THEORETICAL CEILING ON QUARK MASSES IN THE STANDARD MODEL

M.J. DUNCAN [1], R. PHILIPPE [1]

Physics Department, University of Michigan, Ann Arbor, MI 48109, USA

and

Marc SHER [1]

Physics Department, University of California, Santa Cruz, CA 95064, USA

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An unavoidable condition for consistency of the standard model is that all quarks must be lighter than some critical mass. A precise determination of this bound necessitates a complete renormalization group analysis of the scalar potential. Our results, that this critical condition is $M_q < 80$ GeV + 0.54 M_{Higgs}, differs markedly from previous investigations.

It has been known for some time that radiative corrections to the scalar potential of a quantum field theory can play an important role in determining the properties of its vacuum. In their pioneering paper, Coleman and Weinberg [1] developed the techniques necessary for the evaluation of the $O(\hbar)$ terms in the effective potential. In addition to directly writing down the leading log approximation (valid for small couplings and small field fluctuations), they showed how one can use the properties of the renormalization group to sum all leading logs, thus obtaining an expression valid over a wider range of field values.

In the context of the perturbative electroweak model, it was subsequently found [2–4] that heavy fermions destabilize our vacuum. For a given Higgs mass, there are corresponding upper bounds on quark and lepton masses. Should states which violate these bounds be observed experimentally at forthcoming machines, then the standard model would be bereft of life and new physics would have to come into play. There are other phenomena which lead to mass restrictions, but these require a precision measurement [5] or are dependent on unknown hadronic matrix elements [6]. The advantage of focusing our attention on the vacuum is that the only inputs are the Fermi constant and electroweak couplings, which are well known.

The original studies [2–4] only examined the first-order leading log term in the effective potential and also neglected the consequences of the negative mass-squared parameter in evaluating the radiative corrections. A subsequent investigation [7] summed all leading logs involving gauge and Yukawa couplings and found significantly different bounds. More recently, the effects of the negative mass-squared parameter on the radiative corrections have been considered [8] and shown to be crucial. The desire to finally settle the question of the value of the maximum permissible fermion masses in the standard model warrants a complete analysis of the stability of the vacuum. As we shall demonstrate in the rest of this paper, such an analysis is straightforward and leads to bounds which are much more reliable.

At the tree level of perturbation theory, the classical potential of the electroweak model is the familiar

$$V(\phi)_{\text{cl}} = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4, \qquad (1)$$

where $\mu^2, \lambda > 0$ and the tree-level minimum is at

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$\langle\phi\rangle^2 = \mu^2/\lambda$. We have written $\frac{1}{2}\phi^2 = \Phi^\dagger\Phi$, with Φ being the usual Higgs doublet. Utilizing a mass-independent subtraction prescription, the leading log approximation [1] to the $O(\hbar)$ corrections can be written

$$V(\phi)_1^{\rm LL} = (\hbar/64\pi^2)\,{\rm Tr}[(-1)^F M^4 \ln(M^2/\kappa^2)] - c, \quad (2)$$

where κ is the arbitrary mass scale introduced by renormalization and M^2 is the mass matrix for each sector. c is a constant chosen such that $V_1^{\rm LL}$ vanishes when all couplings are set to zero, since there could then be no radiative corrections. With the mass matrices of the standard model, $V_1^{\rm LL}$ becomes, in the Landau gauge,

$$V(\phi)_1^{\rm LL} = (\hbar/64\pi^2)[A\phi^4 \ln(\phi^2/\kappa^2) + M_0^4 \ln(M_0^2/\kappa^2) - \mu^4 \ln(-\mu^2/\kappa^2)], \quad (3)$$

where $M_0^2 = 3\lambda\phi^2 - \mu^2$. The coefficient A is given by

$$A = \tfrac{3}{16}(g_1^4 + 3g_2^4 + 2g_1^2 g_2^2) - \sum_f g_f^4, \quad (4)$$

with g_1, g_2 the usual hypercharge and isospin couplings and g_f are the fermion Yukawas, the sum running over color and flavour. In what follows, we assume that one quark dominates, so we replace Σg_f^4 with $3g_f^4$.

