Radiation-Affected Laminar Flame Quenching

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Under the influence of radiation, the increase in Peclet number characterizing the flame quench distance $\Delta$,

$$\text{Pe} = \frac{\rho \cdot c_p \cdot S_u \cdot T_b^0}{\lambda \cdot T_b^0 / \Delta}$$

and the decrease in flame temperature are shown in terms of an original radiation number

$$\mathcal{R}_u = \frac{\eta (1 - \epsilon_u / 2) B_b^0}{1 + 3 \tau^2 (2 / \epsilon_u - 1) / (1 - \omega)}$$

where $\rho$ is the density, $c_p$ the specific heat at constant pressure, $S_u$ the laminar flame speed, $T_b$ the flame temperature, subscript $u$ the unburned gas and superscript $0$ the adiabatic gas, $\lambda$ the thermal conductivity, $\eta = (\kappa_p / \kappa_R)^{1/2}$ the weighted nongrayness, $\kappa_p$ and $\kappa_R$ being the Planck mean and the Rosseland mean of the absorption coefficient, $\epsilon_u$ the wall emissivity, $\tau = \kappa_M / l$ the optical thickness, $\kappa_M = (\kappa_p \kappa_R)^{1/2}$ being the mean absorption coefficient and $l$ a characteristic length (related to geometry or quench distance), $\omega$ the albedo of single scattering, and $B_b^0$ the adiabatic flame Boltzmann number.

$$B_b^0 = \frac{4 E_b^0}{\rho \cdot c_p \cdot S_u \cdot T_b^0}$$

where $E_b$ is the blackbody emissive power.

It is qualitatively shown that the contribution of radiation to the heat transfer and the laminar flame quenching in small diesel engines can be as much as 35%.

I. INTRODUCTION

A recent study by Arpaci and Tabaczynski [1], hereafter called P1, introduces a radiative heat flux which includes the emission, absorption, and scattering effects of radiation far from boundaries. The study then utilizes this flux for the effect of radiation on combustion by considering, in particular, laminar flame propagation. The present study generalizes P1 by including
the effect of a boundary on the radiative heat flux. Thus, it incorporates into a dimensionless radiation number the effect of wall emissivity in addition to the emission, absorption, and scattering aspects of radiation. The objective of the present study is to show, in terms of the radiation number, the effect of radiation on combustion near a wall which may be important for actual problems. The particular problem considered is the quenching of a laminar flame near a wall.

The outline of the study consists of five sections. Following this introductory section, Section II discusses, on dimensional grounds, the thin gas and thick gas limits of the radiative heat flux, and, by inspection, deduces from these limits the flux for an arbitrary optical thickness. Section III investigates the effect of radiation on the quenching of a laminar flame near a wall by simply adding this effect to previous studies by Ferguson [2] and Ferguson and Keck [3, 4]. Section IV shows the effect of radiation on the laminar flame quenching in small diesel engines. Section V concludes the study with a qualitative discussion of the radiation affected flame temperature and quenching.

II. RADIATIVE HEAT FLUX

The primary effect of thermal radiation of high temperature problems is in the modification of the heat flux. The secondary effects of radiation, that is, the contribution of radiative (internal) energy and radiative (Maxwell) stress, are known to be negligible (see, for example, Sampson [5]). The thin gas and thick gas limits of the one-dimensional radiation flux near a wall are, respectively,

\[ \frac{dq_x^R}{dx} = 2\kappa_p [2(E_b - E_{b,\infty}) - \varepsilon_w (E_{bw} - E_{b,\infty}) e^{-\left(3\kappa_p\beta_R^R\right) x}] \]  

(1)

and

\[ q_x^R = -\frac{4}{3\beta_R^R} \left[ 1 - \left( \frac{1 - \varepsilon_w}{2} \right) \right] dE_b \frac{dE_b}{dx}, \]  

