SUPERSYMMETRIC CONTRIBUTIONS TO THE $\epsilon'$ AND $\epsilon$ CP VIOLATING PARAMETERS

J.-M. FRÈRE

Randall Laboratory of Physics, University of Michigan, Ann Arbor, MI 48109, USA

and

M. Belén GAVELA

Physics Department, Brandeis University, Waltham, MA 02254, USA

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We consider the contributions to $\epsilon$, $\epsilon'/\epsilon$ and the electric dipole moments due to flavor blind mechanisms of CP violation, found in supersymmetric models.

Many mechanisms have been proposed to account for one piece of experimental data, namely CP violation in the K$^0$-K$^0$ system. Most of these mechanisms, like the Lee-Weinberg [1] or the Kobayashi-Maskawa [2] models, can also be incorporated in supersymmetric extensions of the "standard model". Such supersymmetric extensions, however, present us with new sources of CP violation, whose origin is to be found in the exchange of gauge fermions, both charged and neutral [3,4].

In this paper we concentrate on such new sources and investigate the possibility that they be the only cause of CP violation. To achieve consistency with experiments, this not only requires us to be able to account for the measured value of the parameter $\epsilon$, \[ \epsilon = (1.62 \pm 0.09) \times 10^{-3} \] but also to satisfy the present experimental bounds which affect other CP violating quantities such as $\epsilon'/\epsilon$, (which measures the departure from pure $\Delta S = 2$ CP violation), and the various electric dipole moments $d_e^{(e)}$ [3], $d_e^{(n)}$ [3,4], $d_\mu^{(e)}$ ... . Note that a contribution to $d_e^{(n)}$ also arises from the "strong CP" $\theta$ parameter, to which radiative corrections have been examined in ref. [5] in a supersymmetric framework.

We will consider models allowing for explicit soft breaking. This may either be seen as a fundamental characteristic, or as a low-energy approximation. To be more specific, we shall consider such soft breaking terms as are induced by supergravity in the presence of spontaneous supersymmetry breaking in a hidden sector [6].

Such soft breaking terms are found to be of the type:

\[ m_g^2 \phi_i^2 \;, \quad Am_g \Phi (\{ \phi \}) \theta = 0 \;,
\]

where $g$ is the superpotential of the chiral superfields $\Phi_i(x, \theta)$, of which $\phi_i$ are the scalar components. $A$ is a numerical constant, while $m_g$ stands for the "gravitino" mass. (If no dimensional parameter is allowed, $g$ is trilinear in the fields, otherwise, bilinear terms will also be included.)

In addition to the above, mass terms for the Majorana partners of the gauge bosons appear, at least as radiative corrections. In order to establish the notation, we recall the minimal particle content of a supersymmetric extension of the "standard" [SU(2)$_L$ X U(1)] model.

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Standard model scalars:

\[
\begin{pmatrix}
H_1^+ \\
H_1^0
\end{pmatrix},
\begin{pmatrix}
H_2^0 \\
H_2^-
\end{pmatrix}.
\]

Their fermionic partners:

\[
\begin{pmatrix}
\tilde{H}_1^+ \\
\tilde{H}_1^0
\end{pmatrix},
\begin{pmatrix}
\tilde{H}_2^0 \\
\tilde{H}_2^-
\end{pmatrix}.
\]

Standard model fermions:

\[
\begin{pmatrix}
u_L^{(2/3)} \\
d_L^{(-1/3)}
\end{pmatrix},
\begin{pmatrix}
u_L^{(-2/3)} \\
d_L^{(1/3)}
\end{pmatrix}.
\]

Their scalar partners:

\[
\tilde{u}_L, \tilde{d}_L, \tilde{u}_L, \tilde{d}_L.
\]

To which we must add the gauge bosons \(W^\pm, W^0, B\) and their fermionic counterparts \(\tilde{W}^-_L, \tilde{W}^+_L, \tilde{W}^0, \tilde{B}\). We have mentioned explicitly the charge of the particle, using only left-handed fermions this far; in the case of the bosons \(\tilde{u}, \tilde{d}\), the index \(L\) simply keeps track of the chirality of the corresponding superfield.

