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THE NN INTERACTION IN A QUARK MODEL WITH QUARK-ANTIQUARK EXCITATIONS

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The $(q\bar{q})$ and $(q\bar{q})^2$ excitations generated by off-shell terms of a Breit quark-gluon interaction are explicitly incorporated into the quark model of the NN system. Coupled channel RGM solutions lead to a semiquantitative zero adjustable-parameter fit of the NN scattering data.

In the simple quark model the nucleon is treated as a pure three quark (3q) system. However, (3q)-(3q) models of the NN interaction could elucidate only the extreme short range part of this interaction[1)] and failed to make a natural connection with the meson exchange picture responsible for the long range terms. A number of models have therefore been created[1)] in which the quark degrees of freedom of a nucleon interior are coupled to various meson fields. In such models quarks and meson fields are treated as separate entities. It is our philosophy that a model in which both baryons and mesons are described in terms of their common constituents is to be preferred, particularly when quark exchange effects become important, since the Pauli principle among the quark constituents can be respected only in this way. An improved quark model of the NN system has therefore been built[2)] in which the $(q\bar{q})$ and $(q\bar{q})^2$ excitations inherent in the quark-gluon interaction lagrangian[3)] have been explicitly incorporated into the model space. By studying the NN interaction within the framework of the resonating group method very explicit coordinate space results have been attained which make it possible to isolate the interaction corresponding to the exchange of a $(q\bar{q})$-pair or a $(q\bar{q})^2$-cluster between the two nucleons. This has led to a unified picture in which both baryons and the exchanged mesons appear on an equal footing in a pure quark model. This unified picture is versatile enough to lead to an understanding of the extreme short range part of the interaction in terms of the quark exchange mechanism and at the same time takes at least a first step toward the conventional meson exchange description through the possible mechanism of $(q\bar{q})$-pair exchange between nucleons.

Since low energy hadron phenomena confront QCD with formidable problems, an effective quark-quark interaction must perforce form the starting point of any quark model. Following the successes of the 3q models of the nucleon, our

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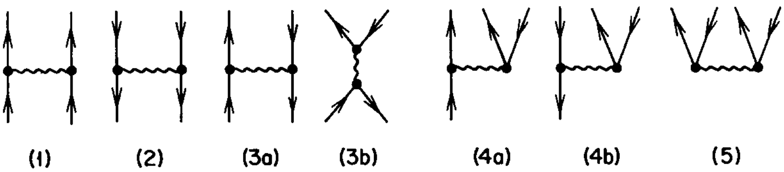

$$H_{int} = H_{qq} + H_{\bar{q}\bar{q}} + \left(H_{q\bar{q}} + H_{q\bar{q}}^{(ann)}\right) + \left(H_{q\to qq\bar{q}} + \text{c.c.}\right) + \left(H_{\bar{q}\to \bar{q}q\bar{q}} + \text{c.c.}\right) + \left(H_{\to (q\bar{q})^2} + \text{c.c.}\right)$$

FIGURE 1 The full Breit interaction hamiltonian

quark interaction is built by combining a phenomenological quark confining potential of simple power law form with a gluon exchange interaction of the general form of a one gluon exchange potential through the color analog of the Fermi-Breit interaction. Although these potential models may not be easy to justify when used for the prediction of single baryon properties, such as the absolute value of the baryon mass, which depend on the exact nature and strength of the confining potential, the final NN interaction has the fortunate property of being almost completely independent of the strength of the confining potential. The model therefore passes the crucial test of being insensitive to the details of the confinement phenomenology. This insensitivity to the strength and nature of the confinement potential, however, is dependent on the choice of a properly constrained model space. If both p-wave and color octet excitations were permitted in the 3q internal wave functions, the resultant color deformability would lead to long-range Van der Waals forces. We consider it one of the advantages of the RGM method that such pathological effects can be automatically excluded by a proper choice of the RGM nucleon internal wave functions.