(2)

where \( x \) is the distance normal to the wall, \( E_b \) is the blackbody emissive power, subscript \( w \) and \( \infty \) denote the wall and far from wall values of the emissive power, \( \kappa_p \) is the Planck mean of the absorption coefficient, \( \beta_R^R \) is the Rosseland mean of the extinction coefficient, and \( \varepsilon_w \) is the emissivity of the wall. The details of the development leading to the limit of Eq. (1) for \( \exp\left[-\left(3\kappa_p\beta_R^R\right) x\right] \to 1 \) may be found in Cess [6], and those leading to expressions similar to Eq. (2), but with neglected scattering, may be found in Arpaci [7] and Arpaci and Larsen [8] (see also Lord and Arpaci [9], Arpaci [10], Arpaci and Gözüm [11], Arpaci and Bayazitoglu [12], Phillips and Arpaci [13], and Arpaci et al. [14]). On the wall, the foregoing radiation fluxes reduce to

\[ \frac{dq_x^R}{dx} \bigg|_{w} = 4\kappa_p \left( 1 - \frac{\varepsilon_w}{2} \right) (E_{bw} - E_{b,\infty}) \]  

(3)

for thin gas, and

\[ q_x^R \bigg|_{w} = \left( \frac{\varepsilon_w}{2} \right) \left( -\frac{4}{3\beta_R^R} \right) \frac{dE_b}{dx} \bigg|_{w} \]  

(4)

for thick gas. The part of Eqs. (3) and (4) in brackets shows the wall effect on the radiation fluxes discussed in P1. Now, split the extinction coefficient so that \( \beta_R^R = \kappa_R + \sigma_R \), where \( \kappa_p \) and \( \sigma_R \) are the Rosseland mean of the absorption and scattering coefficients, respectively, and introduce the Rosseland albedo of single scattering \( \omega = \sigma_R/\beta_R^R \). Also, define the mean absorption coefficient as \( \kappa_M = (\kappa_p + \kappa_R) / 2 \), the weighted nongrayness as \( \eta = \kappa_p/\kappa_M \), and the optical thickness as \( \tau = \kappa_M l \), where \( l \) is a characteristic length (related to geometry or quench distance). Then in terms of these definitions, and on dimensional grounds, Eqs. (3) and (4) may be replaced by the qualitative heat fluxes

\[ q^R - \eta \tau \left( \frac{1 - \varepsilon_w}{2} \right) 4\Delta E_b, \quad \tau \to 0, \]  

(5)

\[ q^R - \eta(1 - \omega) \left( \frac{\varepsilon_w}{2} \right) \frac{4\Delta E_b}{3\tau}, \quad \tau \to \infty. \]  

(6)

Inspection of Eqs. (5) and (6) reveals the radia-
RADIATION AND FLAME QUENCHING

The heat flux for an arbitrary optical thickness,

$$q^R = \frac{4\pi(1 - \epsilon_w/2)\Delta E_b}{1 + 3\pi^2(2/\epsilon_w - 1)/(1 - \omega)}$$

(7)

which reduces to the radiative flux for thin gas and thick gas as $\tau \to 0$ and $\tau \to \infty$, respectively. The same result may be obtained directly from a formal approach for an arbitrary optical thickness involving the assumption of isotropic radiative stress. However, formal considerations, in view of the qualitative nature of the present study, appear to be unwarranted.

Effects of the emission, absorption, and scattering on the radiation heat flux, in the absence of any boundary, were discussed in P1. Figures 1 and 2, borrowed from P1 for completeness of the present discussion, show these effects. According to these figures, the effects of emission and scattering are monotonic, while the effect of absorption diminishes for the thin gas and thick gas limits, and it assumes the maximum $q^R/\eta \Delta E_b = \frac{1}{2}\sqrt{3}$ at about $\tau = 1/\sqrt{3}$. Also the radiation heat flux of Eq. (7) for a black wall is $\frac{1}{2}$ of the flux which excludes any boundary effect (Dessler [15]).

The effect of wall emissivity on the maximum of the radiation flux,

$$\frac{\sqrt{3}q_{max}^R}{\eta \Delta E_b} \sim 2 \left( \frac{1 - \omega}{2} \right) \left( 1 - \frac{\epsilon_w}{2} \right) \frac{1}{2}$$

(8)

is monotonic, as shown in Fig. 3 for various $\omega$. The same effect is also shown in Fig. 4 for an arbitrary optical thickness and negligible scattering. The foregoing considerations on the wall affected radiation heat flux are applied below to the quenching of a laminar flame.

