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THE INTERACTION BETWEEN A DISLOCATION AND A CRACK: CLOSURE CONSIDERATIONS

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ABSTRACT

The interaction between an edge dislocation and a crack in the presence of uniform applied tension and shear is considered. Emphasis is placed on the effects of partial closure and crack-face friction. Representative results are given.

1. INTRODUCTION

The interaction between a dislocation with an arbitrary Burgers vector and a crack in an anisotropic elastic medium was studied first by Atkinson [1]. Considering the case when the elastic fields are independent of one of the coordinate axes, he obtained closed form expressions for the stresses and the crack-opening displacement. Another relevant study is due to Solovev [2], who obtained the stress field near general dislocation pile-ups, including cracks, in presence of line defects in anisotropic elasticity.

The generality of the previously cited papers obscures the effect a dislocation might have on crack closure. Consider, for instance, a crack placed in a uniform tensile field. The presence of a dislocation near the crack, perturbs the applied tension, and may cause the crack to be totally or partly placed in a compressive field. Thus the location and strength of the dislocation dictates whether crack closure will occur. In the present paper we restrict attention to isotropic elastic materials and focus on the crack closure problem allowing Coulomb friction to be transmitted through the crack-face contact. It should be noted that the solution involving a partially closed crack and a dislocation cannot be used as a Green's function for the study of more complex interactions. On the other hand, it exposes the nonlinear nature inherent to such interactions, and shows that general solutions, such as in [1], may not be always applicable. Since closure becomes important when the dislocation is near the crack, the same approach can be used to model the damage zone surrounding a crack tip, or to study the interaction of dislocation pile-ups and a crack.

2. FORMULATION

A crack of length $2a$ is considered in an infinite, isotropic elastic solid. With no loss of generality, the axis x is taken along the crack, which is symmetric about the axis y. An edge dislocation with Burgers vector (b_x, b_y) is located at a point with coordinates (ξ, η), as shown in Fig. 1. The normal and shear tractions due to the dislocation on $y = 0$ are obtained from Dundurs [3] as

$$\sigma^D = \frac{2\mu}{\pi(\kappa+1)} \frac{1}{\left[(x-\xi)^2+\eta^2\right]^2} \left[b_y(x-\xi)\left[(x-\xi)^2+3\eta^2\right] + b_x\eta\left[\eta^2-(x-\xi)^2\right]\right] \tag{1}$$

$$\tau^D = \frac{2\mu}{\pi(\kappa+1)} \frac{1}{\left[(x-\xi)^2+\eta^2\right]^2} \left[b_y\eta\left[\eta^2-(x-\xi)^2\right] + b_x(x-\xi)\left[(x-\xi)^2-\eta^2\right]\right] \tag{2}$$

The crack is represented by a distribution of glide, $B_x(x)$, and a distribution of climb,$B_y(x)$, dislocations. The climb distribution is zero everywhere except along the open part of the crack L_g, and the glide distribution is zero everywhere except along the slipping regions of the crack, L_s. Denoting the applied normal and shear tractions at infinity by σ^∞ and τ^∞, we can express the total normal $N(x)$ and shear $S(x)$ tractions on $y = 0$ as

$$N(x) = \sigma^\infty + \sigma^D + \frac{2\mu}{\pi(\kappa+1)} \int_{L_g} \frac{B_y(\varsigma)}{x-\varsigma} d\varsigma \tag{3}$$

$$S(x) = \tau^\infty + \tau^D + \frac{2\mu}{\pi(\kappa+1)} \int_{L_s} \frac{B_x(\varsigma)}{x-\varsigma} d\varsigma \tag{4}$$

