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RESEARCH NOTE

THE COLOR-MATCHING FUNCTIONS FOR $\lambda \geq 540$ nm: INCOMPATIBILITY WITH THE BLUE FUNDAMENTAL

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Abstract—The question of the blue fundamental contribution to foveal color matches for $\lambda \geq 540$ nm is examined. It is shown that, with two almost universally accepted assumptions, such a blue fundamental contribution leads to a contradiction with the observed color-matching functions. It is concluded that the blue fundamental does not contribute to foveal color matches for $\lambda \geq 540$ nm.

Psychophysics Foveal vision Color vision Color matching

Does the blue fundamental contribute to foveal color matches for $1/\lambda \leq 18{,}500\ \text{cm}^{-1}$ ($\lambda \geq 540$ nm)? Given the commonly held assumption that the spectrum locus is a straight line for $1/\lambda \leq 18{,}500\ \text{cm}^{-1}$, color matching is necessarily dichromatic in that spectral region (Wyszecki and Stiles, 1982, Section 3.2); the usual inference drawn from this presumed dichromacy is that the blue fundamental does *not* contribute to color matches for $1/\lambda \leq 18{,}500\ \text{cm}^{-1}$.

However, the assumption that the spectrum locus is a straight line for $1/\lambda \leq 18{,}500\ \text{cm}^{-1}$ appears unsupportable: Estevez (1979) has shown that both the Stiles and Burch (1955) and nonsmoothed values of the CIE (1931) [average of the original Guild (1931) and Wright (1928–1929) data] chromaticities appear to be straight lines only between about 560 and 600 nm ($\sim$ 17,750–16,500 cm^{-1}). Deviations in other spectral regions are both systematic and marked, as can be seen by the plot of the Stiles and Burch (1955) b and r chromaticities given in Fig. 1.

The presumed linearity of the spectrum locus thus remains at best questionable, and hence there appears

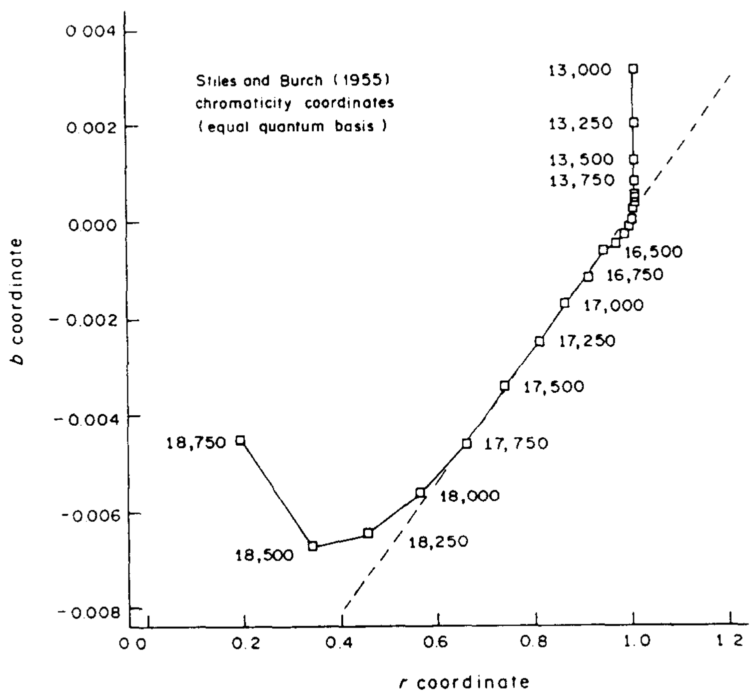

Fig. 1. Stiles and Burch (1955) $b(1/\lambda)$ and $r(1/\lambda)$ chromaticity coordinates for $18{,}750 \geq 1/\lambda \geq 13{,}000\ \text{cm}^{-1}$, as corrected and renormalized by Wyszecki and Stiles (1982, Table 5.5.3), and converted to an equal-quantum basis. The dashed line is fit to the linear portion of the curve from 17,750 to 16,500 cm^{-1}.

to be no basis for concluding that the blue fundamental does not participate in foveal color matches for $1/\lambda \leq 18{,}500\ \text{cm}^{-1}$. While the rate of photon absorption in the blue fundamental will be relatively low at $18{,}500\ \text{cm}^{-1}$ compared to that for the red and green fundamentals, that rate will certainly be non-zero, particularly at higher (but not significantly bleaching) intensities, whereas owing to the invariance of color matches, the color-matching functions (CMF) remain essentially unchanged [Alpern (1979); Wyszecki and Stiles (1980)]. One must *assume* a "threshold-nonlinearity" in the color-matching process, i.e. that the number of photons caught do not have to be equated for the blue fundamental for $1/\lambda \leq 18{,}500\ \text{cm}^{-1}$, because that number is too small to elicit a detectable signal.

