INDIRECT MEASUREMENT OF VECTOR BOSON SCATTERING AT HIGH ENERGIES

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The possibility of an indirect study of longitudinal gauge boson scattering and its relevance to new physics in the TeV region is analyzed.

In recent years the characteristic features of the scalar Higgs sector of the standard model have been extensively investigated with particular emphasis to the case where the Higgs mass is very large. The first step in this direction goes back to the work of Veltman (and also Linde) [1] who pointed out the unnaturalness of the cosmological constant in relation to spontaneous symmetry breaking, therefore suggesting to investigate the possibility that the Higgs be made very heavy and removed from the theory [2]. As a consequence of this limit we are entering a strong coupling regime of weak interactions. As we well know the limit $m_H \to \infty$ of the standard model becomes the massive Yang–Mills lagrangian, which gives raise to infinities. We can regard the Higgs mass as a regulator cutoff and inquire on the range of validity of the theory. However, it turns out that there are no observable corrections at the one-loop level, the so called "screening theorem" [2,3]. Another approach to the large-m_H limit is the inverse unitarity limit [4]. If a relatively light Higgs particle is not found vector boson scattering cannot be described by perturbation theory at energies above the 1–2 TeV region. The need for measurements of WW scattering above 1 TeV and their relation to the Higgs system was first emphasized by Veltman, and Lee, Quigg and Thacker [2,5] and more recently by the Berkeley group, Chanowitz, Gaillard, Cahn and Dawson [6]. We mention that Appelquist and Bjorken [7] also suggested among other possibilities that vector bosons show strong interactions with each other and with a new family of particles, the "sthenons".

In this letter we concentrate on WW scattering at very high energies. The corresponding measurements can be used to derive information on the Higgs mass and even more important they open a window on the new physics we are expecting if indeed there is no such a thing as a Higgs particle. In particular we have in mind the growth of partial-wave amplitudes as well as the possible formation of resonances in the WW channel when weak interactions enter a strong regime. More precisely the question we reexamine is the following. Is there any possibility of measuring the WW cross section by using the next generation of experimental facilities?

As pointed out by the Berkeley group [6] the most promising source of longitudinally polarized vector bosons is their bremsstrahlung from fast beams of standard weak coupling particles. Therefore we use the idea of ref. [6] with an alternative approach and a somewhat different perspective. We are not looking for the predictions of any particular model and we are not even assuming the existence of the Higgs boson. Our concern is to link the WW cross section to something that hopefully will be measurable in a near future. The idea behind our approach can be traced to a similar situation in π–π physics. As suggested long time ago the π–π scattering cross section can be investigated by extrapolating the result of the π–nucleon collision, namely $\pi + p \to N + \pi + \pi$ [8]. In this way, for in

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stance, it was possible to show a marked resonance in the π–π cross section from the π–nucleon data extrapolated at $(q_N - q_P)^2 = -m_\pi^2$. In the same spirit we put emphasis on a relation valid at very high energies between the longitudinal WW cross section and the invariant WW mass distribution in $\bar{\ell}\ell \to \bar{\ell}\ell W_L W_L$. The final result will be the consequence of an approximation equivalent to the one used in ref. [6] which is expected to be good typically to 10–15%.

The process we consider is $e^+e^- \to \bar{\ell}\ell VV$ ($\ell = e, \nu$; $V = W, Z^0$) and the situation we have in mind is that of a hypothetical multi-TeV e^+e^- collider. We choose to describe an e^+e^- experiment instead of a pp (or $p\bar{p}$) experiment largely because of the presence of QCD background in the latter, which may contain also many-body parton interactions. However, there is nothing peculiar to e^+e^- and should flavour identification be much better in the future we could use the same results with few obvious changes for $pp \to VVX$ at a multi-TeV collider. In our calculation we assume that only the diagram of double W-bremsstrahlung gives a contribution therefore neglecting e^+e^- annihilation background, which is justifiable in the region of interest $\sqrt{s} \geqslant 10$ TeV and $M(VV) = 1$–2 TeV.