We will first consider the charged particle exchanges. The mass matrix for the charged 2 components \(\tilde{W}^+\) and \(\tilde{H}^+\) reads:

\[
\begin{pmatrix}
\mu^+ \\
\gamma^+
\end{pmatrix} \begin{pmatrix}
\tilde{H}^+ \\
\tilde{H}^0
\end{pmatrix} + \text{h.c.,}
\]

\[
M = \begin{pmatrix}
\mu^+ & g v^*/\sqrt{2} \\
g v^*/\sqrt{2} & m
\end{pmatrix},
\]

(1)

Here \(m\) stems either from an explicit \(m H_1 H_2\) coupling in the potential, or from the vacuum expectation value of a suitable singlet. In both cases, the effective scalar potential contains an effective contribution: \(A m m H_1 H_2\).

\(\mu\) represents the self-mass of the wino's. This is, in general, found to be logarithmically divergent in the present context; depending upon one's prejudices, such divergences will either be seen as a need for renormalization, leaving the value arbitrary, or as a consequence of the sensitivity of \(\mu\) to the higher scales of some cut-off theory.

What is essential for us is that the phases of \(\mu\) and \(m\) are therefore in general unrelated to those of \(v_1\) and \(v_2\).

We now want to discuss those phases in some detail.

Since we are mainly interested in new sources of CP violation, we will assume the K-M matrix to be real. If \(\lambda_u\) and \(\lambda_d\) represent the "generic" Yukawa couplings of all the up and down quarks respectively, ("generation" indices are left implicit) we have:

\[
\lambda_u v_1 \text{ real}, \quad \lambda_d v_2 \text{ real}.\]

(2)

Let us set:

\[
\rho^2 = A m m^* = |\rho|^2 e^{i\phi}, \quad m = |m| e^{ix}.
\]

(3, 4)

Due to the presence of the above mentioned \(\rho^2 H_1 H_2\) term, the minimization of the Higgs potential then brings:

\[
v_1 v_2 = |v_1 v_2| \exp[-i\phi + (2n + 1)\pi]
\]

(5)

(Note that \(v_1\) and \(v_2\) are not necessarily relatively real: this results from the presence of explicit \(T\) violating terms in the effective lagrangian.) The matrix (1) is in general not hermitian, and a biunitary transformation is required to diagonalize it. In particular, the phases appearing in (1) cannot be simply absorbed in redefinitions of \(\tilde{W}_\pm, \tilde{H}_\pm\),
[unless arg(ν2) + arg(ν1) = arg(μ+ -) + arg(m)]. We therefore write:

\[
\begin{pmatrix}
\bar{f}_1 - \\
\bar{f}_2 -
\end{pmatrix} = U_+ \begin{pmatrix} \bar{i} \bar{W}_- \\
\bar{H}_2-
\end{pmatrix}, \quad \begin{pmatrix}
\bar{f}_1 + \\
\bar{f}_2 +
\end{pmatrix} = U_+ \begin{pmatrix} -i \bar{W}_+ \\
\bar{H}_1+
\end{pmatrix}.
\]

Independent unitary 2 × 2 matrices \(U_+\) and \(U_-\) are used in general (we cannot impose an unimodularity condition):

\[U_+ = e^{i \epsilon +} \begin{pmatrix} e^{i \chi_+} & -e^{i \phi_+} \\
e^{-i \chi_+} & e^{i \phi_+}
\end{pmatrix} \begin{pmatrix} c_{\theta_+} & -s_{\theta_+} \\
s_{\theta_+} & c_{\theta_+}
\end{pmatrix} \begin{pmatrix} e^{i \phi_+} \\
e^{-i \phi_+}
\end{pmatrix}.
\]

