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EXAMINATION OF HILL'S LATEST YIELD CRITERION USING EXPERIMENTAL DATA FOR VARIOUS ANISOTROPIC SHEET METALS

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Summary—Using experimental data published earlier [Vial *et al.*, *Int. J. Mech. Sci.* **25**, 899 (1983)], values of the exponent m in the Hill (1979) yield criterion [Hill, *Math. Proc. Cambridge Phil. Soc.* **85**, 179 (1979)] are calculated for each of the four special cases suggested. With these findings, stress–strain relations for plane-strain compression are derived and predictions using these derived equations are compared with experimental results. Comparisons between prediction and experiment are reasonable in all cases and it is suggested that the discrepancies could arise because of the assumption of planar isotropy (via the use of an average r-value) and because the exponent m apparently varies with induced strain.

NOTATION

$\sigma_1, \sigma_2, \sigma_3$	principal stresses, where $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$
f, g, h, a, b, c	anisotropic parameters in Hill (1979) yield criterion
r	average strain ratio
$\sigma_u = \sigma_1$	yield stress in uniaxial tension, $\sigma_2 = \sigma_3 = 0$
$\sigma_b = \sigma_1 = \sigma_2$	yield stress in balanced biaxial tension, $\sigma_3 = 0$
$\sigma_p = \sigma_3$	yield stress in plane-strain compression, $\sigma_1 = 0$, and $0 > \sigma_2 > \sigma_3$
$x = \dfrac{\sigma_2}{\sigma_p}$	stress ratio in plane-strain compression, for $\dot{\varepsilon}_2 = 0$ and $\sigma_1 = 0$
m	exponent in Hill (1979) yield criterion
$\dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3$	principal strain-rates
ε_u	true strain in uniaxial tension
ε_p	true thickness strain in plane-strain compression
K_u	strength coefficient in uniaxial tension
n_u	strain-hardening exponent in uniaxial tension
K_p	strength coefficient in plane-strain compression
n_b	strain-hardening exponent in balanced biaxial tension
K_b	strain-hardening exponent in balanced biaxial tension

1. INTRODUCTION

In 1979, Hill [2] proposed the following anisotropic yield criterion:

$$f|\sigma_2-\sigma_3|^m+g|\sigma_3-\sigma_1|^m+h|\sigma_1-\sigma_2|^m+a|2\sigma_1-\sigma_2-\sigma_3|^m$$
$$+b|2\sigma_2-\sigma_3-\sigma_1|^m+c|2\sigma_3-\sigma_1-\sigma_2|^m=\sigma^m, \tag{1}$$

where loading is coaxial with the orthotropy, the coefficients f g, etc., characterize the anisotropy, σ is a scaling factor that regulates the unit of stress, and $m > 1$ to insure convexity.

Following Hill's procedure, and for the most general situation, increments of stress tangential to the yield surface are given by

$$f|\sigma_2-\sigma_3|^{m-1}\,\mathrm{d}(\sigma_2-\sigma_3)-g|\sigma_3-\sigma_1|^{m-1}\,\mathrm{d}(\sigma_3-\sigma_1)+h|\sigma_1-\sigma_2|^{m-1}\,\mathrm{d}(\sigma_1-\sigma_2)$$
$$+a|2\sigma_1-\sigma_2-\sigma_3|^{m-1}\,\mathrm{d}(2\sigma_1-\sigma_2-\sigma_3)\pm b|2\sigma_2-\sigma_3-\sigma_1|^{m-1}\,\mathrm{d}(2\sigma_2-\sigma_3-\sigma_1)$$
$$-c|2\sigma_3-\sigma_1-\sigma_2|^{m-1}\,\mathrm{d}(2\sigma_3-\sigma_1-\sigma_2)=0. \tag{2}$$

Note that in equation (2) the ordering of stresses is $\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3$ and the $\pm$signs are determined according to $2\sigma_2-\sigma_3-\sigma_1 \gtrless 0$.

