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A SEMI-REDUCED FORM EQUATION FOR THE STERLING'S REAL EFFECTIVE EXCHANGE RATE

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This paper derives and estimates a semi-reduced form expression for the UK's real effective exchange rate. The expression was derived following Blanchard and Kahn. Non-linear least squares and non-linear instrumental variables were used in estimation.

This paper derives and estimates a semi-reduced form expression for the real exchange rate and estimates using UK data.

Consider the following model with two assets, money and foreign bonds, with wealth effects in the money demand and consumption functions, and sticky prices and agents possessing national expectations:

$$M/P + L(i^* + \epsilon_t \mathrm{D}E/E,\ y,\ (M+EF)/P), \qquad L_1 < 0, \quad L_2 > 0, \quad 0 < L_3 < 1, \tag{1}$$

$$y = A(Qy/P + i^*EF/P,\ i^* + \epsilon_t \mathrm{D}E/E - \epsilon_t \mathrm{D}P/P,\ (M+EF)/P) + T(A, EQ^*/Q), \tag{2}$$

$$0 < A_1 < 1, \quad A_2 < 0, \quad A_3 > 0, \quad 0 < T_1 < 1, \quad T_2 > 0,$$

$$\mathrm{D}Q/Q = j(y - \bar{y}) + \epsilon_t \mathrm{D}E/E, \qquad j' > 0, \tag{3}$$

$$E\mathrm{D}F = T(A,\ EQ^*/Q) + i^*EF/P, \tag{4}$$

$$P \equiv P(Q,\ EQ^*), \tag{5}$$

$$i^* \equiv r^* + \mathrm{D}Q^*/Q^*, \tag{6}$$

where M = money supply, P = price index, E = exchange rate, y = output, F = stock of foreign bonds (assumed positive), Q = price of domestic output, $i^*(r^*)$ = the foreign nominal (real) rate of interest, Q^* = foreign currency price of the foreign good. D denotes a time derivative ($\equiv \mathrm{d}/\mathrm{d}t$) and ϵ_t is the expectation of a variable at time t.

Eq. (1) is the money market equilibrium condition. Eq. (2) is the goods market equilibrium

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condition. Eq. (3) is a Phillips curve. Eq. (4) gives the accumulation of foreign bonds. Eqs. (5) and (6) define the price index and the foreign real rate of interest respectively.

The model can be reduced to a system of differential equations in three state variables ($m \equiv M/Q$, $C \equiv E/Q$ and F). Linearising around the long run equilibrium we have

$$DX = A(X - \bar{X}), \tag{7}$$

where $X \equiv [m, C, f]'$.

Sufficient conditions for A to have two stable roots are:

(a) $(1 - A_1(1 + T_1))Q/Pj' + A_2(1 + T_1)P_1Q/P < 0$,
which is a requirement that an increase in output should create an excess supply in the goods market,

(b) $|T_1A_3F| > (1 - A_1)i^*F$,
which requires the interest service account to be small, and

(c) L_3 should be close to unity.

An increase in real balances requires an increase in its rate of change, *ceteris paribus* (i.e., a fall in the expected rate of inflation). If L_3 is high then this destabilizing effect is small.

We can then write the semi-reduced form for the real exchange rate following Blanchard and Kahn (1980) (in discrete time) as

$$C_t = \alpha_0 + \alpha_1 F_t + \alpha_2 m_t + \alpha_3 \sum_{i=0}^{\infty} \delta^i \epsilon(Z_{t+i}(I(t))), \tag{8}$$

where Z_t are all the exogenous variables and $I(t)$ is the information set at date t.

Table 1 [a]

Parameter	Estimate NLS	Estimate NLIV
α_0	−0.0061 (−0.51)	−0.0124 (−0.56)
$1/\delta$	1.004 (186.78) [b]	1.005 (159.43) [b]
α_1	−0.824 (−14.22) [b]	−0.907 (−3.80) [b]
α_2	0.429 (4.63) [b]	0.415 (2.32) [b]
α_3	0.00232 (3.55) [b]	0.00236 (3.27) [b]
ECD	−0.00296 (−0.45)	−0.00294 (−0.43)
MMD	0.00889(1.75)	0.0119 (1.28)
RSS	0.0135	0.0138
SE of regression	0.0123	0.0124
Log likelihood	289.47	–
TM test		
1st order TR^2	1.94	2.02
12th order TR^2	17.51	13.87

[a] Figures in parentheses are t-statistics. The number of observations for NLS estimation was 96 and for NLIV it was 95.

$\chi^1_{1(0.05)} = 3.84, \quad \chi^2_{12(0.05)} = 21.03.$

Additional instruments for NLIV, c_{t-2}, c_{t-3}, m_{t-2}, F_{t-2}. All variables other than r^*, *ECD* and *MMD* are in logs.

[b] Significant at 95%.

In this paper we use only one exogenous variable – the foreign real rate of interest. To get a tractable expression for C_t we can express it as

$$C_t = \beta_0 + \delta^{-1}C_{t-1} + \alpha_1 F_t - \alpha_1\delta^{-1}F_{t-1} + \alpha_2 m_t - \alpha_2\delta^{-1}m_{t-1} - \alpha_3\delta^{-1}r^*_{t-1}$$
$$+ \left(\alpha_3 \sum_{i=0}^{\infty} \delta^i \epsilon\left(r^*_{t+i}|I(t) - r^*_{t+i}|I(t-1)\right)\right). \qquad (9)$$

The last term in the parenthesis is the new information which arrived between $t-1$ and t on the time path of the exogenous variables and is uncorrelated with the other right-hand side variables.

We estimated eq. (9) using non-linear least squares (NLS) and non-linear instrument variable (NLIV) methods. We require NLIV if either (a) F_t and/or m_t are not observable contemporaneously, or (b) eq. (8) had a white noise error term which would show up as a moving average error term in (9).

The results are shown in table 1. Neither of the two dummies one for exchange control and another for monetary targets used were significant (*ECD* and *MMD* respectively). The other coefficients are significant and of the correct sign. The value of δ is too high, though (almost equal to unity). LM tests were performed for serial correlation/moving average of first and twelfth order and these were found to be absent. The starting values were recovered from OLS estimates. Monthly data from December 1972 to January 1980 was used.

The results are encouraging in view of the fact that the thorough empirical study of the Sterling effective sale by Hacche and Townend (1981) reported inability to explain its behavior by any plausible theoretical model.

References

Blanchard, O.J. and C.M. Kahn, 1980, The solution of linear difference models under rational expectation, Econometrica 48, 1305–1311.

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