In order to sum *all* leading logs, to obtain a more reliable potential at large field values, we must explicitly solve the renormalization group equations. The effective potential, to all orders in perturbation theory obeys

$$\left(\kappa\frac{\partial}{\partial\kappa} + \beta_\lambda\frac{\partial}{\partial\lambda} + \beta_{\mu^2}\mu^2\frac{\partial}{\partial\mu^2} + \sum_g \beta_g\frac{\partial}{\partial g} - \gamma\phi\frac{\partial}{\partial\phi}\right)V_{\rm eff}(\phi) = 0, \quad (5)$$

where the sum runs over all gauge and Yukawa couplings. The β-functions for these couplings are given by the usual $\beta_1 = 41g_1^3/96\pi^2$, $\beta_2 = -19g_2^3/96\pi^2$, $\beta_3 = -7g_3^3/16\pi^2$ and $\beta_y = g_y(9g_y^2/2 - 8g_3^2)/16\pi^2$. In the last case we have neglected g_1 and g_2 compared with g_y and g_3 (this approximation can be relaxed without changing our results). To find β_λ and β_{μ^2}, we follow Einhorn and Jones [9]. Since the β, λ are implicitly $O(\hbar)$, we can separate $V_{\rm eff} = V_{\rm cl} + V_1^{\rm LL}$ and equate $O(\hbar)$ terms in (5), viz.

$$\left(\beta_\lambda\frac{\partial}{\partial\lambda} + \beta_{\mu^2}\mu^2\frac{\partial}{\partial\mu^2} - \gamma\phi\frac{\partial}{\partial\phi}\right)V(\phi)_{\rm cl} = -\kappa\frac{\partial}{\partial\kappa}V(\phi)_1^{\rm LL}, \quad (6)$$

from which we obtain, upon equating ϕ^2 and ϕ^4 coefficients,

$$\beta_{\mu^2} = 2\gamma + 3\lambda/8\pi^2, \quad \beta_\lambda = 4\lambda\gamma + (9\lambda^2 + A)/8\pi^2. \quad (7)$$

The anomalous dimension of ϕ in the Landau gauge is easily derived from the two-point function

$$\gamma = (3g_y^2 - 9g_2^2/2 - 3g_1^2/4)/16\pi^2, \quad (8)$$

from which we find β_{μ^2} and β_λ. The complete eq. (5) can be solved by the method of characteristics (or by the method of educated guessing) to yield the solution

$$V(\phi)_{\rm eff} = -\tfrac{1}{2}\mu^2(t)G^2(t)\phi^2 + \tfrac{1}{4}\lambda(t)G^4(t)\phi^4, \quad (9)$$

where the running coefficients are given by

$$d\lambda(t)/dt = \beta_\lambda(g(t), \lambda(t)),$$

$$d\mu^2(t)/dt = \mu^2(t)\beta_{\mu^2}(g(t), \lambda(t)),$$

$$G(t) = \exp\left(-\int_0^t \gamma(g(t'), \lambda(t'))\,dt'\right), \quad (10)$$

and the parameter $t = \ln(\phi/\kappa)$ [‡1]. All that remains is to address the question of the initial values of the couplings at $t = 0$.

The minimum condition and the Higgs mass are given by

$$\partial V_{\rm eff}/\partial\phi|_{\phi_0} = 0, \quad m_{\rm H}^2 = \partial^2 V_{\rm eff}/\partial\phi^2|_{\phi_0}, \quad (11)$$

and simply choosing the subtraction point to be the vacuum expectation value of the scalar, $\kappa = \phi_0 = (\sqrt{2}G_{\rm F})^{-1/2} = 246$ GeV leads to relation [‡2] between the initial values

‡1 This does not quite match eq. (3) at $t = 0$, but the discrepancy is $O(\lambda^2/16\pi^2)$. Since for Higgs masses below 250 GeV, λ is always less than 0.5, this is negligible.

