THE LOCALLY SUPERSYMMETRIC GEOMETRICAL HIERARCHY MODEL

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We examine the locally supersymmetric geometrical hierarchy model at energies below the unification mass, M_{GU}. The superfield content, for which the theory has acceptable values of M_{GU} and $\sin^2\theta_W$, is discussed. We show that radiative corrections induce the spontaneous breakdown of SU(2) × U(1) to U(1)$_{EM}$ for a top quark mass of 100–200 GeV. In theories with two color triplet Higgs superfields a lower bound of $O(10^{15}-10^{16}$ GeV) on the supersymmetry breaking parameter, μ, is derived.

In a recent paper [1] we extended the geometrical hierarchy model [2] to include N = 1 supergravity (SUGRY). Our main results were the following.

(A) The value of the potential energy at its absolute minimum (the cosmological constant) can be fine tuned to vanish.

(B) In this case SU(5) is spontaneously broken, at tree level, to SU(3) × SU(2) × U(1). This breaking is a purely supergravitational effect, the scale of which is of $O(M)$ where $M = M_{Planck}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV.

(C) SUGRY does not effect supersymmetry (SUSY) breaking. SUSY is spontaneously broken at a scale of $O(\mu^2)$, where μ is the dimension 1 parameter in the O'Raifeartaigh sector of the theory.

(D) We derived the tree level, low energy potential. It divides naturally into two pieces. The first is globally supersymmetric and is independent of supergravitational effects. The second piece explicitly breaks SUSY and is induced by SUGRY. The entire low energy lagrangian is renormalizable.

(E) The parameters of the tree level, low energy potential are such that its absolute minimum occurs at the zero of field space. Furthermore, all scalar

masses are of $O(\mu(\mu/M))$.

We conclude from (E) that, at tree level, SU(2) × U(1) is unbroken. How, then, is the electroweak group to be broken to U(1)$_{EM}$? The answer lies in radiative corrections to the potential. These corrections introduce a renormalization point that must be chosen near M for perturbation theory to be valid. It follows that the tree level, low energy potential is only meaningful for field amplitudes of $O(M)$. To examine the potential energy for small field amplitudes it must be "improved" using the renormalization group (RG). In this paper we carry out the RG improvement of the low energy potential to the one-loop level. We find that [1]

(a) Higgs fields can have negative (mass)^2 near the origin of field space and, hence, develop vacuum expectation values (VEV's) that break SU(2) × U(1) → U(1)$_{EM}$.

(b) Squarks and sleptons, on the other hand, always have non-negative (mass)^2 near the origin of

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[1] The breaking of SU(2) × U(1) → U(1)$_{EM}$, through radiative corrections induced by the top quark, has been discussed by the authors of ref. [3]. Our results differ from theirs mainly due to the large threshold effects inherent in the Geometrical Hierarchy Model.

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field space and, hence, have vanishing VEV's. Thus charge, baryon, and lepton numbers remain unbroken.

(c) The size of the electroweak symmetry breaking is set by $\mu(\mu/M)$ and the top quark Yukawa coupling (evaluated near 10^2 GeV). Hence, given the electroweak scale, one can calculate the top quark mass. We do this for a wide choice of threshold parameters.

A prerequisite to carrying out these calculations is to evaluate the grand unification mass, M_{GU}, and $\sin^2\theta_w$ to the one-loop level. In this paper we evaluate these quantities for a wide choice of threshold parameters. We find that

(d) The values of M_{GU} and $\sin^2\theta_w$ are, in general, too large. This problem is easily overcome by adding families of Higgs superfields that transform as **10** and $\overline{\mathbf{10}}$ under SU(5). These superfields restore M_{GU} and $\sin^2\theta_w$ to acceptable values.

Finally, we discuss the possibility of reducing, from four to two, the number of Higgs superfields transforming as **5** or $\overline{\mathbf{5}}$ under SU(5). To have only two such superfields we find that

(e) scale μ must be of $O(10^{15}-10^{16}\,\text{GeV})$, or larger, to sufficiently suppress the amplitude for nucleon decay to kaons + leptons. All calculations will be performed for both two and four Higgs superfields.

The gauge group of our model is $G = SU(5)$. The chiral superfields are

(1) A, Z, X. A, Z are 24's and X a 1 under SU(5). These fields form the O'Raifeartaigh sector of the model.

(2) $H, H_1, \overline{H}, \overline{H}_1$. H, H_1 are 5's and $\overline{H}, \overline{H}_1$ are $\overline{\mathbf{5}}$'s under SU(5). The extra superfields $H_1, \overline{H}_1$ are introduced to eliminate certain dimension 5 operators which cause the proton to decay too rapidly [2].

(3) $\overline{5}_J, 10_J, \overline{5}_J$ and 10_J are $\overline{\mathbf{5}}$'s and **10**'s respectively under SU(5). We assume there are three leptoquark families $(J = 1, 3)$.

The superpotential of our model is then given by

$$W = \beta\mu^2 M + \lambda_1 X(\text{tr } A^2 - \mu^2) + \lambda_2 \text{tr } ZA^2$$

$$+ \lambda_3 \overline{H}(A + m1)H_1 + \lambda_4 \overline{H}_1(A + m1)H + \lambda_{IJ}^U H10_I 10_J$$

$$+ \lambda_{IJ}^D \overline{H}10_I \overline{5}_J . \qquad (1)$$

Parameters λ_i, λ^U, λ^D, and β are dimensionless and μ, m, and M have dimension 1. The potential energy, V, can be calculated from W [4]. To $O(\mu^3(\mu/M))$, V has

its absolute minimum at

$$\langle X \rangle = \lambda_2(\lambda_2^2 + 30\lambda_1^2)^{-1/2}(\sqrt{3} - 1)M ,$$

$$\langle Z \rangle = \lambda_1(\lambda_2^2 + 30\lambda_1^2)^{-1/2}(\sqrt{3} - 1)M \text{ diag}(2,2,2,-3,-3),$$

$$\langle A \rangle = \lambda_1(\lambda_2^2 + 30\lambda_1^2)^{-1/2}\mu \text{ diag}(2,2,2,-3,-3) . \qquad (2)$$