By the normality flow rule, the principal components of strain-rate satisfy

$$\sigma_1\dot{\varepsilon}_1+\sigma_2\dot{\varepsilon}_2+\sigma_3\dot{\varepsilon}_3>0 \quad \text{and} \quad \dot{\varepsilon}_1\mathrm{d}\sigma_1+\dot{\varepsilon}_2\mathrm{d}\sigma_2+\dot{\varepsilon}_3\mathrm{d}\sigma_3=0 \tag{3}$$

for all increments in equation (2). By direct comparison of the differential forms,

$$\dot{\varepsilon}_1+\dot{\varepsilon}_2+\dot{\varepsilon}_3=0 \tag{4}$$

(which implies constancy of volume) with

$$\frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_3}=\frac{\begin{array}{l}\{f|\sigma_2-\sigma_3|^{m-1}-h|\sigma_1-\sigma_2|^{m-1}-a|2\sigma_1-\sigma_2-\sigma_3|^{m-1}\pm 2b|2\sigma_2-\sigma_3-\sigma_1|^{m-1}\\ \quad +c|2\sigma_3-\sigma_1-\sigma_2|^{m-1}\}\end{array}}{\begin{array}{l}\{-f|\sigma_2-\sigma_3|^{m-1}-g|\sigma_3-\sigma_1|^{m-1}-a|2\sigma_1-\sigma_2-\sigma_3|^{m-1}\mp b|2\sigma_2-\sigma_3-\sigma_1|^{m-1}\\ \quad -2c|2\sigma_3-\sigma_1-\sigma_2|^{m-1}\}.\end{array}} \tag{5}$$

For uniaxial tension (σ_u, 0, 0), equation (1) gives

$$(g+h+a2^m+b+c)\sigma_u^m=\sigma^m \tag{6}$$

and equation (5) reduces to

$$r=\frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_3}=\frac{a2^{m-1}+h+2b-c}{a2^{m-1}+g-b+2c}. \tag{7}$$

Under balanced biaxial tension (σ_b, σ_b, 0), equation (1) gives

$$(f+g+a+b+c2^m)\sigma_b^m=\sigma^m. \tag{8}$$

Combining equations (6) and (8) gives

$$\left(\frac{\sigma_b}{\sigma_u}\right)^m=\frac{g+h+a2^m+b+c}{f+g+a+b+c2^m}. \tag{9}$$

With plane-strain compression (say, $\dot{\varepsilon}_2=0$) it is reasonble to assume that the stress state (σ_1, σ_2, σ_3) can be taken as ($0>\sigma_2>\sigma_p=\sigma_3$). Using equation (5) with $\dot{\varepsilon}_2=0$ gives

$$f|x-1|^{m-1}-h|x|^{m-1}-a|x+1|^{m-1}\pm 2b|2x-1|^{m-1}+c|2-x|^{m-1}=0, \tag{10}$$

where $x=\sigma_2/\sigma_3=\sigma_2/\sigma_p$ and the $\pm$ sign corresponds to $0<x<\frac{1}{2}$ and $\frac{1}{2}<x<1$. Using equations (1) and (5), and noting that σ_p is always negative, we find

$$\left(-\frac{\sigma_p}{\sigma_u}\right)^m=\frac{(g+h+a2^m+b+c)}{f|x-1|^m+g+h|x|^m+a|x+1|^m+b|2x-1|^m+c|2-x|^m}. \tag{11}$$

2. RELATIONS FOR IN-PLANE ISOTROPY

With planar isotropy, we note that $f=g$ and $a=b$, then equations (7), (9), (10), and (11) reduce to

$$r=\frac{(2^{m-1}+2)a-c+h}{(2^{m-1}-1)a+2c+f} \tag{12}$$

$$\left(\frac{\sigma_b}{\sigma_u}\right)^m=\frac{(2^m+1)a+c+f+h}{2a+c2^m+2f} \tag{13}$$

$$f|x-1|^{m-1}-h|x|^{m-1}-a|x+1|^{m-1}\pm 2a|2x-1|^{m-1}+c|2-x|^{m-1}=0 \tag{14}$$

$$\left(-\frac{\sigma_p}{\sigma_u}\right)^m=\frac{(2^m+1)a+c+f+h}{f\{|x-1|^m+1\}+h|x|^m+a\{|x+1|^m+|2x-1|^m\}+c|2-x|^m}. \tag{15}$$

