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A THEORETICAL ANALYSIS AND CONTROL STUDY OF OPEN-CIRCUIT GRINDING

G.T. ADEL[1], A.G. ULSOY[2] and K.V.S. SASTRY[1]

[1] *Department of Materials Science and Mineral Engineering, University of California at Berkeley, Berkeley, CA. 94720 (U.S.A.)*
[2] *Department of Mechanical Engineering and Applied Mechanics, University of Michigan, Ann Arbor, MI 48109 (U.S.A.)*

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ABSTRACT

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Regrinding is an essential step in many mineral processing flowsheets as a final preparatory step for mineral separation. It is normally carried out in open-circuit ball mills or rod mills. Wide fluctuations occurring in the hardness and size distribution of feed materials to regrinding mills result in a nonuniform product fineness and processing inefficiency. Using a previously developed regrinding model, this paper presents the dynamic analysis and design considerations of a control system for open-circuit grinding, using the traditional P-, PI-, and PID-control algorithms. This study shows the utility of simple analysis techniques in designing a mineral process control system with special attention given to the effect of sampling and analysis time on control system performance.

INTRODUCTION

Grinding is a size-reduction operation of mineral processing that is used to liberate valuable minerals from an ore. Mineral feeds prepared for subsequent processing by flotation or pelletization often require further grinding, with this regrinding being carried out in open-circuit ball or rod mills. Hardness and size distribution fluctuations in the mill feed result in product-fineness nonuniformity and subsequent processing-operation inefficiency. Proper circuit control is required to ensure product-fineness uniformity.

Design considerations of Sastry and Wakeman (1980) include the use of pre- and post-mix tanks to smooth rapid fluctuations in the feed characteristics. Control is based on feed-rate manipulation. Simulation results show that:

(a) Pre- and post-mix tanks significantly reduce adverse effects of feed hardness and size fluctuations on system performance.

(b) Measuring the mill product-fineness and automatically adjusting the circuit feed-rate results in substantial improvements in circuit performance.

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(c) Sampling and analysis time delays affect control system performance.

It is recognized that a complete analysis of regrinding is possible by employing the phenomenological models of the grinding process (e.g., Herbst et al., 1974; Austin et al., 1975; Herbst and Rajamani, 1979), however, such models require extensive computation to obtain practical solutions. Therefore, the purpose of this study is to present a formal analysis and design of a control system for open-circuit grinding based on the simple model equations developed by Sastry and Wakemen (1980). Furthermore, this paper is intended to illustrate the application of the more simple control analysis and design techniques to mineral processing systems.

PROCESS ANALYSIS

The basic model equations describing a completely mixed grinding mill operated in an open circuit are given by (Sastry and Wakeman, 1980):

$$H\left(\frac{dS_1}{dt}\right) = F(S_o - S_1) + PG_1 \tag{1}$$

$$H\left(\frac{dG_1}{dt}\right) = F(G_o - G_1) \tag{2}$$

The subscript, 1, refers to the mill and product contents, and the subscript, o, refers to the mill feed. System control is provided by manipulating the feed rate to the mill based on measurements of the mill product surface area as shown in Fig. 1. Note that the variables — surface area (S) and grindability (G) — correspond respectively to the fineness and hardness properties of the material being ground.

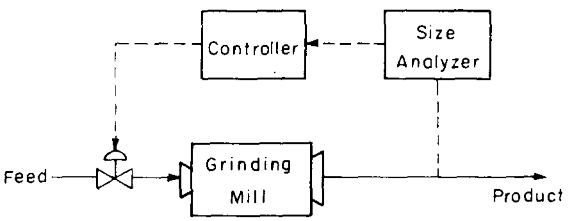

Fig. 1. Schematic of an open-circuit grinding mill control system.

Assume that the mill operates at the steady-state conditions characterized by the equilibrium values s_{oe}, s_{1e}, f_e, g_{1e}, and g_{oe}. Assume also that H and P are constants for the mill system. A set of linear equations about the equilibrium state is then obtained in terms of incremental variables — s_o, s_1, f, g_o, and g_1 — which are given by:

$$S_o = s_{oe} + s_o \tag{3}$$

$$S_1 = s_{1e} + s_1 \tag{4}$$

$$F = f_e + f \tag{5}$$

$$G_o = g_{oe} + g_o \tag{6}$$

$$G_1 = g_{1e} + g_1 \tag{7}$$

Equations 1 through 7 can be combined and the following two equations are obtained after using the definition of the equilibrium state:

$$[f_e(s_{oe} - s_{1e}) + P(g_{1e})]/H = 0 \tag{8}$$

$$f_e(g_{oe} - g_{1e})/H = 0 \tag{9}$$

From eq. 9:

$$g_{oe} = g_{1e} = g_e \tag{10}$$

Equations 1 and 2 can be linearized in terms of the incremental variables by making use of eqs. 3—10 and by neglecting terms containing products of incremental variables, thus obtaining:

$$\frac{d}{dt}\begin{Bmatrix} s_1 \\ g_1 \end{Bmatrix} = \begin{bmatrix} -(f_e/H) & (P/H) \\ 0 & -(f_e/H) \end{bmatrix}\begin{Bmatrix} s_1 \\ g_1 \end{Bmatrix} + \begin{bmatrix} f_e/H & 0 \\ 0 & f_e/H \end{bmatrix}\begin{Bmatrix} s_o \\ g_o \end{Bmatrix} + \begin{bmatrix} (s_{oe} - s_{1e})/H \\ 0 \end{bmatrix} \tag{11}$$

The system of linear first-order differential equations in eq. 11 describes the behavior of the regrinding mill in the neighborhood of the equilibrium state. The state variables of the system are s_1 and g_1; the disturbance variables are s_o and g_o; the control input (manipulated variable) is f; and the controlled variable is s_1.

