BARYON–BARYON SPIN–ORBIT AND TENSOR INTERACTIONS FROM QUARK-EXCHANGE KERNELS

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Abstract: Dibaryon quark-exchange kernels are constructed in explicit analytic form for the tensor and spin–orbit terms of the one-gluon-exchange quark–quark Breit interaction and for spin–orbit terms generated by quark-confinement mechanisms. The spin operators needed are defined through their spin-reduced matrix elements including those needed for interactions coupling NN, NΔ, and $\Delta\Delta$ channels. Effective baryon–baryon spin–orbit potentials, generated through the Wigner transforms of the quark-exchange kernels with the use of a local momentum approximation, show that the NN spin–orbit interaction derived from the symmetric spin–orbit term of the one-gluon-exchange quark–quark interaction is in general agreement with the short-range part of phenomenological potentials derived from NN scattering. With the inclusion of the antisymmetric spin–orbit one-gluon-exchange terms and spin–orbit terms generated by confining potentials the full triplet-odd NN spin–orbit potential is greatly reduced in the 0.5–1 fm range. The uncertainties associated with spin–orbit terms generated by quark-confinement mechanisms are emphasized. The relative importance of various possible quark–gluon exchange terms is studied and shows that models which neglect some types of exchange terms are open to question. An SU(3)-flavor symmetric model for N-hyperon spin–orbit potentials leads to an NΛ spin–orbit potential only slightly weaker than the NN spin–orbit potential.

1. Introduction

Attempts at a fundamental understanding of the baryon–baryon interaction through the interaction among quarks in terms of the underlying quantum chromodynamics are complicated by the unsolved problem of quark confinement, and the medium- and long-range part of the baryon–baryon interaction is at present not understood in terms of the underlying quark–gluon physics. Since this part of the interaction is explained well in terms of the meson-exchange mechanism, recent investigations in terms of quark models have focused on the short-range part of the baryon–baryon interaction. Of the many models and methods used the nonrelativistic resonating group or generator coordinate method [1–9] has the great advantage that the quark exchange kernels needed can be evaluated in very explicit analytic form

including the necessary color, spin, isospin factors which make it possible to couple the NN and $\Delta\Delta$ systems, and the various hidden color channels. With the RGM formalism verý specific quantitative results can be obtained directly and easily. Despite the possible shortcomings of nonrelativistic quark models, RGM models may therefore be very useful, particularly in a study of the relative importance of specific terms of the quark–quark interaction or specific quark–gluon exchange mechanisms.

Most recent quark model studies of the NN interaction have focused on the central parts of the quark–quark interactions; but spin–orbit and tensor terms may be crucial for some aspects of the short-range part of the baryon–baryon interaction, and may be particularly important for the coupling of NN to NΔ channels. Possible dibaryon resonances in the $^1D_2$, $^3F_3$, $^1G_4$ NN channels [10]) appear at energies very close to the NΔ threshold, and it has been suggested [11]) that these dibaryon resonances can be interpreted as arising from NΔ threshold effects. Tensor interactions coupling channels such as $^5S_2$(NΔ) to $^1D_2$(NN), or $^5P_3$(NΔ) to $^3F_3$(NN), thus play a crucial role for the appearance of such resonances, and it has been suggested [12]) that these may be very sensitive to the short- and medium-range parts of such tensor interactions. A specific form of the potential needed may therefore be obtainable from the tensor terms in the quark exchange kernels. A phenomenological Δ-nucleus spin–orbit interaction has been shown [13]) to improve the fit to π-nucleus scattering. Due to the peripheral nature of the π-nucleus interaction such spin–orbit terms may again be explainable in terms of the spin–orbit terms in the quark–quark interaction.

It has also been suggested [14]) that the nuclear spin–orbit potential may arise largely from the short-range part of the NN interaction and may thus be explainable in terms of quark–quark spin–orbit exchange kernels. In the relativistic treatment of ref. [14]), in which quark exchange is handled through a Fierz transformation [15,16]), the analysis is based on a single exchange process in which only two quarks participate and is thus limited to only one of several possible quark–gluon exchange mechanisms of the six-quark system. It is the advantage of the nonrelativistic RGM treatment that all quark–gluon exchange terms can be treated on an equal footing so that it is possible to assess quantitatively the relative importance of the various quark–gluon exchange terms. It has been claimed [17]) that a nonrelativistic treatment of quark–gluon exchange gives rise to a spin–orbit interaction of the wrong sign and a very small magnitude, at least in the energy regime where the spin–orbit interaction begins to play a role. It will, however, be shown that both the sign and the strength of the spin–orbit interaction derived from nonrelativistic RGM quark-exchange kernels are sensitive functions of both range and energy and the method used to represent the confinement mechanism. In the relevant range–energy domain the spin–orbit interaction derived from nonrelativistic RGM quark-exchange kernels has a sign and magnitude in agreement with the phenomenological potentials derived from NN scattering data, if only the symmetric spin–orbit term of the one-gluon-

exchange quark–quark interaction is considered. Inclusion of the antisymmetric spin–orbit term and possible spin–orbit terms generated by a confining potential may, however, be responsible for a cancellation of most of this spin–orbit strength.

The most detailed RGM treatment to date, including tensor and spin–orbit terms, has been given by Warke and Shanker [3]. Their method of constructing the NN potential, however, is a variant of a Born–Oppenheimer approximation, and their treatment of the quark-exchange kernels is not quite complete. It is one of the purposes of this contribution to give the needed tensor and spin–orbit quark-exchange kernels in explicit analytic form [completing the earlier tabulations for the central terms [8,9])]. Particular emphasis is given to the coupling of NN to NΔ and ΔΔ channels with the above applications in mind. The relative importance of the various possible quark–gluon exchange terms in the spin–orbit interaction is studied in some detail. In sect. 3 the spin–orbit quark-exchange kernels are converted into equivalent spin–orbit potentials through the Wigner transforms of these kernels and the use of the local momentum (WKB) approximation employed in ref. [8]. The relative importance of the symmetric and antisymmetric spin–orbit terms of the one-gluon-exchange quark–quark interaction and spin–orbit terms generated by quark-confinement mechanisms can then be examined in detail. Finally, a brief discussion is given of the N-hyperon spin–orbit potential using an SU(3)-flavor approach in which strange and nonstrange quarks are treated on an equal footing.

2. Quark-exchange kernels for tensor and spin–orbit interactions

The nonrelativistic RGM wave function for the six-quark dibaryon system can be written [in the notation of ref. [8])]

$$\psi = \mathscr{A}\{[\phi_{B_1}(123) \times \phi_{B_2}(456)]_{ST}\chi(\boldsymbol{R}_{12})\}, \tag{1}$$

where the internal three-quark baryon functions ϕ_B include a totally antisymmetric color, a totally symmetric spin–isospin (flavor) component, and a totally symmetric internal orbital function made up of 0s oscillator functions, e.g.

$$\phi_{B_1}(\text{orbital}) = (\pi b^2)^{-3/2} \exp\left[-\frac{1}{2b^2}\{\tfrac{1}{2}(r_1-r_2)^2 + \tfrac{1}{6}(r_1+r_2-2r_3)^2\}\right]$$
$$= (\pi b^2)^{-3/2} \exp\left[-(r_1^2+r_2^2+r_3^2-r_1\cdot r_2 - r_1\cdot r_3 - r_2\cdot r_3)/3b^2\right], \tag{2}$$

where b is the oscillator length parameter, $b^2 = \hbar/m\omega$. The square bracket in eq. (1) denotes spin and isospin coupling. For $B_1 = B_2$, e.g. for the NN or ΔΔ system, the symmetry of the dibaryon spin–isospin function is given by $S + T = \text{ə}$. For $B_1 \neq B_2$, e.g. for the NΔ system, it will be convenient to let the square bracket in eq. (1) denote a symmetrically (antisymmetrically) spin–isospin coupled function for $\text{ə} =$

even (odd), and therefore let the square bracket in eq. (1) be defined by

$$\frac{1}{\sqrt{1+\delta_{B_1 B_2}}}\sqrt{\tfrac{1}{2}}\{[\phi_{B_1}(123)\times\phi_{B_2}(456)]_{ST}$$

$$+(-1)^s(-1)^{S_1+S_2-S+T_1+T_2-T}\times[\phi_{B_2}(123)\times\phi_{B_1}(456)]_{ST}\}. \qquad (3)$$

(Note that for $B_1 = B_2$ only one of the s-values survives for a given S and T; note also that these internal functions are normalized in both cases $B_1 = B_2$ and $B_1 \neq B_2$.)

The antisymmetrizer $\mathscr{A}$ in eq. (1) makes ψ totally antisymmetric under exchange of quarks between the two baryons and can be reduced in terms of double coset generators to the simple form

$$\mathscr{A} = \tfrac{1}{2}(1-9P_{36})(1-\mathscr{P}). \qquad (4)$$

The normalization for $\mathscr{A}$ is such that the norm kernel approaches the unit operator in the limit of infinite baryon separation. In eq. (4) P_{36} exchanges quarks 3 and 6, and $\mathscr{P} = P_{14}P_{25}P_{36}$ exchanges the two baryons. (The P_{ij} act on color, spin, isospin and space variables.) The RGM kernel for the quark–quark interaction can be split into a direct and an exchange part:

$$K_V^{(D)}(\boldsymbol{R},\boldsymbol{R}')=\tfrac{1}{2}\langle[\phi_{B_1}\times\phi_{B_2}]_{ST}\delta(\boldsymbol{R}_{12}-\boldsymbol{R})\Big|\sum_{i<j}^6 v_{ij}(1-\mathscr{P})\Big|[\phi_{B_1'}\times\phi_{B_2'}]_{S'T}\delta(\boldsymbol{R}_{12}-\boldsymbol{R}')\rangle,$$

$$K_V^{(E)}(\boldsymbol{R},\boldsymbol{R}')=\tfrac{1}{2}(-9)\langle[\phi_{B_1}\times\phi_{B_2}]_{ST}\delta(\boldsymbol{R}_{12}-\boldsymbol{R})\Big|\sum_{i<j}^6 v_{ij}P_{36}(1-\mathscr{P})\Big|$$

$$\times[\phi_{B_1'}\times\phi_{B_2'}]_{S'T}\delta(\boldsymbol{R}_{12}-\boldsymbol{R}')\rangle, \qquad (5)$$

where $\boldsymbol{R}_{12}=\tfrac{1}{3}(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3)-\tfrac{1}{3}(\boldsymbol{r}_4+\boldsymbol{r}_5+\boldsymbol{r}_6)$. With baryon internal functions defined as in eqs. (1)–(3), $\mathscr{P}$ can be replaced by $(-1)^s\,\mathscr{P}(\text{space})$, where $\mathscr{P}(\text{space})$ acts on the orbital parts of the functions only. Only the exchange term of the color quark–quark interaction leads to an interaction between colorless nucleons. The 15 quark pairs (ij) thus lead to the five basic types of quark–gluon exchange terms shown in fig. 1 when the interaction, v, is mediated by one-gluon exchange. Exchange kernels

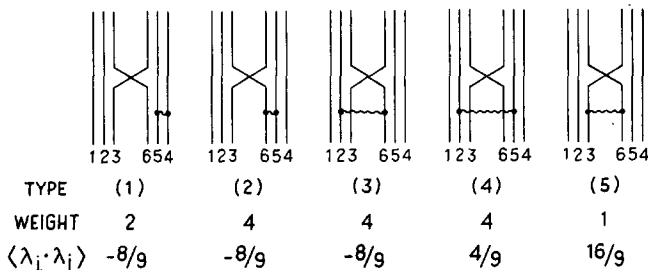| TYPE	(1)	(2)	(3)	(4)	(5)
WEIGHT	2	4	4	4	1
$\langle\lambda_i\cdot\lambda_j\rangle$	-8/9	-8/9	-8/9	4/9	16/9

Fig. 1. The 5 types of quark–gluon exchange terms. The weights count the number of quark pairs which are equivalent to the pairs ij singled out in the figure.

arising from terms of type (1) and the central part of v are proportional to the exchange part of the norm kernel [8]). Quark–gluon exchange terms of type (1) are thus found not to contribute to any baryon scattering processes. Exchange kernels for the spin–orbit and tensor terms gain contributions from the exchange terms of types (2)–(5) only. Type (1) vanishes for baryons built from S-wave quarks.