In the following we compute the cross section to produce a pair of gauge bosons accompanying a pair of fermions starting from the Feynman rules relative to a diagram of double bremsstrahlung without any reference to the folding procedure of ref. [6] and without assuming real vector bosons in the intermediate states. Our result will agree with the luminosity distributions of the Berkeley group.

To study $e^+(p_+) + e^-(p_-) \to \bar{\ell}(q_+) + \ell(q_-) + V(k_1) + V(k_2)$ we introduce $L_\mu^\pm$, the amplitude for $\ell(p_\pm) \to \ell'(q_\pm) + V(p_\pm - q_\pm)$:

$$L_\mu^+ = -\bar{v}(p_+)\,\gamma_\mu(a_+ + b_+\gamma^5)\,v(q_+)\,, \quad L_\mu^- = \bar{u}(q_+)\,\gamma_\mu(a_- + b_-\gamma^5)\,u(p_-)\,.$$

Let also $V_{\mu\nu\alpha\beta}$ be the amplitude for $V_1V_2 \to V_3V_4$. The total amplitude is

$$A(\sigma_1,\sigma_2) = -g^2 L_\mu^+ \Delta^{\mu\alpha}(K_1)\, L_\nu^- \Delta^{\nu\beta}(K_2)\, V_{\alpha\beta\delta\lambda}\, e^\delta(k_1,\sigma_1)\, e^\lambda(k_2,\sigma_2)\,,$$

with $K_{1,2} = p_\pm - q_\pm$ and $\Delta^{\mu\nu}(K_i) = (\delta^{\mu\nu} + K_i^\mu K_i^\nu/M^2)/(K_i^2 + M^2)$. M is the V mass and σ_i denotes the polarization of the ith outgoing vector boson. $e^\mu(k,\sigma)$ is the corresponding wave function $k\cdot e(k,\sigma) = 0$. The cross section will be

$$\sigma(e^+e^- \to \bar{\ell}\ell VV) = \frac{1}{(2\pi)^8 8s} \int \mathrm{dPS} \sum_{\ell\text{-spins}} |A|^2\,.$$

Introducing vectors K_1, K_2 and $Q = k_1 + k_2$, the phase space integral is conveniently written as

$$\int \mathrm{dPS} = \int \mathrm{d}q_+ \mathrm{d}q_- \mathrm{d}^4Q \int \mathrm{d}^4K_1\, \mathrm{d}^4K_2\, \delta^4(p_+ - q_+ - K_1)\, \delta^4(p_- - q_- - K_2)\, \delta^4(Q - k_1 - k_2)$$

$$\times \int \mathrm{d}k_1\, \mathrm{d}k_2\, \delta^4(K_1 + K_2 - k_1 - k_2)\,,$$

where $\int \mathrm{d}p = \int \mathrm{d}^4p\,\theta(p)\,\delta(p^2 + m^2)$. The crucial step in our procedure is to replace

$$\delta_{\mu\nu} + \frac{1}{M^2} K_\mu K_\nu \to \sum_\lambda e_\mu(K,\lambda)\, e_\nu^*(K,\lambda) \quad (K^2 > 0)\,.$$

This replacement is correct only for momenta on-mass-shell $K^2 = -M^2$. However, terms proportional to $K_{i\mu}K_{i\nu}$ $(i = 1,2)$ give contributions to the cross section of order m_e^2 and therefore the substitution is correct since we are neglecting lepton masses. Notice that this does not imply that only transverse vector bosons give a contribution when fermions are taken to be massless. Indeed $e_L^\mu = K^\mu/M$ only for $K \to \infty$. Next we write

$$L^\pm(\lambda_i) = L_\mu^\pm e^\mu(K_i,\lambda_i)\,, \quad V(\lambda_1\lambda_2\sigma_1\sigma_2) = e_\mu^*(K_1,\lambda_1)\, e_\nu^*(K_2,\lambda_2)\, V^{\mu\nu\alpha\beta} e_\alpha(k_1,\sigma_1)\, e_\beta(k_2,\sigma_2)\,,$$

thus

$$\sigma(e^+e^- \to \ell\ell VV) = \frac{g^4}{(2\pi)^8} \frac{1}{8s} \sum_{\ell\text{-spins}} \sum_{\lambda_1 \ldots \lambda_4} \int dPS[L^+(\lambda_3)]^* L^+(\lambda_1)[L^-(\lambda_4)]^* L^-(\lambda_2)$$
$$\times V^*(\lambda_3\lambda_4\sigma_1\sigma_2)\, V(\lambda_1\lambda_2\sigma_1\sigma_2)\,.$$

