THE GEOMETRICAL HIERARCHY MODEL AND $N = 1$ SUPERGRAVITY ☆

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We incorporate the geometrical hierarchy model of Dimopoulos and Raby into $N = 1$ supergravity. Supersymmetry is spontaneously broken at a scale of order 10^{11} GeV and the cosmological constant is fine tuned to zero. A grand unification mass of order M_{Planck} is induced at tree level. We discuss the low energy ($E \sim$ gravitino mass) theory and the ramifications of our model for proton decay.

Recently, many authors [1] ‡1 have incorporated globally supersymmetric unified models into $N = 1$ supergravity ‡2. Supersymmetry (SUSY) [5] is spontaneously broken at an intermediate scale of order 10^{10}–10^{11} GeV ‡3. These authors assume that this breaking occurs in a "hidden" super Higgs sector which, in the global SUSY limit ($M_{\text{Planck}} \to \infty$, and all other scales fixed), decouples from the rest of the theory.

In this paper we discuss a model where the super Higgs sector is not "hidden". In fact, in the global SUSY limit we obtain the geometrical hierarchy model of ref. [6], the super Higgs sector becoming the usual O'Raifeartaigh sector of that model. We find that SUSY is spontaneously broken at an intermediate scale, μ, which we adjust to be of order 10^{11} GeV. The cosmological constant is fine tuned to be zero. In the globally supersymmetric geometrical hierarchy model the grand unification mass (GUM) must be induced by radiative corrections. However, when that model is embedded in $N = 1$ supergravity, and the cosmological constant cancelled, we find that the GUM is induced at tree level and is of order M_{Planck}. We have evaluated the effective low energy potential and obtain results similar to those of the "hidden" sector scenarios. The characteristic features of our model are (1) its non-minimal low energy spectrum which includes [in addition to the usual SUSY quark, lepton and $SU(3)_C \times SU(2)_L \times U(1)$ gauge multiplets] four Higgs doublets, a color octet, and a weak $SU(2)_L$ triplet and (2) color triplet Higgs scalars with mass of order 10^{11} GeV. These color triplet scalars induce nucleon decay with the dominant modes being $p \to K^0\mu^+$, $K^+\bar{\nu}$ and $n \to K^0\bar{\nu}$.

This paper is organized as follows. We first introduce the model and discuss the tree level minimization of the potential as an expansion in the small parameter $\mu/M_{\text{Planck}} \simeq 10^{-8}$. We then describe the low energy potential and discuss some of the physical implications of the theory.

The gauge group of our model is $G = SU(5)$. The chiral superfields are

(1) A.Z, and X. A and Z are **24**'s and X a singlet under SU(5). These fields form the O'Raifeartaigh sector of the model. A and Z develop vacuum expectation values (VEV) which break $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)$.

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‡1 See also ref. [2] for low energy local SUSY models.
‡2 For recent work on coupling general matter multiplets in $N = 1$ supergravity, see ref. [3]. For a review of supergravity, see ref. [4].
‡3 Intermediate scale SUSY breaking in global models was discussed in ref. [6].

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(2) $H, H_1, \bar{H}, \bar{H}_1$. H, H_1 are **5**'s and $\bar{H}, \bar{H}_1$ are $\bar{\mathbf{5}}$'s under SU(5). The $SU(2)_L$ doublet components of H, $\bar{H}$ act as Higgs fields, these VEV's giving mass to quarks and leptons. The extra superfields $H_1, \bar{H}_1$ are introduced to eliminate certain dimension 5 operators which cause the proton to decay too rapidly [7].

(3) $\bar{5}_J, 10_J$. $\bar{5}_J$ and 10_J are $\bar{\mathbf{5}}$'s and **10**'s respectively under SU(5). We assume there are three lepto-quark families ($J = 1, 2, 3$).