These angles are determined from \(M\) in eq. (1):

\[M = e^{i (\epsilon - \xi +)} \begin{pmatrix} e^{-i \phi_+} \\
e^{i \phi_+}
\end{pmatrix}
\]

\[
\begin{pmatrix}
-m_1 c_+ e^{i (x_- - x_+)} + m_2 s_+ e^{-i (x_- - x_+)} & -m_1 c_+ e^{-i (x_- - x_+)} + m_2 s_+ e^{i (x_- - x_+)} \\
-m_1 c_+ e^{-i (x_- - x_+)} + m_2 s_+ e^{i (x_- - x_+)} & -m_1 c_+ e^{i (x_- - x_+)} + m_2 s_+ e^{-i (x_- - x_+)}
\end{pmatrix}
\]

(7)

(where we have used the shorthand \(c_+ = \cos \theta_+, \ldots\)). Since \(\chi_+\)'s and \(\xi_+\)'s only enter (7) through the differences, \(\chi_- = x_+\) and \(\xi_- = \xi_+\), we may without loss of generality set \(x_+ = \xi_+ = 0\). This reduces the number of independent phases in (6) to 4, in agreement with (1). The relevant Yukawa couplings now read:

\[g \tilde{u} \tilde{d} (c \theta_+ R f_1 + s \theta_+ R f_2) e^{-i \phi_+} + \lambda_u^* \tilde{u} \tilde{d} (s \theta_+ R f_1 + c \theta_+ R f_2) e^{i \phi_+} \]

\[= \lambda_d \tilde{u} \tilde{d} (-s \theta_+ e^{-i \chi_+} - L f_1 + c \theta_+ e^{-i \chi_-} - L f_2) e^{i \phi_+} e^{-i \phi_+} + h.c.
\]

where we have used the usual four-component notation for the quark fields, and

\[f_1 = \begin{pmatrix} f_1 - \\
f_1 +
\end{pmatrix}; \quad L = \frac{1}{2} (1 - \gamma_5), \quad R = \frac{1}{2} (1 + \gamma_5).
\]

The effective lagrangian also contains the couplings:

\[-A \lambda_u m_1 \tilde{U} \tilde{Q}^\text{up}_L v_1/\sqrt{2} + A m_2 \tilde{D} \tilde{Q}^\text{down}_L (v_2/\sqrt{2}) \lambda_d.
\]

(9)

Once again, generation indices are implicit.) Since these terms are supposed to be small compared to the mass of the scalar partners of the quarks, we will treat them perturbatively, instead of diagonalizing the scalar–pseudoscalar mass matrix.

The first graph we consider corresponds to fig. 1.1 and may be cast in the form:

\[\langle \bar{K}^0 | H | K^0 \rangle \big|_{1,1} = g \lambda S \left[ m_1 (-s \theta_+ c \theta_+ e^{i \chi_+}) F(m_1) + m_2 s \theta_+ c \theta_+ e^{-i \phi_+} F(m_2) \right] e^{-i (\phi_+ - \phi_+)} e^{i \xi_+}.
\]

(10)

If the fermions \(f_1\) and \(f_2\) were degenerate, (or if we could neglect \(m_1\) and \(m_2\) in \(F\), which we know is experimentally unjustified, since neither \(m_1\) nor \(m_2\) is negligible compared to the relevant momenta inside the loop), we would have:

\[\langle \bar{K}^0 | H | K^0 \rangle \big|_{1,1, m_1 = m_2} = g \lambda S (g v_2/\sqrt{2}) F = m_2 g^2.
\]

(11)

There is however no compelling reason to impose \(m_1 = m_2\). In the following, we will assume \(m_1 \ll m_2\), and neglect the \(m_2\) contribution.

One more question arises at this stage: how different is this from other “global” phase models, e.g. the Lee–
Weinberg case. As shown explicitly in (5), \( v_1 \) and \( v_2^* \) are in general relatively complex, as in the Lee–Weinberg model; and therefore the contribution (10) has to be added to the usual one using scalar exchanges. However, even if we set \( \phi = 0 \) in eq. (5) [i.e., \( v_1 \) and \( v_2^* \) relatively real] we still get a non-trivial \( CP \) violating contribution from (10). The statement that \( \phi = 0 \) indeed only amounts [in the language of eq. (7)] to setting:

\[
e^{2i\alpha}(-m_1^2 \theta_+ e^{i\chi-} + m_2^2 \theta_+ e^{-i\chi-})(-m_1^2 \theta_+ e^{i\chi-} + m_2^2 \theta_+ e^{-i\chi-}) = \text{real},
\]

and does not impose \( \chi = 0 \) or \( \xi = 0 \). These points being made, we now attempt the evaluation of \( \varepsilon \) and \( \varepsilon' \). The \( \Delta S = 1 \) \( (\pi\pi)|H|K^0 \) amplitude being in general complex, we introduce the auxiliary parameters \( \varepsilon_m \) and \( \xi \), following the notations of Sanda [7].