The one-gluon-exchange quark–quark Breit interaction has spin–orbit and tensor terms of the form

$$v_{ij}^{\text{g,s.o.}} = \tfrac{1}{4}\alpha_s \hbar c (\lambda_i \cdot \lambda_j) \frac{-\hbar}{8m^2c^2} \left\{ 3\frac{1}{r_{ij}^3}[r_{ij} \times (p_i - p_j)] \cdot (\sigma_i + \sigma_j) \right.$$

$$\left. -\frac{1}{r_{ij}^3}[r_{ij} \times (p_i + p_j)] \cdot (\sigma_i - \sigma_j) \right\}, \tag{6}$$

$$v_{ij}^{\text{T}} = \tfrac{1}{4}\alpha_s \hbar c (\lambda_i \cdot \lambda_j) \frac{-3\hbar^2}{4m^2c^2} \left\{ \frac{(\sigma_i \cdot r_{ij})(\sigma_j \cdot r_{ij})}{r_{ij}^5} - \frac{(\sigma_i \cdot \sigma_j)}{3r_{ij}^3} \right\}, \tag{7}$$

where the quarks have been taken of equal mass, m, and $r_{ij} = r_i - r_j$. The SU(3) color generators for the ith quark λ_i^α ($\alpha = 1, \ldots, 8$) are normalized such that $\langle (\lambda_i \cdot \lambda_i) \rangle = \tfrac{16}{3}$, where $(\lambda_i \cdot \lambda_j) = \sum_{\alpha=1}^8 \lambda_i^\alpha \lambda_j^\alpha$.

In the nonrelativistic RGM treatments the one-gluon-exchange potential is augmented by phenomenological confinement terms in the quark–quark interaction of the form

$$v_{ij}^{\text{c}} = (\lambda_i \cdot \lambda_j) v_{\text{c}}(r_{ij}) \tag{8}$$

with various radial dependences, e.g. $v_{\text{c}}(r) = -a_{\text{c}} r^2$ (quadratic confinement) or $v_{\text{c}}(r) = -a_{\text{c}}' r$ (linear confinement potential). If such a confinement potential arises from a scalar coupling, $(\bar{\psi}\psi)$, it will generate additional spin–orbit terms. To order v^2/c^2 the spin-dependent terms of such a coupling are [18–20])

$$v_{ij}^{\text{c,s.o.}} = (\lambda_i \cdot \lambda_j) \frac{-\hbar}{8m^2c^2} \frac{1}{r_{ij}} \frac{dv_{\text{c}}}{dr_{ij}} \{[r_{ij} \times (p_i - p_j)] \cdot (\sigma_i + \sigma_j)$$

$$+ [r_{ij} \times (p_i + p_j)] \cdot (\sigma_i - \sigma_j)\}. \tag{9}$$

Since the problem of quark confinement constitutes an unsolved problem it is not completely clear whether these additional spin–orbit terms generated by such a simple-minded quark-confinement mechanism should be included. If confinement is built into the quark model in the spirit of the MIT bag through a mass term in the relativistic single-particle problem (a mass which rises to infinity outside the bag) the single-particle Thomas terms of such a model may generate spin–orbit terms in the NN interaction through the quark-exchange mechanism. This possibility is discussed in sect. 3, where it is shown that such a model of confinement leads to spin–orbit terms of the opposite sign from those generated by $v^{\text{c,s.o.}}$.

In eqs. (6) and (9) the spin–orbit terms have been split into a symmetric $(\sigma_i + \sigma_j)$ and an antisymmetric $(\sigma_i - \sigma_j)$ spin–orbit contribution. (Note that the symmetric

term in $v^{\text{c.s.o.}}$ has the opposite sign from the symmetric spin–orbit term arising from one-gluon exchange.) The antisymmetric spin–orbit term, through its dependence on $p_i + p_j$, seems to add spurious c.m. contributions and thus seems to be outside the realm of permissible interactions. These galilean non-invariant two-body terms have been discussed in detail [see, in particular, refs. [18,21])]. They are precisely of the form needed to cancel the spin-momentum-dependent terms generated in the two-particle barycentric frame if the two-body central potential v_{ij}, color coulombic or confining, is expressed in terms of appropriate two-particle barycentric relative coordinates, $\hat{r}_{ij}$, which differ from the r_{ij} by additional spin-momentum dependent terms which arise from the Wigner rotation of the intrinsic spins of the two particles [18,21]). In the c.m. frame of the six-particle system, however, the usual coordinates r_{ij} are the most natural, and since the color coulombic and confining potentials are expressed in terms of these, the full spin–orbit potential, symmetric *and* antisymmetric contributions, must be used. In the six-quark center-of-mass frame, where the c.m. momentum P can be set equal to zero, the $(p_i + p_j)$ can be replaced by $p_i + p_j - \frac{2}{6}P$. (The c.m. component of $(p_i + p_j)$ vanishes automatically if a 0s harmonic oscillator c.m. motion function is included as part of the full six-particle wave function.) In all, therefore, exchange kernels are needed for three types of interaction terms, the symmetric and antisymmetric spin–orbit terms and the tensor term.

It will be convenient to express the exchange kernels, $K^{(\text{E})}(R, R')$, in terms of dimensionless relative position variables, a, with conjugate dimensionless momenta q, where

$$a = \sqrt{\tfrac{3}{2}}\, R/b, \qquad q = \sqrt{\tfrac{2}{3}}\, bP/\hbar . \tag{10}$$

The physical kernels, $K(R, R')$ are then related to kernels $K(a, a')$, expressed in terms of dimensionless a, a', through

$$K(R, R') = (3/2b^2)^{3/2} K(a, a') . \tag{11}$$

For the exchange kernels of eq. (5) these always occur in the combination

$$K^{(\text{E})}(a, a') = -\tfrac{9}{2}[\tilde{K}(a, a') - (-1)^{\mathfrak{a}} \tilde{K}(a, -a')] , \tag{12}$$

where $\mathfrak{a}$ gives the symmetry of the spin–isospin (or spin–flavor) dibaryon function, see eq. (3). Note that the relative motion function must have the symmetry, $\chi(-R_{12}) = (-1)^{\mathfrak{a}+1}\chi(R_{12})$. Consequently, only the $\tilde{K}(a, a')$ need be given. The calculation of these kernels has been carried out by straightforward integrations. For the tensor and spin–orbit terms it will be convenient to express these $\tilde{K}(a, a')$ in the following form:

$$\tilde{K}(a, a') = \begin{cases} \sum_{ij} c^{(ij)}(k^{(2)}(a, a') \cdot [\sigma \times \sigma]^{(2)}_{\text{eff}})_{(ij)} & \text{(tensor) ,} \tag{13a} \\[2ex] \sum_{ij} c^{(ij)}(k^{(1)}(a, a') \cdot \sigma_{\text{eff}})_{(ij)} & \text{(spin–orbit) ,} \tag{13b} \end{cases}$$

where the $k(\mathbf{a}, \mathbf{a}')$ give the kernels of dimensionless orbital operators (see subsects. 2.1 and 2.2), the $c^{(ij)}$ contain the overall strength coefficients, and the effective $\boldsymbol{\sigma}$-operators, designated $\boldsymbol{\sigma}_{\text{eff}}^{(ij)}$ in the following, are determined through the spin-reduced matrix elements of $[\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]^{(2)}$ and $(\boldsymbol{\sigma}_i \pm \boldsymbol{\sigma}_j)$, tabulated in tables 1 and 2. These have been evaluated by the recoupling techniques explained in ref. [22]. The summations over (ij) can be reduced to a sum over a small number of properly weighted characteristic quark–gluon exchange terms of the type illustrated in fig. 1 (four terms for the tensor, and two terms each for the symmetric and antisymmetric spin–orbit potentials).

2.1. TENSOR TERMS

For the tensor potentials it will be convenient to consider the dimensionless factor

$$\frac{b^3}{r_{ij}^5}\{(\boldsymbol{\sigma}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\sigma}_j \cdot \mathbf{r}_{ij}) - \tfrac{1}{3}r_{ij}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\} = \frac{b^3}{r_{ij}^5}([\mathbf{r}_{ij} \times \mathbf{r}_{ij}]^{(2)} \cdot [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]^{(2)}) \qquad (14a)$$

and write its orbital part in cartesian tensor form

$$(O_{ij}^{\text{T}})_{\alpha\beta} \equiv \frac{b^3}{r_{ij}^5}((r_{ij})_\alpha(r_{ij})_\beta - \tfrac{1}{3}r_{ij}^2\delta_{\alpha\beta}) \qquad \alpha(\beta) = x, y, z. \qquad (14b)$$

Exchange terms of type (2), (3), (4), and (5) (see fig. 1) give contributions to the kernels. For terms of types (2), (3), and (4) the exchange kernels for $(O_{ij}^{\text{T}})_{\alpha\beta}$ have the form

$$k^{(2)}(\mathbf{a}, \mathbf{a}')_{\alpha\beta} = C^{(i)} \exp\left[-\tfrac{5}{8}(a^2 + a'^2) + \tfrac{3}{4}(\mathbf{a} \cdot \mathbf{a}')\right]$$

$$\times \frac{1}{V^2}(V_\alpha V_\beta - \tfrac{1}{3}V^2\delta_{\alpha\beta})[\mathscr{W}(V^2) - \tfrac{2}{3}e^{-V^2}], \qquad (15)$$

where

$$\mathscr{W}(V^2) \equiv \frac{1}{V^2}[\tfrac{1}{2}\sqrt{\pi}\, h(V) - e^{-V^2}] = -\tfrac{1}{2}\sqrt{\pi}\, \frac{1}{V}\frac{\mathrm{d}h(V)}{\mathrm{d}V}, \qquad (16a)$$

$$h(V) = \text{erf}\,(V)/V = \frac{2}{\sqrt{\pi}}\int_0^1 e^{-V^2t^2}\,\mathrm{d}t. \qquad (16b)$$

For type-(2) exchange terms, with characteristic $(ij) = (56)$:

$$C^{(2)} = 27/5\sqrt{10}\ \pi^2, \qquad V = \sqrt{\tfrac{3}{40}}(\mathbf{a} - 3\mathbf{a}'). \qquad (17a)$$

For type (3), with characteristic $(ij) = (26)$:

$$C^{(3)} = 27/5\sqrt{10}\ \pi^2, \qquad V = \sqrt{\tfrac{3}{40}}(\mathbf{a}' - 3\mathbf{a}). \qquad (17b)$$

For type (4), with characteristic $(ij) = (25)$:

$$C^{(4)} = 27/8\sqrt{2}\ \pi^2, \qquad V = \sqrt{\tfrac{3}{8}}(\mathbf{a} + \mathbf{a}'). \qquad (17c)$$

TABLE 1

Spin-reduced matrix elements $\langle \alpha ST \| (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) P_{36}^{ST} \| \beta S'T \rangle$