The superpotential of our model is then gives by [‡4]

$$W = \lambda_1 X(\mathrm{tr}\, A^2 - \mu^2) + \lambda_2\, \mathrm{tr}\, ZA^2 + \lambda_3 \bar{H}(A + m\mathbf{1})\, H_1 + \lambda_4 \bar{H}_1 (A + m\mathbf{1})\, H + \lambda^U_{IJ} H 10_I 10_J + \lambda^D_{IJ} \bar{H} 10_I \bar{5}_J + \beta\mu^2 M. \tag{1}$$

The parameters λ_i, λ^U, λ^D and β are dimensionless and μ, m and M have dimension 1. Parameter μ sets the scale of SUSY breaking ($\mu \approx 10^{11}$ GeV), m will be fine tuned to give the $SU(2)_L$ doublets in $H, H_1, \bar{H}, \bar{H}_1$ vanishing tree level mass in the $M \to \infty$ limit ($m \approx 10^{11}$ GeV), and $M = (1/\sqrt{8\pi}) M_p$ where M_p is the Planck mass ($M \approx 10^{18}$ GeV). Note that the $\beta\mu^2 M$ term vanishes in a globally supersymmetric theory but not when it is coupled to $N = 1$ supergravity. Parameter β will be fine tuned so as to give zero cosmological constant. Without the $\beta\mu^2 M$ term our model would have an R-invariance [8]. The $\beta\mu^2 M$ term explicitly breaks this symmetry.

When W is coupled to $N = 1$ supergravity the potential energy becomes [3,4]

$$V = \exp(K/M^2) \left(\sum_i |D_{\phi_i} W|^2 - \frac{3}{M^2} |W|^2 \right) + \tfrac{1}{2}\, \mathrm{tr}\, D^2, \tag{2}$$

and

$$K = \sum_i |\phi_i|^2, \qquad D_{\phi_i} W = \partial W/\partial \phi_i + (\phi_i^\dagger / M^2)\, W. \tag{3}$$

[‡4] We have, for the sake of simplicity, ignored the terms in W of the form [6]

$$Y(\bar{H}_1 H_1 + C^2 + \tilde{\mu}^2)$$

These are necessary to break the global symmetry

$$H \to e^{2i\alpha} H, \quad \bar{H} \to e^{2i\alpha} \bar{H},$$

$$H_1 \to e^{-2i\alpha} H_1, \quad \bar{H}_1 \to e^{-2i\alpha} \bar{H}_1,$$

$$10 \to e^{-i\alpha} 10, \quad \bar{5} \to e^{-i\alpha} \bar{5},$$

where all other fields are unchanged.

We now find the absolute minimum of V, fine tuning β so as to make the value of V at this minimum (the cosmological constant) vanish. That is, we solve

$$\langle \partial V/\partial \phi_i \rangle = 0, \tag{4}$$

$$\sum_i \langle |D_{\phi_i} W|^2 \rangle = (3/M^2) \langle |W|^2 \rangle, \tag{5}$$

simultaneously. This is most easily done by expanding the VEV of ϕ_i as

$$\langle \phi_i \rangle = \langle \phi_i \rangle_0 \sum_{n=0}^{\infty} c_n \epsilon^n, \tag{6}$$

where $c_0 = 1$ and $\epsilon = \mu/M$, and realizing that

$$\langle X \rangle_0 \sim O(M), \quad \langle Z \rangle_0 \sim O(M),$$

$$\langle A \rangle_0 \sim O(\mu), \quad \langle H \rangle_0\text{'s} \sim O(\mu(\mu/M)). \tag{7}$$

We also expand the parameter β in a manner similar to (6). Let us solve eq. (4) first. Keeping terms of $O(\mu M^2)$ only we find that

$$\lambda_1 \langle X \rangle_0 \langle A \rangle_0 + \lambda_2 \langle Z \rangle_0 \langle A \rangle_0 - \tfrac{1}{5} \lambda_2 (\mathrm{tr} \langle Z \rangle_0 \langle A \rangle_0)\, \mathbf{1} = 0. \tag{8}$$

At $O(\mu^2 M)$ we learn nothing new, but at $O(\mu^3)$ we find that $x_1 = z_1$ and

$$\lambda_1^2 \langle A \rangle_0 (\mathrm{tr} \langle A \rangle_0^2 - \mu^2) + \lambda_2^2 [\langle A \rangle_0^3 - \tfrac{1}{5} \langle A \rangle_0\, \mathrm{tr} \langle A \rangle_0^2 - \tfrac{1}{5} (\mathrm{tr} \langle A \rangle_0^3)\, \mathbf{1}] = 0. \tag{9}$$