The cross section is the sum of three terms $\sigma = \sigma_L + \sigma_T + \sigma_{LT}$ where σ_L has all the λ_i corresponding to longitudinal polarizations and similarly σ_T contains transverse polarizations while σ_{LT} gives the interference. Consider now the bremsstrahlung of a virtual vector boson from a massless fermion line, $\ell(p) \to \ell'(p') + V(k)$. The corresponding amplitude is proportional to $p \cdot e(k)$ and we can easily derive that both $|p\cdot e_L(k)|^2$ and $|p\cdot e_\perp(k)|^2$ are $O(k^2)$ for $k^2 = (p-p')^2 \to 0$ leading to a suppression in the forward region. To see it explicitly we introduce the following longitudinal polarization vectors (continued to $K^2 > 0$) [9]:

$$e^L_\mu(K_1) = -N_1(K_1\cdot K_2 K_{1\mu} - K_1^2 K_{2\mu})\,, \quad e^L_\mu(K_2) = -N_2(K_1\cdot K_2 K_{2\mu} - K_2^2 K_{1\mu})\,,$$

where $N_i^{-2} = K_i^2[K_1^2K_2^2 - (K_1\cdot K_2)^2]$. From these expressions we explicitly see that $e^L_\mu(K_1) \propto K_{1\mu}$ only for $K_1\cdot K_2 \neq 0$ and $K_1^2 = 0$. The above relations reduce to the more familiar ones in the K_1, K_2 CM system. The whole result becomes more transparent by using our covariant formalism. For this reason we also construct transverse polarization vectors. According to ref. [9] we first introduce

$$n_\mu(K_1) = n_1\epsilon_{\mu\nu\alpha\beta}p_+^\nu K_1^\alpha K_2^\beta\,, \quad n_\mu(K_2) = n_2\epsilon_{\mu\nu\alpha\beta}p_-^\nu K_2^\alpha K_1^\beta\,,$$

with $n_1^{-2} = 2K_1\cdot K_2\, p_+\cdot K_1\, p_+\cdot K_2 - K_1^2(p_+\cdot K_2)^2 - K_2^2(p_+\cdot K_1)^2$ and similarly for n_2. Next we define

$$n'_\mu(K_i) = (-K_i^2)^{-1/2}\epsilon_{\mu\nu\alpha\beta}n^\nu(K_i)\, e^\alpha_L(K_i) K_i^\beta\,.$$

Transverse polarization vectors (K_i off-mass-shell) are thus given by

$$e^\perp_\mu(K_i, \pm) = (1/\sqrt{2})[n_\mu(K_i) \mp n'_\mu(K_i)]\,.$$

The lepton part of the cross section is

$$\mathcal{L}^+_{L,\perp} = 4(a_+^2 + b_+^2)[2|p_+\cdot e_{L,\perp}(K_1)|^2 - p_+\cdot q_+]\,.$$

Using the previous results we get

$$\mathcal{L}^+_L = 4(a_+^2+b_+^2)p_+\cdot q_+\{4/\lambda(\mu^2, 2p_+\cdot q_+, 2p_-\cdot q_-)[\tfrac{1}{2}\mu^2 + 2p_+\cdot p_- - p_+\cdot q_+ - p_-\cdot q_- - 2p_+\cdot q_-]^2 - 1\}\,,$$

and

$$\mathcal{L}^+_\perp = 4(a_+^2+b_+^2)p_+\cdot q_+\{4/\lambda(\mu^2, 2p_+\cdot q_+, 2p_-\cdot q_-)[(p_+\cdot p_- - p_+\cdot q_-)(\mu^2 - 2p_+\cdot q_+ - 2p_-\cdot q_-)$$
$$+ 2p_+\cdot q_+\, p_-\cdot q_- + 2(p_+\cdot p_- - p_+\cdot q_-)^2] - 1\}\,,$$

where $(k_1+k_2)^2 = -\mu^2$ is the VV invariant mass and similar expressions with + and − subscripts interchanged hold for $\mathcal{L}^-$. In the above equations λ denotes the Kallen-function.