(A) Symmetric combinations			(B) Antisymmetric combinations		
α/β $(ST)(S'T)$	$(ij)=(25)$	$(ij)=(36)$	α/β $(ST)(S'T)$	$(ij)=(25)$	$(ij)=(36)$
(1) NN/NN			(1) NN/NN		
(11)(11)	$10\sqrt{6}/81$	$14\sqrt{6}/27$	(10)(10)	$2\sqrt{6}/27$	$-2\sqrt{6}/9$
(2) NN/NΔ			(2) NN/NΔ		
(11)(11)	$34\sqrt{6}/81$	$4\sqrt{6}/27$	(01)(11)	$4\sqrt{3}/27$	$-8\sqrt{3}/27$
(11)(21)	$2\sqrt{30}/27$	$-4\sqrt{30}/27$			
(3) NN/ΔΔ			(3) NN/ΔΔ		
(11)(11)	$20\sqrt{6}/81$	0	(10)(10)	$4\sqrt{30}/27$	0
(4) NΔ/NΔ			(4) NΔ/NΔ		
(11)(11)	$35\sqrt{6}/162$	$\sqrt{6}/3$	(11)(11)	$\sqrt{6}/6$	$\sqrt{6}/3$
(12)(12)	$-5\sqrt{6}/18$	$\sqrt{6}/3$	(12)(12)	$-7\sqrt{6}/54$	$\sqrt{6}/3$
(11)(21)	$-5\sqrt{30}/54$	$5\sqrt{30}/27$	(11)(21)	$-5\sqrt{30}/54$	$5\sqrt{30}/27$
(12)(22)	$\sqrt{30}/18$	$-\sqrt{30}/9$	(12)(22)	$\sqrt{30}/18$	$-\sqrt{30}/9$
(21)(21)	$-5\sqrt{30}/54$	$-\sqrt{30}/27$	(21)(21)	$\sqrt{30}/18$	$-\sqrt{30}/27$
(22)(22)	$\sqrt{30}/2$	$5\sqrt{30}/9$	(22)(22)	$\sqrt{30}/18$	$5\sqrt{30}/9$
(5) NΔ/ΔΔ			(5) NΔ/ΔΔ		
(12)(02)	$-2\sqrt{3}/9$	$4\sqrt{3}/9$	(11)(01)	$-2\sqrt{15}/27$	$4\sqrt{15}/27$
(11)(11)	$5\sqrt{6}/81$	$10\sqrt{6}/27$	(12)(12)	$\sqrt{30}/27$	$2\sqrt{30}/9$
(12)(22)	$-\sqrt{30}/9$	$2\sqrt{30}/9$	(11)(21)	$-5\sqrt{6}/27$	$10\sqrt{6}/27$
(21)(11)	$-\sqrt{30}/27$	$2\sqrt{30}/27$	(22)(12)	$-\sqrt{6}/9$	$2\sqrt{6}/9$
(22)(22)	$\sqrt{30}/3$	$2\sqrt{30}/9$	(21)(21)	$5\sqrt{6}/9$	$10\sqrt{6}/27$
(21)(31)	$-2\sqrt{105}/27$	$4\sqrt{105}/27$	(22)(32)	$-2\sqrt{21}/9$	$4\sqrt{21}/9$
(6) ΔΔ/ΔΔ			(6) ΔΔ/ΔΔ		
(11)(11)	$4\sqrt{6}/81$	$-2\sqrt{6}/27$	(10)(10)	$4\sqrt{6}/27$	$-2\sqrt{6}/9$
(13)(13)	$-4\sqrt{6}/9$	$2\sqrt{6}/3$	(12)(12)	$-4\sqrt{6}/27$	$2\sqrt{6}/9$
(20)(20)	0	$-2\sqrt{30}/9$	(21)(21)	0	$-2\sqrt{30}/27$
(22)(22)	0	$2\sqrt{30}/9$	(23)(23)	0	$2\sqrt{30}/3$
(31)(31)	$-4\sqrt{21}/27$	$-4\sqrt{21}/27$	(30)(30)	$-4\sqrt{21}/9$	$-4\sqrt{21}/9$
(33)(33)	$4\sqrt{21}/3$	$4\sqrt{21}/3$	(32)(32)	$4\sqrt{21}/9$	$4\sqrt{21}/9$

Spin-reduced matrix elements for $(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j) P_{36}^{ST}$ can be read from this table through the relation

$$\langle \alpha ST \| (\boldsymbol{\sigma}_5 - \boldsymbol{\sigma}_6) P_{36}^{ST} \| \beta S'T \rangle = \langle \alpha ST \| (\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_6) P_{36}^{ST} \| \beta S'T \rangle$$

$$= \tfrac{1}{2} \langle \alpha ST \| (\boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_5) P_{36}^{ST} \| \beta S'T \rangle$$

$$- \tfrac{1}{2} \langle \alpha ST \| (\boldsymbol{\sigma}_3 + \boldsymbol{\sigma}_6) P_{36}^{ST} \| \beta S'T \rangle.$$

The spin-reduced matrix elements of tables 1, 2, 4 are defined through

$$\langle \alpha S M_S T M_T | \mathcal{O}_m^k | \beta S' M_S' T M_T \rangle = (2S+1)^{-1/2} \langle S' M_S' km | S M_S \rangle \langle \alpha ST \| \mathcal{O}^k \| \beta S'T \rangle.$$

Under bra/ket interchange the spin-reduced matrix elements of table 1 change by the factor $(-1)^{S+S'}$.

Finally, for exchange term (5), with $(ij) = (36)$:

$$k^{(2)}(\boldsymbol{a}, \boldsymbol{a}')_{\alpha\beta} = \frac{27}{64\pi^{3/2}} \exp\left[-\tfrac{5}{8}(a^2 + a'^2) + \tfrac{3}{4}(\boldsymbol{a} \cdot \boldsymbol{a}')\right]$$

$$\times \frac{1}{V^5}(V_\alpha V_\beta - \tfrac{1}{3}V^2\delta_{\alpha\beta}) \tag{18}$$

with

$$\boldsymbol{V} = \sqrt{\tfrac{3}{4}}(\boldsymbol{a} - \boldsymbol{a}') .$$

The $[\boldsymbol{\sigma} \times \boldsymbol{\sigma}]^{(2)}_{\text{eff}}$ of eq. (13a) can be expressed in terms of dibaryon spin operators. For potentials coupling different dibaryon channels (e.g. NN to NΔ coupling terms or S to $S' \neq S$ coupling terms) such expressions may be complicated, and it may be most convenient to express these dibaryon spin operators solely through their spin-reduced matrix elements as given in table 2. For tensor potentials for a single channel composed of two spin-$\tfrac{1}{2}$ baryons the operators $[\boldsymbol{\sigma} \times \boldsymbol{\sigma}]^{(2)}_{\text{eff}}$ can be related to the dibaryon spin tensor operator $[\boldsymbol{\sigma}_{B_1} \times \boldsymbol{\sigma}_{B_2}]^{(2)}$ through

$$[\boldsymbol{\sigma} \times \boldsymbol{\sigma}]^{(2)}_{\text{eff}} = \frac{\langle[B_1B_2]ST\|[\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]^{(2)}P^{ST}_{36}\|[B_1B_2]ST\rangle}{\langle[B_1B_2]ST\|[\boldsymbol{\sigma}_{B_1} \times \boldsymbol{\sigma}_{B_2}]^{(2)}\|[B_1B_2]ST\rangle}[\boldsymbol{\sigma}_{B_1} \times \boldsymbol{\sigma}_{B_2}]^{(2)} , \tag{19}$$

where the spin reduced matrix element of the numerator can be read from table 2; and e.g.

$$\langle[\tfrac{1}{2}\tfrac{1}{2}]1\,T\|[\boldsymbol{\sigma}_{B_1} \times \boldsymbol{\sigma}_{B_2}]^{(2)}\|[\tfrac{1}{2}\tfrac{1}{2}]1\,T\rangle = 2\sqrt{5} . \tag{20}$$

For the NN channel with $(ST) = (11)$, e.g., the first entry of table 2A gives $\langle NN11\|[\boldsymbol{\sigma}_5 \times \boldsymbol{\sigma}_6]^{(2)}P^{ST}_{36}\|NN11\rangle = \tfrac{2}{81}\sqrt{5}$. Thus, in this case, $[\boldsymbol{\sigma} \times \boldsymbol{\sigma}]^{(2)}_{\text{eff}\,(ij)=(56)} = \tfrac{1}{81}[\boldsymbol{\sigma}_{N_1} \times \boldsymbol{\sigma}_{N_2}]^{(2)}$. In this case, therefore, exchange terms of type (2) give the following contribution to the full $\tilde{K}(\boldsymbol{a}, \boldsymbol{a}')$ of eq. (13a):

$$\tfrac{1}{4}\alpha_s\hbar c\frac{-3\hbar^2}{4m^2c^2b^3} \times -\tfrac{8}{9} \times 4 \times \frac{27}{5\sqrt{10}}\frac{1}{\pi^2}\exp\left[-\tfrac{5}{8}(a^2 + a'^2) + \tfrac{3}{4}(\boldsymbol{a} \cdot \boldsymbol{a}')\right]$$

$$\times [\mathcal{W}(V^2) - \tfrac{2}{3}e^{-V^2}]\frac{1}{V^2}([V \times V]^{(2)} \cdot [\boldsymbol{\sigma}_{N_1} \times \boldsymbol{\sigma}_{N_2}]^{(2)})(\tfrac{1}{81}) , \tag{21}$$

where $\boldsymbol{V} = \sqrt{\tfrac{3}{40}}(\boldsymbol{a} - 3\boldsymbol{a}')$. The color matrix element $\langle(\lambda_5 \cdot \lambda_6)\rangle = -\tfrac{8}{9}$ can be read from fig. 1, the weighting factor of 4 accounts for the fact that terms with $(ij) = (56)$, (46), (13), (23) give equal contributions to this exchange term. By adding the analogous contributions of exchange terms of types (3), (4), and (5) to the above, the full kernel $\tilde{K}(\boldsymbol{a}, \boldsymbol{a}')$ for the tensor term is constructed in explicit analytic form.

TABLE 2

Spin-reduced matrix elements $\langle \alpha ST \| [\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j]^{(2)} P_{36}^{ST} \| \beta S' T \rangle$

(A) Symmetric combinations

α/β $(ST)(S'T)$[b]	CM[a]	$(ij) = (56)$	$(ij) = (26)$	$(ij) = (25)$	$(ij) = (36)$
(1) NN/NN					
(11)(11)	$\sqrt{5}/81$	2	2	5	34
(2) NN/NΔ					
(11)(11)	$\sqrt{5}/81$	-7	23	2	-20
(11)(21)	$\sqrt{5}/27$	-3	7	-2	-20
(3) NN/$\Delta\Delta$					
(00)(20)	$\sqrt{30}/27$	-2	4	-3	-4
(11)(11)	$\sqrt{5}/81$	10	-14	22	8
(11)(31)	$\sqrt{70}/27$	0	2	-1	-4
(4) NΔ/NΔ					
(11)(11)	$\sqrt{5}/324$	29	29	-58	-8
(12)(12)	$\sqrt{5}/36$	1	1	-2	8
(11)(21)	$\sqrt{5}/108$	-15	25	-26	40
(12)(22)	$\sqrt{5}/12$	5	-3	-2	-8
(21)(21)	$\sqrt{105}/108$	1	1	-2	-8
(22)(22)	$\sqrt{105}/108$	13	13	6	8
(5) NΔ/$\Delta\Delta$					
(11)(11)	$\sqrt{5}/162$	41	-37	14	52
(12)(22)	$\sqrt{5}/6$	1	-1	-2	4
(11)(31)	$\sqrt{70}/54$	-3	1	-2	4
(22)(02)	$\sqrt{30}/54$	-5	7	-6	-4
(21)(11)	$\sqrt{5}/54$	-1	9	2	-4
(22)(22)	$\sqrt{105}/54$	5	-1	6	4
(21)(31)	$\sqrt{105}/27$	1	1	-2	4
(6) $\Delta\Delta$/$\Delta\Delta$					
(00)(20)	$\sqrt{30}/27$	-2	1	0	2
(02)(22)	$\sqrt{30}/27$	2	-1	0	-2
(11)(11)	$\sqrt{5}/405$	1	1	16	-34
(13)(13)	$\sqrt{5}/45$	-1	-1	-16	34
(11)(31)	$\sqrt{70}/135$	-3	2	2	2
(13)(33)	$\sqrt{70}/15$	3	-2	-2	-2
(20)(20)	$\sqrt{105}/27$	-1	-1	0	-2
(22)(22)	$\sqrt{105}/27$	1	1	0	2
(31)(31)	$\sqrt{70}/45$	-2	-2	-2	-2
(33)(33)	$\sqrt{70}/5$	2	2	2	2

TABLE 2 (*cont.*)

(B) Antisymmetric combinations

α/β $(ST)(S'T)$ [b]	CM [a]	$(ij)=(56)$	$(ij)=(26)$	$(ij)=(25)$	$(ij)=(36)$
(1) NN/NN					
(10)(10)	$\sqrt{5}/27$	4	4	1	-22
(2) NN/NΔ					
(01)(21)	$\sqrt{30}/81$	-5	-5	0	20
(3) NN/$\Delta\Delta$					
(01)(21)	$\sqrt{6}/81$	-10	20	-15	-20
(10)(10)	$1/27$	10	-14	22	8
(10)(30)	$\sqrt{14}/9$	0	2	-1	-4
(4) NΔ/NΔ					
(11)(11)	$\sqrt{5}/36$	1	1	-6	8
(12)(12)	$\sqrt{5}/108$	23	23	-10	-56
(11)(21)	$\sqrt{5}/36$	-1	7	-10	8
(12)(22)	$\sqrt{5}/36$	3	-5	-2	-8
(21)(21)	$\sqrt{105}/324$	7	7	6	-40
(22)(22)	$\sqrt{105}/36$	3	3	-2	8
(5) NΔ/$\Delta\Delta$					
(12)(12)	$1/54$	41	-37	14	52
(11)(21)	$1/18$	5	-5	-10	20
(12)(32)	$\sqrt{14}/18$	-3	1	-2	4
(21)(01)	$\sqrt{6}/162$	-25	35	-30	-20
(22)(12)	$1/18$	-1	9	2	-4
(21)(21)	$\sqrt{21}/162$	25	-5	30	20
(22)(32)	$\sqrt{21}/9$	1	1	-2	4
(6) $\Delta\Delta$/$\Delta\Delta$					
(01)(21)	$\sqrt{30}/81$	-2	1	0	2
(03)(23)	$\sqrt{30}/9$	2	-1	0	-2
(10)(10)	$\sqrt{5}/135$	1	1	16	-34
(12)(12)	$\sqrt{5}/135$	-1	-1	-16	34
(10)(30)	$\sqrt{70}/45$	-3	2	2	2
(12)(32)	$\sqrt{70}/45$	3	-2	-2	-2
(21)(21)	$\sqrt{105}/81$	-1	-1	0	-2
(23)(23)	$\sqrt{105}/9$	1	1	0	2
(30)(30)	$\sqrt{70}/15$	-2	-2	-2	-2
(32)(32)	$\sqrt{70}/15$	2	2	2	2

[a]) CM = common multiplier (multiply the numbers in each row by CM to obtain the spin-reduced matrix element).