Note that in the contact region of the crack it is possible to have a combination of slip and stick zones. If we use the symbol L to denote the entire crack region, we represent the stick region by $L - L_s$, since L_g is contained in L_s. We now write the boundary conditions for the general case in which the crack has separation, slip and stick zones:

$$N(x) = 0 \quad \text{in} \quad L_g, \tag{5}$$

$$g(x) \geq 0 \quad \text{in} \quad L_g, \tag{6}$$

$$N(x) \leq 0 \quad \text{in} \quad L - L_g, \tag{7}$$

$$S(x) = \rho N(x) \quad \text{in} \quad L_s, \tag{8}$$

$$|S(x)| < -fN(x) \quad \text{in} \quad L - L_s, \tag{9}$$

where

$$\rho = -f \operatorname{sgn} h(x), \tag{10}$$

f is the coefficient of friction, $g(x)$ is the gap and $h(x)$ the relative tangential shift between the crack faces. The two latter quantities are related to the dislocation densities of the crack by

$$B_y = -dg(x)/dx, \tag{11}$$

$$B_x = -dh(x)/dx. \tag{12}$$

The boundary conditions must also be complemented by two auxiliary conditions requiring that the crack contains no net dislocations:

$$\int_{L_g} B_y(x)dx = 0, \tag{13}$$

$$\int_{L_s} B_x(x)dx = 0. \tag{14}$$

Note that for more general line defects, such as those considered by Solovev, the right sides of (13) and (14) need not vanish. Moreover, these conditions must be applied to *each* disjoint gap and slip zone.

To solve the problem we must determine the unknown dislocation distributions and the unknown locations and extents of the various zones along the crack. We observe that $B_y(x)$ and $B_x(x)$ are only coupled through the friction law (8). It is, therefore, possible to obtain the normal traction, the gap and L_g independently using (1), (3), (5), (6), (7) and (11). Having obtained this part of the solution, we use the remaining boundary conditions to determine the shear tractions and L_s. Although we can still proceed with generality, the method of solution we employ actually assumes an arrangement of zones, computes their extents, and determines limits within which the dimensionless combinations of the given parameters of the problem should lie so that the assumed arrangement is possible. Some of the arrangements that are possible in this problem are illustrated in Fig. 2.

2. METHOD OF SOLUTION

Assuming the extents of the various zones as given, the problem reduces to the determination of B_y and B_x by inversion of a Cauchy singular integral equation, respectively. The unknown ends of the zones are then determined by using the auxiliary conditions iteratively. Since the problem has a unique solution, Fichera [4], there is always a sufficient number of conditions for the computation of all the unknowns. Consider, for instance, the arrangement of Fig. 2(a). In addition to the dislocation densities, the parameters b and c are unknown; they will be determined by using (13) and (14) iteratively. The dislocation densities are completely determined by (5) and (8), because they are bounded at their left end and unbounded at their right end, [6]. Consider now the case in which there is no stick zone, i.e c coincides with $-a$. Then B_x is singular at both ends, and inversion of the corresponding Cauchy integral equation introduces an unknown constant, Muskhelishvili [5]. This constant is determined by using (14); there is no other unknown associated with slip. At this point one might ask what is the role of the inequalities. Their role is twofold: on one hand, they dictate the behavior of the dislocation densities and of the elastic fields at the endpoints of the various zones, and on the other, they delineate the various configurations of the zones by placing limits in their range of applicability.

To illustrate, let us consider an example for which $\eta = 0$, that is the dislocation is on the x axis, but outside the crack. Using (3) and (5) we compute B_y:

$$B_y(x) = \left(\frac{x-b}{a-x}\right)^{1/2} \left[\frac{(\kappa+1)\sigma^\infty}{2\mu} + \frac{b_y}{(x-\xi)\pi}\left|\frac{a-\xi}{\xi-b}\right|^{1/2}\right]. \qquad a < x < b \quad (15)$$

Using (15) and (13), we obtain a relation between b and the applied normal traction σ^∞:

$$\frac{\pi(\kappa+1)\sigma^\infty}{2\mu b_y} = \frac{2}{a-b}\left(1 - \left|\frac{a-\xi}{\xi-b}\right|^{1/2}\right). \quad (16)$$

The normal tractions outside the gap are then obtained from (15) and (3):

$$N(x) = -\frac{2\mu}{\pi(\kappa+1)}\frac{b_y}{x-\xi} + \left|\frac{x-b}{x-a}\right|^{1/2}, \qquad x > a, x < b \quad (17)$$

It is not worthwhile to solve (16) for b. Instead, since the applied tractions appear necessarily in a linear manner, it is computationally simpler to take b as given, and solve for σ^∞. To proceed, let us fix c: assume, for instance, that we have an arrangement with no slip zone, or $b = c$ in Fig. 2a. Using (4) and (8) we obtain an expression for B_x which is analogous to (15). Similarly, (14) produces a relation between b_x and σ^∞, which simplifies to

$$\tau^\infty/\sigma^\infty = b_x/b_y, \quad (18)$$

when (16) is taken into account. Whether this configuration is possible depends on whether there is a feasible choice of the remaining parameters that will satisfy the inequalities (6), (7) and (9).