Both $\mathcal{L}^+_L$ and $\mathcal{L}^-_L$ as well as the corresponding transverse terms are suppressed in the forward region by the factors $p_+\cdot q_+$ and $p_-\cdot q_-$. However, the longitudinal vector bosons have a chance to give the leading contribution to the cross section as their $K^2 \to 0$. For instance if the VV scattering amplitude $V_{\mu\nu\alpha\beta}$ contains a factor $\delta_{\mu\nu}$, as in the case of an intermediate Higgs exchange, then σ_L will be also proportional to

$$|e_L(K_1)\cdot e_L(K_2)|^2 = (K_1\cdot K_2)^2/K_1^2K_2^2 = \tfrac{1}{4}(\mu^2 + K_1^2 + K_2^2)^2/K_1^2K_2^2\,.$$

A similar situation is not present for transverse vector bosons. As we have checked using their explicit expressions $|e_\perp(K_1)\cdot e_\perp(K_2)|^2$ remains finite as $K_1^2, K_2^2 \to 0$. The same considerations apply for the interference terms. Therefore in the region $M^2 \ll \mu^2 \ll s$ where the cross section peaks at small angles due to the propagator factors $(K_i^2 + M^2)^{-2}$, σ_L dominates. This defines the approximation; only longitudinal intermediate vector bosons contribute and moreover they are taken to be virtual ($K_i^2 > 0$) when produced by fermion lines and real ($K_i^2 = -M^2 \ll s$) when rescattered. To achieve this and to insure the proper behavior at small angles we replace the incoming wave functions in VV scattering by $(M^2/K^2)^{1/2} \times$ wavefunction computed on-mass-shell. At very high energies masses become less and less important and in the forward region where the cross section peaks we may identify virtual with real vector bosons (both with $K^2 \approx 0$). In computing σ_L we replace V^*V with $V = |V_L|^2$ which is a function of the invariant mass μ^2 and of the momentum transfer $t = -(K_1 - k_1)^2$. Therefore going to the $K_1 K_2$ CM system we get

$$\int \mathrm{d}k_1\, \mathrm{d}k_2\, \delta^4(K_1 + K_2 - k_1 - k_2)\, \delta(\mu^2 + (k_1 + k_2)^2)\, \delta(t + (K_1 + k_1))^2 V = \tfrac{1}{2}(\pi/\mu^2)\, \delta(Q^2 + \mu^2)\, V(\mu^2, t),$$

where once more we have neglected masses. Finally we fix the outgoing polarizations to be longitudinal. Even if there are not measurable we are expecting a strong signal only from $V_L V_L$ scattering. Collecting the results we obtain

$$\frac{\partial^2}{\partial\mu^2\,\partial t}\,\sigma(\mathrm{e^+e^-} \to \bar{\ell}\ell V_L V_L) = \frac{\alpha^2 A_+ A_-}{16\pi^5 \sin^4\theta_\mathrm{w}}\,\frac{M^4}{s} \int \mathrm{d}^4Q\, \mathrm{d}q_+\, \mathrm{d}q_-\, \delta(Q^2 + \mu^2)\, \delta^4\!\left(\sum p - \sum q - Q\right) \mathcal{L}\, \frac{V(\mu^2, t)}{\mu^2},$$

where $A_\pm = (a_\pm^2 + b_\pm^2)$ and $\mathcal{L}$ denotes collectively the leptonic contribution. The vector Q is timelike with positive time component and we can replace $\mathrm{d}^4Q\, \delta(Q^2 + \mu^2)$ with $\mathrm{d}Q = \mathrm{d}^4Q\, \theta(Q)\, \delta(Q^2 + \mu^2)$. The phase space is reduced to a three-body problem which we evaluate in the p_+, p_- CM system with p_- along the positive z-axis, q_- in the x–z plane and θ_-, $\pi - \theta_+$ being the q_-, q_+ polar angles. The Q-integration is performed by using the last δ-function and $Q^2 = -\mu^2$ evaluated at $\theta_+ = \theta_- = 0$ gives the relation