However, a *proof* of such lack of involvement of the blue fundamental for $1/\lambda \leq 18{,}500\ \text{cm}^{-1}$ can be made that is independent of the straightness of the spectrum locus and also of the shape of the blue fundamental for $1/\lambda \leq 18{,}500\ \text{cm}^{-1}$. The proof is by contradiction; the assumptions are that:

1. Foveal color matches result from the equation of the rate of photon absorptions within 3 fundamentals. A fundamental may refer only to the response arising from photon absorptions in a distinct, homogeneous photopigment or, if that photopigment is assumed to be isolated in a separate cone class, to that cone class as well.

2. When plotted on a log sensitivity vs wavenumber basis, the log sensitivity of the blue fundamental declines in a roughly linear manner by more than 0.5 log units over the range $18{,}500 \geq 1/\lambda > 16{,}750\ \text{cm}^{-1}$ (540.5–597.0 nm).

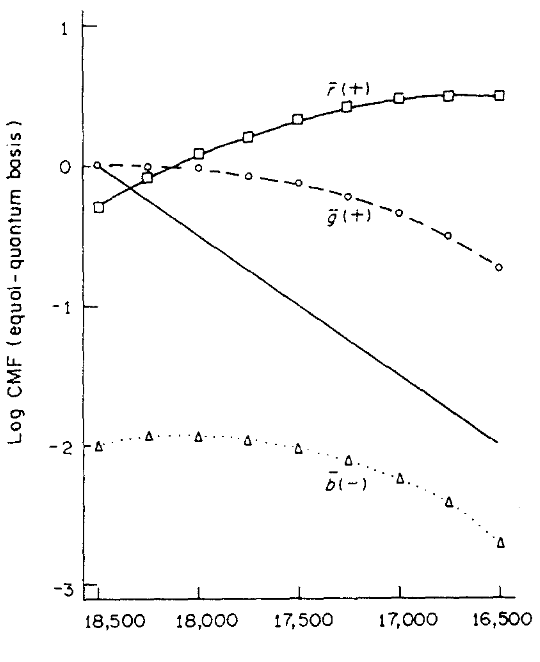

Fig. 2. Stiles and Burch (1955) log $|\bar{r}(1/\lambda)|$, $|\bar{g}(1/\lambda)|$ and $|\bar{b}(1/\lambda)|$ color-matching functions (CMF) for $18{,}500 \geq 1/\lambda \geq 16{,}500\ \text{cm}^{-1}$, corrected and converted as for Fig. 1. + or − symbol gives the sign of the respective $\bar{r}$, $\bar{g}$ and $\bar{b}$.

Assumption 1 is the almost universally accepted basis of Grassmann's laws (Wyszecki and Stiles, 1982, pp. 587–588; Brindley, 1970, pp. 222–223). Current knowledge of photopigment and receptor sensitivities indicates that assumption 2 should also be easily satisfied: e.g. a fundamental with peak wavenumber at approximately $23{,}500\ \text{cm}^{-1}$ and the spectral-sensitivity shape of either (a) Stiles' π_3 mechanism, (b) the Dartnall nomogram, (c) the relative rod response, i.e. the quantized CIE $V'(\lambda)$ corrected for lens absorption, or (d) the red limb of the cone curve (derived from color matching; Wyszecki and Stiles, 1982, p. 599), have at $18{,}500\ \text{cm}^{-1}$ all reached their limiting gradients of 1.0, 1.05, 1.44, and 1.726 log units/$1000\ \text{cm}^{-1}$, respectively (Wyszecki and Stiles, 1982, pp. 533, 600). These gradients predict declines much greater than 0.5 log units (1.75, 1.84, 2.52 and 3.02 log units, respectively) between 18,500 and $16{,}750\ \text{cm}^{-1}$. The gradient of 1.0 is the slope of the straight line plotted in Fig. 2.