The VEV's of $H, \overline{H}, \overline{5}$, and 10 fields are undetermined at this order. Note that $\langle Z \rangle$ breaks $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ with $M_{GU} = \lambda_1(\lambda_2^2 + 30\lambda_1^2)^{-1/2}(\sqrt{3} - 1)M$. It follows that, for any choice of λ_1 and λ_2, $M_{GU} \leqslant [(\sqrt{3} - 1)/\sqrt{30}]M \simeq 3.2 \times 10^{17}$ GeV. This vacuum state has vanishing cosmological constant as long as we take

$$\beta = -\lambda_1\lambda_2(\lambda_2^2 + 30\lambda_1^2)^{-1/2}(2 - \sqrt{3}) . \qquad (3)$$

The VEV's of Kahler derivatives $D_{\phi_i}W = \partial W/\partial\phi_i + (\phi_i^\dagger/M^2)W$, evaluated to $O(\mu^2)$ at (2), are

$$\langle D_X W \rangle = -\sqrt{3}\lambda_1\lambda_2(\lambda_2^2 + 30\lambda_1^2)^{-1}\mu^2 ,$$

$$\langle D_Z W \rangle = -\sqrt{3}\lambda_1^2\lambda_2(\lambda_2^2 + 30\lambda_1^2)^{-1}\mu^2 \text{ diag}(2,2,2,-3,-3). \qquad (4)$$

All other $\langle D_{\phi_i}W \rangle$ vanish to this order. Therefore, SUSY is spontaneously broken by this vacuum state at a scale of $O(\mu^2)$. Define low energy superpotentials

$$\widetilde{g}(y_a, \langle A \rangle) = e^{(2-\sqrt{3})}(\lambda_3\overline{H}(\langle A \rangle + m1)H_1$$

$$+ \lambda_4\overline{H}_1(\langle A \rangle + m1)H + \lambda_{IJ}^U H10_I 10_J + \lambda_{IJ}^D \overline{H}10_I\overline{5}_J) \qquad (5)$$

and

$$\widetilde{f}(Z_3, Z_8) = [m_{3/2}/(\sqrt{3}-1)](\tfrac{9}{2}\text{tr } Z_3^2 - \tfrac{4}{3}\text{tr } Z_8^2) , \qquad (6)$$

where Z_3 and Z_8 are the SU(2) triplet and SU(3) octet components of Z respectively, and $m_{3/2}$ is the gravitino mass given by

$$m_{3/2} = e^{(2-\sqrt{3})}\lambda_1\lambda_2(\lambda_2^2 + 30\lambda_1^2)^{-1/2}\mu(\mu/M) . \qquad (7)$$

Then the tree level, low energy potential is found to be

$$V_{LE} = V_{LE}^{(1)}(y_a) + V_{LE}^{(2)}(Z_3, Z_8) + \tfrac{1}{2}D_{y,Z,\alpha}^2 , \qquad (8)$$

where

$$V_{LE}^{(1)}(y_a) = |\partial\widetilde{g}/\partial y_a|^2 + (m_{3/2}A\widetilde{g} + \text{h.c.})$$

$$+ [m_{3/2}(y_a\partial\widetilde{g}/\partial y_a - 3\widetilde{g}) + \text{h.c.}] + m_{3/2}^2|y_a|^2 , \qquad (9)$$

$$V_{LE}^{(2)}(Z_3, Z_8) = \text{tr}|\partial \tilde{f}/\partial Z_3|^2 + \text{tr}|\partial \tilde{f}/\partial Z_8|^2$$

$$+ (m_{3/2}A'\tilde{f} + \text{h.c.}) + m_3^2(\text{tr}|Z_3|^2 + \text{tr}|Z_8|^2) \quad (10)$$

and

$$A = 3 - \sqrt{3}, \quad A' = -\tfrac{5}{2}(\tfrac{3}{5}\sqrt{3} - 1), \quad (11)$$

D_{y,Z_α}^2 is the sum of the squares of SU(3), SU(2), and U(1) D terms. Since the D term associated with U(1) arises from the spontaneous breakdown of SU(5), it contains no dimensional parameter. Parameter m in (5) is chosen so that the masses of the SU(3) $\mathbf{3}$ ($\mathbf{\bar{3}}$) components of H, $H_1(\bar{H}, \bar{H}_1)$ are of $O(\mu)$. Since $|A| < 3$ and $|A'| < 3$ the absolute minimum of potential (8) occurs at the origin of field space [5]. The masses of all SU(2) doublet Higgs fields, squarks, sleptons, Z_3, and Z_8 are of $O(m_{3/2})$. We will assume that the top quark mass is $\lesssim 10^2$ GeV. Parameters λ_3, λ_4, and m are chosen so that the SU(2) doublet higgsinos have mass between 0 and 10^2 GeV. We use the "minimal" version of SUGRY in which all gaugino masses vanish at tree level. Gauginos are expected to receive masses of $O(\alpha m_{3/2})$ from radiative corrections. We will assume these masses are $\lesssim 10^2$ GeV. It follows that there are four mass thresholds in our model.

(1) 10^2 GeV. Particles with masses of $\lesssim 10^2$ GeV are quarks, leptons, SU(2) doublet higgsinos, gauge fields and gauginos.

(2) $m_{3/2}$. Particles with masses of $O(m_{3/2})$ are squarks, sleptons, SU(2) doublet Higgs fields, Z_3 (scalars and fermions) and Z_8 (scalars and fermions).