$$E_+ = \tfrac{1}{2}(s - \mu^2 - 2\sqrt{s}E_-)/(\sqrt{s} - 2E_-), \quad 0 \leqslant E_- \leqslant (1/2\sqrt{s})(s - \mu^2),$$

where $E_\pm$ are the $q_\pm$ time components. Proceeding in the approximation scheme we set $\theta_+ = \theta_- = 0$ everywhere but in the two propagator factors. In this way the integrations over θ_+, θ_- and E_- can be done exactly.

$$\frac{\partial^2}{\partial\mu^2\,\partial t}\,\sigma(\mathrm{e^+e^-} \to \bar{\ell}\ell V_L V_L) = \frac{\alpha^2 A_+ A_-}{16\pi^2 \sin^4\theta_\mathrm{w}}\,\frac{\mu^2}{s^2}\,\frac{V(\mu^2, t)}{16\pi\mu^4} \int_0^{E_{\max}} \frac{F(E)}{\sqrt{s} - 2E}\,\mathrm{d}E,$$

$$F(E) = [(4/\mu^4)(\tfrac{1}{2}\mu^2 - s + 2\sqrt{s}E)^2 - 1]\,\{[(\sqrt{s} + 2E)/(\sqrt{s} - 2E)]^2 - 1\}.$$

The last step is to recognize in $V(\mu^2, t)/16\pi\mu^4$ the expression for $\mathrm{d}\sigma(V_L V_L \to V_L V_L)/\mathrm{d}t$ computed at the center of mass energy μ. Integrating over t we obtain

$$\mu^2\, \mathrm{d}\sigma(\mathrm{e^+e^-} \to \bar{\ell}\ell V_L V_L)/\mathrm{d}\mu^2 = (\alpha^2 A_+ A_-/2\pi^2 \sin^4\theta_\mathrm{w})\, f(\mu^2/s)\, \sigma(V_L V_L \to V_L V_L),$$

where $f(x) = (1 + x)\ln(1/x) - 2(1 - x)$ is essentially the luminosity factor of ref. [6]. The scattered leptons will be in the forward region and therefore it will not be possible to distinguish between electrons and neutrinos. By measuring the W^+W^- and Z^0Z^0 (or even the exotic channels WZ) invariant mass distributions we can study combinations of cross sections as a function of their center of mass energy μ. For instance

$$\mu^2\, \mathrm{d}\sigma(\mathrm{e^+e^-} \to \mathrm{X} W_L^+ W_L^-)/\mathrm{d}\mu^2 = (\alpha^2/32\pi^2 \sin^4\theta_\mathrm{w})\, f(\mu^2/s)\, [h(\theta_\mathrm{w})\, \sigma(Z_L^0 Z_L^0 \to W_L^+ W_L^-) + \sigma(W_L^+ W_L^- \to W_L^+ W_L^-)],$$

$$h(\theta_\mathrm{w}) = (1/16 \cos^4\theta_\mathrm{w})\, [(4 \sin^2\theta_\mathrm{w} - 1)^2 + 1]^2.$$

In conclusion, following the idea introduced by the Berkeley group we have verified the existence of a relation between certain combinations of cross sections for longitudinal vector boson scattering and distributions which hopefully will be measurable in a near future. In general we are expecting a quite low statistics but at the same time we are waiting for some spectacular effect coming from the short-range strong part of the Yang–Mills force between massive vector bosons. For this reason the invariant mass distributions are the relevant objects to study. If we use a partial-wave expansion for $V_L V_L$ scattering we can in principle infer the behavior of $|a_j(\mu^2)|^2$ in a region of μ around 1–2 TeV thereby opening an indirect window on the dynamics of the strongly interacting Higgs sector and on its consequences, of which the most appealing is the formation of bound states of gauge bosons. It is perhaps worthwhile to mention at this point that double bremsstrahlung of W_L's as mechanism of production for vector boson bound states (called S and V) has been advocated and debated in the past [10].

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