The main point of departure of our model comes from the inclusion of the $(q\bar{q})$ and $(q\bar{q})^2$-creation terms which are a natural part of the one gluon exchange diagrams and are explicitly incorporated into our model. Fig. 1 shows the five types of interaction terms of the Breit quark-gluon interaction. Off shell contributions of type (4a) and of the RPA-type (5) can lead to additional interactions in a multi-quark system through the quark exchange mechanism. As a first step in the RGM calculation, improved single nucleon internal wave functions are calculated in which the dominant (3q) component is augmented by the $(3q)(q\bar{q})$ components generated by interactions (4a) as well as by the $(3q)(q\bar{q})^2$ components generated by interactions of type (5). The improved single nucleon wave function (resulting from a diagonalization of our interaction in a 31-dimensional basis), then has the form

$$\Psi_N = c_0\Psi_0((3q)) + \sum_{\alpha=1}^{24} c_\alpha\Psi_\alpha((3q)(q\bar{q})) + \sum_{\beta=1}^{6} c_\beta\Psi_\beta((3q)(q^2\bar{q}^2)) \quad (1)$$

The Ψ_α, (similarly Ψ_β), are described in an RGM cluster basis. Even if the (3q)-($q\bar{q}$) relative motion function of the Ψ_α is restricted to the energetically lowest p-wave, there are 24 possible spin, isospin, color combinations of the (3q) and ($q\bar{q}$) clusters which can couple to the resultant quantum numbers of the nucleon, including 15 hidden color combinations in which both (3q) and ($q\bar{q}$) clusters carry color octet quantum numbers. These are shown, together with their final amplitudes in table 1 where the spin, isospin, color labels of the (3q) and ($q\bar{q}$) clusters are abbreviated through their standard baryon and meson labels, and η^C, e.g., indicates that the ($q\bar{q}$)-pair carries the S,T quantum numbers of an η meson but is a color octet; (N^C, C_1, C_2) are color octet (3q) clusters with ST = $\frac{1}{2}\frac{1}{2}$, $\frac{3}{2}\frac{1}{2}$ and $\frac{1}{2}\frac{3}{2}$. The resultant spin, S_{12}, couples with the ℓ=1 orbital excitation to the J=$\frac{1}{2}$ value of the nucleon. Due to the intrinsic p-wave character associated with the ($q\bar{q}$)-pair creation process the fully antisymmetrized cluster functions $\Psi_\alpha((3q)(q\bar{q}))$ form a nearly orthonormal set, with overlaps $\langle\Psi_{\alpha'}|\Psi_\alpha\rangle$ for $\alpha' \neq \alpha$ generally much less than 0.15, so that real physical significance can be ascribed to the spin, isospin quantum numbers of the ($q\bar{q}$)-pairs. The ψ_β which are generated by the RPA-type terms of the Breit interaction are given in a (3q)($q^2\bar{q}^2$) cluster basis with two color