(3) μ. Particles with masses of $O(\mu)$ are SU(3) $\mathbf{3}$ and $\mathbf{\bar{3}}$ Higgs fields and higgsinos.

(4) M. Everything else.

The values of α_3 and α_{EM} at 10^2 GeV are chosen to be 10 and $\frac{1}{127}$ respectively. The values of M_{GU} and $\sin^2\theta_w$ (evaluated at 10^2 GeV) can be determined from the equations [6]

$$\alpha_{EM}(Q)/\alpha_3(Q)$$

$$= \tfrac{3}{8} \{1 - [\alpha(Q)/\pi] [(\tfrac{5}{6}b_1 + \tfrac{1}{2}b_2 - \tfrac{4}{3}b_3) \ln(M_{GU}/\mu)$$

$$+ (\tfrac{5}{6}b_1' + \tfrac{1}{2}b_2' - \tfrac{4}{3}b_3')\ln(\mu/m_{3/2})$$

$$+ (\tfrac{5}{6}b_1'' + \tfrac{1}{2}b_2'' - \tfrac{4}{3}b_3'')\ln(m_{3/2}/Q) - \tfrac{1}{2}]\}, \quad (12)$$

$$\sin^2\theta_w(Q) = \tfrac{3}{8} \{1 - [5\alpha(Q)/6\pi] [(b_1 - b_2)\ln(M_{GU}/\mu)$$

$$+ (b_1' - b_2')\ln(\mu/m_{3/2})$$

$$+ (b_1'' - b_2'')\ln(m_{3/2}/Q) - \tfrac{1}{3}]\}, \quad (13)$$

where $Q = 10^2$ GeV. The coefficients b_i (threshold dependent) are defined by

$$d\alpha_i/dt = (b_i/2\pi)\alpha_i^2, \quad (14)$$

where α_3, α_2, and α_1 are the running coupling parameters for SU(3), SU(2), and U(1) $[\hat{Y} = (\tfrac{3}{5})^{1/2}Y]$. The results, for a range of values of μ and $m_{3/2}$, are given in table 1. In all cases the values of M_{GU} exceed the bound of 3.2×10^{17} GeV derived earlier. It is, therefore, impossible to adjust parameters λ_1 and λ_2 to account for these values. Furthermore, these unification masses are so large that corrections due to quantum gravitation (incalculable) cannot be ignored. Similarly, in all cases the values of $\sin^2\theta_w$ at 10^2 GeV exceed the experimental upper bound of 0.229 [6]. Finally, for several choices of μ and $m_{3/2}$, the value of α_3 at M_{GU} is so large that perturbation theory cannot be trusted. We conclude that the theory described by superpotential (1) does not properly account for M_{GU} and $\sin^2\theta_w$.

Fortunately, there is a simple solution to this problem [7]. It is clear from eqs. (12) and (13) that both M_{GU} and $\sin^2\theta_w$ will decrease if one can increase coefficient b_1 without changing b_2 and b_3. Any superfield which transforms as a $\mathbf{10}$ under SU(5) has the SU(3) $\times$ SU(2) $\times$ U(1) decomposition

Table 1
M_{GU}, $\sin^2\theta_w$, and $\alpha_3(M_{GU})$ as functions of $\mu, m_{3/2}$, and the number of Higgs doublets, N_H.

μ (GeV)	$m_{3/2}$ (GeV)	N_H	M_{GU} (GeV)	$\sin^2\theta_w$	$\alpha_3(M_{GU})$
10^{11}	10^2	4	7.1×10^{21}	0.245	0.48
10^{11}	10^3	4	2.9×10^{21}	0.243	0.24
10^{11}	10^4	4	1.2×10^{21}	0.242	0.16
10^{14}	10^2	4	7.1×10^{20}	0.256	0.20
10^{14}	10^4	4	1.2×10^{20}	0.254	0.11
10^{14}	10^6	4	2.0×10^{19}	0.251	0.07
10^{16}	10^2	2	3.3×10^{22}	0.237	0.13
10^{16}	10^4	2	4.2×10^{21}	0.236	0.09
10^{16}	10^6	2	5.5×10^{20}	0.234	0.06

$$10 = (\bar{3}, 1, -\tfrac{2}{3}) + (3, 2, \tfrac{1}{6}) + (1, 1, 1) . \qquad (15)$$

If one can give the $(\bar{3}, 1, -2/3)$ and $(3, 2, 1/6)$ components mass of $O(M)$, and the $(1,1,1)$ component a mass of $O(\mu)$, then the addition of such a super-field to the theory will appropriately change the b_i coefficients. Henceforth we add n families of $\mathbf{10}, \overline{\mathbf{10}}$ representations to the theory. For each $\mathbf{10}$ we must introduce a $\overline{\mathbf{10}}$ to make the theory anomaly free. We denote these new superfields as $\widetilde{10}_i$, $\widetilde{\overline{10}}_i$ where $i = 1$, n. To superpotential (1) we add the interaction

$$\widetilde{\lambda}_i \widetilde{10}_i (Z + \widetilde{m}1) \widetilde{\overline{10}}_i , \qquad (16)$$