[b]) Under bra/ket interchange $(ij)=(25)$ or (36) terms are changed by the factor $(-1)^{S+S'}$; also

$$\langle\beta S'T\|[\boldsymbol{\sigma}_5\times\boldsymbol{\sigma}_6]^{(2)}P_{36}^{ST}\|\alpha ST\rangle=(-1)^{S+S'}\langle\alpha ST\|[\boldsymbol{\sigma}_2\times\boldsymbol{\sigma}_6]^{(2)}P_{36}^{ST}\|\beta S'T\rangle.$$

2.2. SPIN–ORBIT TERMS

For the symmetric and antisymmetric spin–orbit terms it is convenient to define the dimensionless vector operators

$$O_{ij}^{(s)} = \boldsymbol{L}_{ij} \equiv \left(\frac{b}{r_{ij}}\right)^{n} \frac{1}{\hbar} [\boldsymbol{r}_{ij} \times (\boldsymbol{p}_i - \boldsymbol{p}_j)],$$

$$O_{ij}^{(a)} = \boldsymbol{K}_{ij} \equiv \left(\frac{b}{r_{ij}}\right)^{n} \frac{1}{\hbar} [\boldsymbol{r}_{ij} \times (\boldsymbol{p}_i + \boldsymbol{p}_j)]. \tag{22}$$

The cases $n = 3$ (one-gluon exchange terms), $n = 1$ (linear confinement spin–orbit terms) and $n = 0$ (quadratic confinement spin–orbit terms) are of greatest interest. The symmetric spin–orbit terms gain contributions only from exchange terms of type (4) and (5), (see fig. 1; the orbital integrals for exchange terms (1), (2), (3) are identically zero). The antisymmetric spin–orbit terms gain contributions only from exchange terms of type (2) and (3). [It should perhaps be mentioned that Warke and Shanker [3]) consider only exchange terms of type (3), (4) and (5) and omit type (2).]

The orbital factors of the exchange kernels are given by the vectors (see eq. (13b))

$$k^{(1)}(\boldsymbol{a}, \boldsymbol{a}') = F\{\exp[-\tfrac{5}{8}(a^2 + a'^2) + \tfrac{3}{4}(\boldsymbol{a} \cdot \boldsymbol{a}')]\}[\boldsymbol{a} \times \boldsymbol{a}'] \tag{23}$$

where the factors F are the following:

(*i*) *Symmetric spin–orbit terms.* For $O_{ij}^{(s)}$, type-(4) exchange, with characteristic $(ij) = (25)$:

$$n = \begin{cases} 3: & F = -i\dfrac{81}{16\sqrt{2}} \dfrac{1}{\pi^2} \mathcal{W}(V^2) & \text{(24a)} \\[4mm] 1: & F = -i\dfrac{81}{16\sqrt{2}} \dfrac{1}{\pi^2}[\tfrac{1}{2}\sqrt{\pi}\, h(V) - \tfrac{1}{2}\mathcal{W}(V^2)] & \text{(24b)} \\[4mm] 0: & F = -i\dfrac{81}{32\sqrt{2}} \dfrac{1}{\pi^{3/2}} \times 1 & \text{(24c)} \end{cases}$$

with

$$V = \sqrt{\tfrac{3}{8}}(\boldsymbol{a} + \boldsymbol{a}'). \tag{24d}$$

For $O_{ij}^{(s)}$, type-(5) exchange, with $(ij) = (36)$:

$$n = \begin{cases} 3: & F = i\dfrac{9}{16} \dfrac{\sqrt{3}}{\pi^{3/2}} \dfrac{1}{|\boldsymbol{a} - \boldsymbol{a}'|^3} & \text{(25a)} \\[4mm] 1: & F = i\dfrac{27}{32} \dfrac{\sqrt{3}}{\pi^{3/2}} \dfrac{1}{|\boldsymbol{a} - \boldsymbol{a}'|} & \text{(25b)} \\[4mm] 0: & F = i\dfrac{81}{32\sqrt{2}} \dfrac{1}{\pi^{3/2}} \times 1. & \text{(25c)} \end{cases}$$

(*ii*) *Antisymmetric spin–orbit terms.* For $O_{ij}^{(a)}$, type-(2) exchange, with characteristic $(ij) = (56)$:

$$
n = \begin{cases}
3: & F = i\dfrac{81}{20\sqrt{10}}\dfrac{1}{\pi^2}\mathcal{W}(V^2) & (26\text{a}) \\[3mm]
1: & F = i\dfrac{81}{16\sqrt{10}}\dfrac{1}{\pi^2}[\tfrac{1}{2}\sqrt{\pi}\,h(V) - \tfrac{1}{2}\mathcal{W}(V^2)] & (26\text{b}) \\[3mm]
0: & F = i\dfrac{81}{64\sqrt{2}}\dfrac{1}{\pi^{3/2}}\times 1 & (26\text{c})
\end{cases}
$$

with

$$
V = \sqrt{\tfrac{3}{40}}(a - 3a') \, . \tag{26d}
$$

For $O_{ij}^{(a)}$, type-(3) exchange, with characteristic $(ij) = (26)$, the factors F are given by eqs. (26a–c) but now with

$$
V = \sqrt{\tfrac{3}{40}}(a' - 3a) \, . \tag{27}
$$

The vectors $k^{(1)}(a, a')$ must be combined with the appropriate spin-vector operators (see eq. (13b)) to construct the full $\tilde{K}(a, a')$. The σ_{eff} are expressible in terms of the spin-reduced matrix elements of $(\sigma_i \pm \sigma_j)P_{(36)}^{ST}$. Those for $(\sigma_i + \sigma_j)P_{36}^{ST}$ are given explicitly in table 1; those for $(\sigma_i - \sigma_j)P_{36}^{ST}$ can be deduced from this table (see the first footnote to the table).

3. The baryon–baryon spin–orbit interaction

It is the great advantage of the nonrelativistic RGM formalism that very specific quantitative results can be obtained directly, and all quark–gluon exchange terms can be treated on an equal footing. Since some earlier quark-model treatments of the baryon–baryon spin–orbit interaction, both relativistic and nonrelativistic, have been incomplete or have singled out specific quark–gluon exchange mechanisms, it is worthwhile to give a more detailed quantitative treatment of the spin–orbit terms. The kernels of sect. 2 are nonlocal operators acting on the baryon–baryon relative motion function. To get an intuitive picture of their role it is useful to recast them in the form of equivalent local potentials. The simplest possible method of constructing such potentials, which does not require the solution of the full RGM equation, is based on the local momentum approximation used in ref. [8]) to study the short-range central part of the NN interaction. This method may not simulate the relative motion function as accurately as more elaborate techniques [23]), but it gives a quick and direct way of arriving at an effective potential and has the advantage that it is relatively insensitive to other (short- and long-range) terms of the interaction. Since the emphasis in this investigation is on one part of the NN interaction, this method is particularly suitable for a study of the short-range spin–orbit interaction.

The spin–orbit quark-exchange kernels are first converted into momentum-dependent operators. These are then approximated by their Wigner transforms, K_W, where the $K_W^{(E)}$ are related to the $\tilde{K}(a, a')$ of eqs. (12) and (13b) by

$$K_W^{(E)}(a^2, q^2, (a \cdot q)^2, ([a \times q] \cdot \sigma_{\text{eff}}))$$

$$= -\tfrac{9}{2} \times 2 \sum_{ij} c^{(ij)} \int dt \, e^{it \cdot q} (k^{(1)}(a - \tfrac{1}{2}t, \, a + \tfrac{1}{2}t) \cdot \sigma_{\text{eff}})_{(ij)}. \tag{28}$$

For the symmetrized dibaryon functions of the form of eq. (3) and the exchange kernels of the form of eq. (12), the factors $\tilde{K}(a, a')$ and $\tilde{K}(a, -a')$ give equal contributions to the full Wigner transform [cf. ref. [8]], accounting for the factor 2 in eq. (28). In the final expression the c-numbers q^2 and $(a \cdot q)^2$ are to be replaced by

$$q^2 = \frac{2}{3} \frac{b^2}{\hbar^2} M_B(E - U(R)),$$

$$(a \cdot q)^2 = \frac{M_B R^2}{\hbar^2}\left(E - U(R) - \frac{\hbar^2(L + \tfrac{1}{2})^2}{M_B R^2}\right), \tag{29}$$

where the baryon reduced mass $M_{B_1}M_{B_2}/(M_{B_1} + M_{B_2})$ has been approximated by $\tfrac{1}{2}M_B$, cf. also eq. (10). In this local momentum approximation, the $U(R)$ are the equivalent local potentials for the full interaction. These can be evaluated in this WKB approximation from simple transcendental equations [8]. To estimate the strength of the spin–orbit terms it will be sufficient to evaluate the Wigner transforms of the exchange kernels for the quark–quark spin–orbit interaction as functions of a and q. (Note that $|q^2|$ ranges from $0 \to \sim 2$ as $|E - U(R)|$ ranges from $0 \to \sim 350$ MeV.)

The spin–orbit parts of the one-gluon-exchange quark–quark interaction give rise to the following Wigner transforms: For the symmetric spin–orbit interaction, $(L_{ij} \cdot (\sigma_i + \sigma_j)$ terms), eqs. (28), (13b), (24a), (25a) give

$$K_W^{(E)} = \frac{\alpha_s}{m^2 c^4}\left(\frac{\hbar c}{b}\right)^3 \exp\left[-\tfrac{1}{2}(a^2 + q^2)\right]$$

$$\times \left\{\frac{243}{16\sqrt{\pi}} \mathcal{W}(\tfrac{3}{2}a^2)([a \times q] \cdot \sigma_{\text{eff}}^{(25)}) - \frac{27}{16}\sqrt{\frac{3}{\pi}} \mathcal{W}(-\tfrac{1}{2}q^2)([a \times q] \cdot \sigma_{\text{eff}}^{(36)})\right\}, \tag{30}$$

where $\mathcal{W}(V^2)$ is given by eq. (16). Note that $\mathcal{W}(V^2)$ is positive and monotonically decreasing.