Combining eqs. (8) and (9) gives

$$\langle A \rangle_0 = [\lambda_1 \mu / (\lambda_2^2 + 30\lambda_1^2)]\, \mathrm{diag}(2, 2, 2, -3, -3), \tag{10}$$

which breaks SU(5) to $SU(3)_C \times SU(2)_L \times U(1)$ at mass scale μ. Combining (8) and (10) implies that

$$\langle Z \rangle_0 = (\lambda_1/\lambda_2) \langle X \rangle_0\, \mathrm{diag}(2, 2, 2, -3, -3), \tag{11}$$

which gives the same pattern of breaking but at scale $\langle X \rangle_0$. Keeping terms of $O(\mu^3 (\mu/M))$ we find

$$x_2 - z_2 = \tfrac{1}{2} - \beta_0 M / 2\lambda_1 \langle x \rangle_0. \tag{12}$$

Solving the remaining equations simultaneously with the lowest order equation of (5) gives

$$\langle X \rangle_0 = \lambda_2 (\lambda_2^2 + 30\lambda_1^2)^{-1/2} (\sqrt{3} \pm 1) M,$$

$$\beta_0 = \pm \lambda_1 \lambda_2 (\lambda_2^2 + 30\lambda_1^2)^{-1/2} (2 \pm \sqrt{3}). \tag{13}$$

To this order the $\langle H \rangle_0$'s, $\langle \bar{5} \rangle_0$ and $\langle 10 \rangle_0$ are undetermined. The lower solution in (13) is the minimum. Therefore, putting everything together we find that, to $O(\mu^3(\mu/M))$, the potential V has its absolute minimum at

$$\langle X\rangle_0 = \lambda_2(\lambda_2^2+30\lambda_1^2)^{-1/2}(\sqrt{3}-1)M,$$

$$\langle Z\rangle_0 = \lambda_1(\lambda_2^2+30\lambda_1^2)^{-1/2}(\sqrt{3}-1)M \times \mathrm{diag}(2,2,2,-3,-3),$$

$$\langle A\rangle_0 = \lambda_1\mu(\lambda_2^2+30\lambda_1^2)^{-1/2}\,\mathrm{diag}(2,2,2,-3,-3),$$

$$\langle H\rangle_0\text{'s},\ \langle\bar{5}\rangle_0,\ \langle 10\rangle_0 \quad \text{undetermined.} \tag{14}$$

This solution has vanishing cosmological constant as long as we take

$$\beta_0 = -\lambda_1\lambda_2(\lambda_2^2+30\lambda_1^2)^{-1/2}(2-\sqrt{3}). \tag{15}$$

Note that $\langle Z\rangle_0$ breaks SU(5) down to $SU(3)_C \times SU(2)_L \times U(1)$ at a mass scale of $O(M)$. SUSY is spontaneously broken at this minimum if and only if $\langle D_{\phi_i} W\rangle \neq 0$ for at least one field ϕ_i. The values of $\langle D_{\phi_i} W\rangle$ evaluated to $O(\mu^2)$ at (14) are

$$\langle D_X W\rangle = -\sqrt{3}\lambda_1\lambda_2\mu^2(\lambda_2^2+30\lambda_1^2)^{-1},$$

$$\langle D_Z W\rangle = -\sqrt{3}\lambda_1^2\lambda_2\mu^2(\lambda_2^2+30\lambda_1^2)^{-1} \times \mathrm{diag}(2,2,2,-3,-3). \tag{16}$$

All other $\langle D_{\phi_i} W\rangle$ vanish to this order. Therefore, SUSY is spontaneously broken by this vacuum state at a scale of $O(\mu^2)$. Keeping terms of $O(\mu^3(\mu/M)^2)$ in (4) we find that

$$x_1 = z_1 = a_1 = 0, \qquad x_3 = z_3,$$

$$\lambda_3\langle\bar{H}\rangle_0\langle H_1\rangle_0 + \lambda_4\langle\bar{H}\rangle_0\langle H\rangle_0 = 0. \tag{17}$$