where $\widetilde{\lambda}_i$ are dimensionless and $\widetilde{m}$ has dimension 1. We chose parameter $\widetilde{m}$ in such a way that the SU(3) and SU(2) components of $\langle Z \rangle + \widetilde{m}1$ are of $O(M)$ and $O(\mu)$ respectively. Parameters $\widetilde{\lambda}_i$ and $\widetilde{m}$ can be taken so that $\langle \widetilde{10}_i \rangle = \langle \widetilde{\overline{10}}_i \rangle = 0$ for $i = 1, n$. Hence, the $(\bar{3}, 1, -2/3), (3, 2, 1/6), (3, 1, 2/3), (\bar{3}, 2, -1/6)$ component fields get masses of $O(M)$ and can be ignored. The $(1, 1, 1), (1, 1, -1)$ component fields get masses of $O(\mu)$ and contribute to the scaling behavior of U(1) at energies above μ. They both increase coefficient b_1 and leave b_2, b_3 unchanged. We again calculate M_{GU} and $\sin^2\theta_w$, this time including the $\widetilde{10}_i$, $\widetilde{\overline{10}}_i$ families in the theory. The results, for a range of values of $\mu, m_{3/2}$, and n, are given in table 2. In most cases the values of M_{GU} are below the bound of 3.2×10^{17} GeV derived earlier. It is, therefore, possible to adjust λ_1 and λ_2 to account for these values. Similarly, in most cases the values of $\sin^2\theta_w$ at 10^2 GeV lie between the experimental bound of $0.201 -$
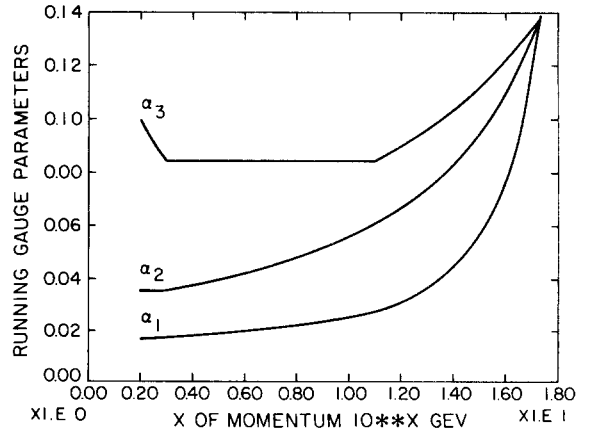
Table 2
M_{GU}, $\sin^2\theta_w$, and $\alpha_3(M_{GU})$ as functions of $\mu, m_{3/2}, N_H$, and the number of $\widetilde{10}, \widetilde{\overline{10}}$ families, n.

μ (GeV)	$m_{3/2}$ (GeV)	N_H	n	M_{GU} (GeV)	$\sin^2\theta_w$	$\alpha_3(M_{GU})$
10^{11}	10^2	4	4	3.2×10^{17}	0.220	0.19
10^{11}	10^3	4	4	1.9×10^{17}	0.219	0.14
10^{11}	10^4	4	4	1.1×10^{17}	0.219	0.10
10^{14}	10^2	4	6	2.7×10^{17}	0.237	0.13
10^{14}	10^4	4	6	1.1×10^{17}	0.236	0.08
10^{14}	10^6	4	6	4.4×10^{16}	0.236	0.06
10^{16}	10^2	2	15	7.2×10^{17}	0.210	0.11
10^{16}	10^4	2	15	4.0×10^{17}	0.213	0.08
10^{16}	10^6	2	15	2.3×10^{17}	0.215	0.06

Fig. 1. Running gauge parameters α_3, α_2, and α_1 as functions of momentum, in a theory with four Higgs doublets, $\mu = 10^{11}$ GeV, $m_{3/2} = 10^3$ GeV, and $n = 4$.

0.229. (Recent work indicates that the upper bound might be larger than 0.229. Hence, all the values for $\sin^2\theta_w$ are probably acceptable.) Finally, for all choices of $\mu, m_{3/2}$, and n, the value of α_3 at M_{GU} is sufficiently small that one can trust perturbation theory. We conclude that the modified theory described by superpotential (1), along with interaction (16), properly accounts for M_{GU} and $\sin^2\theta_w$. A graph of the running gauge coupling parameters for a theory with four Higgs doublets, $\mu = 10^{11}$ GeV, $m_{3/2} = 10^3$ GeV, and $n = 4$ is shown in fig. 1. Note the large changes in the β-functions at each mass threshold. It is these changes that tend to make M_{GU} and α_3 large and necessitate the introduction of the $\widetilde{10}_i$, $\widetilde{\overline{10}}_i$ families.

We now consider the question of electroweak symmetry breaking. One can ignore Z_3, Z_8 and the SU(3) $3 (\bar{3})$ components of $H, H_1 (\bar{H}, \bar{H}_1)$ in the low energy lagrangian. The reasons are that (1) the associated superfields do not couple directly to the SU(2) doublet Higgs superfields, (2) Z_3 and Z_8 have vanishing hypercharge, and (3) $H, H_1, \bar{H}, \bar{H}_1$ have identical tree level mass. Also, it is easy to show that these fields cannot get non-vanishing VEV's from radiative corrections. Hence, they do not contribute to electroweak symmetry breaking to the one-loop level. We absorb the factor $e^{(2-\sqrt{3})}$ into the definition of all parameters λ, and define

$$m_i = \lambda_i[-3\lambda_1(\lambda_2^2 + 30\lambda_1^2)^{-1/2}\mu + m] , \qquad (17)$$

for $i = 3, 4$. Then the most general, low energy lagrangian derivable from the geometric hierarchy model is

$$\mathcal{L}_{LE} = \int d^2\theta(\hat{m}_3\overline{H}H_1 + \hat{m}_4\overline{H}_1 H + \hat{\lambda}_{IJ}^E \overline{E}_I \overline{H}L_J$$

$$+ \hat{\lambda}_{IJ}^D \overline{D}_I \overline{H}Q_J + \hat{\lambda}_{IJ}^U \overline{U}_I HQ_J) + \tfrac{1}{2}\hat{m}_{y_a}^2 |y_a|^2$$

$$+ m_{3/2}(\hat{m}_3\hat{C}_3\overline{H}H_1 + \hat{m}_4\hat{C}_4\overline{H}_1 H)$$