‡2 Since we are not summing non-logarithmic corrections, the higher order terms in eq. (12) should be taken cum grano salis. These terms will affect the Higgs mass by ~ 1%, and can thus be ignored.

$$\mu_0^2 = \kappa^2 [\lambda_0 + (3\lambda_0^2 + A_0)/32\pi^2] ,$$

$$m_{\rm H}^2 = \kappa^2 [2\lambda_0 - (6/32\pi^2)(7\lambda_0^2 + A_0)] . \quad (12)$$

[We have dropped $O(\lambda^4)$ terms which are very small.] The initial values of the gauge couplings are easily obtained from experiment ($\Lambda_{\rm QCD} = 0.2$ GeV) and the Yukawa coupling is fixed by [10] $g_y(2M_q) = M_q/175$ GeV. Sensitivity to two-loop beta-functions can be estimated by varying $\Lambda_{\rm QCD}$ from 0.1 to 0.4 GeV, and is found to be negligible, affecting the bound by less than 5 GeV. In practice, we choose λ_0, g_{y_0} and numerically integrate the running couplings up to $O(10^{15}$ GeV), checking that all couplings remain perturbative throughout (only λ may become nonperturbative, and whether one defines "nonperturbative" to mean $\lambda > 1/2, 1, 4\pi$ or $16\pi^2$ makes no practical difference in our results). These couplings are then put into eq. (9) and the potential is examined to determine whether our vacuum is stable. The requirement that our vacuum be stable leads to an upper bound on a quark mass for a given Higgs mass.

The original calculations of this bound [2–4] considered only the first-order leading log corrections, i.e. eqs. (1)–(3). They neglected two critical effects: the running coupling constants, and the correct expression for M_0^2 in eq. (3). It is easy to see that the first effect will weaken the bounds. Since the Yukawa coupling falls as the scale increases, its effect on the potential ar large ϕ is smaller than if its running were ignored, thus larger Yukawa couplings are needed to have a similar effect and the upper bound thus increases significantly. The second effect will be crucial when scalar loops are important, i.e. when the scalar is fairly heavy ($\geqslant$100 GeV), and it is necessary to reliably calculate the bound for heavy Higgs scalars. In ref. [7], the first effect was included, with significant changes in the previous bounds, but the second effect, plus the beta functions for μ^2 and λ were ignored. In ref. [8], the second effect was included, but the first effect was ignored. In the present analysis, by explicitly solving the full renormalization group equation, all leading logs have been included.

The results are summarized in fig. 1 which plots the maximum permissible quark mass for a given value of the Higgs mass, which we vary from 10 to 250 GeV. Also plotted are the bounds from previous analyses, which are seen to be quite different for Higgs masses below 250 GeV, the scalar self-coupling be-

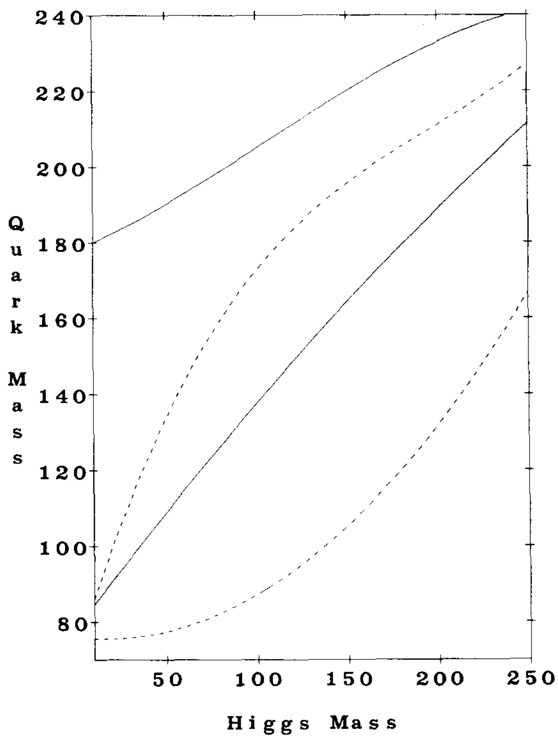

Fig. 1. Ceiling on the quark mass as a function of the Higgs mass. The lower (upper) dashed curve is the previous limit of refs. [2–4,8] ([7]). Below the lower solid curve, the present vacuum is absolutely stable. Between the two solid curves the vacuum is unstable but with a lifetime $> 10^{10}$ yr.

comes nonperturbative before the region of instability is reached, thus any instability lies outside the realm of perturbation theory; one can say nothing about quark mass limits for Higgs masses above 150 GeV. It turns out that our result can be fit rather well to a straight line, and we conclude that if our vacuum is stable, then there is an upper bound on the mass of a heavy quark given by $M_q < 80$ GeV + $0.54\, M_{\rm H}$, valid if $M_{\rm H} < 250$ GeV.