Since eq. 11 is to be used in subsequent control system design and analysis, the following substitutions are made to simplify the analysis and to conform with standard control terminology;

$$a = f_e/H \tag{12}$$

$$b = (s_{1e} - s_{oe})/H \tag{13}$$

$$c = P/H \tag{14}$$

$$\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} s_1 \\ g_1 \end{Bmatrix} \tag{15}$$

$$\mathbf{V} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} s_o \\ g_o \end{Bmatrix} \tag{16}$$

$$\mathbf{U} = f \tag{17}$$

$$\mathbf{Y} = s_1 \tag{18}$$

$$A = \begin{Bmatrix} -a & c \\ 0 & -a \end{Bmatrix} \tag{19}$$

$$B = \begin{Bmatrix} -b \\ 0 \end{Bmatrix} \tag{20}$$

$$C = [1 \quad 0] \tag{21}$$

$$D = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \tag{22}$$

Substituting eqs. 12—22 into eq. 11 results in the matrix state equation:

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} = A\,\mathbf{X} + B\,\mathbf{U} + D\,\mathbf{V} \tag{23}$$

and the output equation:

$$\mathbf{Y} = C\,\mathbf{X} \tag{24}$$

The operator $\mathrm{d}/\mathrm{d}t$ can be replaced by the Laplace operator s and eqs. 23 and 24 combined to give the following input-output relationships:

$$\mathbf{Y}(s) = C(sI - A)^{-1}\,B\,\mathbf{U}(s) + C(sI - A)^{-1}\,D\,\mathbf{V}(s) \tag{25}$$

where I is the identity matrix. In eq. 25, the terms $C(sI - A)^{-1}B$ and $C(sI - A)^{-1}D$ are the plant (regrind mill) transfer function, G_p, and the disturbance transfer function, N, respectively (Takahashi et al., 1972).

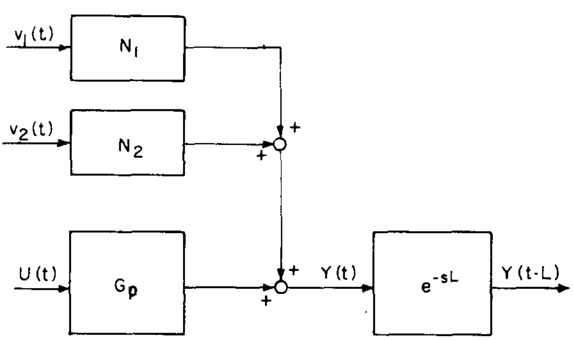

Fig. 2. Block diagram representation of the matrix input-output equation (eq. 25).

Solving these two terms for the plant and disturbance transfer functions one obtains:

$$G_p = -b/(s+a) \tag{26}$$

$$N = \left[\frac{a}{s+a} \quad \frac{ca}{(s+a)^2}\right] \tag{27}$$

The block diagram representation of eq. 25 is shown in Fig. 2. A pure time delay, e^{-sL}, with a lag of L time units has been included to model the effect of analysis time on the control performance.

CONTROL SYSTEM ANALYSIS

The purpose of the proposed control system, shown in Fig. 1, is to maintain the surface area (fineness) of the mill product (S_1) at the desired operating value (s_{1e}). When the feed surface area (S_o) and grindability (G_o) deviate from their desired operating values (i.e., s_{oe} and g_e, respectively) then the feed rate (F), and consequently the value of the deviation variable, f, must be adjusted by the controller to ensure $S_1 = s_{1e}$ (i.e., $s_1 = 0$). While many control algorithms are possible which will meet this goal, we consider here only the traditional linear algorithms such as proportional (P), proportional plus integral (PI), and proportional plus integral plus derivative (PID) feedback control (Takahashi et al., 1972). In terms of the controlled input, f, these algorithms are given respectively by:

$$f = K_P\, s_1 \tag{28}$$

$$f = K_P\, s_1 + K_I \int_0^t s_1(\tau)\,d\tau \tag{29}$$

$$f = K_P\, s_1 + K_I \int_0^t s_1(\tau)\,d\tau + K_D \frac{ds_1}{dt} \tag{30}$$

Continuous analysis

The block diagram of the closed-loop feedback control system is shown in Fig. 3. In this figure G_c is the transfer function of the controller, R^* is the reference input and E is the error. For the time being, the delay has been neglected. In the system under consideration, the output variable, $\mathbf{Y}$, which is equal to s_1, corresponds to the deviation of the product surface area from equilibrium, and thus, is equal to the error, E. Therefore, the reference input, R^*, for the system becomes zero.

