SOFT BREAKING OF $N = 4$ SUPERSYMMETRY

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We consider the addition of soft breaking terms to the $N = 4$ supersymmetric Yang–Mills lagrangian. $N = 1$ supersymmetric mass terms and mass terms of the form $(A^2 - B^2)$ do not give rise to any new divergence, which implies that in a gauge in which the massless theory is completely finite, this finiteness is preserved. All other soft terms give rise to at most logarithmic divergences.

1. Introduction. The $N = 4$ extended supersymmetric Yang–Mills theory [1] may be the first example of a completely finite theory in four dimensions. It has recently been shown [2] in the light cone gauge superfield formalism that there is no renormalization of the coupling constant in this theory. Because the wave function renormalization is gauge dependent, this has gone most of the way in establishing the complete finiteness of the theory in some gauge. Actually, calculations have been performed up to three loops [3], in a corrected supersymmetric Fermi–Feynman gauge, and verified that the wave function renormalization is also finite.

Now, it is known that adding soft interaction terms (with mass dimension $\leqslant 3$) to a lagrangian often does not change the divergence structure of a theory [4]. (In particular, the divergences of the wave function and dimensionless coupling constant renormalizations are not affected by these terms.) Therefore, we have made a systematic investigation of the effects of soft breaking on the theory.

In section 2 of this paper, we consider the basic lagrangian and discuss the effects of adding terms which will preserve ($N = 1$) supersymmetry. In section 3, we consider the effects of adding non-supersymmetric soft terms. A simple SU(2) model is given in section 4 to illustrate the possible mass spectra.

2. Breaking by a $N = 1$ supersymmetric mass term. The $N = 4$ supersymmetric Yang–Mills lagrangian describes one Yang–Mills field, four Weyl spinors and six scalars, all in the adjoint representation of a gauge group and having zero mass. The lagrangian density can be found in ref. [1]. We just want to mention the existence of an SU(4) "family" invariance. One can also describe the theory in terms of $N = 1$ superfields. It then consists of one gauge multiplet and three scalar multiplets, whereby the lagrangian is

$$\mathcal{L} = \mathrm{tr}\Bigg(\int \mathrm{d}^4\theta\, \mathrm{e}^{-gV}\bar{\Phi}_i \mathrm{e}^{gV}\Phi_i + \frac{1}{64g^2}\int \mathrm{d}^2\theta\, W^\alpha W_\alpha + \frac{\mathrm{i}g}{3!}\epsilon_{ijk}\int \mathrm{d}^2\theta\, \Phi_i[\Phi_j, \Phi_k] + \frac{\mathrm{i}g}{3!}\epsilon_{ijk}\int \mathrm{d}^2\bar{\theta}\, \bar{\phi}_i[\bar{\Phi}_j, \bar{\Phi}_k]\Bigg)$$

$$i, j, k = 1, 2, 3\,, \qquad (1)$$

where

$\Phi_i = \phi_i^a \tau_a/\sqrt{2}$, $W_\alpha = \bar{D}^2(\mathrm{e}^{-gV}D_\alpha \mathrm{e}^{gV})$, $V = V^a\tau_a/\sqrt{2}$ $(\mathrm{tr}\,\tau_a\tau_b = 2\delta_{ab})$.

This lagrangian is invariant under the set of gauge transformations

$$\mathrm{e}^{gV} \to \mathrm{e}^{\mathrm{i}g\bar{\Lambda}}\mathrm{e}^{gV}\mathrm{e}^{-\mathrm{i}g\Lambda}\,, \quad \mathrm{e}^{-gV} \to \mathrm{e}^{\mathrm{i}g\Lambda}\mathrm{e}^{-gV}\mathrm{e}^{-\mathrm{i}g\bar{\Lambda}}\,,$$
$$\Phi_i \to \mathrm{e}^{\mathrm{i}g\Lambda}\Phi_i\mathrm{e}^{-\mathrm{i}g\Lambda}\,, \quad \bar{\Phi}_j \to \mathrm{e}^{\mathrm{i}g\bar{\Lambda}}\bar{\Phi}_j\mathrm{e}^{-\mathrm{i}g\bar{\Lambda}}\,, \qquad (2)$$
$$D_\alpha\bar{\Lambda} = \bar{D}^{\dot{\alpha}}\Lambda = 0\,.$$