For the antisymmetric spin–orbit interaction, $(K_{ij} \cdot (\sigma_i - \sigma_j)$ terms), eqs. (26a), (27) give

$$K_W^{(E)} = -\frac{\alpha_s}{m^2 c^4}\left(\frac{\hbar c}{b}\right)^3 \frac{81}{8\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}(a^2 + q^2)\right]$$

$$\times \left\{\int_0^1 du \, u^2 \exp\left[-\tfrac{3}{16}(a^2 - q^2)u^2\right] \cos\left[\tfrac{3}{8}(a \cdot q)u^2\right]\right\}([a \times q] \cdot \sigma_{\text{eff}}^{(26)}). \tag{31}$$

A spin–orbit potential generated by a linear confining potential (with assumed scalar coupling) gives rise to a Wigner transform of similar form. For the symmetric part of this spin–orbit interaction, ($\boldsymbol{L}_{ij} \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)$ terms), eqs. (24b), (25b) give

$$K_{\mathrm{W}}^{(\mathrm{E})} = \frac{a_c'\hbar^2 c^2}{b} \frac{1}{m^2 c^4} \exp\left[-\tfrac{1}{2}(a^2 + q^2)\right]$$

$$\times \left\{ -\frac{81}{4\sqrt{\pi}} f(\tfrac{3}{2}a^2)([\boldsymbol{a} \times \boldsymbol{q}] \cdot \boldsymbol{\sigma}_{\mathrm{eff}}^{(25)}) + \frac{27}{4}\sqrt{\frac{3}{\pi}} f(-\tfrac{1}{2}q^2)([\boldsymbol{a} \times \boldsymbol{q}] \cdot \boldsymbol{\sigma}_{\mathrm{eff}}^{(36)}) \right\}, \qquad (32)$$

where $f(V^2) = [\tfrac{1}{2}\sqrt{\pi}\, h(V) - \tfrac{1}{2}\mathcal{W}(V^2)]$.

(The strength factor $(a_c'\hbar^2 c^2/b)$ is determined by the potential constant a_c' of the linear confining potential $v_c(r) = -a_c' r$, see eq. (8).)

For the antisymmetric part of this spin–orbit interaction ($\boldsymbol{K}_{ij} \cdot (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)$ terms) eqs. (26b), (27) give

$$K_{\mathrm{W}}^{(\mathrm{E})} = -\frac{a_c'\hbar^2 c^2}{b} \frac{1}{m^2 c^4} \frac{81}{2\sqrt{2\pi}} \exp\left[-\tfrac{1}{2}(a^2 + q^2)\right]$$

$$\times \left\{ \int_0^1 du(1 - u^2) \exp\left[-\tfrac{3}{16}(a^2 - q^2)u^2\right] \cos\left[\tfrac{3}{8}(\boldsymbol{a} \cdot \boldsymbol{q})u^2\right] \right\}([\boldsymbol{a} \times \boldsymbol{q}] \cdot \boldsymbol{\sigma}_{\mathrm{eff}}^{(26)}).$$

$$(33)$$

A spin–orbit potential generated by a quadratic confining potential gives rise to the simple Wigner transform (see eqs. (24c), (25c), (26c), (27))

$$K_{\mathrm{W}}^{(\mathrm{E})} = \frac{a_c\hbar^2 c^2}{m^2 c^4} \frac{81}{4} \exp\left[-\tfrac{1}{2}(a^2 + q^2)\right]\{[\boldsymbol{a} \times \boldsymbol{q}] \cdot (\boldsymbol{\sigma}_{\mathrm{eff}}^{(36)} - \boldsymbol{\sigma}_{\mathrm{eff}}^{(25)} - \boldsymbol{\sigma}_{\mathrm{eff}}^{(56)} - \boldsymbol{\sigma}_{\mathrm{eff}}^{(26)})\}. \qquad (34)$$

(The strength factor $(a_c\hbar^2 c^2)$ is determined by the potential constant of the quadratic confining potential, $v_c(r) = -a_c r^2$.) Eq. (34) contains both the symmetric and anti-symmetric parts of this spin–orbit term through the $\boldsymbol{\sigma}_{\mathrm{eff}}$ with $(ij) = (36)$, (25) and (56), (26), respectively.

3.1. THE NN SPIN–ORBIT INTERACTION

The dibaryon spin vectors, $\boldsymbol{\sigma}_{\mathrm{eff}}^{(ij)}$ in eqs. (30)–(34) are determined through the spin-reduced matrix elements of the operators $(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)P_{36}^{ST}$ for $(ij) = (25)$ and (36) and $(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)P_{36}^{ST}$ for $(ij) = (26)$ and (56); (see table 1, and note that $\boldsymbol{\sigma}_{\mathrm{eff}}^{(56)} = \boldsymbol{\sigma}_{\mathrm{eff}}^{(26)}$). For spin–orbit potentials coupling different dibaryon channels (e.g. NN to NΔ coupling terms or S to $S' \neq S$ coupling terms) these spin operators may be compli-cated. For the spin–orbit potentials for a single channel composed of two spin-$\tfrac{1}{2}$ baryons (e.g. $B_1 B_2 = NN$) the operators $\boldsymbol{\sigma}_{\mathrm{eff}}^{(ij)}$ can be related to the baryon spin operators $\boldsymbol{s}_{B_1} + \boldsymbol{s}_{B_2} \equiv \boldsymbol{S}$ through the relation

$$\boldsymbol{\sigma}_{\mathrm{eff}}^{(ij)} = \frac{\langle B_1 B_2 ST \| (\boldsymbol{\sigma}_i \pm \boldsymbol{\sigma}_j) P_{36}^{ST} \| B_1 B_2 ST \rangle}{\langle B_1 B_2 ST \| \boldsymbol{\sigma}_{B_1} + \boldsymbol{\sigma}_{B_2} \| B_1 B_2 ST \rangle} 2(\boldsymbol{s}_{B_1} + \boldsymbol{s}_{B_2}). \qquad (35a)$$

For two spin-$\frac{1}{2}$ baryons, coupled to $S = 1$, the baryon spin-reduced matrix element has the simple value

$$\langle \tfrac{1}{2}\tfrac{1}{2} S = 1 \, T \| \boldsymbol{\sigma}_{B_1} + \boldsymbol{\sigma}_{B_2} \| \tfrac{1}{2}\tfrac{1}{2} S = 1 \, T \rangle = 2\sqrt{6} . \tag{35b}$$

For the spin–isospin symmetric NN channel with $S = 1$ $T = 1$ the first entry of table 1 together with eqs. (35a–b) thus gives

$$\boldsymbol{\sigma}_{\text{eff}}^{(25)} = \tfrac{10}{81}\boldsymbol{S}, \qquad \boldsymbol{\sigma}_{\text{eff}}^{(36)} = \tfrac{42}{81}\boldsymbol{S}, \qquad \boldsymbol{\sigma}_{\text{eff}}^{(26)} = \boldsymbol{\sigma}_{\text{eff}}^{(56)} = -\tfrac{16}{81}\boldsymbol{S} . \tag{35c}$$

With these relations and eqs. (30)–(34) it is possible to evaluate the strength coefficients of the various $[\boldsymbol{a} \times \boldsymbol{q}] \cdot \boldsymbol{S} = (1/\hbar)[\boldsymbol{R} \times \boldsymbol{P}] \cdot \boldsymbol{S} = \boldsymbol{L} \cdot \boldsymbol{S}$ terms as functions of $a(R)$, q^2, and $(\boldsymbol{a} \cdot \boldsymbol{q})^2$; $[\boldsymbol{R} \times \boldsymbol{P}]$ is reinterpreted as the orbital angular momentum operator.

Figs. 2a–d give characteristic examples for two quark–quark interaction parameters used in recent RGM calculations for NN scattering. The parameters selected are those for the most recent calculations of Ohta et al.[24] and Faessler et al.[6] (see the figure caption and table 3). The figures are drawn for $q^2 = 0$ and show curves for both the $(ST) = (11)$ NN channel $(L = 1)$ and the $(ST) = (10)$ NN channel $(L = 2)$. The symmetric spin–orbit potentials (from one-gluon exchange terms, designated $L \cdot S^{(+)}$) are negative in the 0.3 to 1 fm range, in the triplet-odd $(ST) = (11)$ channel, and positive in the triplet-even $(ST) = (10)$ channel; that is they have the correct sign for a nuclear spin–orbit interaction. The antisymmetric one-gluon-exchange spin–orbit potential, the $K \cdot S^{(-)}$ term of fig. 2, has the opposite (wrong) sign, as does the spin–orbit term generated by a confining potential. The one-gluon-exchange $(L \cdot S^{(+)}$ and $K \cdot S^{(-)})$ terms are insensitive to a wide selection of recent quark–quark interaction parameters. This is a reflection of the fact that the parameter combination $(\alpha_s/m^2 b^3)$, has been fitted to the observed Δ-N mass difference by all recent investigators. The parameter sets of table 3, in agreement with most recent choices, have also been based on the "consistency condition" whereby the potential constants are chosen to minimize the nucleon mass as a function of b, the oscillator length parameter. However, the choice of b is made arbitrarily, usually to be consistent with nucleon size and/or the philosophy of the little bag. In ref.[24] some consideration has also been given to the position of the Roper resonance, the nucleon breathing mode excitation [see the state $|B^*\rangle$ of ref.[24]]. The above considerations do not lead to a unique set of interaction parameters and also fail to fit the nucleon mass (see table 3). Since the interactions do not attempt to account for the long-range parts of the NN interaction, and the effects of the mesonic cloud are not included in the quark models, it is not clear that a fit to the nucleon mass should be required. However, entry III of table 3 shows that a set of parameters intermediate to OOAY and FFLS, also based on the consistency condition, gives a fit to both the nucleon mass and the Δ-N difference. Since it involves a small decrease in b relative to OOAY, the approximate fit to the Roper resonance is also preserved (and actually somewhat improved). Table 3 shows that a small change in the philosophy of fitting
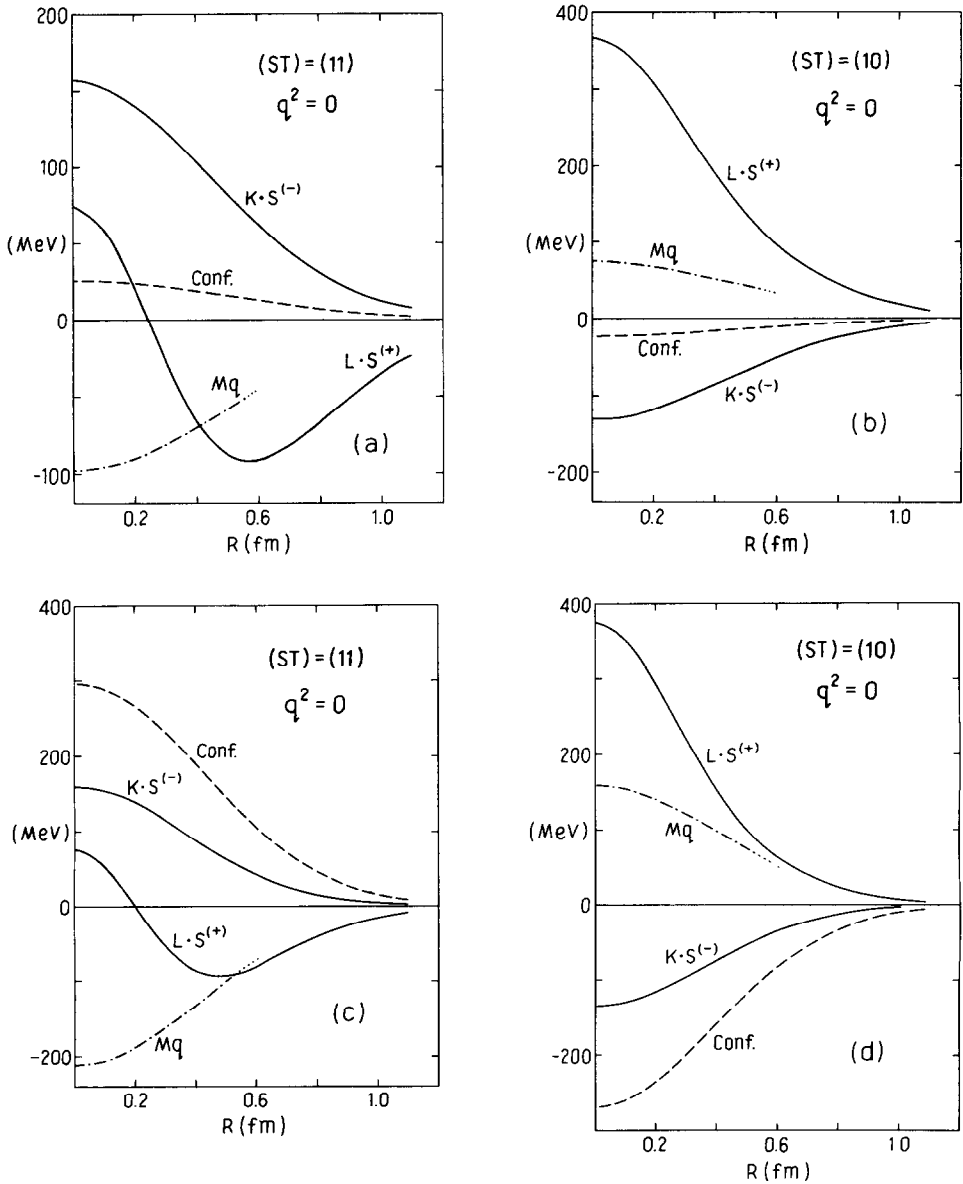
Fig. 2. Strength of the spin–orbit potentials. Curves marked $L \cdot S^{(+)}$ and $K \cdot S^{(-)}$ give the contributions arising from the symmetric and antisymmetric spin–orbit terms of the one-gluon-exchange quark–quark interaction. Curves marked Conf. give the strengths of the spin–orbit terms generated by the confining potentials. Curves marked M_q give the strengths due to an alternate mass-confinement mechanism arising from one-body Thomas dm'/dr terms, according to a simple model defined through eqs. (36)–(39). Curves (a) and (b) are for the recent parameters of Ohta *et al.* [24]), see OOAY of table 3. Curves (c) and (d) are for the LOGEP potential of ref. [6]), see FFLS entry of table 3. (Note the different energy scale of fig. 2(a).) Both OOAY and FFLS use linear confining potentials. Curves marked M_q use the parameters mc^2 and b of table 3.