$$+ m_{3/2}(\hat{A}_{IJ}^E \hat{\lambda}_{IJ}^E \overline{E}_I \overline{H}L_J + \hat{A}_{IJ}^D \hat{\lambda}_{IJ}^D \overline{D}_I \overline{H}Q_J + \hat{A}_{IJ}^U \hat{\lambda}_{IJ}^U \overline{U}_I HQ_J)$$

$$+ \text{h.c.} + \text{kinetic energy terms} , \qquad (18)$$

where the field in the integrand of $\int d^2\theta$ are superfields and all other fields are scalars. $H, H_1, \overline{H}, \overline{H}_1$ are SU(2) doublet Higgs fields, L_J and Q_J are SU(2) doublet left chiral leptons and quarks, and $\overline{E}_I, \overline{U}_I$, and $\overline{D}_I$ are SU(2) singlet right chiral leptons and quarks. All parameters with "$\hat{}$" run under the renormalization group. For field amplitudes just below M_{GU} the potential energy associated with (18) becomes identical to (8). Hence, at this scale

$$\hat{m}_i = m_i , \quad \hat{m}_{y_a} = m_{3/2} , \quad \hat{\lambda}_{IJ}^{E,D} = \lambda_{IJ}^D , \quad \hat{\lambda}_{IJ}^D = \lambda_{IJ}^U ,$$

$$\hat{C}_3 = \hat{C}_4 = A - 1 , \quad \hat{A}_{IJ}^E = \hat{A}_{IJ}^D = \hat{A}_{IJ}^U = A , \qquad (19)$$

where A is given in (11). We first consider the case where $\hat{m}_3 = \hat{m}_4 = 0$. The Yukawa coupling parameters of the first two lepto-quark families are sufficiently small that they cannot induce electroweak symmetry breaking. We henceforth consider the third lepto-quark generation only. The potential energy associated with (18) is then given by

$$V_{LE} = \hat{m}_H^2|H|^2 + \hat{m}_{H_1}^2|H_1|^2 + \hat{m}_H^2|\overline{H}|^2 + \hat{m}_{H_1}^2|\overline{H}_1|^2$$

$$+ \hat{m}_{\overline{E}}^2|\overline{E}|^2 + \hat{m}_{\overline{U}}^2|\overline{U}|^2 + \hat{m}_{\overline{D}}^2|\overline{D}|^2 + \hat{m}_L^2|L|^2 + \hat{m}_Q^2|Q|^2$$

$$+ |\hat{\lambda}^E \overline{E}L + \hat{\lambda}^D \overline{D}Q|^2 + |\hat{\lambda}^U \overline{U}Q|^2 + |\hat{\lambda}^E \overline{H}L|^2 + |\hat{\lambda}^D \overline{H}Q|^2$$

$$+ |\hat{\lambda}^U HQ|^2 + |\hat{\lambda}^E \overline{E}\overline{H}|^2 + |\hat{\lambda}^D \overline{D}H + \hat{\lambda}^U \overline{U}H|^2$$

$$+ m_{3/2}(\hat{A}^E \hat{\lambda}^E \overline{E}\overline{H}L + \hat{A}^D \hat{\lambda}^D \overline{D}\overline{H}Q + \hat{A}^U \hat{\lambda}^U \overline{U}HQ + \text{h.c.})$$

$$+ \frac{1}{2}\sum_{i=1}^3 D_i^2 , \qquad (20)$$

where

$$D_1 = -\hat{e}_1(\tfrac{3}{5})^{1/2}(-\tfrac{1}{2}|\overline{H}|^2 + \tfrac{1}{2}|H|^2 - \tfrac{1}{2}|\overline{H}_1|^2 + \tfrac{1}{2}|H_1|^2$$

$$+ |\overline{E}|^2 + \tfrac{1}{3}|\overline{D}|^2 - \tfrac{2}{3}|\overline{U}|^2 - \tfrac{1}{2}|L|^2 + \tfrac{1}{6}|Q|^2) ,$$

$$D_2^a = -\tfrac{1}{2}\hat{e}_2(\overline{H}^\dagger \sigma^a \overline{H} + H^\dagger \sigma^a H + \overline{H}_1^\dagger \sigma^a \overline{H}_1 + H_1^\dagger \sigma^a H_1$$

$$+ L^\dagger \sigma^a L + Q^\dagger \sigma^a Q) \qquad (21)$$

and we have dropped the subscripts indicating the third family. Assume that $\hat{m}_H^2 < 0$, and that all other $\hat{m}_{y_a}^2$ are non-negative. Then V_{LE} has its absolute minimum at

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = 2|\hat{m}_H|^2(\hat{e}_1^2 \tfrac{3}{5} + \hat{e}_2^2)^{-1/2} . \qquad (22)$$

All other VEV's vanish. The W and Z boson masses are given by

$$m_W^2 = \tfrac{1}{2}\hat{e}_2^2 v^2, \quad m_Z^2 = \tfrac{1}{2}(\hat{e}_1^2 \tfrac{3}{5} + \hat{e}_2^2)v^2 . \qquad (23)$$