\[
\varepsilon_m = \text{Im}(K^0|H|\bar{K}^0)/\text{Re}(K^0|H|\bar{K}^0), \quad \xi = \text{Im} A_0/\text{Re} A_0\ ,
\]

\[
A_0 = e^{i\delta_0}((2\pi)\delta_{I=0}|H|K^0), \quad \delta_0 = (\pi\pi)_{I=0} \text{ phase shift},
\]

and

\[
|\varepsilon| = 2^{-3/2}(\varepsilon_m + 2\xi), \quad |\varepsilon'| = 2^{-1/2}|\xi| |A_2/A_0|,
\]

where we have assumed \( A_2 \) to be nearly real, as is the case in the present context. For the graphs of fig. 1,1, we have [we neglect the contribution due to \( f_2 \)]:

\[
A_{1,1}^{ab} = \theta_-(c\theta_+)^3 \lambda_s^\ast e^2 s^2 \tilde{\theta} (\pm) \exp[i(\xi_+ + \chi_+ - \phi_+ - \phi_-)] m_1 I(m_1, \tilde{M}_a, \tilde{M}_b),
\]

\[
I(m_1, \tilde{M}_a, \tilde{M}_b) = \frac{1}{(2\pi)^4} \int \frac{d^4k}{(k^2 - M_1^2)((k - p_3)^2 - m_1^2)} \frac{(k - p_3)^2 - \tilde{M}_a^2}{(k - p_3 + p_4)^2 - \tilde{M}_b^2} |(k - p_1)^2 - m_1^2| ,
\]

where \( \tilde{M}_a \) and \( \tilde{M}_b \) represent the masses of the exchanged scalars \( \bar{u}_a, \bar{u}_b \) (which are in general mixtures of current eigenstates \( \bar{u}, \bar{c}, \bar{t} \)). For simplicity, we will only take into account two generations; \( \tilde{\theta} \) then stands for the generalized “Cabibbo angle” acting between \( \bar{u} \) and \( \bar{c} \) and the \( (\pm) \) refers to the \( (aa, ab, ba, bb) \) exchanges. We must of course sum (13) over all \( a, b \) combinations, and this implies, as usual for the box diagram, a double GIM-type cancellation. This fact proves crucial, since the “GIM” suppression involving squarks is expected to be more severe than the one involving quarks. The value of \( \varepsilon_m \) being given by the ratio of the imaginary part of the \( K^0\bar{K}^0 \) transition to the real part therefore receives a suppression factor:

\[
\frac{[(\tilde{M}_a^2 - \tilde{M}_b^2)]^2}{[(m_a^2 - m_b^2)/m_a m_b]^2}.
\]

(One should also take into account a small enhancement of the imaginary graph due to its chiral structure; this is however not sufficient to alter our conclusions.) The contribution to \( \xi \), however only receives one power of this ratio of masses, and therefore we will in general have:

\[
|\xi/\varepsilon_m| \sim \frac{1}{(\tilde{M}_a^2 - \tilde{M}_b^2)/\tilde{M}_a \tilde{M}_b}^{-1} \gg 1.
\]
where we have assumed that \((\tilde{\mathcal{M}}_u^2 - \tilde{\mathcal{M}}_d^2) \sim (m_u^2 - m_d^2)\) in agreement with current prejudices (6). The limitation (22) might be somewhat imperilled if the t contribution were dominant, and the squarks light. There is however no reason to assume that the corresponding mixing angles favor the t exchange.

These remarks are easily shown to apply as well to the contributions resulting from graphs 1.2–1.4.