The block diagram of Fig. 3 represents the following equation:

$$\mathbf{Y} = \frac{v_1 N_1 + v_2 N_2}{1 - G_c\, G_p} \tag{31}$$

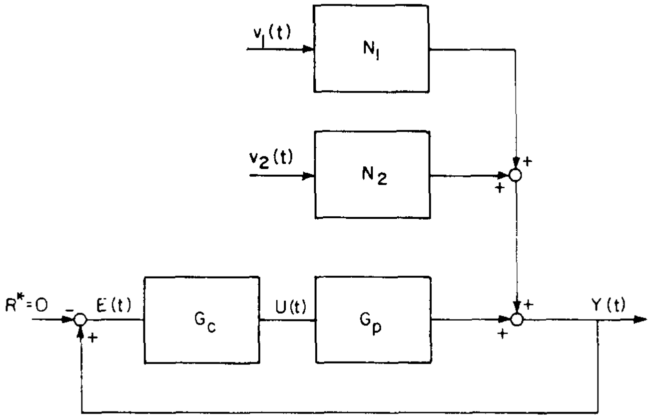

Fig. 3. Block diagram representation of the feedback control system.

where the denominator, $1 - G_c G_p$, set equal to zero is known as the closed-loop characteristic equation (Takahashi et al., 1972).

For P-control (eq. 28):

$$G_c = K_p \tag{32}$$

Substituting this relationship and eqs. 16, 18, 26, and 27 into eq. 31, an equation for the output variable, s_1, in the Laplace domain is obtained:

$$s_1 = s_o \frac{a}{s + a + b K_p} + g_o \frac{ca}{(s + a)(s + a + b K_p)} \tag{33}$$

The steady-state error of $s_1(t)$ can be investigated by applying the final-value theorem (Takahashi et al., 1972). Consider a unit-step change of magnitude s_o^* in the fineness and no change in the grindability. The final-value theorem yields:

$$s_1(t = \infty) = \frac{a s_o^*}{a + b K_p} \tag{34}$$

Equation 34 indicates that P-control has a steady-state error for all finite values of K_p. As the proportional gain K_p is increased, the steady-state error is reduced. Similarly, setting $s_o = 0$ and letting g_o be a unit step of magnitude g_o^*, the final-value theorem gives the following:

$$s_1(t = \infty) = \frac{c g_o^*}{a + b K_p} \tag{35}$$

Again, a steady-state error exists for all finite K_p, and this error is reduced as K_p is increased.

To investigate the stability under P-control, the Routh test (Takahashi et al., 1972) is used. Application of the Routh test gives the condition that the system reaches asymptotic stability whenever:

$$K_p > -a/b \tag{36}$$

To eliminate the steady-state error of the system, integral action is needed. Using PI control (eq. 29), the controller transfer function, G_c, becomes:

$$G_c = K_p + \frac{K_I}{s} \tag{37}$$

where K_I is the integral control gain. With PI-control, therefore, eq. 33 becomes:

$$s_1 = s_o \frac{as}{s(s+a) + K_p bs + K_I b} + g_o \frac{cas}{s(s+a) + K_p bs + K_I b} \tag{38}$$

Applying the final-value theorem gives:

$$s_1(t = \infty) = 0 \tag{39}$$

for either a step input in s_o or g_o. The Routh test gives, in addition to eq. 36, the conditions:

$$K_I > 0 \tag{40}$$

and

$$2a^3 + 3a^2 bK_p + abK_I + b^2 K_p K_I + ab^2 K_p^2 > 0 \tag{41}$$

for asymptotic stability of the system under PI-control. It can be seen that it is possible for the system to be unstable for certain values of the parameters. However, since a and b are always positive from physical considerations, if the control gains K_p and K_I are chosen to be positive (which is the usual case in practice), then the system is always stable.

Finally, for PID-control (eq. 30) where:

$$G_c = K_p + \frac{K_I}{s} + K_D s \tag{42}$$

eq. 33 becomes:

$$s_1 = \frac{s_o\, as + g_o\, cas}{s(s+a) + K_p bs + K_I b + K_D s^2} \tag{43}$$

application of the final-value theorem once again indicates that the PID-control system will eliminate the steady-state error. Applying the Routh test for stability gives, in addition to eqs. 35 and 40 the conditions that:

$$K_D > -\frac{1}{b} \tag{44}$$

and

$$[2a + b(K_D + K_p)]\,[a^2 + b(aK_p + K_I)] > (1 + bK_D)abK_I \tag{45}$$

Sampled analysis

As was mentioned previously, Sastry and Wakeman (1980) have found the sampling interval to exhibit a significant effect upon the control of a regrind mill circuit. Therefore, it was deemed necessary to further conduct the stability analysis for a sampled system and to establish a criterion for choosing the sampling frequency. This analysis is conducted by converting the plant transfer function in the Laplace domain, G_p, to the Z-domain and applying a slightly modified version of the Routh test (Takahashi et al., 1972).

For P-control the control transfer function in the Z-domain is once again simply:

$$G_c = K_p \tag{46}$$

The plant transfer function, G_p, now becomes:

$$G_p = \frac{b\,(e^{-aT} - 1)}{a\,(z - e^{-aT})} \tag{47}$$

As before the characteristic equation is given by:

$$1 - G_c\,G_p > 0 \tag{48}$$

which is used in the modified Routh test for determining the stability limits of a sampled system. For sampled P-control, the condition given by eq. 36 was still found to be true, along with the following additional condition:

$$K_p < \frac{a\,(1 + e^{-aT})}{b\,(1 - e^{-aT})} \tag{49}$$

where T is the sampling interval.