Eqs. (22) and (23) imply that

$$\hat{m}_H^2 = -\tfrac{1}{2}m_Z^2 . \qquad (24)$$

It follows that SU(2) × U(1) is spontaneously broken to $U(1)_{EM}$ if $\hat{m}_H^2$, and only $\hat{m}_H^2$, becomes negative. To see if this is possible we must discuss the RG equations for the parameters in (20) and (21). The RG equations for $\hat{\alpha}_1, \hat{\alpha}_2$, and $\hat{\alpha}_3$ are given by (14). The RG equations, that are relevant to the value of $\langle H \rangle$ to the one-loop level, are

$$\frac{d}{dt}\begin{pmatrix} \hat{m}_H^2 \\ \hat{m}_{\overline{U}}^2 \\ \hat{m}_Q^2 \end{pmatrix} = \frac{(\hat{\lambda}^U)^2}{8\pi^2}\begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} \hat{m}_H^2 \\ \hat{m}_{\overline{U}}^2 \\ \hat{m}_Q^2 \end{pmatrix}$$

$$+ \frac{|\hat{A}^U|^2(\hat{\lambda}^U)^2 m_{3/2}^2}{8\pi^2}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \qquad (25)$$

$$d\hat{A}^U/dt = [3(\hat{\lambda}^U)^2/4\pi^2]\,\hat{A}^U , \qquad (26)$$

$$d\hat{\lambda}^U/dt = 3(\hat{\lambda}^U)^3/8\pi^2 - (1/2\pi)\hat{\lambda}^U(\tfrac{8}{3}\hat{\alpha}_3 + \tfrac{3}{2}\hat{\alpha}_2 + \tfrac{13}{30}\hat{\alpha}_1) , \qquad (27)$$

where $t = \ln(|y_a|/M_{GU})$. Note that $t = 0$ when $|y_a| = M_{GU}$ and that $t \to -\infty$ as $|y_a|$ approaches the origin of field space. The boundary conditions for $\hat{m}_H, \hat{m}_U, \hat{m}_Q, \hat{\lambda}^U$, and $\hat{A}^U$ at $t = 0$ are given in (19). The solution to eq. (25) is

$$\hat{m}_{\rm H}^2(t) = \left\{ -\tfrac{1}{2} + \tfrac{3}{2}\exp\left[-\frac{3}{4\pi^2} \int_t^0 dt'\, \hat{\lambda}^U(t')^2 \right. \right.$$

$$\left. \left. \times \left(\frac{1 + |\hat{A}^U(t')|^2 m_{3/2}^2}{(2\hat{m}_{\rm H}^2(t') + m_{3/2}^2)} \right) \right] \right\} m_{3/2}^2 , \qquad (28)$$

$$\hat{m}_{\rm U}^2(t) = \tfrac{2}{3}\hat{m}_{\rm H}^2(t) + \tfrac{1}{3}m_{3/2}^2 , \qquad (29)$$

$$\hat{m}_{\rm Q}^2(t) = \tfrac{1}{3}\hat{m}_{\rm H}^2(t) + \tfrac{2}{3}m_{3/2}^2 . \qquad (30)$$

At $t = 0$, $\hat{m}_{\rm H}^2(0) = (-\tfrac{1}{2} + \tfrac{3}{2})m_{3/2}^2 = m_{3/2}^2$, the correct boundary condition. For $t \to -\infty$, $\hat{m}_{\rm H}^2(t) \to (-\tfrac{1}{2} + 0)m_{3/2}^2 = -\tfrac{1}{2}m_{3/2}^2$, so that $\hat{m}_{\rm H}^2$ can indeed become negative. Similarly, $\hat{m}_{\rm U}^2(0) = \hat{m}_{\rm Q}^2(0) = m_{3/2}^2$, the correct boundary conditions. For $t \to -\infty$, $\hat{m}_{\rm U}^2(t) \to 0$ and $\hat{m}_{\rm Q}^2(t) \to \tfrac{1}{2}m_{3/2}^2$, so that these parameters are always non-negative. It follows that squark VEV's vanish and, hence, baryon number and charge remain unbroken. The solution to eq. (26) is

$$\hat{A}^U(t) = (3 - \sqrt{3})\exp\left(-\frac{3}{4\pi^2} \int_t^0 dt'\, \hat{\lambda}^U(t')^2 \right). \qquad (31)$$

It follows that the value for $\hat{A}^U$ decreases as $t \to -\infty$. Eq. (27) is most easily solved numerically. However, it is clear that $\tfrac{1}{3}(\tfrac{8}{3}\hat{\alpha}_3 + \tfrac{3}{2}\hat{\alpha}_2 + \tfrac{13}{30}\hat{\alpha}_1)$ acts as an ultra-violet (UV) attractive fixed point for $\hat{\alpha}^U = \hat{\lambda}^{U2}/4$. Finally, the RG equations for the remaining masses are

$$d\hat{m}_{y_a}^2/dt = 0 . \qquad (32)$$

Hence, $\hat{m}_{\rm H_1}^2(t) = \hat{m}_{\bar{\rm H}}^2(t) = \hat{m}_{\bar{\rm H}_1}^2(t) = \hat{m}_{\rm E}^2(t) = \hat{m}_{\bar{\rm D}}^2(t) = \hat{m}_{\rm L}^2(t) = m_{3/2}^2 > 0$. Therefore no Higgs field, other than H, has a non-vanishing VEV. Furthermore, all slepton VEV's vanish and lepton number and charge remain unbroken. It follows that $\hat{m}_{\rm H}^2$, and only $\hat{m}_{\rm H}^2$, can be non-negative. From eqs. (22), (27), (28), and (31) we see that the size of the electroweak symmetry breaking, v, is determined by $m_{3/2}$ and the top quark Yukawa coupling, $\hat{\lambda}^U$, evaluated at 10^2 GeV. Therefore, given v and $m_{3/2}$, one can evaluate the top quark mass using the equation

$$m_{\rm top} = \hat{\lambda}^U(10^2 \text{ GeV})v , \qquad (33)$$

VEV v is related to m_Z by eq. (23). Henceforth, take $m_Z = 93.8$ GeV [6]. Using eqs. (12), (14) and (23) we can determine v. The results for v, $\hat{\alpha}^U$ at 10^2 GeV, and $m_{\rm top}$, for a range of values of μ, $m_{3/2}$, and n, are given in table 3. Parameter $\hat{\alpha}^U$ is driven toward UV

Table 3
VEV v, $\tilde{\alpha}^U(10^2$ GeV), and $m_{\rm top}$ as functions of μ, $m_{3/2}$, $N_{\rm H}$, and n.