III. FLAME QUENCHING

The developments in this section follow qualitatively Ferguson [2] and Ferguson and Keck [3, 4], with the augmentation of the heat flux by

![Fig. 1. Effect of emission and absorption on the radiation flux (Arpaci and Tabaczynski [1]).](image1)

![Fig. 2. Effect of scattering and absorption on the radiation flux (Arpaci and Tabaczynski [1]).](image2)

![Fig. 3. Effect of wall emissivity on the maximum of radiation flux for various cases of scattering.](image3)
radiation. Before proceeding, however, a brief review of the adiabatic flame temperature is useful.

Consider the steady flame stabilized near a porous flat flame burner, as shown in Fig. 5. The first law of thermodynamics applied to the control volume whose thickness is determined by an arbitrarily selected ignition distance gives, in the absence of any heat loss,

\[ p_u S_u (C_p T_u + Q) - p_u S_u C_p T_b^0 = 0, \]  

(10) readily yields the adiabatic flame temperature,

\[ T_b^0 = T_u + Q/c_p. \]  

(11)

In the actual case involving heat losses by conduction and radiation, the flame temperature falls below the adiabatic flame temperature. For this case, the first law of thermodynamics applied to the control volume bounded by the burner surface and the flame gives (Fig. 6),

\[ \rho_u S_u (C_p T_u + Q) - \rho_u S_u C_p T_b - (q^K + q^R) = 0, \]  

(12)

where \( q^K \) and \( q^R \) indicate, respectively, the contribution of conduction and radiation to the heat loss. By Eq. (11), Eq. (12) may be expressed in terms of the adiabatic flame temperature,

\[ \rho_u S_u (C_p T_b^0 - T_b) - (q^K + q^R) = 0. \]  

(13)

Also by introducing the Arrhenius relation between the flame speed and temperature (see, for example, Mayer [16])

\[ \frac{S_u}{S_u^0} = \exp \left( -\frac{E}{2R} \left( \frac{1}{T_b} - \frac{1}{T_b^0} \right) \right), \]  

(14)

\( E \) being the activation energy and \( R \) the gas constant, Eq. (13) may be further rearranged as

\[ \rho_u S_u (C_p T_b^0 - T_b) \exp \left( -\frac{E}{2R} \left( \frac{1}{T_b} - \frac{1}{T_b^0} \right) \right) \]

\[ - (q^K + q^R) = 0. \]  

(15)

In the literature, Eq. (14) is often attributed to Kaskan [17], and assumed to be an empirical
relation correlating the flame speed on flat plate burners. The same result is available also from the original Mallard and Le Chatelier thermal flame model (see, for example, Williams [18] or Glassman [19]).

Finally, by employing the Fourier law of conduction for \( q^K \) and Eq. (7) for \( q^R \), and by noting that \( E_b = \sigma T^4 \), Eq. (15) may be rearranged as

\[
\rho u S_u^0 C_p (T_b^0 - T_u) \exp \left[ -\frac{E}{2RT_b^0} \left( 1 - \frac{1}{T_b^0} \right) \right] \]

\[ -\frac{T_b - T_u}{\Delta} + \frac{4\eta\tau(1 - \varepsilon_u/2)\sigma(T_b^4 - T_u^4)}{1 + 3\tau^2(2/\varepsilon_u - 1)/(1 - \omega)}, \]

(16)

where \( \Delta \) is the quench distance including the effect of both radiation and conduction. In view of the qualitative nature of this investigation, the radiation flux may be linearized without much concern. However, since Eq. (16) would require a numerical computation, this linearization need not be considered.