In the case of sampled PI-control the control algorithms is given by the following:

$$G_c = K_p + \frac{K_I\,z}{z - 1} \tag{50}$$

The modified Routh test gives in addition to eqs. 36 and 40, the condition:

$$2K_p + K_I < \frac{2a\,(1 + e^{-aT})}{b\,(1 - e^{-aT})} \tag{51}$$

Finally, for sampled PID-control, the control algorithm is given by:

$$G_c = K_p + \frac{K_I\,z}{z - 1} + \frac{K_D(z - 1)}{z} \tag{52}$$

and the asymptotic stability conditions are eq. 40 along with:

$$2K_p + K_I > -2a/b \tag{53}$$

$$2K_p + K_I + 4K_D < \frac{2a(1 + e^{-aT})}{b(1 - e^{-aT})} \tag{54}$$

and

$$4a^2 + 2K_I ab(1 + ce^{-aT}) + 4K_p ab + [K_D K_p b - 4K_D a - 2K_p K_I b - K_I^2 b] \times$$
$$\times \; (1 - e^{-aT})b > 0 \tag{55}$$

Thus, it can be seen that the sampling interval does play a role in determining the stability of the control system.

CONTROL SYSTEM DESIGN

The system response was simulated for both a step disturbance input and a pulse train disturbance input as used by Sastry and Wakeman (1980). This pulse train disturbance input is shown in Fig. 4. The standard values of the system parameters used to generate the simulation results are as follows:

Mill holdup, H = 10,000 kg
Power input, P = 10 kW
Equilibrium values for:
  Feed rate, f_e = 10,000 kg/h
  Feed grindability, g_e = 10,000 (m²/kg)/(kg/kWh)
  Feed surface area, s_{oe} = 75 m²/kg
  Product surface area, s_{1e} = 175 m²/kg

The traditional control algorithms of P, PI and PID control were simulated for a variety of control gains, sampling times, and disturbance inputs. To

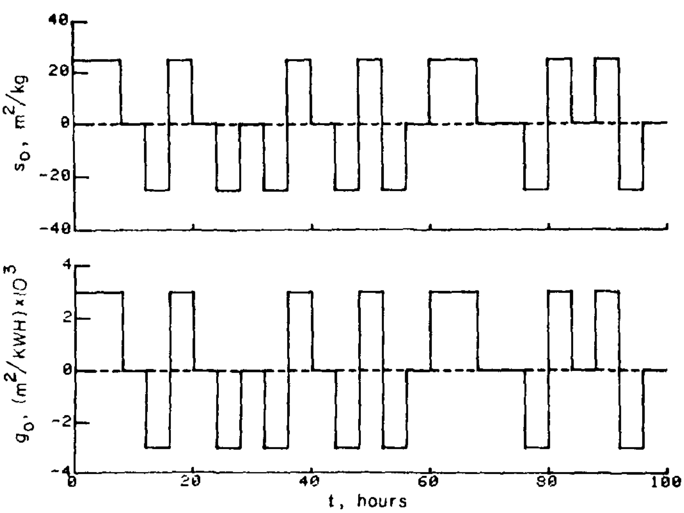

Fig. 4. Pulse train input disturbances in the grindability, g_o, and feed surface area, s_o.

evaluate the performance of each control system, the time averaged value of the absolute error defined by Sastry and Wakeman (1980) was utilized. In terms of the parameters used in this paper, this time averaged error is as follows:

$$\bar{E}(t) = \left\{\int_0^t F(\tau)\,|S_1(\tau)|\,\mathrm{d}\tau\right\} \Big/ \left\{\int_0^t F(\tau)\mathrm{d}\tau\right\} \tag{56}$$

Tables I and II summarize the relevant run parameters used in the computer simulations. All control gains, K_p, K_I, K_D, were tested for stability using the limits established by the Routh test analysis in the previous section. The response of the grinding process under these different run conditions is presented in Figs. 5 through 12 and the values of the time-averaged error are summarized in Tables I and II.

In the first set of computer simulations (Runs 2 through 7), sampling and analysis time were assumed to be negligible and the system was considered to be under continuous control with a step disturbance input at time equal

TABLE I

Control gains, simulation parameters and results for a step disturbance input with s_o = 22.5 m²/kg and g_o = 2500 m²/kWh

Run No.	Control strategy (*) k_p	k_I	k_D	Sampling freq. (h)	Analysis delay (h)	Time-averaged error, $\bar{E}$(m²/kg)	Fig. No.
1	Open Loop			0	0	18.37	5
2	200	0	0	0	0	7.52	5
3	400	0	0	0	0	4.68	5
4	800	0	0	0	0	2.66	5
5	400	312.5	0	0	0	1.84	—
6	400	625	0	0	0	0.98	—
7	400	1250	0	0	0	0.50	—
8	400	0	0	0	0.2	4.66	6
9	400	0	0	0	0.4	4.84	6
10	400	0	0	0	0.6	unstable	6
11	400	625	0	0	0.2	1.03	—
12	400	625	0	0	0.4	unstable	—
13	400	625	0	0	0.6	unstable	—
14	100	0	0	0	1.0	9.70	8
15	90	27.3	0	0	1.0	9.57	8
16	120	60	60	0	1.0	8.17	8, 9
17	120	60	60	0.25	1.0	8.85	9
18	120	60	60	0.50	1.0	9.54	9

(*)Control gains for Runs 14 through 18 calculated by Ziegler-Nichols Tuning Rules.