μ (GeV)	$m_{3/2}$ (GeV)	$N_{\rm H}$	n	v (GeV)	$\hat{\alpha}^U(10^2$ GeV)	$m_{\rm top}$ (GeV)
10^{11}	10^2	4	4	174.67	0.109	204.84
10^{11}	10^3	4	4	173.82	0.058	148.61
10^{11}	10^4	4	4	173.99	0.054	143.46
10^{14}	10^2	4	6	179.24	0.107	208.13
10^{14}	10^4	4	6	179.86	0.053	146.61
10^{14}	10^6	4	6	180.04	0.049	141.35
10^{16}	10^2	2	15	171.77	0.108	200.29
10^{16}	10^4	2	15	172.93	0.053	141.19
10^{16}	10^6	2	15	174.53	0.049	136.83

attractive fixed point $\tfrac{1}{3}(\tfrac{8}{3}\hat{\alpha}_3 + \tfrac{3}{2}\hat{\alpha}_2 + \tfrac{13}{30}\hat{\alpha}_1) > \tfrac{8}{9}\hat{\alpha}_3 = 0.089$ at 10^2 GeV. It follows that $\hat{\alpha}^U$ at 10^2 GeV and the top quark mass are relatively large. In all cases, however, $\hat{\alpha}^U$ is sufficiently small that one can trust perturbation theory. A graph of $m_{\rm top}$ as a function of $m_{3/2}$, for four Higgs doublets, $\mu = 10^{11}$ GeV, and $n = 4$, is shown in fig. 2. We conclude that when $\hat{m}_3 = \hat{m}_4 = 0$, H, and only H, can get a non-vanishing VEV. This VEV breaks $SU(2) \times U(1) \to U(1)_{\rm EM}$ at the correct scale as long as the top quark is sufficiently massive. VEV $\langle H \rangle$ gives mass to "up" quarks only. Therefore, when $\hat{m}_3 \to \hat{m}_4 = 0$, "down" quarks and leptons remain massless.

In order to give masses to "down" quarks and leptons, H must develop a non-vanishing VEV. In a
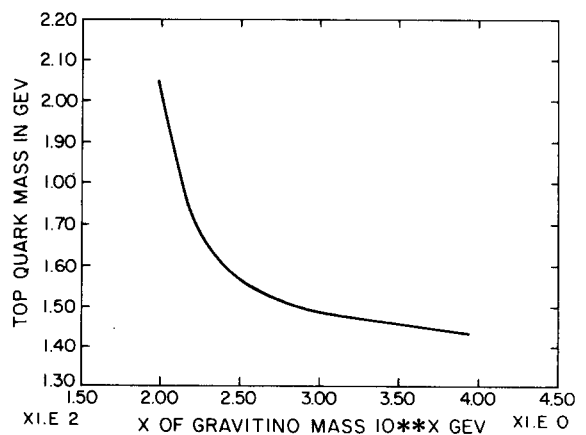

Fig. 2. The top quark mass as a function of $m_{3/2}$, in a theory with four Higgs doublets, $\mu = 10^{11}$ GeV, and $n = 4$.

theory with four Higgs doublets this can only occur beyond the one-loop level in perturbation theory, or if we consider "non-minimal" versions of SUGRY. These possibilities will be explored elsewhere. Consider a theory with two Higgs doublets ($\hat{m}_3, \hat{m}_4$ are replaced by $\hat{m}$, and $\hat{C}_3$, $\hat{C}_4$ are replaced by $\hat{C}$, in (18)). Assume $\hat{m} \neq 0$. Then, from the potential energy associated with (18), we find that

$$\langle \overline{H} \rangle = \begin{pmatrix} \tilde{v} \\ 0 \end{pmatrix}, \quad \tilde{v} = -m_{3/2} v \hat{m} \hat{C}/(\hat{m}_H^2 - |\hat{m}_H^2|) . \quad (34)$$

The RG equations associated with $\hat{m}$ and $\hat{C}$ are

$$d\hat{m}/dt = (\hat{m}/4\pi)(3\hat{\alpha}^U - \hat{\alpha}_1 - 3\hat{\alpha}_2) , \quad (35)$$

$$d\hat{C}/dt = -3\hat{\alpha}^U/2\pi - \hat{A}^U , \quad (36)$$

where the boundary condition for $\hat{C}$ at $t = 0$ is given in (19). The solutions to eqs. (35) and (36) are

$$\hat{m}(t) = \hat{m}(0) \exp\left(-\frac{1}{4\pi} \int_t^0 dt'\right.$$

$$\left. \times [3\hat{\alpha}^U(t') - \hat{\alpha}_1(t') - 3\hat{\alpha}_2(t')]\right) \quad (37)$$

and

$$\hat{C}(t) = 2 - \sqrt{3} + \int_t^0 dt' [(3/2\pi)\hat{\alpha}^U(t') + \hat{A}^U(t')] , \quad (38)$$

respectively. If one chooses $\hat{m}(0) \ll m_{3/2}$, our previous results will not be substantially altered. Henceforth, we take $\hat{m}(0) = 10^{-2} m_{3/2}$, which satisfies this criterion. Using our previous results, and eqs. (34), (37), and (38), we can determine $\tilde{v}$. The results for $\tilde{v}$, $\hat{m}$ at 10^2 GeV, and $\hat{C}$ at 10^2 GeV, for a range of values of $m_{3/2}$, are given in table 4. We conclude that in a theory with two Higgs doublets, $\overline{H}$ can de-

Table 4
VEV $\tilde{v}$, $\hat{m}(10^2$ GeV$)$, $\hat{C}(10^2$ GeV$)$ as functions of $m_{3/2}$ and $\hat{m}(M_{GU})$, in a theory where $\mu = 10^{16}$ GeV, $N_H = 2$, and $n = 15$.