Table 1. Single nucleon amplitudes

(3q)	c_0 = 0.849			(3q)($q\bar{q}$) Hidden Color	S_{12}	c_α
(3q)($q\bar{q}$)	S_{12}		c_α			
$N\eta$	$\frac{1}{2}$		-0.053	$N^C\eta^C$	$\frac{1}{2}$	-0.020
$N\omega$	$\frac{1}{2}$		-0.052	$N^C\omega^C$	$\frac{1}{2}$	-0.044
$N\pi$	$\frac{1}{2}$		-0.169	$N^C\pi^C$	$\frac{1}{2}$	-0.015
$N\rho$	$\frac{1}{2}$		0.126	$N^C\rho^C$	$\frac{1}{2}$	0.004
$\Delta\rho$	$\frac{1}{2}$		-0.101	$C_1\omega^C$	$\frac{1}{2}$	0.019
$N\omega$	$\frac{3}{2}$		-0.216	$C_1\rho^C$	$\frac{1}{2}$	0.003
$N\rho$	$\frac{3}{2}$		-0.239	$C_2\pi^C$	$\frac{1}{2}$	0.005
$\Delta\pi$	$\frac{3}{2}$		0.190	$C_2\rho^C$	$\frac{1}{2}$	-0.031
$\Delta\rho$	$\frac{3}{2}$		-0.241	$N^C\omega^C$	$\frac{3}{2}$	0.044
(3q)($q^2\bar{q}^2$)	ST	$\overline{ST}$	c_β	$N^C\rho^C$	$\frac{3}{2}$	-0.049
$N\sigma_1$	00	00	-0.004	$C_1\eta^C$	$\frac{3}{2}$	0.023
$N\sigma_2$	11	11	-0.100	$C_1\omega^C$	$\frac{3}{2}$	0.054
$N\sigma_3$	01	01	-0.195	$C_1\pi^C$	$\frac{3}{2}$	-0.010
$N\sigma_4$	10	10	0.043	$C_1\rho^C$	$\frac{3}{2}$	0.023
$N\delta_1$	11	11	-0.053	$C_2\rho^C$	$\frac{3}{2}$	-0.012
$N\delta_2$	01	01	-0.138			

antisymmetric q^2 states with ST values of 11 or 00 and two color symmetric ones with ST values of 01 or 10, with similar $\bar{S}\bar{T}$ restrictions for $\bar{q}^2$. Since the relative motion function of the (3q) vs. $(q^2\bar{q}^2)$ cluster motion is assumed to be in its energetically lowest 0s wave, the ψ_β span a smaller part of the quark model space, and the physical significance of the $(q\bar{q})^2$ quantum numbers is partly washed out by quark antisymmetrization effects. To avoid problems of overcompleteness a smaller Ψ_β-basis has been selected. Hidden color components have been excluded, and the resultant spin, isospin quantum numbers of the $(q\bar{q})^2$ cluster have been restricted to 00 and 01, the quantum numbers of a σ or δ meson. The final two-nucleon calculations show that those parts of the exchange kernels which correspond to the exchange between two nucleons of such $(q\bar{q})^2$-clusters do indeed lead to interactions with the basic characteristics of conventional σ and δ meson exchange potentials.

The improved single nucleon wave functions of eq. (1) are used to evaluate the exchange kernels for the two nucleon system

$$G(\underset{\sim}{R},\underset{\sim}{R}') = c_0^4\, G_0(\underset{\sim}{R},\underset{\sim}{R}') + c_0^3 \sum_{\alpha=1}^{24} c_\alpha\, G_\alpha\, (\underset{\sim}{R},\underset{\sim}{R}') + c_0^3 \sum_{\beta=1}^{6} c_\beta\, G_\beta(\underset{\sim}{R},\underset{\sim}{R}') + \ldots \qquad (2)$$

where terms of second order in the c_α and c_β have been neglected. These kernels have been converted into equivalent local potentials through the Wigner transform-WKB approximation. The Wigner transform of a particular component of the kernel can also be used as a first measure of its characteristics and importance for the full potential. To isolate the part of the interaction corresponding to the exchange of a $(q\bar{q})$-pair between nucleons a further analysis of the exchange terms of a particular G_α must be made. The pure (3q)-(3q) part of the kernel, G_0, gains contributions from 5 types of exchange terms. Due to the antisymmetry of the NN system these involve only a single quark permutation operator, (exchanging a single quark between the two nucleons), with 5 possible inequivalent placements of the potential lines corresponding to the gluon exchange interactions. A particular coupling kernel, G_α, connecting the (3q)-(3q) component of the NN system to a particular $(3q)$-$(3q)(q\bar{q})$ component, gains contributions from 25 distinct types of exchange terms, built from six types of quark permutation operators each with several possible placements of the potential lines. Most of these, however, involve complicated multi-quark exchange processes. It was shown[2b)] that the exchange of a $(q\bar{q})$-pair between two nucleons is effected by only two of the 25 possible types of exchange terms. These two dominate the long range parts of the potentials. They contribute only through those G_α for which the (3q) cluster has the quantum numbers of a real nucleon and the $(q\bar{q})$ cluster the color singlet character of a real pseudoscalar or vector meson. The