TABLE II

Control gains, simulation parameters and results for a pulse train disturbance input (Fig. 4)

Run No.	Control strategy (*)			Sampling freq. (h)	Analysis delay (h)	Time-averaged error, $\bar{E}$(m²/kg)	Fig. No.
	k_p	k_I	k_D				
19	Open Loop			0	0.125	15.63	10
20	267	0	0	0.25	0.125	4.89	12
21	160	0	0	0.50	0.125	6.84	—
22	89	0	0	1.00	0.125	9.43	—
23	47	0	0	2.00	0.125	12.49	—
24	24	0	0	4.00	0.125	15.69	—
25	186	108	0	0.25	0.125	5.29	—
26	96	96	0	0.50	0.125	7.28	—
27	45	69	0	1.00	0.125	11.57	—
28	21	43	0	2.00	0.125	17.27	—
29	9.9	24	0	4.00	0.125	21.03	—
30	200	240	240	0.25	0.125	3.13	11
31	85	213	120	0.50	0.125	5.63	—
32	30	154	60	1.00	0.125	13.22	—
33	9.1	95	30	2.00	0.125	20.26	—
34	2.5	53	15	4.00	0.125	27.63	—

(*) Control gains calculated by Modified Ziegler-Nichols Tuning Rules.

to zero. As predicted by the final-value theorem and the Routh test, P-control showed an offset and was found to be stable for all positive values of K_p. Figure 5 shows the results for the first four simulation runs, and it is clear that the offset can be reduced by increasing K_p. This requires an increase in the feed rate which is, of course, a desirable requirement (since it leads to increased throughput from the grinding circuit). We also note that P-control provides a significant improvement over the open-loop response (Run 1).

The final-value theorem analysis conducted previously indicated that the steady-state error could be completely eliminated if PI-control was used. This was simulated for the conditions used in the previous P-control analysis, a proportional gain, K_p, of 400, and various values of integral gain, K_I. Table I shows that the integral action in Runs 5, 6 and 7 improves the control performance over that of P-control in Runs 2, 3 and 4 as well as over the open-loop response (Run 1).

A number of simulations (Runs 8 through 16) were carried out to account for the analysis delay. Figure 6 (for P-control with K_p = 400) demonstrates that the analysis delay has quite a significant effect on the regrinding mill operation. When L, the delay time, is increased, the control performance degrades until the system ultimately becomes unstable. Similar results were obtained for PI-control (Table II, Runs 11, 12 and 13). Thus, it is clear

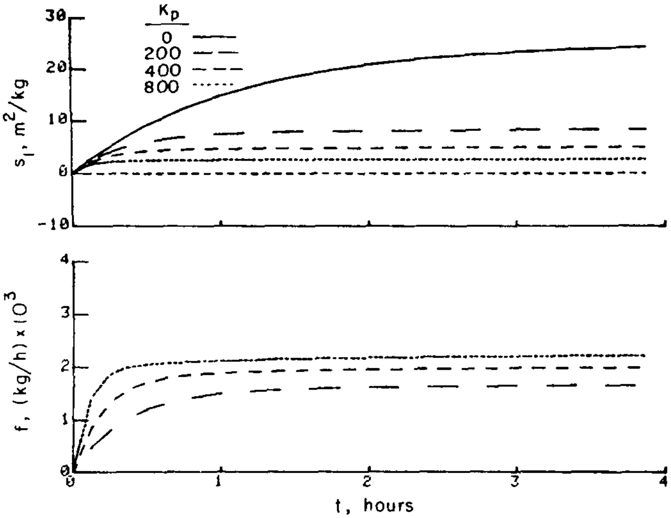

Fig. 5. The effect of proportional gain, K_p, on continuous P-control for a step change disturbance in the feed surface area (run nos. 1, 2, 3, and 4).

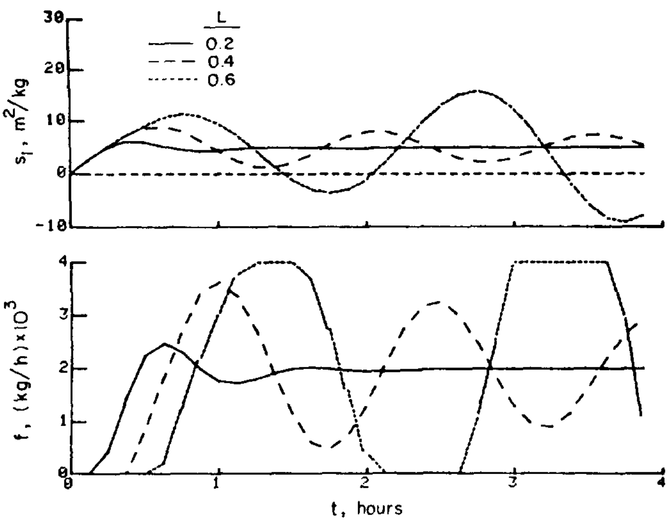

Fig. 6. The effect of analysis delay, L, on continuous P-control during a step change disturbance in the feed surface area (run nos. 8, 9 and 10).

from these results that unless L is very small, its value must be accounted for in the controller design.