We therefore get to the first conclusion: if supersymmetric forces linked to the charged gaugino exchanges are sufficiently strong to account for all CP violating effects, they will induce a large \(\xi/e_m\) ratio, resulting in a large value of \(e'/e\). Namely if we accept the vacuum saturation approximation, we obtain, from (17) and (18)

\[
|e'/e| = |A_2/A_0| \sim 5% ,
\]

which is known to be in contradiction with experiment. It has however been argued that the vacuum saturation approximation was inappropriate for this evaluation. In particular, ref. [8] suggests that the contribution of the vacuum saturation amplitude is actually 3 times larger than the total contribution, the difference being accounted for by soft exchanges, which are not expected to violate CP. This then leads to a decrease in the estimation of \(e'/e\). We have indeed (\(\bar{\alpha}\) and \(\bar{\pi}\) are the quantities estimated in this framework):

\[
\bar{\alpha} = \text{Im}(K^0/H/\bar{K}^0)|_{vs}/(\text{Re}(K^0/H/\bar{K}^0)|_{vs} + \text{Re}(K^0/H/\bar{K}^0)|_{soft}) , \quad \bar{\pi} = e' ,
\]

in a choice of phases where \(A_0\) is made real, and this leads to a ratio \(|\bar{\alpha}/\bar{\pi}| = \frac{1}{2} |e'/e|_{\text{vacuum saturation}}\). This somewhat contrived possibility would bring \(e'/e\) at the limit of agreement with present experimental data; a decisive test will be provided by currently planned experiments.

Although we have already stressed some unfortunate characteristics of this CP violation mechanism, we have not yet answered the question as to whether sufficient CP violation simply can be generated in this way.

This requires the evaluation of the penguin diagrams pictured in figs. 2.1, 2.2. The first graph gives (a similar contribution arises from 2.2; we do not give it explicitly since we are merely interested in orders of magnitude):

\[
\left| \text{Im} \left( A_1 \right) \right| \sim \left| g_s(q^2)g_s(\overline{M}^2)/4\pi q^2 \right| \frac{1}{6} \bar{m}_Y \sum_{\nu_s} \left[ (M_{\nu_u}^2 - \frac{1}{2} \bar{m}_1^2/M_{\nu_u}^2 + \frac{1}{2} \bar{m}_1^2/M_{\nu_u}^2) \ln(\bar{m}_1^2/M_{\nu_u}^2) \right] - [\nu_u \leftrightarrow \nu_s] \right| \times \left| \text{Im}(\bar{\alpha}_s) \right| \times \left| \text{Im}(\bar{\pi}_s) \right| \times \left| \text{Im}(\bar{\pi}_d) \right| \times \left| \text{Im}(\bar{\pi}_d) \right|
\]

\[
Y_s = g_2 c_\theta s_\theta - s_\theta c_\theta \left( \text{Im}(\bar{\alpha}_s) \exp[i(\chi_+ + \xi_+ - \phi_+ - \phi_-)] \right) .
\]

We then follow closely Sanda [7] to obtain a lower bound on \(\xi\). Note that the sign \(\leq\) essentially comes from replacing \(g_s(q^2)\) by \(g_s(\overline{M}^2)\) and that therefore (22) still correctly represents the order of magnitude of \(\xi\). We have:

\[
|\xi| \lesssim \left( Y_s/s_\theta c_\theta \right)/[16 \sqrt{2}] g_s(\overline{M}^2) \left[ (M_{\nu_u}^2/M_{\nu_u}^2) - \frac{1}{2} \left( \bar{m}_1^2/M_{\nu_u}^2 \right) \right] D ,
\]

\[
D = \{ M_s + M_d [f_K + f_{+}(1 - m_K^2/m_{\sigma}^2)] / [-f_K + f_{+}(1 - m_K^2/m_{\sigma}^2)] \} g_s^2(\overline{M}^2) ,
\]

where \(M_d\) and \(M_s\) are quarks constituent masses. One more constraint arises from the necessity to satisfy simultaneously the upper bound on the value of the electric dipole moment of the neutron (< \(6 \times 10^{-25}\) cm, ref. [9]).