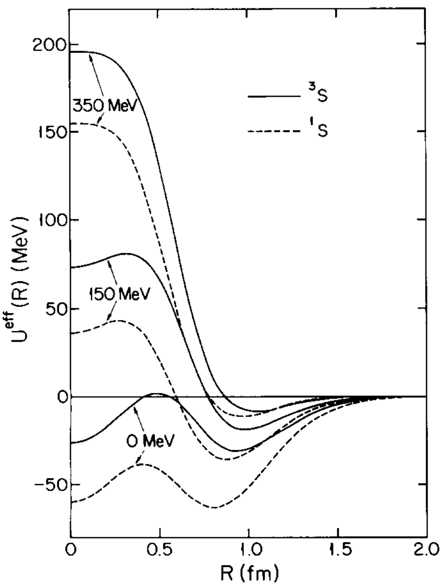

FIGURE 2
S-wave equivalent local potentials

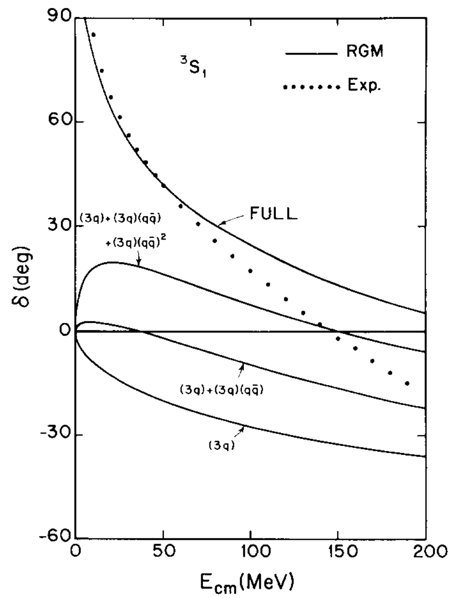

FIGURE 3
$^3S_1$ phase shift analysis.
Exp. points from ref. 4)

potentials arising from these simple $(q\bar{q})$ exchanges have all the characteristics of conventional OBEP's in their dependence on nucleon $(\underset{\sim}{\sigma}_1 \cdot \underset{\sim}{\sigma}_2)$ and $(\underset{\sim}{\tau}_1 \cdot \underset{\sim}{\tau}_2)$ factors and the relative importance and signs of spin-spin, spin-independent central, LS, and tensor terms. For R > 1.2 fm they are in remarkably good agreement with conventional OBEP's and generally have the same qualitative radial features over an even wider range.

The full kernels of eq. (2) have been converted into equivalent local potentials. The S-wave potentials are plotted in fig. 2 showing that an attractive ^1S potential at 0 MeV, gains a weak repulsive core at E_{cm} = 350 MeV. Similar strongly energy dependent ^3S central potentials show somewhat higher repulsive cores. It is also interesting to note that the ^1S E_{cm} = 0 potential is roughly similar to the square well potentials of historical interest. (The square well binding rule $M_N U_0 a^2/\hbar^2 \gtrsim \pi^2/4$ with a range parameter, a, of about 1.4 fm for U_0 = 50 MeV shows at once that the $^1S_0$ state is unbound).

Since the binding in the $^3S_1$ state gains important contributions through coupling to the $^3D_1$ channel via tensor force terms the solutions of the RGM equations have been carried out in a coupled channel formalism. The predicted RGM phase shifts are compared with the experimental numbers in figs. 3-7.