Since the delay could not be taken into account in the Routh test, any instability arising from this delay time would not be predicted. The work of Ziegler and Nichols (1942, 1943), however, provides a method for proper selection of the controller gains for systems where the delay time L is significant. Their method is based on two values, R and L, which are obtained from the open-loop unit step input response of the process to be controlled. Figure 7 shows a typical unit-step process response curve, and defines the varibles R and L. Ziegler and Nichols recommended the following rules for setting the control gains:

(a) P-control: $K_p = 1/RL$
(b) PI-control: $K_p = 0.9/RL$
$K_I = 0.273/RL^2$
(c) PID-control: $K_p = 1.2/RL$
$K_I = 0.6/RL^2$
$K_D = 0.6/R$

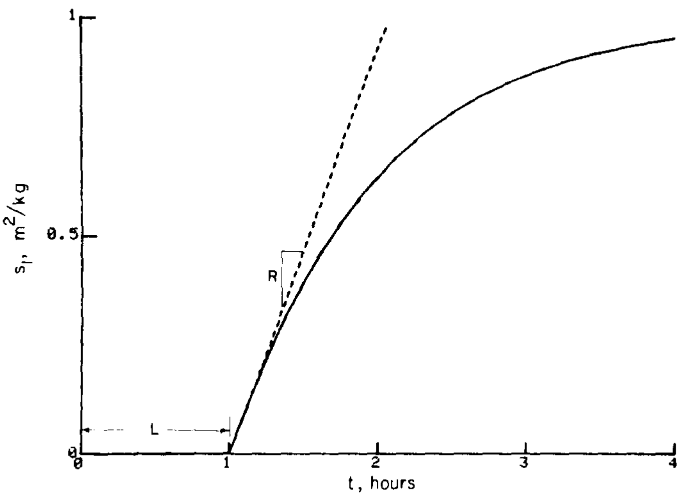

Fig. 7. Unit step response for obtaining control parameters by Ziegler-Nichols tuning.

In this investigation, the analysis delay, L, was chosen as 1 hour and R was determined from Fig. 7 to be 0.01. Thus, using the above rules, $K_p = 100.0$ for P-control, $K_p = 90.0$ and $K_I = 27.3$ for PI-control, and $K_p = 120.0$, $K_I = 60.0$, and $K_D = 60.0$ for PID-control. The performance of these controllers is shown in Fig. 8 and Runs 14 to 16 of Table I. Even with a fairly large sampling and analysis time of $L = 1.0$, the control systems are stable. The performance of the controllers, while not as good as with $L = 0.0$, is quite satisfactory. The PID-controller exhibits the best control but the PI-controller performance is also acceptable and requires less control effort. The P-control results in a steady-state error, and is least effective of the three.

The control system analysis presented thus far has been conducted assuming that control action was being taken continuously. From a plant operation point-of-view, this is not very practical since the mill discharge samples are taken only periodically. Clearly, no control action can be taken

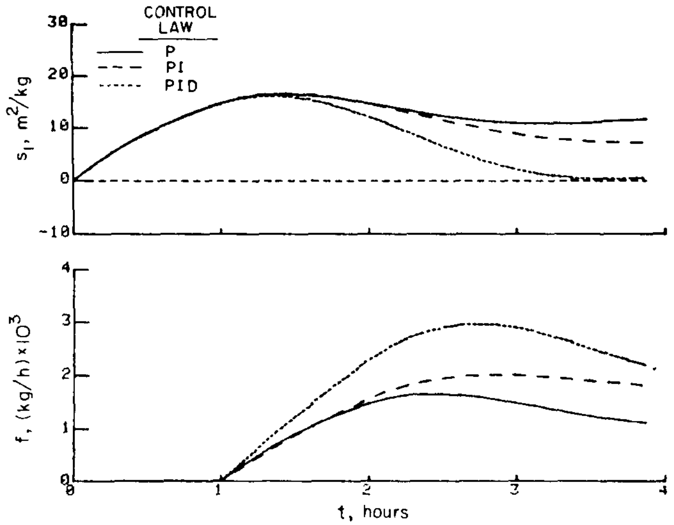

Fig. 8. Continuous P, PI and PID-control tuned by the Ziegler-Nichols rules (run nos. 14, 15 and 16).

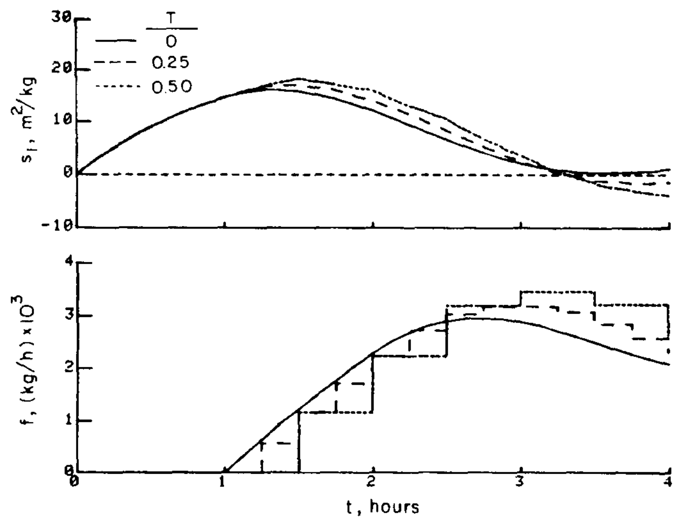

Fig. 9. The effect of sampling interval, T, on PID-control tuned by the Ziegler-Nichols rules (run nos. 16, 17 and 18).