Plotted in Fig. 2 are the CMF for the range $18{,}500 \geq 1/\lambda \geq 16{,}500\ \text{cm}^{-1}$ of Stiles and Burch (1955), as modified slightly and renormalized by Wyszecki and Stiles (1982, p. 334) and transformed to an equal-quantum basis (Pugh and Sigel, 1978, p. 319). The Stiles and Burch (1955) CMF are preferred for their precision and lack of dependence on heterochromatic brightness matching (Stiles, 1955; Estevez, 1979), but these CMF must be only very approximately correct for the argument given below to hold. Exactly the same argument as given below can be made for the 1931 CIE and Judd-modified CIE CMF—cf. Stiles and Burch (1955, p. 179).

Assumptions 1 and 2 yield that if

$$S(1/\lambda) = a_1\bar{r}(1/\lambda) + a_2\bar{g}(1/\lambda) + a_3\bar{b}(1/\lambda),$$

then

$$\log S(18{,}500) - \log S(16{,}750) > 0.5 \qquad (1)$$

where $1/\lambda$ is wavenumber, $S(1/\lambda)$ is the spectral sensitivity of the blue fundamental (quantum basis), $\bar{r}(1/\lambda)$, $\bar{g}(1/\lambda)$ and $\bar{b}(1/\lambda)$ are the CMF (Fig. 2) and a_1, a_2 and a_3 are constants. Equation (1) is simply an expression of the equation of the rate of photon absorptions in the blue fundamental for both sides of the matching field; a_1, a_2, a_3 are simply the sensitivities of the blue fundamental at the respective red, green and blue primary wavenumbers. Note that a_1, a_2, a_3 are >0 since $S(1/\lambda) > 0$ for all λ.

Examining the CMF in Fig. 2, first note that $\bar{r}(1/\lambda)$ is increasing in this region as $1/\lambda$ decreases. Since $a_1 > 0$, the effect of the $\bar{r}$ term in equation (1) is in a direction opposite to that needed to fit the >0.5 log unit sensitivity decline of the blue fundamental. Thus a_1 should be made as small as possible.

With essentially no contribution from $\bar{r}$, it is only necessary to determine if coefficients a_2 and a_3 can be found for $\bar{g}$ and $\bar{b}$ in equation (1). Now $\bar{b}$ does decrease (nearly monotonically) in its absolute value between 18,500 and 16,750 cm^{-1}, but $\bar{b}$ is negative in that region of the spectrum. Since a_3 must be positive, any contribution from $a_3\bar{b}$ in equation (1) would cause a relative *increase* in sensitivity, a change in the same direction as that for $\bar{r}$, and one in the wrong direction.

This leaves the $\bar{g}$ term in equation (1), which is positive and decreasing from 18,500 to 16,750 cm^{-1}. This change is the only change in the correct direction (decreasing sensitivity), but is less than 0.5 log units. Thus there cannot exist a_1, a_2, a_3 such that equation (1) holds, and assumptions 1 and 2 are inconsistent. Most importantly, note that the decrease of 0.5 log units for $\bar{g}$ is very gradual and spread over the entire range of 18,500–16,750 cm^{-1}: *no* range of wavenumbers within the bounds of 18,500–16,750 cm^{-1} will exhibit the required decline of sensitivity.

Without accepting a very unlikely photopigment sensitivity (rejection of assumption 2), or without rejecting the photopigment basis of foveal color matching (assumption 1), the alternative is a modification of assumption 1 which allows for the "threshold nonlinearities" of the blue fundamental as discussed above: the blue fundamental does not participate in foveal color matches for $18{,}500 \geq 1/\lambda \geq 16{,}750$ cm^{-1} because the photons caught by the blue fundamental in that spectral region are too few in number. It then follows that, since the photons caught are too few in number for $18{,}500 \geq 1/\lambda \geq 16{,}750$ cm^{-1} and even fewer photons will be caught for $1/\lambda < 16{,}750$ cm^{-1}, the blue fundamental will not contribute to foveal color matches for all $1/\lambda \leq 18{,}500$ cm^{-1}.

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