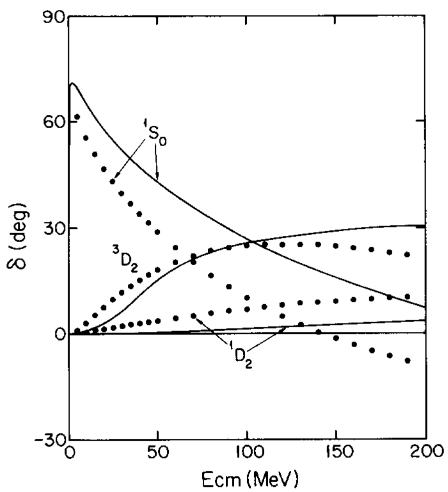

FIGURE 4
Phase shifts given by single channel RGM calculations

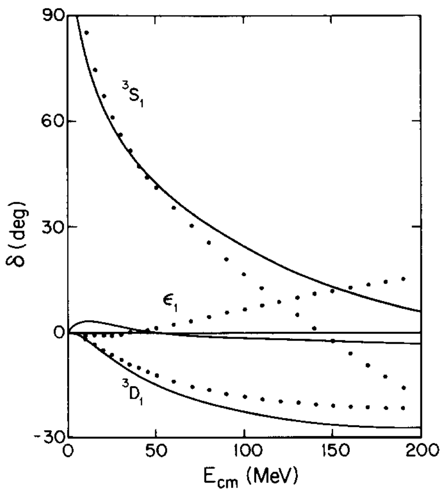

FIGURE 5
$^3S_1$ and $^3D_1$ phase shifts and mixing angle, ε_1

It is important to point out that the four parameters of our model have been determined from single nucleon properties. The quark mass, m, and the gluon coupling constant, or α_s, are considered as parameters of our effective quark interaction. Together with a quadratic confinement potential constant, a_c, and the oscillator length parameter, b, the four parameters of the model

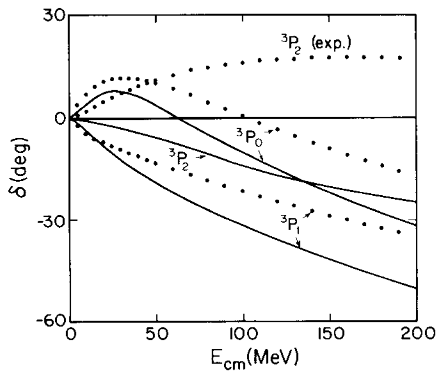

FIGURE 6
^3P phase shifts

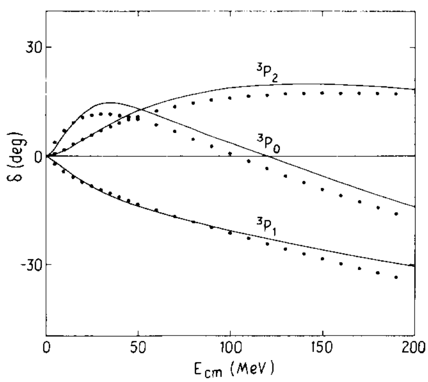

FIGURE 7
^3P phase shifts with central (C) kernels turned off

have values of

$$mc^2 = 471.2 \text{ MeV}, \ \alpha_s = 2.973, \ a_c = 335.7 \text{ MeV fm}^{-2}, \ b = 0.5235 \text{ fm}$$