Now, by dividing each side of Eq. (16) by \( T_b - T_u \), introducing the dimensionless temperatures \( \theta_u = T_u/T_b^0 \) and \( \theta_b = T_b/T_b^0 \), and incorporating the quench distance into a Peclet number based on the adiabatic flame, Eq. (16) may be rearranged as

\[
\left( \frac{1 - \theta_b}{\theta_b - \theta_u} \right) \exp \left[-\frac{E}{2RT_b^0} \left( 1 - \frac{1}{\theta_b} \right) \right] \]

\[ -\frac{1}{\text{Pe}} + \mathcal{B}_w \left( \frac{\theta_b^4 - \theta_u^4}{\theta_b - \theta_u} \right), \]

(17)

Here

\[
\text{Pe} = \frac{\rho u S_u^0 c_p T_b^0}{\lambda T_b^0/\Delta}
\]

\[
\frac{\text{adiabatic flame enthalpy flow}}{\text{conduction}} \]

(18)

and

\[
\mathcal{B}_w = \frac{\eta\tau(1 - \varepsilon_u/2)E_b^0}{1 + 3\tau^2(2/\varepsilon_u - 1)/(1 - \omega)}
\]

(19)

is a radiation number describing, in the neighborhood of a wall, all (emission, absorption, and scattering) effects of radiation relative to the adiabatic flame enthalpy flow and \( E_b^0 \) is the adiabatic flame Boltzmann number.

\[
B_b^0 = \frac{4E_b^0}{\rho u S_u^0 C_p T_b^0}
\]

\[
\frac{\text{emission}}{\text{adiabatic flame enthalpy flow}}, \]

(20)

\( E_b^0 \) being the blackbody emissive power at the adiabatic flame temperature. The characteristic length to be used in the optical thickness is related to geometry (when \( \tau \to 0 \)) or to conductive quench distance \( D \) (when \( \tau \to \infty \)). This distance is readily obtained from Eq. (17) by letting \( \mathcal{B}_w \to 0 \). The result in terms of the Peclet number,

\[
\frac{\rho u S_u^0 c_p D}{\lambda}
\]

\[
\sim \left( \frac{T_b^K - T_u}{T_b^0 - T_b^K} \right) \exp \left[-\frac{E}{2RT_b^0} \left( 1 - \frac{1}{T_b^0} \right) \right] \]

(21)

has been plotted against the flame temperature \( T_b \) for \( E/RT_b^0 = 10 \) and \( \theta_u = 0.2 \) (see Fig. 2 of Ferguson and Keck [3]). The effect of radiation on this figure, now plotted against the dimensionless flame temperature \( \theta_b \), is shown in Fig. 7 for various values of the radiation number \( \mathcal{B}_w \).

Under the influence of radiation, the Peclet number continues to remain as a U-shaped function of temperature. That is, for sufficiently large quench distances, there exists a high-temperature solution and a low-temperature solution, and both solutions converge to one solution, corresponding to the minimum quench distance. The physics behind this behavior is explained in Ferguson and Keck [3] and need not be repeated here. The added effect of radiation does quantitatively, but not conceptually, alter this behavior. The minimum quench distance is
available from the usual extrema condition,
\[ \frac{\partial}{\partial P_e} (P_e) = 0. \] (22)

The critical temperature corresponding to the minimum quench distance, obtained from the insertion of Eq. (17) into Eq. (22), satisfies
\[ \frac{\theta_b - 1}{\theta_b - \theta_a} + \frac{E}{2RT_b^0} \left( \frac{1}{\theta_b^2} - \frac{1}{\theta_b} \right) \times \exp \left[ -\frac{E}{2RT_b^0} \left( \frac{1}{\theta_b} - 1 \right) \right] = 0 \]
\[ \theta_b^k - \theta_b = \theta_w (\theta_b - \theta_a)(3\theta_b^2 + 2\theta_b\theta_a + \theta_a^2). \] (23)

Figure 8 shows the computational results for this temperature. Also shown in the same figure is the effect of linearized radiation. Since the linearization approximates the effect of radiation on the quench distance by a term directly proportional to the flame temperature, \( \partial(P_e)/\partial \theta_b = 0 \) becomes independent of \( \theta_b \), and the linearized radiation does not affect the critical flame temperature.