In this formulation, it is simple to see that adding a term

$$\mathcal{L}_{\text{break}} = \int d^2\theta\, m_{ij}\phi_i^a\phi_j^a + \text{h.c.} \tag{3}$$

to the superpotential does not give rise to new divergences. This follows, because the superpotential does not get renormalized [5] and because the divergences of the wave function and the dimensionless coupling constant renormalizations are mass independent [4], for example in the minimal subtraction scheme.

3. Soft supersymmetry breaking terms. In this section, we make an analysis of soft supersymmetric breaking terms. Clearly, any term added should not break the local gauge symmetry of eq. (2), otherwise the theory will not even be renormalizable. We follow the analysis of Girardello and Grisaru [6], where supersymmetry is broken through the coupling to fixed spurion fields. The possible divergences can in this way be found by superfield power counting. We have the following cases:

Case a:

$$\mathcal{L}_{\text{break}} = \int d^4\theta\, U_{ij}\bar\phi_i^a\phi_j^a = \tfrac{1}{2}\mu_{ij}^2(A_i^aA_j^a + B_i^aB_j^a)\,, \tag{4}$$

where $U_{ij} = \mu_{ij}^2\theta^2\bar\theta^2$. This expression has been written in the Wess–Zumino gauge. This may have a logarithmically divergent renormalization of the term itself. An explicit calculation in the supersymmetric Fermi–Feynman gauge [‡1] shows that at least at the one-loop level the effects are finite if $\Sigma_i\mu_{ii}^2 = 0$. The diagrams which contribute are shown in figs. 1a–1d.

In a general supersymmetric model, there will also be a logarithmically divergent term linear in ϕ_i^a. This is forbidden by gauge invariance, because we have no singlets.

Case b:

$$\mathcal{L}_{\text{break}} = \int d^2\theta\, \chi_{ij}\phi_i^a\phi_j^a + \text{h.c.} = \bar\mu_{ij}^2(A_i^aA_j^a - B_i^aB_j^a)\,, \tag{5}$$

where $\chi_{ij} = \bar\mu_{ij}^2\theta^2$. In a general supersymmetric model,

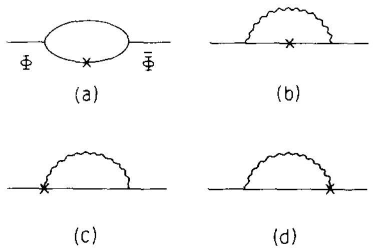

Fig. 1. Mass insertions due to eq. (4) and footnote one. Solid and wavy lines are scalar and vector propagators, respectively.

this generates a new logarithmic divergence

$$\Delta\mathcal{L} \sim \int d^4\theta\, \bar\chi_{ij}\phi_j^a + \text{h.c.}$$

$$\sim F^a\,. \tag{6}$$

But this is impossible because of gauge invariance. *Therefore the addition of such a term keeps the theory finite, even though supersymmetry is broken.*

Case c[1]:

$$\mathcal{L}_{\text{break}} = \int d^4\theta\, \eta W^\alpha W_\alpha + \text{h.c.}\,,$$

$$\eta = \mu\theta^2\,, \tag{7}$$

corresponding to giving the gauge fermions an explicit mass. This causes logarithmic divergences in

$$\Delta\mathcal{L} \sim \int d^4\theta\, \eta\bar\phi_i^a\phi_i^a + \text{h.c.}\,,$$

$$\Delta\mathcal{L} \sim \int d^4\theta\, \eta\bar\eta\bar\phi_i^a\phi_i^a\,. \tag{8}$$

Case c[2]:

$$\mathcal{L}_{\text{break}} = \int d^4\theta\, \bar U_{ij}D^\alpha\phi_i^aD_\alpha\phi_j^a + \text{h.c.}\,,$$

$$\bar U_{ij} = \bar\mu_{ij}\theta^2\bar\theta^2 \tag{9}$$

(again, this is written in the Wess–Zumino gauge.) This corresponds to fermion mass terms of the scalar multiplets. Because of the SU(4) invariance, this is equivalent to case c[1] and so gives rise to only logarithmic divergences. This is in contrast to the general supersymmetric case where a quadratically divergent term linear in A may be generated.