![Fig. 2. As fig. 1.](image-url)
This electric dipole moment is estimated from the one of the individual quarks. The analytic form being similar to the expression leading \( e \) (26) we obtain a direct relation between the contributions to \( e \) and to \( d_e \) (d quark), due to charged gaugino exchange:

\[
|e|_{\text{charged gauginos}} \approx 2^{-1/2} (\sin \theta_c \cos \phi_c / \sin \theta \cos \theta) |\lambda_d / \lambda_u| |\sqrt{2} G_F \ln(M^2_u/M^2_c)|^{-1} 2\pi^2 d_e^{(d)} D(M^2_c - M^2_u) / M^2_c
\]

\[
\sim 10^{-22} (d_e^{(d)}/e) (\text{cm}) (M^2_c - M^2_u) / M^2_c.
\]  

(27)

This shows that the limits arising for the experimental bounds on the electric dipole moments are actually much stronger than those arising from the K system for reasonably split \( \tilde{c} \) and \( \tilde{u} \) masses.

It might be argued that the value of the electric dipole moment of hadrons is a bad bound to use, since it is influenced by the value of the \( \theta \) parameter controlling strong CP violation, which may in principle be adjusted in an ad-hoc way. This claim however faces comparable limits arising from the electric dipole moment of the electron, which are not sensitive to \( \theta \), and differ only by the relative value of the masses of \( M_e \) and \( M_u \). The present limit on \( (e) \) is \( d_e^{(e)} < 10^{-24} e \text{ cm} \) [10], but one may expect considerable improvement in the near future [11].

We now turn to the exchange of neutral gauginos. We have to consider both higgsinos-winos, which are the counterpart of the above charged fermions, and gluinos. Little more is brought by the consideration of neutral H-winos, except that new phases will in general be introduced by the diagonalization of the neutral mass matrix, and that more (Majorana) fermions contribute [3]. We find it unnecessary to pursue this issue here, since the results are qualitatively comparable to the above.

Some new features appear when we consider the exchange of gluinos. At first sight this should not bring phases into play, since the same coupling plays at each vertex. However the possibility of \( d \rightarrow \tilde{d} \) transitions brings such phases back. If \( 2\phi_g \) is the phase associated to the gluino mass:

\[
\tilde{g} \sim \tilde{g}' m_g e^{i2\phi_g},
\]

we define \( \tilde{g} = e^{i\phi}\tilde{g} \) to make the mass real, and introduce the four-component notation \( \tilde{g} = \begin{pmatrix} \tilde{g} \\ \tilde{g}' \end{pmatrix} \) to obtain the couplings:

\[
g_{\tilde{g}} \tilde{g}^a (\tilde{u}^* e^{i\phi} \frac{1}{2} \lambda^a L u + \tilde{u} e^{-i\phi} \frac{1}{2} \lambda^a R u). \]  

(28)

As was stressed in refs. [4,12], strangeness violation occurs due to the fact that the squarks are in general not diagonalized at the same time as the quarks.

The above remarks concerning the smallness of \( e_m \) relative to \( \xi \) still obtain; however some new graphs (fig. 2.3) have to be considered which further increase the value of \( \xi \), since the gluon of the "penguin" diagram can now attach to the gluino line. Since no such contribution is associated with the electric dipole moment, eq. (28) has to be modified accordingly. This however does not affect our conclusions, which we briefly restate:

- Supersymmetric extensions of the "standard model" introduce new possible mechanisms of CP violation.
- One such mechanism relies on the "overall" phases (i.e. flavor blind) between the various gauginos and squarks and is therefore a peculiarity of supersymmetric models.
- The value of \( \epsilon'/\epsilon \) resulting for considering this parameter alone is at best at the limit of present experimental accuracy.
- The ratio of \( \epsilon \) to the electric dipole moments of hadrons or leptons shows that these, in general, impose better bounds on the phases and mixing angles, except perhaps in the case of extreme splitting between squarks, where some contribution to \( \epsilon \) and \( \epsilon' \) might still be observed.

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