The results presented in the next section were obtained by numerical integration of the Cauchy integrals using the method of Erdogan et al [7], and by monitoring all the inequalities.

4. RESULTS

To present the results in dimensionless form we have used the following normalisations:

- normalised applied normal stress $= \sigma^\infty a(\kappa+1)/(2\mu b_y)$,
- normalized applied shear stress $= \tau^\infty a(\kappa+1)/(2\mu b_x)$,
- normalized stress intensity factor $K_I = K_I/(\sigma^\infty\sqrt{(a-b)/2})$,
- normalised stress intensity factor $K_{II} = K_{II}/(\sigma^\infty\sqrt{(a-c)/2})$.

The results also depend on the parameters ξ, η, b_x/b_y and f. An exhaustive parametric study was not attempted, but the results are meant to be representative. Fig. 3 shows a map of the various regions in the normalized σ^∞ and τ^∞ plane for $\xi/a = -2$, $\eta/a = 1$, $b_x/b_y = 0$ and $f = 0.5$. It is important to note that in our normalization the applied normal stress is positive. The negative normalized value necessarily implies that b_y is negative. All the inequalities were verified with this assumption in mind. It is certainly possible to obtain results for the the case of a pre-compressed crack which is partially opened by a dislocation using the same analysis. In Fig. 3 the gap region is confined between the two vertical lines marked $b/a = -1$ and $b/a = 1$. The lines of constant c, i.e of fixed slip extent, are straight lines with slope equal to f. Negative slope corresponds to positive (forward) slip direction and positive slope corresponds to backslip. It is easy to prove that the loci of constant c are straight lines if there is no gap by examining the auxiliary condition (14), in which B_x is substituted after solving (8). The curve marked $b = c$ markes the boundary between slip and backslip and corresponds to a configuration of stick-gap. Only the lines $c/a = 1$ and $c/a = -1$ have been drawn. The lines $c/a = 1$ bound the region of complete stick, and the lines $c/a = -1$ bound the region of slip (with or without gap). Between the two pairs we have regions corresponding to stick- (back)slip-gap or stick-(back)slip.

A map of the type of Fig. 3 is obtained if we fix η and ξ, and vary b_x/b_y in the range $(0, 1.5)$ approximately. An example with $b_x/b_y = -0.2$ is shown in Fig. 4. The map is similar to the previous one in the region of forward slip. However, forward slip is not replaced by backslip. Instead a new backslip zone starts from the left tip of the crack. The two zones of opposite slip co-exist in the shaded region, until forward slip disappears and another backslip zone starts from the left end of the gap. The region of two backslip zones lies above the envelope of $b = c$ and $c/a = -0.6$. For b_x/b_y positive and greater than approximately 1.5 the region of two backslips starts near the center of the curve $b = c$ and expands in both directions.

Some examples of the stress intensity factors are given in Figs. 5 and 6. Fig. 5 shows the variation of K_I at a as a function of the gap extent for three values of b_x/b_y. Fig. 6 shows the variation of K_{II} at a with c/a for three different values of gap length. The three curves marked with symbols correspond to backslip, and the three plain ones to forward slip. The two sets meet at the points $b = c$ (stick-gap configuration).

Changing the location of the dislocation changes the configuration of the zones. One important difference is that for some locations the contact zone is detached from both ends leaving both crack tips open and creating two disjoint gaps, Fig.2(d). The boundary between the region of one gap and two gaps is shown in Fig. 7 in the ξ, η plane. Two boundaries are shown, one for the case $b_y < 0$ and the other for the case $b_y > 0$. The results are given for the case $b_x = 0$, for which there is symmetry about the axis η.