Finally, strictly speaking, there is no requirement that our vacuum be stable, but only that its lifetime be longer than 10^{10} yr. The calculation of the lifetime of our vacuum was discussed in detail in ref. [7], where it was noted that the results are, unlike the stability bound, very *insensitive* to running couplings. We have checked and found that the bounds of ref. [7] are not significantly altered. The resulting bound, obtained by assuming only that our vacuum is sufficiently long-lived, is also plotted in fig. 1. As $M_{\rm H}$ increases from 10 to 250 GeV, the bound increases from 180 to 240 GeV. In the region between this bound and our stability bound, our vacuum is unstable, but lives longer than 10^{10} yr. (In most of this region, the lifetime is *much* longer than 10^{10} yr, so there is no cause for alarm).

It is easy to generalize this bound to more complicated models. If there are many heavy quarks, M_q is replaced by $(\Sigma M_q^4)^{1/4}$. If there are more scalars, there are many more parameters in the Higgs potential, but a bound can still be found. For the moment, consider the two-doublet model. There are two neutral (0^+) scalars, ϕ and η, a neutral (0^-), χ^0 and a charged scalar, $\chi^{\pm}$. To avoid tree-level flavor changing neutral currents, one can only couple one of the doublets to the heavy quark; call that doublet Φ_2. Since only Φ_2 couples to the quark, the instability should occur in the Φ_2-direction, i.e. in the direction $\Phi_1 = 0$ (clearly, if an instability develops in another direction, our bound will only be tightened). One can then [‡3] look at the potential with $\Phi_1 = 0$. Examining the one-loop potential in this case, one sees that the ordinate of fig. 1 is replaced by

$$(M_q^4 - \tfrac{1}{6}M_{\chi^+}^4 - \tfrac{1}{12}M_{\chi^0}^4)^{1/4}.$$

The neutral scalar combination parallel to Φ_2 has already been included, the one orthogonal to Φ_2 drops out since the $\Phi_1 = 0$ direction is being considered. The abscissa of fig. 1, ignoring λ^2 terms, is $(2\lambda_2)^{1/2}\kappa$ where κ is 246 GeV and λ_2 is the self-coupling of Φ_2.

This bound is useless without knowledge of λ_2 (which can only be "measured" through Higgs–Higgs scattering). The masses of ϕ and η satisfy the relation

$$M_\phi^2 + M_\eta^2 = 2\lambda_1 v_1^2 + 2\lambda_2 v_2^2,$$

thus

$$2\lambda_2 v_2^2 < M_\phi^2 + M_\eta^2,$$

and so we have an upper bound of

$$\tfrac{1}{2}(M_\phi^2 + M_\eta^2)/v_2^2$$

on λ_2. Since the curves in fig. 1 are increasing, one can replace λ_2 on the abscissa by this bound; the resulting upper bound on the fermion mass is still valid. All we now need is an expression for v_2. The Yukawa coupling of the quark to the neutral scalars is $g_y\bar{Q}Q \times (\phi\cos\alpha + \eta\sin\alpha)$, where $g_y = \sqrt{2}M_q/v_2$. In the Yukawa couplings to each mass eigenstate are g_ϕ and g_η, then $g_\phi^2 + g_\eta^2 = 2M_q^2/v_2^2$ so the abscissa in fig. 1 is replaced by

$$[\tfrac{1}{2}(M_\phi^2 + M_\eta^2)(g_\phi^2 + g_\eta^2)]^{1/2}\kappa/M_q.$$

This, then, is the generalization of our bound to two-Higgs models.

In multi-doublet models, one can easily show that the ordinate becomes

$$\left(M_q^4 - \tfrac{1}{6}\sum_i M_{\chi_i^+}^4 - \tfrac{1}{12}\sum_i M_{\chi_i^0}^4\right)^{1/4},$$

and the abscissa becomes

$$\left(\tfrac{1}{2}\sum_i M_{\phi_i}^2 \sum_i g_{\phi_i}^2\right)^{1/2}\kappa\Big/M_q.$$

The bound clearly becomes useless as the number of doublets proliferates.

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[‡3] In the work of Georgi, Manohar and Moore [12], there was a large ratio of vacuum values, thus the mass eigenstates were essentially the same as weak eigenstates. This procedure then leads to an effective field theory involving only the light mass eigenstate.

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