersymmetry. While this if true, would clearly have drastic consequences for supersymmetric gauge theories in general, it is not clear from ref. [24] to what extent the conclusion applies to finite theories; perhaps such theories are the only supersymmetric theories that make sense at the quantum level.

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