3. SPECIAL CASES APPLIED TO HILL'S CRITERION (equation 1)

Hill suggested four simple versions of equation (1) which with planar isotropy and particular values of the coefficients, alter equations (12 to 15) as follows:

Case 1

Where $a=b=0$, $h=0$, $f=g$

$$\frac{f}{c}=-\left(\frac{1}{r}+2\right) \tag{16}$$

$$\left(\frac{\sigma_b}{\sigma_u}\right)^m = \frac{1+\frac{f}{c}}{2^m+2\frac{f}{c}} \tag{17}$$

$$\left(-\frac{\sigma_p}{\sigma_u}\right)^m = \frac{1+\frac{f}{c}}{\frac{f}{c}\{|x-1|^m+1\}+|2-x|^m}, \tag{18}$$

where
$$-\frac{f}{c} = \left|\frac{2-x}{x-1}\right|^{m-1}. \tag{19}$$

Case 2

Where $a = b,\ c = 0,\ f = g = 0$

$$\frac{h}{a} = 2^{m-1}(r-1)-r-2 \tag{20}$$

$$\left(\frac{\sigma_b}{\sigma_u}\right)^m = \frac{2^m+1+\frac{h}{a}}{2} \tag{21}$$

$$\left(-\frac{\sigma_p}{\sigma_u}\right)^m = \frac{2^m+1+\frac{h}{a}}{\frac{h}{a}|x|^m+|x+1|^m+|2x-1|^m}, \tag{22}$$

where
$$-\frac{h}{a}|x|^{m-1}-|x+1|^{m-1}\pm 2|2x-1|^{m-1} = 0 \tag{23}$$

according to $x \lesseqgtr \frac{1}{2}$.

Case 3

Where $a = b,\ c = 0,\ f = g,\ h = 0$

$$\frac{f}{a} = \frac{2^{m-1}+2}{r}-(2^{m-1}-1) \tag{24}$$

$$\left(\frac{\sigma_b}{\sigma_u}\right)^m = \frac{2^m+1+\frac{f}{a}}{2+2\frac{f}{a}} \tag{25}$$

$$\left(-\frac{\sigma_p}{\sigma_u}\right)^m = \frac{2^m+1+\frac{f}{a}}{\frac{f}{a}\{|x-1|^m+1\}+|x+1|^m+|2x-1|^m}, \tag{26}$$

where
$$\frac{f}{a}|x-1|^{m-1}-|x+1|^{m-1}\pm 2|2x-1|^{m-1} = 0 \tag{27}$$

according to $x \lesseqgtr \frac{1}{2}$.

Case 4

Where $a = b = 0,\ f = g = 0$

$$\frac{h}{c} = 2r+1 \tag{28}$$

$$\left(\frac{\sigma_b}{\sigma_u}\right)^m = \frac{1+\frac{h}{c}}{2^m} \tag{29}$$

$$\left(-\frac{\sigma_p}{\sigma_u}\right)^m = \frac{1+\frac{h}{c}}{\frac{h}{c}|x|^m+|2-x|^m}, \tag{30}$$

where

$$-\frac{h}{c}|x|^{m-1}+|2-x|^{m-1}=0$$

or

$$\frac{h}{c}=\left|\frac{2-x}{x}\right|^{m-1}. \tag{31}$$

Dodd and Caddell [3] used Hill's criterion [2]; discuss anomalous behavior; previously published data [4–6] were included. Although equations (17), (21), (25) and (29) are *equivalent* to equations for $(\sigma_b/\sigma_u)^m$ used in [3], the forms used here are expressed in terms of the coefficients a, b, c, etc. and the stress ratio, x, instead of the strain ratio, r, as used in [3]. This provides consistency with the expressions for $(-\sigma_p/\sigma_u)^m$ used in this paper; those expressions were not considered in [3]. To assist the reader, the formulation of $(\sigma_b/\sigma_u)^m$ are included here for completeness.