TABLE 3

Interaction parameters and predicted baryon masses

Interaction	a_c' (MeV · fm^{-1})	α_s	b (fm)	mc^2 (MeV)	$M_N c^2$ (MeV)	$(M_\Delta - M_N)c^2$ (MeV)
OOAY [24])	14.317	1.9514	0.6	355	681	293
FFLS [6])	133.07	1.12	0.5	350	1936	299
III	34.66	1.741	0.578	355	938	292

the parameters of the quark–quark interaction can lead to a rather large spread in the confinement parameters a_c' or a_c. This reflects itself in the large differences of the spin–orbit terms arising from confinement potentials. These large differences, however, are due to the different magnitudes of the confining potential constants. Note that both the OOAY and FFLS quark–quark interactions of fig. 2 use a linear confining potential, but with very different magnitudes.

The spin–orbit terms arising from confinement potentials, however, are quite insensitive to the nature of the radial dependence. This is illustrated by fig. 3 which compares such spin–orbit terms for two interactions which differ only in the radial character of the confining potential but use the same quark mass, oscillator length
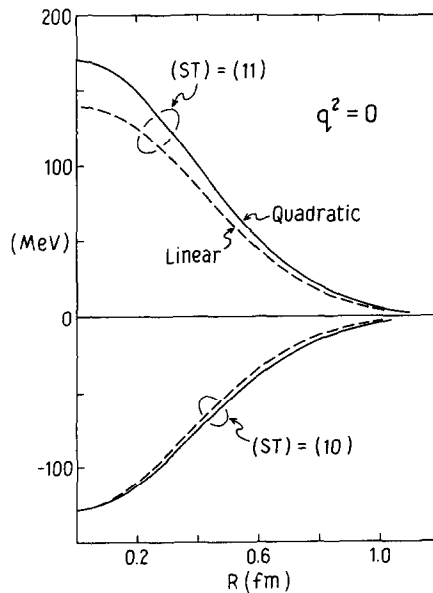

Fig. 3. Spin–orbit terms generated by confining potentials. Comparison of linear and quadratic confining potentials. The parameters are the "Oka-modified" parameters of ref. [6]) with linear confinement, $a_c' = 61.6$ MeV · fm^{-1}, and the parameters of ref. [6]) with quadratic confinement, $a_c = 34.5$ MeV · fm^{-2}. Both use the same $b = 0.475$ fm, $mc^2 = 355$ MeV, and $\alpha_s = 0.97$. Upper curves for NN $(ST) = (11)$, lower curves for NN $(ST) = (10)$.

parameter, b, and α_s. Note that in this case the linear and quadratic confining potentials generate spin–orbit terms of almost identical strength. The uncertainty in the strengths of the confining potentials, however, makes it difficult to draw conclusions regarding the relative importance of the one-gluon-exchange and confinement spin–orbit terms. For the FFLS parameters of fig. 2, e.g., the $K \cdot S^{(-)}$ and confinement spin–orbit terms together overwhelm the $L \cdot S^{(+)}$ term. Apart from the uncertainty in the strengths of the spin–orbit terms generated by the confining potentials, it is also not completely clear whether these additional spin–orbit terms should be included since no satisfactory theory of quark confinement exists.

A quite different confinement spin–orbit effect is obtained if quark confinement is built into the model in the spirit of the MIT bag model through a mass term in the relativistic single-particle equation, with a mass which rises to infinity outside the bag. If both the quark mass, to be denoted by m', and V in the single-particle Dirac equation are permitted to be functions of r, the distance from the bag center, the Thomas term will have the form

$$v^{\text{Th}} = -\frac{\hbar}{4m^2c^2}\frac{1}{r}\left(\frac{\mathrm{d}(m'c^2)}{\mathrm{d}r} - \frac{\mathrm{d}V}{\mathrm{d}r}\right)([r \times p] \cdot \sigma). \tag{36}$$

The requirement $(2m'c^2 - V) > 0$ outside the bag, needed to avoid Klein-paradox difficulties, makes it natural to set $V = 0$ and require $m' \to \infty$ outside the bag. To make contact with nonrelativistic harmonic oscillator quark models, it is convenient to build a model based on the simple assumption

$$V = 0, \qquad m'c^2 = mc^2 + \frac{1}{2}\frac{\hbar^2}{mb^4}r^2, \tag{37}$$

where mc^2 and b are chosen to be consistent with the numerical values of table 3. In eqs. (36) and (37) the single-particle r, and its conjugate p are defined relative to the bag center, and for two separated bags of three quarks v^{Th} does not give rise to a baryon–baryon interaction. In the limit of extremely small separation between the two bags, however, r must be defined relative to the common six-quark bag center, and in this limit the single-particle terms, v^{Th}, will generate spin–orbit terms in the baryon–baryon interaction through the quark-exchange mechanism; that is, through a term $-\frac{9}{2}\sum_{i=1}^{6} v^{\text{Th}}(r_i)P_{36}(1 - \mathscr{P})$ in the exchange kernel, eq. (5).

Assuming that this confinement mechanism is independent of color, and with $P_{36}(\text{color})$ matrix element $= \frac{1}{3}$ for the dibaryon system, this leads to the exchange kernel

$$i\frac{243}{512\sqrt{2}\,\pi^{3/2}}\left(\frac{\hbar c}{b}\right)^4\frac{1}{m^3c^6}\exp\left[-\tfrac{5}{8}(a^2 + a'^2) + \tfrac{3}{4}(a \cdot a')\right]$$

$$\times\{[a \times a'] \cdot (\sigma_3 + \sigma_6 - \tfrac{1}{2}(\sigma_1 + \sigma_2 + \sigma_4 + \sigma_5))_{\text{eff}}\}. \tag{38}$$

In the notation of eqs. (30)–(34) this has a Wigner transform

$$K_{\mathrm{W}}^{(\mathrm{E})} = -\frac{243}{256}\left(\frac{\hbar c}{b}\right)^4 \frac{1}{m^3 c^6} \exp\left[-\tfrac{1}{2}(a^2+q^2)\right]\{[\boldsymbol{a}\times\boldsymbol{q}]\cdot(\boldsymbol{\sigma}_{\mathrm{eff}}^{(36)}-\boldsymbol{\sigma}_{\mathrm{eff}}^{(25)})\} \qquad (39)$$

to be considered valid only in the extreme short-range limit of very small a. This "mass-confinement" spin–orbit term is plotted for $R \lesssim 0.5$ fm in fig. 2 (designated M_{q}) mainly to show that it has a sign opposite to that of the spin–orbit terms arising from a quark-confining scalar coupling term.

To understand this difference in sign, it may be useful to convert the 2-body quark confining potentials, such as $v^{\mathrm{c}}(r_{ij})=(\lambda_i\cdot\lambda_j)(-a_{\mathrm{c}}r_{ij}^2)$, to equivalent 1-body form, such as the $V(r)$ of eq. (36), by performing properly weighted summations over j for each i. For a colorless three-quark system this gives $V(r)=-\frac{8}{3}(-a_{\mathrm{c}})\kappa r^2$, with a positive weighting factor κ. For a colorless six-quark system this gives

$$V(r) = \frac{-\frac{8}{3}n_{\mathrm{a}}+\frac{4}{3}n_{\mathrm{s}}}{n_{\mathrm{a}}+n_{\mathrm{s}}}(-a_{\mathrm{c}})\kappa r^2 = \tfrac{16}{15}a_{\mathrm{c}}\kappa r^2$$

where $n_{\mathrm{a}}=9$, $n_{\mathrm{s}}=6$ are the color-antisymmetrically and symmetrically coupled quark pairs, respectively. That is, the single-particle $V(r)$ equivalent to the confining potential has a derivative $\mathrm{d}V/\mathrm{d}r$ of the same sign as $\mathrm{d}m'/\mathrm{d}r$. Since these terms appear with opposite sign in eq. (36), through the opposite signs of m and V in the Dirac equation for the small components of ψ, the opposite signs of the two types of confining mechanism can be understood. Finally, it should perhaps be mentioned that some recent relativistic quark models [25] use a constant "mass" and a "potential" of the form $\frac{1}{2}(1+\gamma_0)ar^2$, which eliminates the r-dependence in the Dirac equation for the small components of ψ and would thus lead to no spin–orbit terms. This discussion is included mainly to show that even the sign of spin–orbit terms arising from quark-confinement mechanisms may be model-dependent and hence open to question. The subsequent discussion will therefore first focus on the one-gluon-exchange spin–orbit potentials alone.

The energy dependence of the one-gluon-exchange spin–orbit potentials is illustrated in fig. 4 for both the $(ST)=(11)$ (triplet-odd) and $(ST)=(10)$ (triplet-even) NN channels. For the OOAY parameters used in fig. 4, $|q^2|=1$ corresponds to a value of $|E-U(R)|\simeq170$ MeV, see eq. (29). For large R, $E-U(R)>0$, and therefore $q^2>0$. As $R\to0$, q^2 changes sign because of the repulsive short range character of the full equivalent potential, $U(R)$. A more complete picture of the R-dependence of the spin–orbit potential would therefore be given by a curve which starts out with positive q^2 at large R, switches over to $q^2=0$ at $R\sim0.4$–0.5 fm, and finally to negative values of q^2 at very small values of R. In the triplet-odd NN channel, therefore, the spin–orbit term is negative in the 0.6 fm–0.8 fm region, a sign in agreement with phenomenological potentials derived from NN scattering data. However, its magnitude of ~50 MeV is too small by a factor of ~3–5.
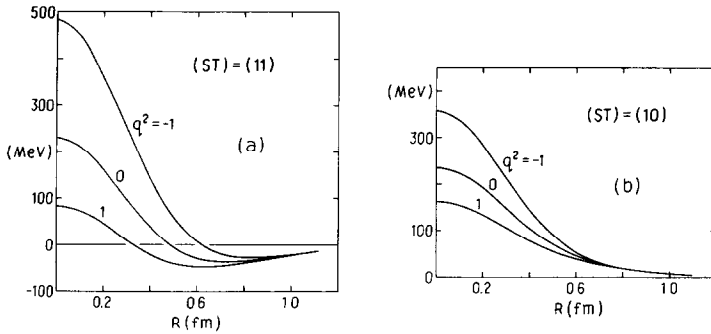
Fig. 4. The q^2-dependence of the one-gluon exchange spin–orbit potentials. The curves give the strength for the full one-gluon exchange terms, $L \cdot S^{(+)} + K \cdot S^{(-)}$. The curves are for the parameters OOAY of fig. 2. For these, $|q^2| = 1$ corresponds to $|E - U(R)| \sim 170$ MeV.