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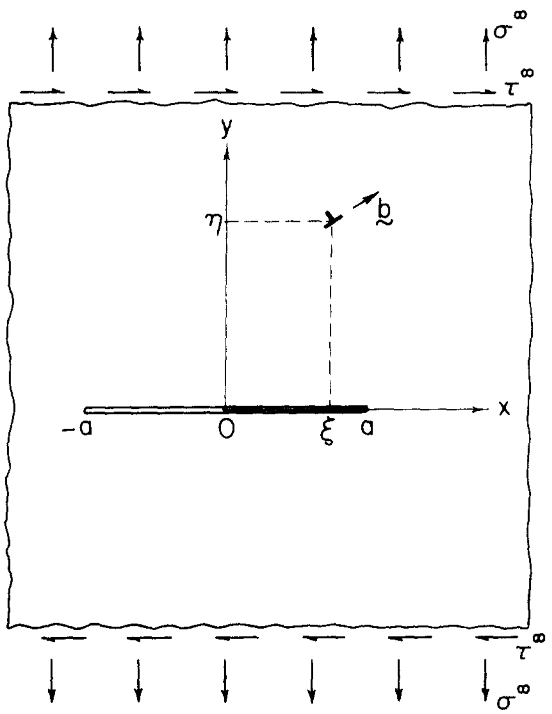

Fig. 1 Geometry of the problem.

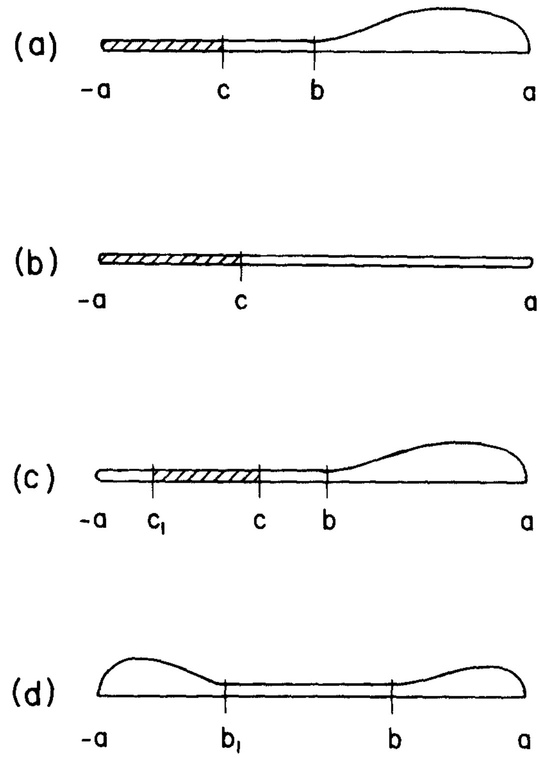

Fig. 2 Configurations of stick (shaded), slip and gap along the crack.