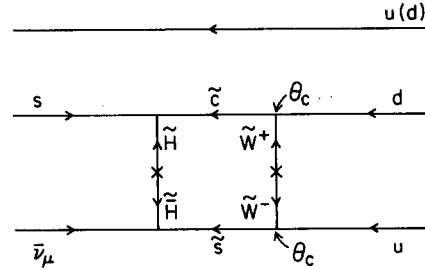| $m_{3/2}$	$\hat{m}(M_{GU})$ (GeV)	$\tilde{v}$(GeV)	$\hat{m}(10^2$ GeV$)$	$\hat{C}(10^2$ GeV$)$
10^2	1	-64.429	9.367×10^{-1}	22.454
10^4	10^2	-85.289	1.346×10^2	36.626
10^6	10^4	-76.467	1.251×10^4	35.023

Fig. 3. The main graph contributing to nucleon decay modes $p(n) \to K^+(K^0)\bar{\nu}_\mu$.

velop a non-vanishing VEV that is sufficiently large to give perturbative masses to all "down" quarks and leptons.

Finally, we want to consider the implications on proton decay of reducing the number of 5 and $\overline{5}$ Higgs superfields from four to two. In theories with four Higgs superfields the decay of nucleons into bosons and leptons is naturally suppressed [2]. In theories with two Higgs superfields this is no longer the case. The main graph contributing to the dominant decay modes, $p(n) \to K^+(K^0)\bar{\nu}_\mu$, is shown in fig. 3. The decay amplitude associated with this graph is [8]

$$A \propto b_0 g_{cuu} , \quad (39)$$

where

$$b_0 = (G_F/\sqrt{2})(m_W^2/32\pi^2) m_{\widetilde{W}}$$

$$\times [f(m_{\tilde{q}}, m_{\tilde{q}}, m_{\widetilde{W}}) + f(m_{\tilde{q}}, m_{\tilde{\varrho}}, m_{\widetilde{W}})] , \quad (40)$$

$$f(m_1, m_2, m_3) = (m_2^2 - m_3^2)^{-1}$$

$$\times [m_2^2(m_1^2 - m_2^2)^{-1} \ln(m_1^2/m_2^2)$$

$$- m_3^2(m_1^2 - m_3^2)^{-1} \ln(m_1^2/m_3^2)] \quad (41)$$

and

$$g_{cuu} = (m_c m_s/\tilde{v}\tilde{v} m_{\widetilde{H}}) \sin^2\theta_c . \quad (42)$$

The "~" denotes the superpartner to a standard particle. $\widetilde{H}$ and $\widetilde{\overline{H}}$ are the higgsinos associated with the color triplet components of Higgs scalars in H and $\overline{H}$. One expects that

$$m_{\widetilde{W}} [f(m_{\tilde{q}}, m_{\tilde{q}}, m_{\widetilde{W}}) + f(m_{\tilde{q}}, m_{\tilde{\varrho}}, m_{\widetilde{W}})] \simeq 1/m_W . \quad (43)$$

Define $\bar{b}_0$ by

$$\bar{b}_0 = (G_F/\sqrt{2}) m_W/32\pi^2 = 2.2 \times 10^6 \text{ GeV}^{-1} . \quad (44)$$

From (39) (with $b_0 \approx \bar{b}_0$), (44), and the present lower bound on the nucleon lifetime ($\tau_n \gtrsim 3 \times 10^{30}$ yr), one can infer that

$$g_{cuu} < 10^{-7}/M . \quad (45)$$

Therefore,

$$b_0 g_{cuu} \simeq \bar{b}_0 g_{cuu} < \bar{b}_0 10^{-7}/M , \quad (46)$$

which, using (42), implies that

$$(b_0/\bar{b}_0)[(m_c m_s/v\tilde{v}) \sin^2\theta_c] 10^7 M < m_{\tilde{H}} . \quad (47)$$

Assuming that $m_{\tilde{q}} = m_{\tilde{\ell}} = \bar{m}$ (a good approximation)

$$\begin{aligned} b_0/\bar{b}_0 &= m_W/\bar{m} , && \text{for } \bar{m} = m_{\tilde{W}} , \\ &= 2m_{\tilde{W}} m_W/\bar{m}^2 , && \text{for } \bar{m} \gg m_{\tilde{W}} , \\ &= (2m_W/m_{\tilde{W}}) \ln(m_{\tilde{W}}^2/\bar{m}^2) , && \text{for } \bar{m} \ll m_{\tilde{W}} . \end{aligned} \quad (48)$$

Clearly, the smallest value of $b_0/\bar{b}_0$ is obtained when $\bar{m} \gg m_{\tilde{W}}$. Taking typical values for $m_W = 10^2$ GeV, $m_c = 1.5$ GeV, $m_s = 0.15$ GeV, $\sin\theta_c = 0.2$, and $v = \tilde{v} = 250/\sqrt{2}$ GeV, we find that

$$m_{\tilde{H}} > 2 \times 10^{15} \text{ GeV}(m_{\tilde{W}}/2 \text{ GeV})(10^6 \text{ GeV}^2/\bar{m}^2) . \quad (49)$$

The tree level expression for $m_{\tilde{H}}$ (in terms of μ) can be derived from (1) and (2). We conclude that, in order for our theory to have only two $\mathbf{5}, \bar{\mathbf{5}}$ Higgs doublets and still be consistent with the lower bound on the nucleon lifetime, it is necessary for

$$\mu > (1/6.83\lambda_3)(M/M_{GU})(2 \times 10^{15} \text{ GeV})$$

$$\times (m_{\tilde{W}}/2 \text{ GeV})(10^{16} \text{ GeV}^2/\bar{m}^2) . \quad (50)$$

For typical parameters (50) implies that $\mu > O(10^{15} - 10^{16} \text{ GeV})$.

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