between samples, and thus, the controller input is held constant during these periods. The effect of sampling interval does not seem significant when a continuous PID-controller and a sampled PID-controller are compared for the step disturbance conditions used previously as shown in Fig. 9. This response, however, is misleading since, from a practical standpoint, the disturbance input is not one pulse but a series of pulses, in which case, the sampling interval becomes important. Therefore, the control gains must be chosen to provide for both analysis delay and sampling interval.

The Ziegler-Nichols tuning equations have been modified (Takahashi et al., 1971) to account for the sampling interval, T. These modified Ziegler-Nichols tuning rules are as follows:

(a) P-control: $$K_P = \frac{1}{R(L+T)}$$

(b) PI-control: $$K_P = \frac{0.9}{R(L+T)} - \tfrac{1}{2}K_I$$

$$K_I = \frac{0.27}{R(L+0.5T)^2}\,T$$

(c) PID-control: $$K_P = \frac{1.2}{R(L+T)} - \tfrac{1}{2}K_I$$

$$K_I = \frac{0.6}{R(L+0.5T)^2}\,T$$

$$K_D = \frac{0.6}{RT}$$

For the system under consideration, $L = 0.125$ h was chosen as a reasonable value for the analysis delay of an automated system. Once again R was 0.01 as previously determined, and various values of sampling interval were chosen for the simulation of the sampled data system.

The P, PI, and PID controllers were simulated for the sampled system using a disturbance input represented by the pulse train shown in Fig. 4. The performance of these controllers is shown in Runs 20 to 34 of Table II. Also the open-loop response caused by this disturbance input is shown in Run 19 and Fig. 10. Table II shows that for sampling intervals of 1 hour or less, all three control algorithms provide substantial improvement over the open-loop response. At sampling intervals greater than 1 hour, however, the control gains become so low that very little improvement is seen and in some cases the response is worse with control. This result is as expected from the sampling theorem (Takahashi et al., 1972) which states that a digital control system can handle a continuous signal with frequencies up to $0.5/T$. Since the frequency of the disturbances is often 1/8 h in the pulse train used for this analysis, the control system is at its limit when sampling interval, T, is 4 hours.

Although all three control algorithms with sampling intervals of 30 minutes or less provide quite adeqate control for most purposes, PID-control with a sampling interval of 15 minutes seems to provide the best performance as shown in Run 30 of Table II and Fig. 11. P-control with the same sampling interval, however, also provides reasonable control and is easier to implement.

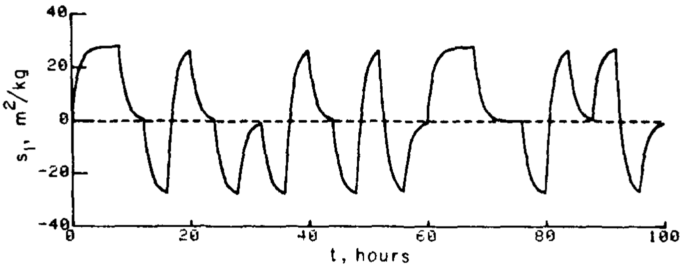

Fig. 10. Open loop response to pulse train disturbance input (run no. 19).

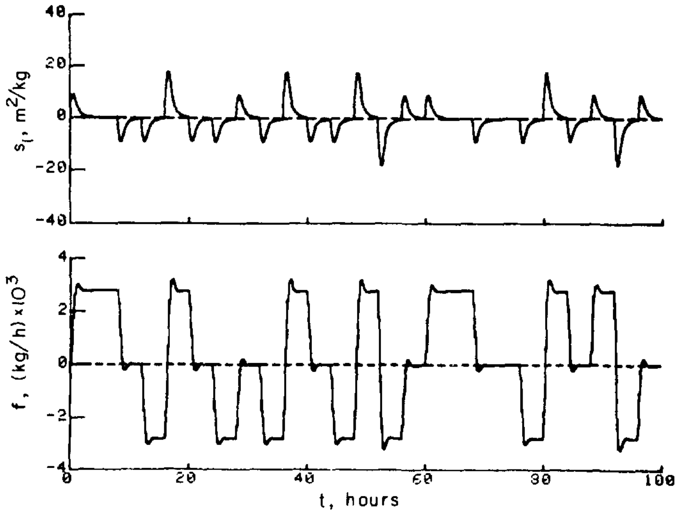

Fig. 11. PID-control with 15-minute sampling interval tuned by the modified Ziegler-Nichols rules (run no. 30).