4. COMPARISON OF PREDICTIONS WITH EXPERIMENTS

For all cases in the previous section, it can be seen that the values of r and m define the yield criterion. Now r is found experimentally and m can be determined from two tests, say, uniaxial and balanced biaxial tension, with the use of equations (17), (21), (25), and (29). Vial *et al.* [1] performed experiments using uniaxial tension, uniaxial (through thickness) compression, balanced biaxial tension, and plane-strain compression using four sheet metals. The relevant experimental results are shown in Tables 1 to 3 for the reader's convenience. Using the average values of r, K_u, and n_u from the uniaxial tensile tests and the values of K_b and n_b from the balanced biaxial tension tests, m values were determined at various strains for all four metals. To do this, the equivalence between the two tests must be considered. Instead of attempting to derive such equivalence in a general form, we have applied directly the

TABLE 1. UNIAXIAL TENSION TESTS

Material	Angle (°)	K (MPa)	n	Range of strain	Strain ratio (R)
Al-killed steel	0	520.2	0.237	0.02–0.24	1.950
	90	511.8	0.235	0.02–0.24	2.318
	45	533.0	0.233	0.02–0.24	1.470
	av	524.9	0.234	0.02–0.24	1.802
Aluminum	0	195.5	0.214	0.02–0.24	0.655
	90	183.2	0.215	0.02–0.24	0.510
	45	187.1	0.222	0.02–0.24	0.753
	av	188.1	0.218	0.02–0.24	0.668
Copper	0	465.7	0.362	0.02–0.28	0.870
	90	433.9	0.364	0.02–0.31	0.818
	45	436.8	0.361	0.02–0.36	0.449
	av	443.3	0.362	0.02–0.28	0.654
Brass 260	0	828.8	0.493	0.08–0.39	0.944
	90	828.6	0.498	0.08–0.39	0.743
	45	809.7	0.495	0.08–0.39	0.841
	av	819.2	0.495	0.08–0.39	0.842

TABLE 2. BIAXIAL TENSION AND UNIAXIAL COMPRESSION TESTS

Material	K (MPa)	n	Range of strain
Biaxial tension stress vs thickness strain curves			
Al-killed steel	712.6	0.278	0.06 –0.25
Aluminum 3003-0	171.3	0.170	0.055–0.33
Copper 110	444.6	0.368	0.09 –0.42
Brass 260	848.0	0.465	0.10 –0.25
Uniaxial compression stress vs thickness strain curves			
Al-killed steel	661.2	0.248	0.013–0.27
Aluminum 3003-0	176.9	0.2034	0.027–0.35
Copper 110	418.8	0.334	0.025–0.32
Brass 260	787.8	0.442	0.065–0.24

TABLE 3. PLANE-STRAIN COMPRESSION TESTS

Material	K (MPa)	n	Range of strain	Lubricant
(a) Plane-strain compression ($\varepsilon_y = 0$)				
Al-killed steel	805.0	0.256	0.055–0.36	MoS_2
	812.0	0.268	0.055–0.35	Teflon
*Aluminum 3003-0	212.7	0.204	0.03–0.35	MoS_2
Copper 110	505.9	0.369	0.04–0.36	MoS_2
	448.9	0.304	0.10–0.37	Teflon
Brass 260	1012.9	0.469	0.07–0.29	MoS_2
	950.5	0.457	0.06–0.31	Teflon
(b) Plane-strain compression ($\varepsilon_x = 0$)				
Al-killed steel	826.9	0.259	0.05–0.31	MoS_2
	836.1	0.266	0.05–0.27	Teflon
*Aluminum 3003-0	197.4	0.214	0.02–0.5	MoS_2
Copper	483.8	0.362	0.075–0.41	MoS_2
	497.2	0.386	0.08–0.33	Teflon
Brass 260	927.4	0.453	0.07–0.34	MoS_2
	951.7	0.471	0.09–0.34	Teflon

*When Teflon was used with aluminum, results were questionable.