[‡1] The breaking term is

$$\mathcal{L}_{\text{break}} = \text{tr}\int d^4\theta\, \mu_{ij}^2\theta^2\bar\theta^2 e^{gV}\Phi_i\cdot e^{-gV}\bar\Phi_j$$

in a general gauge.

Table 1
Various possibilities of eq. (12).

Parameters and solutions		Gauge invariance	Conformal invariance
$m_i = 0$	$s_i = 0$	SU(2)	unbroken
(this is the original theory)	$s_1 \neq 0,\ s_2 = s_3 = 0$	SU(2) → U(1)	spontaneously broken
$m_1 \neq 0;\ m_2 = m_3 = 0$	$s_i = 0$	SU(2)	explicitly broken
	$s_1 = 0;\ s_2 \parallel s_3$	SU(2) → U(1)	explicitly and spontaneously broken
$m_1 = 0;\ m_2, m_3 \neq 0$	$s_i = 0$	SU(2)	explicitly broken
	$s_1 \neq 0;\ s_2 = s_3 = 0$	SU(2) → U(1)	explicitly and spontaneously broken
$m_1, m_2, m_3 \neq 0$	$s_i = 0$	SU(2)	explicitly broken
	$s_1 = -(m_2 m_3)^{1/2}\,\hat{i}$	completely broken	explicitly and spontaneously broken
	$s_2 = -(m_1 m_3)^{1/2}\,\hat{j}$		
	$s_3 = -(m_1 m_2)^{1/2}\,\hat{k}$		

Case d:

$$\mathcal{L}_{\text{break}} = \int d^2\theta\, \eta_{ijk} \epsilon^{abc} \phi_i^a \phi_j^b \phi_k^c + \text{h.c.}$$

$$\sim \mu(A^3 - 3AB^2)\,,$$

where

$$\eta_{ijk} = \mu_{ijk}\, \theta^2\,, \tag{10}$$

gives rise to only logarithmic divergences.

Case e:

$$\mathcal{L}_{\text{break}} \sim \text{tr} \int d^4\theta\, U(\Phi + \bar{\Phi})^3 \sim \gamma A^3\,,$$

$$U \sim \gamma \theta^2 \bar{\theta}^2 \tag{11}$$

(again written in Wess–Zumino gauge) can cause only logarithmic divergences.

In summary, all soft breaking terms are indeed soft, in the sense that they do not cause quadratic divergences.

4. A model. We consider a simple SU(2) model with an $N = 1$ supersymmetric mass term

$$-\int d^2\theta\, \tilde{m}_{ij} \boldsymbol{\phi}_i \cdot \boldsymbol{\phi}_j + \text{h.c.}\,.$$

Then, there are always supersymmetric vacua determined by the conditions

$$\epsilon_{ijk}\, \boldsymbol{s}_j \times \boldsymbol{s}_k + m_{ij} \boldsymbol{s}_j = 0\,, \quad \boldsymbol{s}_i^* \times \boldsymbol{s}_i = 0\,, \tag{12}$$

where

$$m_{ij} = 4\tilde{m}_{ij}/\sqrt{2}g\,, \tag{13}$$

$$\boldsymbol{s}_i \equiv \langle \boldsymbol{s}_i \rangle = 2^{-1/2}(\boldsymbol{A}_i + \mathrm{i}\boldsymbol{B}_i)\,,$$

are the vacuum expectation values of the spin zero components of $\boldsymbol{\phi}_i$. By a rotation in the "family" space, we can make m_{ij} diagonal with eigenvalues m_1, m_2, and m_3. Now, there are a number of inequivalent possibilities which are listed in table 1.

We find that the last case is particularly interesting. All particles are massive here and therefore there is no infrared problem. Besides, the theory is finite and should serve as a useful model to study explicit and spontaneous breaking of conformal invariance. The existence of conformal invariance may be of use in the construction of Green's functions.

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