Clearly, the flame temperature resulting from the heat loss by conduction and radiation is lower than that resulting from the heat loss by conduction alone. Since any heat loss lowers the flame temperature below its adiabatic value, radiation added to conduction increases the heat loss and, consequently, drops the flame temperature below that resulting from conduction alone.

Consider Eq. (13) for conduction alone,
\[ \rho_a S_a c_p (T_b^0 - T_b^k) - q^k = 0, \] (24)
\( T_b^k \) being the flame temperature for this case. Now, subtract Eq. (24) from Eq. (13). Thus,
\[ \rho_a S_a c_p (T_b^k - T_b^l_k) - q^k = 0, \] (25)

Insert Eqs. (7) and (14) into Eq. (25), and, in addition to the already defined \( \theta_a \) and \( \theta_b \), introduce \( \theta_b^k = T_b^k/T_b^0 \). The result is
\[ \frac{\theta_b^k - \theta_b}{\theta_b^4 - \theta_a^4} \times \exp \left[ -\frac{E}{2RT_b^0} \left( \frac{1}{\theta_b} - 1 \right) \right] = 0. \] (26)

Figure 9 shows \( \theta_b \) versus \( \theta_b^k \) for \( \theta_w = 0, 0.04, 0.08 \), and for \( E/RT_b^0 = 10 \) and \( \theta_u = 0.2 \). For a given \( \theta_b^k \), an increase in \( \theta_w \) reduces \( \theta_b \), as expected. In the next section, the foregoing general considerations are applied to diesel engines.

**IV. QUENCHING IN DIESEL ENGINES**

The contributions of radiation to heat transfer and quench distance in gasoline engines are known to be negligible. Also, the heat transfer in
large scale (forest, residential, and commercial buildings) fires and industrial furnaces (including conventional power plants) is known to be controlled by radiation. Between these two limiting cases there exists an intermediate case related to diesel engines in which the quench distance is appreciably affected by radiation. The purpose of this section is to give a qualitative estimate for this case.

It has recently been shown that the soot (which results from incomplete combustion) is the major contributor to the radiative properties of flames, smoke, and combustion products (see, for example, Tien and Lee [20] for the properties of luminous and nonluminous flames and Siegla and Smith [21] for that of particulate emissions from diesel engines). Accordingly, the contribution of gas absorption to these properties is neglected.

In the literature it is generally accepted that the soot particles range in size and shape from single spheres of about 0.01 μm diameter to clusters, often chain-like, extending over 1 μm in length. In view of the fact that the major contribution of thermal (infrared) radiation is at wavelengths in the neighborhood of \( \lambda^* \approx 10 \text{ μm} \),

\[ \frac{d}{\lambda^*} = 10^{-1} - 10^{-3}, \]

\( d \) being a characteristic length for these particles. Accordingly, the contribution of geometry to qualitative calculations is negligible. Furthermore, because of these values of \( d/\lambda^* \), the scattering from soot particles has been shown to be negligible (see, for example, Felske and Tien [22]).

Hubbard and Tien [23] have demonstrated that at high temperatures \( \kappa_p \) and \( \kappa_R \) differ by at most 17%. Thus, \( \eta \approx 1 \) and \( \kappa_M \approx \kappa_p \approx \kappa_R \), and it is reasonable to employ the gray-gas approximation in the qualitative numbers sought here. Also, the experiments by Roessler et al. [24] on single- and multicylinder diesel engines have shown that the mass concentration of soot remains in the range of \( 0 < M < 1.6 \text{ g/m}^3 \) and the mass absorption coefficient in the neighborhood of \( \lambda^* = 10 \text{ μm} \) remains in the range of \( 0.5 < k < 1.5 \text{ m}^2/\text{g} \) for air/fuel ratios \( 15 < \phi < 80 \). The effect of rpm does not appear to be too important.