Since the near cancellation of positive and negative contributions to the spin–orbit potential are crucial for the 0.5–1.0 fm region of the triplet odd NN channel, it is instructive to show the q^2-dependence of the full spin–orbit potential arising from one-gluon-exchange terms and the confining potential terms. For this purpose the parameter set III of table 3 which fits both M_N and M_Δ has been selected. Fig. 5 shows that this full spin–orbit potential is very shallow in the 0.5–1.0 fm region of the triplet-odd $(ST) = (11)$ $(L = 1)$ channel. For $E \simeq 200$–400 MeV, the full R-dependence of the potential can be gleaned from a curve which starts at $q^2 \sim +2$ crosses over toward $q^2 = 0$ at $R \sim 0.4$–0.5 fm and tends toward negative q^2 as $R \to 0$.
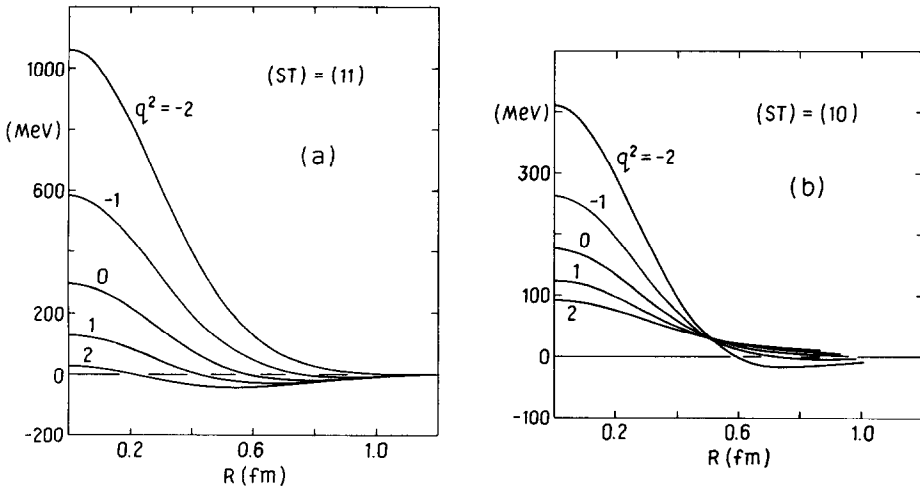

Fig. 5. The q^2-dependence of the full spin–orbit potentials, one-gluon exchange + confining potential terms. The curves are for parameter set III of table 3 which fits both M_N and M_Δ. For these parameters $|q^2| = 1$ corresponds to $|E - U(R)| \sim 185$ MeV.
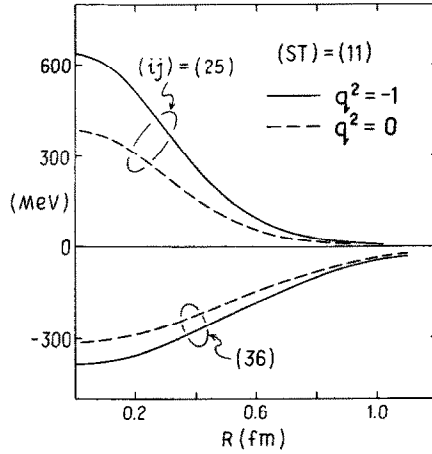
Fig. 6. The symmetric spin–orbit potentials, $L \cdot S^{(+)}$. The curves give the separate contributions of exchange terms of type (4), $(ij) = (25)$ and type (5), $(ij) = (36)$. The curves are for the parameters OOAY of fig. 2.

For the triplet-even channel $(ST) = (10)$ $(L = 2)$ the curves are in reasonable agreement with phenomenological potentials derived from NN scattering data.

The results shown in figs. 4 and 5 gain contributions from both symmetric and antisymmetric parts of the spin–orbit potentials, and therefore include the effects of all types of quark–gluon exchange terms. Since some recent relativistic treatments [14,26]) of the spin–orbit interaction have been based on a single quark–gluon exchange mechanism corresponding to term (5) of fig. 1, with $(ij) = (36)$, it is interesting to show separately the contributions of the $(ij) = (36)$ and $(ij) = (25)$ terms to the symmetric spin–orbit $(L \cdot S^{(+)})$ potential. This is illustrated for the $(ST) = (11)$ NN channel in fig. 6. It is interesting to note that the $(ij) = (36)$ term by itself can give a large negative short-range spin–orbit potential, but for $R \lesssim 0.2$ fm this is completely reversed by the positive contribution of the $(ij) = (25)$ term (see eq. (30)). In the relativistic treatments of refs. [14,26]) the quark exchange of the $(ij) = (36)$ term, through a Fierz transformation, causes the vector coupling of the one-gluon exchange to be converted to a coupling of the form $S - P - \frac{1}{2}V + \frac{1}{2}A$. The $(ij) = (25)$ term corresponds to a four-body quark–quark interaction. It may therefore be appropriate to augment the vector coupling of quarks 2 and 5 with a multiplicative factor $(\psi^+(3)\psi(6))(\psi^+(6)\psi(3)) = (\bar{\psi}(3)\gamma_0\psi(6))(\bar{\psi}(6)\gamma_0\psi(3))$. This would lead to a vector coupling of quarks 2 and 5 augmented by a factor

$$\{1 - 2\langle R_1^2 \rangle + \tfrac{10}{9}\langle R_1^2 \rangle^2\}P_{36}^{ST} + \tfrac{4}{9}\langle R_1^2 \rangle^2 P_{36}^T , \qquad (40)$$

where the relativistic wave function is given by

$$\psi(r) = \begin{pmatrix} R_0(r)\chi \\ -i(\boldsymbol{\sigma} \cdot \hat{r})R_1(r)\chi \end{pmatrix}, \qquad (41)$$

and where χ is a 2-component spinor. The large and small components of the wave function are normalized such that $\langle R_0^2 \rangle + \langle R_1^2 \rangle = 1$. It has been argued [17] that exchange terms other than the simple $(ij) = (36)$ term may become negligible in the relativistic limit. For $\langle R_1^2 \rangle \approx \langle R_0^2 \rangle$ this would be true. Note, however, that the factor of eq. (40) is of order $(1 - 2\langle R_1^2 \rangle)$. In the MIT bag model, with quarks of zero rest mass, $\langle R_1^2 \rangle = 0.26$, so that this factor would be $\sim \frac{1}{2}$, and it is therefore doubtful whether the exchange terms of type $(ij) = (25)$ can be neglected. Note also that the antisymmetric spin–orbit coupling arises solely from exchange terms of type $(ij) = (26)$ and (56) and is therefore also neglected in ref. [14]. Although a nonrelativistic RGM treatment may be open to question, it does give a strong indication that a proper inclusion of all quark–gluon exchange terms is required in a relativistic treatment of the spin–orbit interaction.

3.2. THE N-HYPERON SPIN–ORBIT INTERACTION

The spin–orbit potential of Λ- and Σ-hypernuclei has become a subject of some recent interest. The nonrelativistic model of the present investigation lies closest to a model in which mass differences between strange and u- and d-quarks can be neglected; that is, to a model of good SU(3)-flavor symmetry. In this model, also, N, Λ, and Σ are to be considered as different states of the $S = \frac{1}{2}$ baryon. To extend the investigation of the spin–orbit interaction to NΛ and NΣ states, it will be convenient to transform the symmetrically (antisymmetrically) spin–isospin coupled states of eq. (3) to spin, SU(3)-flavor-coupled states of the same symmetry with the use of simple SU(3) flavor $\supset$ SU(2) isospin $\times$ U(1) hypercharge Wigner coefficients. E.g., for NΛ states coupled to $S = 1$

$$\sqrt{\tfrac{1}{2}}\{[\phi_N(123) \times \phi_\Lambda(456)]_{S=1,T=1/2} + [\phi_\Lambda(123) \times \phi_N(456)]_{S=1,T=1/2}\}$$
$$= 3\sqrt{\tfrac{1}{10}}[\phi^{(11)} \times \phi^{(11)}]^{(22)}_{S=1,Y=1,T=1/2} + \sqrt{\tfrac{1}{10}}[\phi^{(11)} \times \phi^{(11)}]^{(11)\rho=2}_{S=1,Y=1,T=1/2}, \quad (42)$$

while

$$\sqrt{\tfrac{1}{2}}\{[\phi_N(123) \times \phi_\Lambda(456)]_{S=1,T=1/2} - [\phi_\Lambda(123) \times \phi_N(456)]_{S=1,T=1/2}\}$$
$$= -\sqrt{\tfrac{1}{2}}[\phi^{(11)} \times \phi^{(11)}]^{(03)}_{S=1,Y=1,T=1/2} + \sqrt{\tfrac{1}{2}}[\phi^{(11)} \times \phi^{(11)}]^{(11)\rho=1}_{S=1,Y=1,T=1/2}. \quad (43)$$

On the left side of eqs. (42)–(43) the square bracket denotes spin and isospin coupling; on the right side it denotes spin and SU(3)-flavor coupling. The notation $(\lambda\mu)$ is used for the SU(3)-flavor states $((\lambda\mu) = (11), (22),$ and (03) correspond to the 8, 27, and $\overline{10}$ flavor representations, while $\rho = 1$ (and $\rho = 2$) designate the antisymmetric or F-coupling (and the symmetric or D-coupling) of the two 8-dimensional representations to resultant dibaryon 8-dimensional representation with $(\lambda\mu) = (11)$.) Note that the symmetrically coupled spin-flavor states of eq. (42) contain only dibaryon states with $(\lambda\mu) = (22)$ and (11) $\rho = 2$ (D-coupling); (the 1-dimensional representation $(\lambda\mu) = (00)$ cannot accommodate the hypercharge

TABLE 4

Spin-reduced matrix elements $\langle\alpha ST\|(\sigma_i+\sigma_j)P_{36}^{S,f}\|\beta S'T\rangle$ for N-hyperon systems

(A) Symmetric combinations			(B) Antisymmetric combinations		
α/β $(ST)\,(S'T)$	$(ij)=(25)$	$(ij)=(36)$	α/β $(ST)\,(S'T)$	$(ij)=(25)$	$(ij)=(36)$
(1) NΛ/NΛ			(1) NΛ/NΛ		
$(0\tfrac{1}{2})(1\tfrac{1}{2})$	$\sqrt{3}/18$	$-\sqrt{3}/9$	$(0\tfrac{1}{2})(1\tfrac{1}{2})$	$\sqrt{3}/18$	$-\sqrt{3}/9$
$(1\tfrac{1}{2})(1\tfrac{1}{2})$	$\sqrt{6}/9$	$4\sqrt{6}/9$	$(1\tfrac{1}{2})(1\tfrac{1}{2})$	$\sqrt{6}/9$	$-2\sqrt{6}/9$
(2) NΣ/NΣ			(2) NΣ/NΣ		
$(0\tfrac{1}{2})(1\tfrac{1}{2})$	$-\sqrt{3}/6$	$\sqrt{3}/3$	$(0\tfrac{1}{2})(1\tfrac{1}{2})$	$-\sqrt{3}/6$	$\sqrt{3}/3$
$(1\tfrac{1}{2})(1\tfrac{1}{2})$	$\sqrt{6}/81$	$-4\sqrt{6}/27$	$(1\tfrac{1}{2})(1\tfrac{1}{2})$	$\sqrt{6}/9$	$-2\sqrt{6}/9$
$(0\tfrac{3}{2})(1\tfrac{3}{2})$	0	0	$(0\tfrac{3}{2})(1\tfrac{3}{2})$	0	0
$(1\tfrac{3}{2})(1\tfrac{3}{2})$	$10\sqrt{6}/81$	$14\sqrt{6}/27$	$(1\tfrac{3}{2})(1\tfrac{3}{2})$	$-2\sqrt{6}/27$	$2\sqrt{6}/3$

Under bra/ket interchange multiply by $(-1)^{S+S'}$.

$Y=1$, isospin $T=\tfrac{1}{2}$ quantum numbers). Similarly, the antisymmetrically coupled spin–flavor states of eq. (43) contain only dibaryon states with $(\lambda\mu)=(03)$ and (11) $\rho=1$ (F-coupling); $((\lambda\mu)=(30)$ is again missing because of the restriction $Y=1$, $T=\tfrac{1}{2})$.