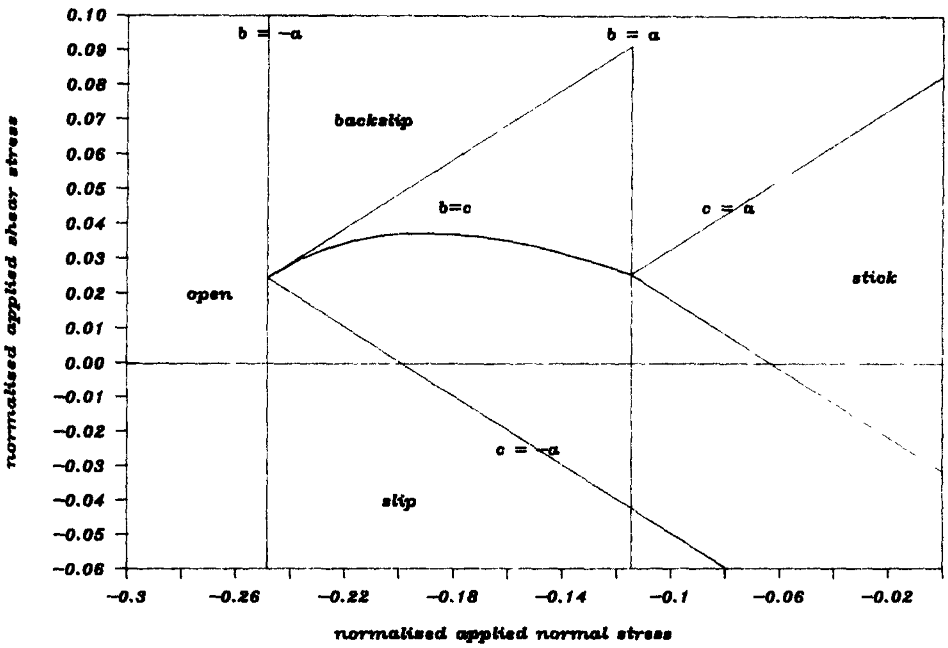

Fig. 3 Map of crack regions for $\eta/a = 1, \xi/a = -2, b_x = 0, f = 0.5$ and $b_y < 0$.

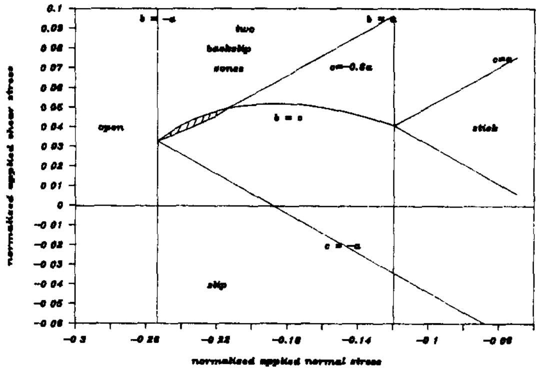

Fig. 4 Map of crack regions for $\eta/a = 1, \xi/a = -2, b_x/b_y = -0.2$, $f = 0.5$ and $b_y < 0$.

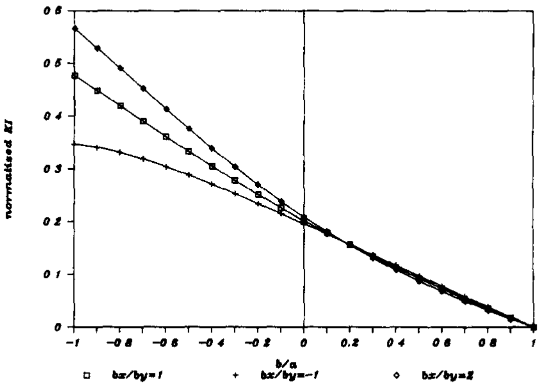

Fig. 5 Variation of Mode I stress intensity factor at a with gap extent for $\eta/a = 1, \xi/a = -2$ and $f = 0.5$.

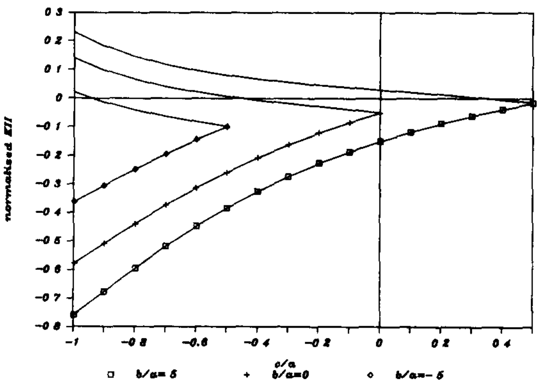

Fig. 6 Variation of mode II stress intensity factor with c/a for $\eta/a = 1, \xi/a = -2, f = 0.5$ and $b_x/b_y = 1$. Curves with symbols correspond to back-slip and plain curves to forward slip.

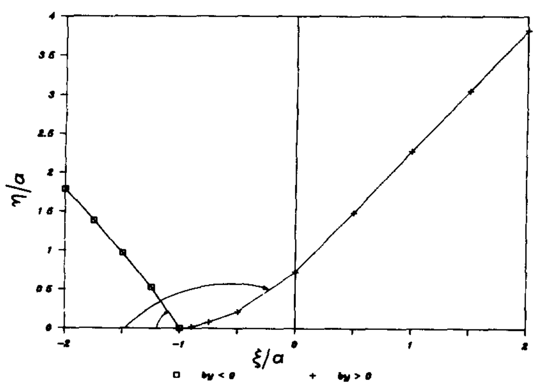

Fig. 7 Curved arrows bound regions with a single gap, $b_x = 0$.