Thus,

\[ \kappa_M = Mk = 1.6 \times 1.5, \]

and

\[ \kappa_M = 2.4 \text{ m}^{-1} \]

may be used as an upper bound for the absorption coefficient in diesel engines. Assuming \( l = 15 \text{ cm} \) as a characteristic length for combustion geometry of small diesel engines, one has for the optical thickness

\[ \tau \leq 0.36. \]  

(27)

Actually, for large (ship or power plant) diesels this number is an order of magnitude larger. Note that the contributions of the quench distance and the flame thickness to optical thickness are negligible. The hot soot which fills almost the entire geometry dictates the characteristic length for the optical thickness in the case of thin gas.

By assuming that the walls are black (\( \epsilon_w = 1 \)) and noting that \( \eta \approx 1, \omega \approx 0 \), and \( T_b \gg T_u \), the right-hand side of Eq. (16) may be rearranged as

\[ \lambda \left( \frac{T_b - T_u}{\Delta} \right) \left[ 1 + \frac{1}{2} \left( \frac{\tau}{1+3\tau^2} \right) \Phi_b \right], \]

(28)

where the dimensionless term in brackets shows the contribution of radiation relative to conduction, and

\[ \Phi_b = \frac{4\sigma T_b^4}{\lambda(T_b/\Delta)} \]

is the Planck number based on the temperature of combustion products. Assume that the quench distance is controlled by conduction and the radiation effect is of second order, and as a first approximation, let

\[ \Delta = D. \]  

(30)

The quench distance \( D \) on flat burners has been experimentally shown not to be appreciably influenced by the type of fuel (see Fig. 5 of Ferguson and Keck [3]). Assume that this fact is also a feature of quenching in engines. The
quench distances measured by Daniel [25] in a propane fueled engine may then approximately be applied to isooctane fueled engines. Thus, as a mean value, let

\[ D \sim 0.1 \text{ mm}. \]  

(31)

For these engines, \( T_b \sim 2,000^\circ\text{K} \) and \( \lambda \sim 0.12 \text{ W/m K} \) for a gas at this temperature. Accordingly,

\[ \vartheta_b \equiv 1.5. \]  

(32)

Insertion of Eqs. (27) and (32) into Eq. (28) yields

\[ \lambda \left( \frac{T_b - T_u}{\Delta} \right) (1 + 0.27), \]  

(33)

which shows the effect of radiation relative to conduction to be 27%. In terms of this result, a better approximation for the quench distance including the effect of both conduction and radiation is

\[ \Delta \sim 1.27D \sim 0.127 \text{ mm}, \]  

(34)

and that for Eq. (29) is

\[ \vartheta_b \sim 1.93. \]  

(35)

Now, Eq. (28) yields

\[ \lambda \left( \frac{T_b - T_u}{\Delta} \right) (1 + 0.35), \]  

(36)

which increases the effect of radiation to 35%. In the literature, the effect of radiation on the heat transfer in small diesel engines is shown to be 30% or even more as determined by experiments, as well as by lumped radiation models (see, for example, Kunimoto et al. [26], Flynn et al. [27], and the references cited in these articles). With the recent and rapidly increasing interest in the uncooled (commonly called adiabatic) diesel engines, a greater contribution from radiation to total heat transfer is expected because of higher levels of temperature.

V. CONCLUSIONS

A radiation number describing all (emission, absorption, and scattering) effects of radiation near a wall has been introduced. This number incorporates the wall (emissivity) effect on the dimensionless number developed in P1. Under the influence of radiation, as well as that of conduction, the quenching of a laminar flame near a wall is investigated in terms of this number. The dimensionless quench distance (Peclet number) is shown to depend on

\[ \text{Pe} = f(\theta_u, \theta_b, E/RT_b^0, \vartheta_w), \]

where \( \theta_u = T_u/T_b^0, \theta_b = T_b/T_b^0, T_u \) being the unburned gas temperature, \( T_b \) the flame temperature, \( T_b^0 \) the adiabatic flame temperature, \( E \) the activation energy, \( R \) the gas constant, and \( \vartheta_w \) the radiation number,

\[ \vartheta_w = f(\eta, \tau, \omega, \epsilon_w, B_b^0). \]

where \( \eta \) is the weighted nongrayness, \( \tau \) the optical thickness, \( \omega \) the albedo, \( \epsilon_w \) the wall emissivity, and \( B_b^0 \) the adiabatic flame Boltzmann number,

\[ B_b^0 \sim \frac{4E_b^0}{\rho_a S_u c_p T_b^0}. \]