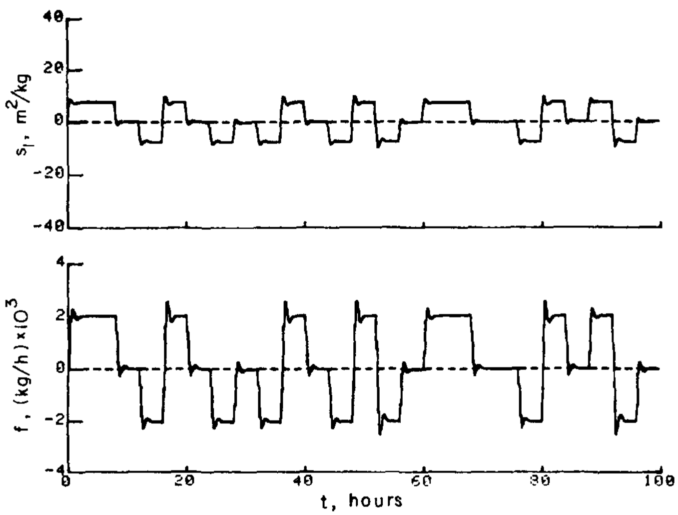

Fig. 12. P-control with 15-minute sampling interval tuned by the modified Ziegler- Nichols rules (run no. 20).

The P-control response is shown in Fig. 12. Once again, the offset is evident under P-control and may ultimately make the PID-control algorithm more desirable if disturbances are not very frequent. For frequent disturbances, however, the less oscillatory nature of P-control may make it more desirable. Thus, the use of pre-mix and post-mix tanks to smooth out disturbances (Sastry and Wakeman, 1980) may ultimately provide the best control by allowing one to use the better response provided by the PID algorithm.

SUMMARY AND CONCLUSIONS

In this paper, the design of traditional feedback control systems for an open-circuit regrinding mill has been discussed. The analysis has been based on the process model equations developed by Sastry and Wakeman (1980), the final-value theorem for determining steady-state error, the Routh test for determining stability, and the Ziegler-Nichols and modified Ziegler-Nichols rules for tuning the control parameters. The concepts and methods used in the present study are quite general and can be effectively applied to other problems in mineral process control.

In addition to the recommendation that pre-mix and post-mix tanks be included in regrinding circuits to smooth out the effects of process disturbances, we can state that:

(1) A traditional feedback control system such as P- or PID-control which manipulates the feed rate to the mill based on measurement of the product fineness will be very effective in improving regrinding mill circuit performance (for a low frequency of disturbance pulses, integral action should be included).

(2) The design of the control system can be based on ensuring stable performance, eliminating steady-state errors, requiring that the control effort not be excessive, and that the time delay due to sampling and analysis of the mill product be properly taken into account.

(3) The sampling and analysis time should be minimized for best control system performance. This can be achieved by employing automatic sampling procedures and the use of on-line particle size analyzers.

As indicated in the introduction, this paper is intended to demonstrate the applicability of a few of the control theory techniques to mineral process control problems by using the regrind mill case as an example. Areas for further study could be the effectiveness of feed forward strategies based on measurements of feed surface area and grindability, the implementation of a digital control scheme, the use of advanced control strategies such as optimal or adaptive control, and the practical aspects of the control system hardware. The usefulness of adapting the simple model equations (equations 1 and 2) for carrying out the design and stability analysis of the control system in a rapid, cost-effective manner should be self-evident.

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NOMENCLATURE

a	a constant (time^{-1})
A	A-matrix
b	a constant (area/mass^2)
B	B-matrix
c	a constant (power/mass)
C	C-matrix
D	D-matrix
E	error input
$\overline{E}(t)$	time-averaged error (area/msss)
F	mass feed rate of solids (mass/time)
f	deviation of mass feed rate from equilibrium (mass/time)
f_e	equilibrium mass feed rate (mass/time)
G_c	control transfer function
G_p	plant (regrind mill) transfer function
G_o	grindability of solids in feed [(area/mass)/(energy/mass)]
G_i	grindability of solids in mill and product [(area/mass)/(energy/mass)]
g_o	deviation of feed grindability from equilibrium [(area/mass)/(energy/mass)]
g_o^*	magnitude of a step change in g_o [(area/mass)/(energy/mass)]
g_{oe}	equilibrium feed grindability [(area/mass)/(energy/mass)]
g_i	deviation of mill and product grindability from equilibrium [(area/mass)/(energy/mass)]
g_{ie}	equilibrium mill and product grindability [(area/mass)/(energy/mass)]
g_e	equilibrium grindability [(area/mass)/(energy/mass)]
H	mass holdup of solids in mill (mass)
k_o	derivative gain
k_i	integral gain
k_p	proportional gain
L	analysis time lag (time)
N	disturbance matrix transfer function
P	power input to the grinding mill (power)
R	Ziegler-Nichols tuning parameter
R^*	reference input
S	Laplace operator
S_o	feed fineness (area/mass)
S_i	mill and product fineness (area/mass)
s_o	deviation of feed fineness from equilibrium (area/mass)
s_o^*	magnitude of a step change in s_o (area/mass)
s_{oe}	equilibrium feed fineness (area/mass)
s_i	deviation of mill and product fineness from equilibrium (area/mass)
s_{ie}	equilibrium mill and product fineness (area/mass)
t	time
T	sampling interval (time)
$\mathbf{U}$	input vector
$\mathbf{V}$	disturbance vector

v_1	disturbance input transfer function
v_2	disturbance input transfer function
X	state vector
Y	output vector
Z	Z-operator
τ	an integration representation of time (time)

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