which were determined from the following single nucleon properties: (1) The Δ-N mass difference, (2) the nucleon magnetic moments (which gain large contributions from c_0c_α cross terms), and (3) the ρ-meson coupling-constant (in particular, $(m_\rho/2M_N)(g_{NN\rho} + f_{NN\rho})$. With this choice for (3) both ρ and ω-coupling constants are in good agreement with experiment. In particular, the predicted tensor to vector coupling constant ratios are $f_{NN\rho}/g_{NN\rho} = 3.98$, $f_{NN\omega}/g_{NN\omega} = 0.18$. Although a_c can in principle be chosen to fit the absolute value of the nucleon mass as well, this parameter is not crucial for the NN scattering data since the a_c-dependent contribution to the (3q)-(3q) kernel, $G_0 = G^{(E)} - 2E_0N^{(E)}$, disappears for a quadratic confinement potential, through exact cancellation of the interaction kernel and internal energy term so that even a very large change in a_c has only a very small indirect effect on the NN interaction through small changes in the c_α. The one failure of the above parameters involves the predicted pion coupling constant, $g^2_{NN\pi}$, which is too small by a factor of ~3. As a result, the predicted Nπ tensor force was too weak by a factor of ~3. Since the exchange terms responsible for this piece of the NN interaction can be identified, it is possible to improve this part of the interaction. The $(q\bar{q})$-exchange terms of Nπ type were removed from the tensor part of the coupling kernel and replaced with a pion tensor term with the experimental value of $g^2_{NN\pi}$ and a gaussian form factor which gives a very good approximation to the form factor predicted by our quark model. With the exception of this one improvement of the interaction, no further adjustments of parameters were permitted.

Figs. 3-5 show that our model gives a satisfying semiquantitative fit of the NN scattering data in the even partial waves. Fig. 3 in particular shows that the repulsive phase shifts predicted by the pure (3q) G_0-components of our kernel gain attractive contributions from the central terms of the $(3q)(q\bar{q})$ and $(3q)(q\bar{q})^2$-coupling kernels of type G_α and G_β. However, the combined effects of the central terms of the $(3q)(q\bar{q})$ and $(3q)(q\bar{q})^2$-coupling kernels do not give sufficient attraction in this case. The important effects of channel coupling to the $^3D_1$ channel through tensor force terms lead to the results for the full coupled channel calculation which are in good agreement with the experimental $^3S_1$ phase shifts in the 0-100 MeV range. A low-k^2 analysis of these predicted phase shifts and a diagonalization of the interior region trial function both lead to a $^3S_1$ binding energy of ~0.7 MeV. The predicted ^3P phase shifts of fig. 6 show that our odd-L interaction is too repulsive. The offending terms are the odd-L central potentials. This is

Table 2. Characteristics of the Quark Model NN Interaction

Components		Even	Odd
	C	Repulsive (~600-700 MeV, predominantly from color-magnetic contact term)	Repulsive
(3q) G_0	LS	Repulsive	Attractive (~70% symm. LS ~30% antis. LS)
	T	~0	~0
	C	Attractive (~-300 to -100 MeV) Strongly energy-dependent	Repulsive
(3q)($q\bar{q}$) G_α	LS	Repulsive; Dominated by ($q\bar{q}$)-exchange of Nρ, Nω type	Attractive (~65% of total LS)
	T	Attractive; Dominated by ($q\bar{q}$)-exchange of Nπ, Nρ type	Repulsive
(3q)$(q\bar{q})^2$ G_β	C	Attractive; Many characteristics of σ meson exchange	Attractive

shown in fig. 7, for which the odd-L central terms are arbitrarily turned off, showing that the pure LS and tensor (T) terms, (with improved Nπ tensor force) do give a good quantitative account of the experimental ^3P phase shifts.

The main features of the nucleon-nucleon interaction of our extended quark model are summarized in table 2. Despite the successes of the model important problems remain. Although the odd-L central potentials may be reduced by refinements in the model, the pion presents a greater challenge since the exchange of a simple ($q\bar{q}$)-pair with the quantum numbers of a pion is clearly not a good model for a realistic OPEP with its long range Yukawa tail.

REFERENCES

1) M. Oka and K. Yazaki in Quarks and Nuclei, W. Weise, ed. (World Scientific, Singapore 1984). See this review and ref. 2a) for many earlier refs.

2) Y. Fujiwara and K. T. Hecht, Nucl. Phys. A444 (1985) 541, A451 (1986) 625, A456 (1986) 669, and Phys. Lett. 171B (1986) 17.

3) Y. W. Yu and Z. Y. Zhang, Nucl. Phys. A426 (1984) 557

4) M. H. MacGregor, R. A. Arndt, and R.M. Wright, Phys. Rev. 182 (1969) 1714