\( E_b^0 \) being the blackbody emissive power at the adiabatic flame temperature and \( S_u \) the flame speed. Also shown, under the influence of radiation, is the flame temperature

\[ \theta_b = f(\theta_u, \theta_b^K, E/RT_b^0, \vartheta_w), \]

where \( \theta_b^K = T_b^K/T_b^0, T_b^K \) being the flame temperature resulting from conduction alone.

It is found that radiation increases the quench distance and lowers the flame temperature. Since the contributions of conduction and radiation to the heat flux are cumulative, these results are expected. The decrease in flame temperature and the increase in quench distance under the influence of radiation are shown qualitatively in Fig. 10.

Following some dimensional arguments in terms of spectrally average properties of radiation, a conceptual understanding of the qualitative effects of radiation on the flame temperature
and the quench distance has been achieved. This understanding has been applied to small diesel engines, and it has been shown that the radiation can contribute as much as 35% to the heat transfer and the quench distance to these engines.

**NOMENCLATURE**

- \( c_p \): specific heat at constant pressure
- \( d \): a characteristic length for soot
- \( D \): quench distance affected by conduction alone
- \( E \): activation energy
- \( E_b \): blackbody emissive power
- \( E_{b,w} \): blackbody emissive power on a boundary
- \( E_{b,\infty} \): blackbody emissive power far from a boundary
- \( \Delta E_b \): difference in blackbody emissive powers
- \( k \): mass absorption coefficient
- \( l \): a characteristic length
- \( M \): mass concentration of soot
- \( q^K \): conduction heat flux
- \( q^R \): radiation heat flux
- \( q^{R,x} \): radiation heat flux in \( x \)
- \( Q \): heat of reaction
- \( R \): gas constant
- \( S_u \): unburned gas laminar flame speed
- \( S_u^0 \): unburned gas laminar adiabatic flame speed
- \( T \): temperature
- \( T_b \): burned gas temperature
- \( T_b^0 \): burned gas adiabatic temperature
- \( T_u \): unburned gas temperature
- \( \chi \): variable

**Greek Letters**

- \( \beta_R \): Rosseland mean of extinction coefficient
- \( \Delta \): quench distance affected by conduction and radiation
- \( \epsilon_w \): wall emissivity
- \( \eta \): weighted nongrayness
- \( \theta_b \): burned gas temperature (dimensionless)
- \( \theta_b^K \): burned gas temperature (dimensionless) affected by conduction alone
- \( \theta_b^0 \): burned gas adiabatic temperature (dimensionless)
- \( \theta_u \): unburned gas temperature (dimensionless)
- \( \kappa \): absorption coefficient
- \( \kappa_M \): mean absorption coefficient
- \( \kappa_P \): Planck mean of absorption coefficient
- \( \kappa_R \): Rosseland mean of absorption coefficient
- \( \lambda \): thermal conductivity
- \( \lambda^* \): electromagnetic wavelength
- \( \rho \): density
- \( \sigma \): Stefan-Boltzmann constant
- \( \sigma_R \): Rosseland mean of scattering coefficient
- \( \tau \): optical thickness
- \( \omega \): Rosseland albedo of single scattering

**Dimensionless Numbers**

\[
B_b^0 = 4E_b / \rho_o c_p S_u^0 T_b^0 \quad \text{adiabatic flame}
\]

\[
Pe = \rho_o c_p S_u^0 \Delta / \lambda \quad \text{Boltzmann number}
\]

\[
\theta_b^0 = 4\sigma T_b^4 / \lambda (T_b / \Delta) \quad \text{burned gas Planck number}
\]

\[
\Omega_w = \eta \tau (1 - \epsilon_w / 2) B_b^0 / [1 + 3\tau^2 (2/\epsilon_w - 1) / (1 - \omega)] \quad \text{radiation number}
\]

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