The cosmological constant will vanish as long as

$$b_1 = 0. \tag{18}$$

Calculating $\langle D_A W\rangle$ to this order $(O(\mu^2(\mu/M)^2))$ we find that

$$\langle D_A W\rangle = 0. \tag{19}$$

We can conclude the following. Prior to coupling to supergravity, the O'Raifeartaigh sector of our model has a flat direction and $\langle X\rangle$, $\langle Z\rangle$ are undetermined at tree level. When the theory is coupled to $N = 1$ supergravity this flat direction is altered and an absolute minimum of the potential develops [eqs. (14)] which breaks SU(5) down to $SU(3)_C \times SU(2)_L \times U(1)$ [at a scale of $O(M)$], and breaks SUSY [at a scale of $O(\mu^2)$]. The cosmological constant is fine tuned to be zero. This vacuum state is determined at tree level. The VEV $\langle X\rangle$ spontaneously breaks R-invariance. However, the $\beta\mu^2 M$ term explicitly breaks this symmetry and gives the pseudo-Goldstone boson a mass of $O(\mu(\mu/M))$.

We now derive the tree level, low energy scalar potential. The starting point for our discussion is eqs. (1) and (2). Superpotential (1) has a natural separation into the super-Higgs sector

$$h(Z_i, A) = \lambda_1 X(\mathrm{tr}\,A^2-\mu^2) + \lambda_2\,\mathrm{tr}\,ZA^2 + \beta\mu^2 M \tag{20}$$

(where the fields X and Z are denoted by Z_i) and the rest

$$g(y_a, A) = W - h \tag{21}$$

(where y_a are all the other fields). Note that the adjoint field A is included in both sectors. The potential energy (2) can then be written as

$$V = \exp(K/M^2)\,[|h_i + (Z_i^\dagger/M^2)(h+g)|^2 + |D_A W|^2 + |g_a + (y_a^\dagger/M^2)(h+g)|^2 - 3M^{-2}|h+g|^2] + \tfrac{1}{2}\,\mathrm{tr}\,D^2, \tag{22}$$

where

$$h_i = \partial h/\partial Z_i, \qquad g_a = \partial g/\partial y_a. \tag{23}$$

The low energy potential, V_{LE}, is obtained by (1) shifting X, A, and Z around their VEV's (14), (2) eliminating the supermassive field A completely using its low energy equation of motion, (3) ignoring all interactions involving only heavy fields, and then (4) taking the limit $M\to\infty$, $\mu\to\infty$ keeping $\mu(\mu/M)$ fixed. We find that the $|D_A W|^2$ term decouples in this limit. The low energy potential is given by

$$V_{LE} = V_{LE}^{(1)}(y_a) + V_{LE}^{(2)}(Z_3, Z_8). \tag{24}$$

The potential $V_{LE}^{(1)}$ for the fields y_a is identical to that obtained previously for the "hidden" sector scenarios. We find

$$V_{LE}^{(1)}(y_a) = |\tilde{g}_a|^2 + (m_{3/2}A\tilde{g} + \text{h.c.}) + [m_{3/2}(y_a\tilde{g}_a - 3\tilde{g}) + \text{h.c.}] + m_{3/2}^2|y_a|^2 + \tfrac{1}{2}D_{y\alpha}D_{y\alpha} \tag{25}$$

where

$$\tilde{g} = \exp(\tfrac{1}{2}\,|b_i|^2)\, g(y_a, \langle A\rangle)$$

$$m_{3/2} = \exp(\tfrac{1}{2}\,|b_i|^2)\, h(\langle Z_i\rangle, \langle A\rangle) M^{-2}$$

$$A = b_i^* (a_i + b_i). \tag{26}$$

The constants a_i and b_i are defined by

$$\langle h_i\rangle = a_i^*\, h(\langle Z_i\rangle, \langle A\rangle) M^{-1}, \quad \langle Z_i\rangle = b_i M, \tag{27}$$

with $|a_i + b_i|^2 = 3$ as required by the vanishing of the cosmological constant. Using (14) and (15) we have

$$\tilde{g} = \exp(2 - \sqrt{3})\, g,$$

$$m_{3/2} = \exp(2 - \sqrt{3})\, \lambda_1 \lambda_2 (\lambda_2^2 + 30\lambda_1^2)^{-1/2} \mu(\mu/M),$$

$$A = 3 - \sqrt{3}. \tag{28}$$

If we now assume that m is fine tuned to make the Higgs doublet masses vanish in the limit that $M \to \infty$, then $g(y_a, \langle A\rangle)$ contains no dimensional parameters. It follows that

$$y_a \tilde{g}_a = 3\tilde{g}, \tag{29}$$

and, since $|A| < 3$, all y_a scalar masses are identical and given by $m_{3/2}$ (ignoring weak interaction breaking and Yukawa couplings). Thus the GIM cancellation required to avoid unobserved flavor changing neutral currents is automatic in our model.