TABLE 4. VARIATIONS OF m-VALUES AT DIFFERENT STRAINS

ε_u	Case I	Case II	Case III	Case IV
	m-values (Al-killed steel)			
0.060	1.984	1.970	1.919	2.044
0.100	1.999	1.998	1.995	2.022
0.140	2.009	2.017	2.044	1.977
0.180	2.016	2.032	2.084	1.956
0.220	2.022	2.046	2.115	1.941
	m-values (aluminum)			
0.060	2.219	2.189	3.080	1.667
0.100	2.184	2.152	2.826	1.717
0.140	2.166	2.128	2.680	1.751
0.180	2.142	2.111	2.578	1.778
0.220	2.127	2.098	2.502	1.780
	m-values (copper)			
0.040	2.160	2.125	2.667	1.754
0.100	2.167	2.132	2.710	1.743
0.180	2.172	2.137	2.738	1.736
0.280	2.175	2.141	2.758	1.731
0.340	2.177	2.202	2.766	1.729
	m-values (brass)			
0.040	2.185	2.202	2.997	1.682
0.100	2.161	2.169	2.814	1.722
0.180	2.143	2.147	2.699	1.749
0.280	2.131	2.131	2.617	1.771
0.340	2.125	2.124	2.583	1.780

equivalence of mechanical work for these two tests as follows, noting that power-law hardening is displayed by both. The work per unit volume is the area beneath the stress–strain curve up to the strain considered. So, if $\sigma_u = K_u\varepsilon_u^{n_u}$ in uniaxial tension, then the work per unit volume is

$$w_u = \frac{K_u\varepsilon_u^{n_u+1}}{n_u+1} \tag{32}$$

TABLE 5. PREDICTIONS OF PLANE-STRAIN COMPRESSION FOR EACH CASE

Case	m^*	x	$-\sigma_p/\sigma_u$	K_p
	Al-killed steel (R_{av} = 1.802)			
I	1.999	0.357	1.305	728.9
II	1.998	0.358	1.305	729.1
III	1.995	0.358	1.305	729.1
IV	2.022	0.358	1.305	729.2
	Aluminum (R_{av} = 0.668)			
I	2.184	0.467	1.150	223.0
II	2.152	0.418	1.113	214.3
III	2.826	0.449	1.135	219.5
IV	1.717	0.469	1.128	217.7
	Copper (R_{av} = 0.654)			
I	2.167	0.486	1.139	529.4
II	2.132	0.455	1.105	507.7
III	2.710	0.474	1.126	521.1
IV	1.743	0.490	1.120	517.1
	Brass (R_{av} = 0.842)			
I	2.161	0.416	1.197	1072.1
II	2.169	0.353	1.161	1024.4
III	2.814	0.385	1.180	1048.8
IV	1.722	0.406	1.173	1039.4

*m-value calculated from $\varepsilon_u = 0.1$.

and, if $\sigma_b = K_b\varepsilon_b^{n_b}$ in balanced biaxial tension (ε_b: magnitude equal to the thickness strains in biaxial tension) is

$$w_b = \frac{K_b\varepsilon_b^{n_b+1}}{n_b+1}. \tag{33}$$

Considering $w_u = w_b$, ε_b can be explicitly expressed as

$$\varepsilon_b = \left[\frac{K_u(n_b+1)}{K_b(n_u+1)}\varepsilon_u^{n_u+1}\right]^{1/(n_b+1)}. \tag{34}$$

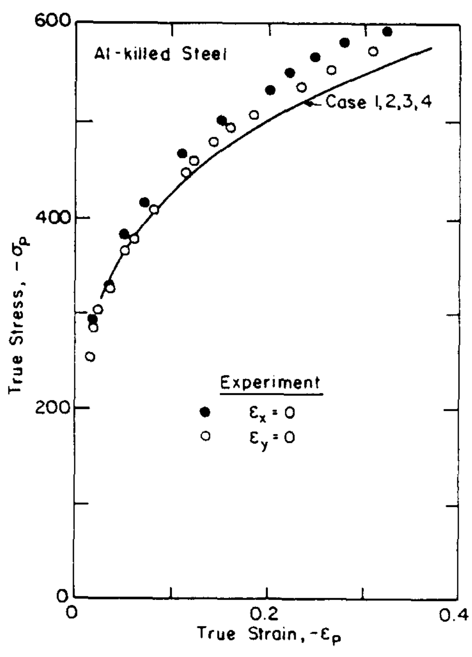

FIG. 1. Plane-strain compression results comparing predictions vs experiment for an Al-killed steel.