Once the N-hyperon states have been converted to the spin, SU(3) (flavor)-coupled form, the SU(2)×SU(2) spin–isospin arithmetic [22]) used to calculate the spin-reduced matrix elements of $(\sigma_i\pm\sigma_j)P_{36}^{ST}$ can be extended to the analogous SU(2)×SU(3) spin–flavor arithmetic to calculate the spin-reduced matrix elements of $(\sigma_i\pm\sigma_j)P_{36}^{S,f}$, where $P_{36}^{S,f}$ now acts on the spin–flavor degrees of freedom of quarks 3 and 6. Results for the NΛ and NΣ states are shown in table 4. The second entry of this table shows, e.g., that the symmetrically coupled spin–flavor combination for NΛ with $S=1$, (eq. (42)), leads to values of $\sigma_{\mathrm{eff}}^{(25)}$ and $\sigma_{\mathrm{eff}}^{(36)}$ which are $\tfrac{9}{10}$ and $\tfrac{12}{14}$ of the corresponding triplet-odd NN states with $(ST)=(11)$. In a model of good SU(3)-flavor symmetry therefore, the NΛ spin–orbit interaction would be expected to be only slightly weaker than the NN spin–orbit interaction. This is shown in more detail in fig. 7.

Fig. 7a compares the NΛ and NΣ spin–orbit interaction for the symmetric spin–flavor states coupled to $S=1$ with the corresponding triplet-odd NN states with $(ST)=(11)$, while fig. 7b makes the comparison of the antisymmetric spin–flavor NΛ and NΣ states with $S=1$ with the corresponding triplet–even NN states with $(ST)=(10)$. The interaction for the NΛ states is somewhat weaker than that for the corresponding NN states. For the NΣ states with $S=1$ the interaction strength for one of the two possible T-values is identical (or nearly equal) with that for the corresponding NN state. For the other T-value it has the opposite sign for the larger R-values (or for the whole range of R-values). The strength of the NΣ
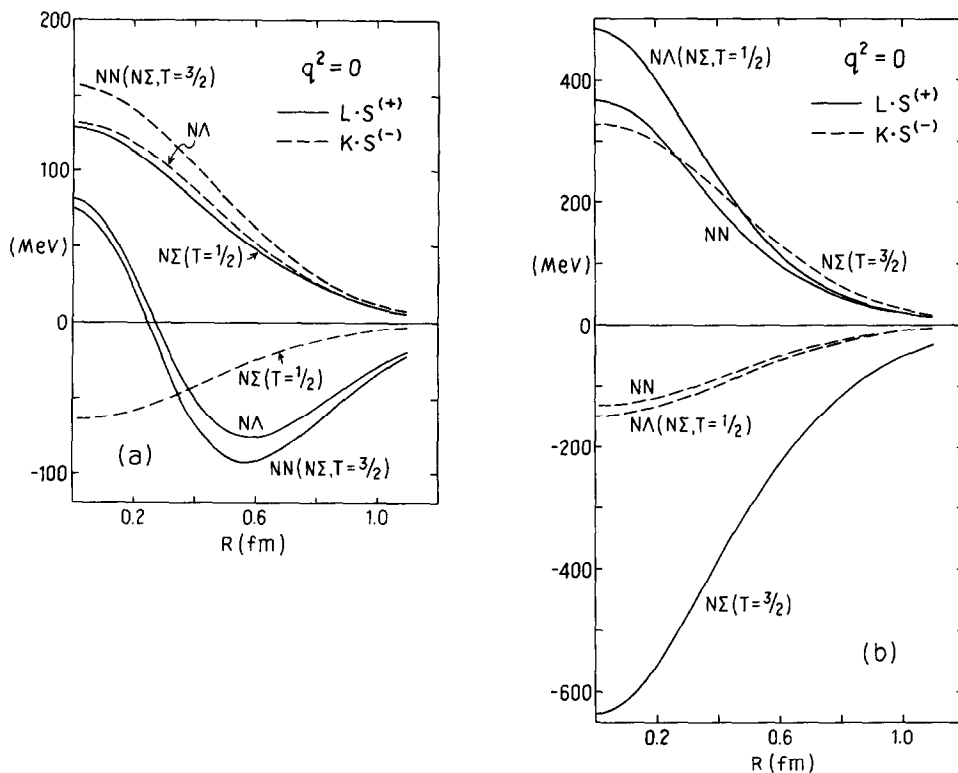
Fig. 7. Comparison of NN and NΛ, NΣ spin–orbit potentials, for triplet states, $S = 1$: (a) is for symmetric ($a =$ even), (b) for antisymmetric ($a =$ odd) dibaryon spin–flavor states. Contributions for symmetric spin–orbit terms (solid lines) and antisymmetric spin–orbit terms (dashed lines) are shown separately. The strength parameter (α_s/m^2b^3) is that of the OOAY potential of fig. 2.

spin–orbit interaction can thus be expected to be weaker than the NN spin–orbit interaction. It is interesting to note that this result, valid for a model of good SU(3)-flavor symmetry, is quite different from the predictions of ref. [14]) which takes into account only one of the possible exchange diagrams of fig. 1. Ref. [14]) is based on a model in which the N-hyperon spin–orbit interaction arises from interactions among nonstrange quarks only. In this model of strongly broken SU(3)-flavor symmetry the ΛN spin–orbit interaction vanishes, in seeming agreement with experimental results [27]), whereas the Σ-nucleus spin–orbit potential is predicted to be stronger than the N-nucleus spin–orbit potential by a factor of $\frac{4}{3}$ [refs. [14,26])]. Since some vestige of SU(3)-flavor symmetry must be expected to be present in a realistic model, the predictions from quark–gluon exchange mechanisms must certainly be expected to lie somewhere between these two extremes. If the Λ-N mass difference is incorporated into the model so that Λ and N become distinguishable particles, the NΛ states can become linear combinations of the spin–flavor symmetric and

antisymmetric states. Since the NΛ potentials of figs. 7b and 7a have the opposite sign this can lead to a reduction of the NΛ spin–orbit strength. The NΣ potentials of figs. 7b and 7a have the same sign, so that this effect would strengthen the NΣ spin–orbit potentials, therefore going toward the experimental results.

4. Summary

Quark-exchange kernels have been constructed in very explicit analytic form for the tensor and spin–orbit terms of the one-gluon-exchange quark–quark Breit interaction and for spin–orbit terms generated by quark-confinement mechanisms. The effective vector and tensor spin operators needed for these kernels are defined through their spin-reduced matrix elements which have been tabulated in tables 1, 2, and 4. Some emphasis is given to the coupling of NN to NΔ and $\Delta\Delta$ channels for which short-range tensor and spin–orbit coupling potentials may be of particular relevance [10-13]. For a full discussion of potentials coupling the NΔ to NN or $\Delta\Delta$ channels the earlier tabulations [8] of the spin–isospin factors of $(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$-dependent central terms have to be extended to include the NΔ channels. These factors are therefore included through table 5.

The very explicit results of the nonrelativistic RGM formalism have made it possible to give a quantitative analysis of the dibaryon spin–orbit interaction arising from quark exchange kernels. Effective spin–orbit potentials are generated through the Wigner transforms of these kernels using a local momentum approximation. This has made it possible to study the relative importance of various spin–orbit terms in the quark–quark interaction and the relative importance of the various possible quark–gluon exchange terms. If only the symmetric spin–orbit term of the one-gluon-exchange quark–quark interaction is retained, the spin–orbit interaction derived from the corresponding RGM quark-exchange kernel has a sign and magnitude in the 0.5–0.8 fm range not in disagreement with the short-range part of phenomenological potentials derived from NN-scattering data. With the inclusion of the antisymmetric spin–orbit one-gluon-exchange terms and spin–orbit terms generated by confining potentials, the strength of the full triplet-odd NN spin–orbit potential is greatly reduced and its sign may even be reversed, the uncertainties in the magnitude of this effect being related to uncertainties in the strengths of the quark-confinement potential constants. The results of this investigation of the baryon–baryon spin–orbit interaction are therefore consistent with studies of P-wave baryons [20] where similar cancellations have been found in the three-quark matrix elements of the full spin–orbit interaction. The uncertainties associated with spin–orbit terms generated by quark-confinement mechanisms are emphasized by an alternate model in which confinement is built into the quark model through a $\mathrm{d}m'/\mathrm{d}r$ term (a mass which rises to infinity outside the quark bag). The single-particle Thomas terms of such a model generate spin–orbit terms in the NN interaction through the quark-exchange mechanism, and it is shown that such a model of

TABLE 5

The spin–isospin coefficients $C_{ST}^{(i)} = \langle \alpha ST | \mathcal{O}^{(i)} P_{36}^{ST} | \beta ST \rangle$

α/β (ST)	$C_{ST}^{(0)}$	$C_{ST}^{(1)}$	$C_{ST}^{(2)}$	$C_{ST}^{(3)}$	$C_{ST}^{(4)}$	$C_{ST}^{(5)}$
(A) Symmetric combinations						
(1) NN/NΔ						
(11)	40/81	−32/81	−44/81	16/81	4/81	−40/81
(2) NΔ/NΔ						
(11)	31/81	22/81	−11/81	−11/81	−29/81	5/81
(12)	−1/9	2/9	−1/9	−1/9	−1/9	13/9
(21)	−1/9	2/9	−1/9	−1/9	−1/9	5/9
(22)	7/9	−2/9	1/9	1/9	1/3	5/9
(3) NΔ/$\Delta\Delta$						
(11)	20/81	20/81	−10/81	20/81	−40/81	−20/81
(22)	4/9	4/9	−2/9	4/9	0	−4/9
(B) Antisymmetric combinations						
(4) NΔ/NΔ						
(11)	1/3	2/9	−1/9	−1/9	−1/3	1/9
(12)	1/27	10/27	−5/27	−5/27	−5/27	35/27
(21)	1/27	10/27	−5/27	−5/27	1/9	11/27
(22)	1/3	−2/3	1/3	1/3	−1/3	1
(5) NΔ/$\Delta\Delta$						
(12)	$4\sqrt{5}/27$	$4\sqrt{5}/27$	$-2\sqrt{5}/27$	$4\sqrt{5}/27$	$-8\sqrt{5}/27$	$-4\sqrt{5}/27$
(21)	$4\sqrt{5}/27$	$4\sqrt{5}/27$	$-2\sqrt{5}/27$	$4\sqrt{5}/27$	0	$-4\sqrt{5}/27$

$\mathcal{O}^{(0)} = 1$; $\mathcal{O}^{(1)} = (\boldsymbol{\sigma}_4 \cdot \boldsymbol{\sigma}_5)$; $\mathcal{O}^{(2)} = (\boldsymbol{\sigma}_5 \cdot \boldsymbol{\sigma}_6)$; $\mathcal{O}^{(3)} = (\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_6)$; $\mathcal{O}^{(4)} = (\boldsymbol{\sigma}_2 \cdot \boldsymbol{\sigma}_5)$; $\mathcal{O}^{(5)} = (\boldsymbol{\sigma}_3 \cdot \boldsymbol{\sigma}_6)$.

For α/β = NN/NN, $\Delta\Delta/\Delta\Delta$, NN/$\Delta\Delta$ see table 2 of ref. [8]).

For bra/ket interchange $C_{ST}^{(2)}$ and $C_{ST}^{(3)}$ must be interchanged; all others are invariant to bra/ket interchange.

confinement leads to spin–orbit terms of the opposite sign from those generated by a confining potential derived from a scalar coupling.

The nonrelativistic RGM quark-exchange kernels gain contributions from four types of quark–gluon exchange terms. In one of these (type (5) of fig. 1) only a single quark pair participates in the quark–gluon exchange process. The remaining three types of exchange terms involve either three or four of the quarks of the six-quark system. Some recent relativistic treatments [14,15,28]) are based on exchange mechanisms of the first kind in which only a single quark pair participates. This exchange term in the RGM formalism by itself leads to an attractive spin–orbit interaction in the triplet–odd NN channel with a strength which ranges from ~400 to 100 MeV in the 0 to 0.8 fm range. It should be noted, however, that the remaining three types of exchange terms all give important contributions of the opposite sign.

Relativistic models in which such exchange terms are neglected are therefore open to question.

The quark-exchange kernels generated in the RGM formalism can give very specific answers regarding the nature of the short-range spin–orbit and tensor parts of the baryon–baryon interaction. To remove the remaining uncertainties a better description of quark confinement is clearly needed.

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