The fields Z_3 and Z_8 are the SU(2) triplet and color octet components of Z (after expanding around $\langle Z\rangle$). We find that $V_{\mathrm{LE}}^{(2)}$ for the fields Z_3 and Z_8 is given by

$$V_{\mathrm{LE}}^{(2)}(Z_3, Z_8) = \mathrm{tr}\,|\partial \tilde{f}/\partial Z_3|^2 + \mathrm{tr}\,|\partial \tilde{f}/\partial Z_8|^2$$

$$+ (m_{3/2} A' \tilde{f} + \mathrm{h.c.})$$

$$+ m_{3/2}^2 (\mathrm{tr}\,|Z_3|^2 + \mathrm{tr}\,|Z_8|^2) + \tfrac{1}{2} D_{Z\alpha} D_{Z\alpha}, \tag{30}$$

where

$$\tilde{f} = m_{3/2} (\sqrt{3} - 1)^{-1} (\tfrac{9}{2}\,\mathrm{tr}\, Z_3^2 - \tfrac{4}{3}\,\mathrm{tr}\, Z_8^2),$$

$$A' = -\tfrac{5}{2} (\tfrac{3}{5}\sqrt{3} - 1). \tag{31}$$

Since $|A'| < 3$, all scalar masses are positive and, since $|A'| \ll 1$, these masses are of $O(m_{3/2})$. Note that all scalar masses in the low energy sector are positive and of $O(m_{3/2})$. Hence, at tree level, $SU(2)_L \times U(1)$ remains unbroken. However, it has been shown in ref. [9] that the weak interaction breaking may be induced naturally if the top quark mass is sufficiently large. In that case radiative corrections to the Higgs scalar can drive symmetry breaking.

The model predicts a non-minimal, low energy spectrum including SUSY chiral multiplets of four Higgs doublets, a color octet and an $SU(2)_L$ triplet. There are also color triplet Higgs scalars whose masses are constrained by the relations

$$m^{\mathrm{c}}_{\bar{\mathrm{H}},\mathrm{H}_1} \leqslant 2.1\, (\lambda_3/\sqrt{\lambda_2})(m_{3/2}M)^{1/2},$$

$$m^{\mathrm{c}}_{\bar{\mathrm{H}}_1,\mathrm{H}} \leqslant 2.1\, (\lambda_4/\sqrt{\lambda_2})(m_{3/2}M)^{1/2}. \tag{32}$$

For $m_{3/2} \approx 10^2$ GeV and $M \approx 10^{18}$ GeV we have

$$m^{\mathrm{c}}_{\mathrm{H}} \lesssim [\lambda_3 (\lambda_4)/\sqrt{\lambda_2}]\, 10^{10}\ \mathrm{GeV}. \tag{33}$$

The parameters λ_i must be smaller than unity since we want to remain in a perturbative regime. The masses $m^{\mathrm{c}}_{\mathrm{H}}$ can be made greater than 10^{10} GeV but only by unnaturally taking $\lambda_2 \ll \lambda_3, \lambda_4$. Hence, we expect $m^{\mathrm{c}}_{\mathrm{H}}$ to be of $O(10^{10}$ GeV). These scalars induce nucleon decay with dominant decay modes

$$\mathrm{p} \to \mathrm{K}^0 \mu^+, \mathrm{K}^+ \bar{\nu}, \quad \mathrm{n} \to \mathrm{K}^0 \bar{\nu}. \tag{34}$$

Since $m^{\mathrm{c}}_{\mathrm{H}}$ are not ultra heavy we expect these decays to be observable. Aesthetically the model has two shortcomings. (1) We have not explained why the Higgs doublet are light. They are arranged to be so via a tree level fine tuning. (2) Since the GUM is of $O(M)$ we may not be able to predict the value of $\sin^2\theta_{\mathrm{W}}$.

Finally, we have not discussed the electroweak symmetry breaking. This must occur via radiative corrections. Recent work has shown that this possibility is very natural [9]. We will discuss this question, and present a detailed discussion of the phenomenology of our model, in future publications.

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