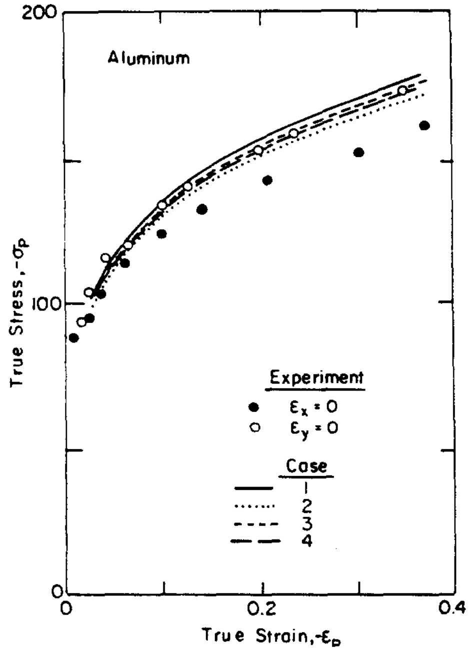

FIG. 2. Plane-strain compression results comparison predictions vs experiment for an aluminum alloy (3003-0).

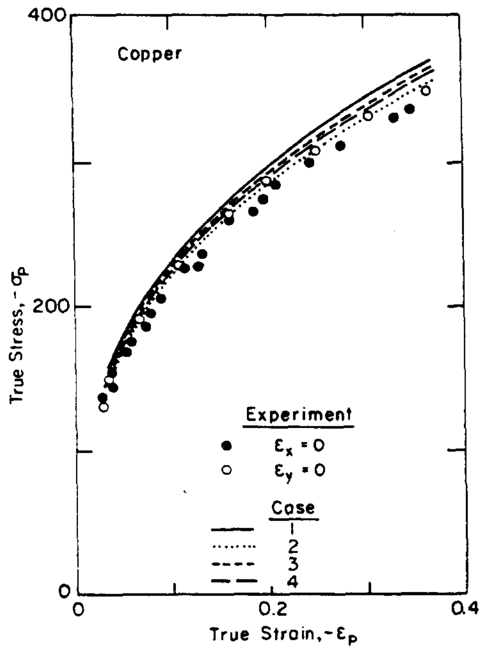

FIG. 3. Plane-strain compression results comparing predictions vs experiment for Cu-110.

A sample calculation indicates how the m-values in Table 4 were determined. Consider the Al-killed steel, where, from Table 1, the *average* values of interest are $r = 1.802$, $K_u = 524.9$, and $n_u = 0.234$; from Table 2, $K_b = 712.6$ and $n_b = 0.278$. For a uniaxial strain of $\varepsilon_u = 0.100$, the balaced biaxial strain that gives the same amount of plastic of work is 0.0876 from equation (34). The stress, σ_u, for $\varepsilon_u = 0.100$, is $\sigma_u = 524.9(0.1)^{0.234} = 306.25$, while the stress σ_b, for $\varepsilon_b = 0.0876$ is $\sigma_b = 712.6(0.0876)^{0.278} = 362.13$. Thus, $\sigma_b/\sigma_u = 1.1825$.

For Case 1, from equation (16), $f/c = -2.5549$ and with the ratio of σ_b/σ_u of 1.1825, m is found, using equation (17), to be 1.999. Each of the four cases has been used for a range of uniaxial tensile stress and the results are tabulated in Table 4. It should be noted that n_u must be equal to n_b in order to have the same m-value for all strains. Once the m-values are calculated, the stress–strain behavior for plane-strain compression can be predicted by theory and the merit of any particular yield criterion can be assessed by comparison with experiment. Again, we use the concept of

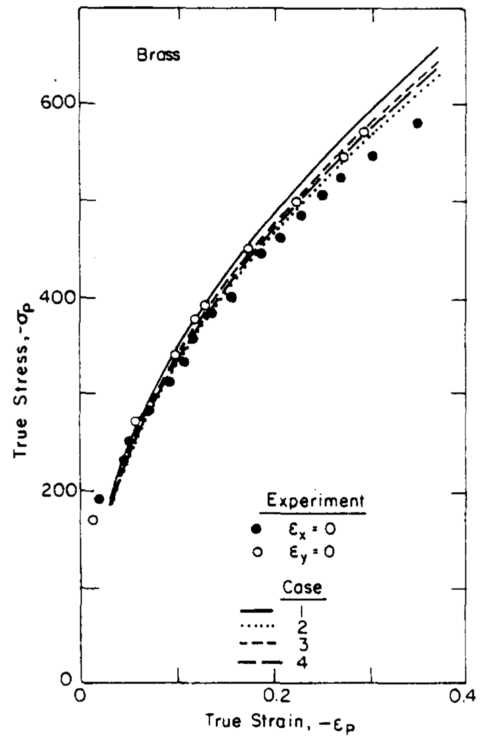

FIG. 4. Plane-strain compression results comparing predictions vs experiment for an alpha brass.

the equivalence of mechanical work for uniaxial tension and plane-strain compression, and assume that the value of m does not vary with strains. Here, the true strain, ε_u, in uniaxial tension and the true *thickness* strain, ε_p, in plane-strain compression are related by

$$\sigma_u d\varepsilon_u = \sigma_p d\varepsilon_p \quad \text{or} \quad \varepsilon_u = \left(-\frac{\sigma_p}{\sigma_u}\right)(-\varepsilon_p). \tag{35}$$

The uniaxial tensile behavior is again given by $\sigma_u = K_u \varepsilon_u^n$ and with equation (35), the predicted stress strain behavior for plane-strain compression is expressed as

$$-\sigma_p = \sigma_u\left(-\frac{\sigma_p}{\sigma_u}\right) = K_u\left(-\frac{\sigma_p}{\sigma_u}\right)^{n+1}(-\varepsilon_p)^n = K_p(-\varepsilon_p)^n, \tag{36}$$

where both σ_p and ε_p are compressive as to sign. Since the m-values for all cases vary with strain, as seen in Table 4, we have arbitrarily chosen the m-value associated with a uniaxial tensile strain of 0.100 to obtain the predicted values summarized in Table 5. To illustrate the procedure, we will analyze the Al-killed steel for Case 1. Here, $r = 1.802$ (Table 1), $m = 1.999$ (Table 4), and $f/c = -2.5549$ as found earlier.

Using equation (19), x is found first. There are two values of x that satisfy this equation, but since x must be less than 1.0 to satisfy the physics of this situation, the value of 0.357 is chosen. With equation (18) the value of $(-\sigma_p/\sigma_u)$ is 1.305. With $K_u = 524.9$ *and* $n_u = 0.234$ (both from Table 1), the use of equation (36) gives $K_p = 728.9$. Using the values from Table 5, the predicted plane-strain compression behavior for all four cases is compared with experimental data in Figs 1 to 4.

5. DISCUSSION

As seen in Figures 1 to 4, the predicted plane-strain compression stress–strain behavior is fairly similar for all four special cases of Hill's new criterion where Case 1 consistently gives the highest level, Case 2 the lowest, while Cases 3 and 4 are not only between the extremes, but themselves are almost identical. Except for the Al-killed steel, stress–strain behavior that is predicted for the other three metals is generally a bit higher than the experimental findings. It is certainly possible that the discrepancies can be attributed to at least two sources. First, as seen in Table 4, the m-value varies with strain and all predictions were based upon that value associated with an arbitrarily chosen strain of 0.100.*

Secondly, the metals themselves do not exhibit planar isotropy so the use of an average r-value remains questionable. Even though this approach is technically incorrect, as pointed out in [1], it has been used in most past studies of the type presented here. Although we have no better suggestion at this time, this remains a questionable point.

Finally, we note that the calculated stress ratios (x) for all cases under plane-strain compression are less than 0.5, whereas, if $m = 2$, they should equal $1/(1+r)$.

* We note that Wagoner [7] and Mohammed-Ali and Mellor [8] have observed similar behavior.

6. CONCLUSIONS

Although discrepancies exist between prediction and experiment, all four cases give reasonable results from a practical point of view. Cases 1 and 4 are somewhat simpler in formulation than are Cases 2 and 3 and from Figs 1 to 4, there is a slight preference for Case 4.

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