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INVESTIGATING SECURITY-PRICE PERFORMANCE IN THE PRESENCE OF EVENT-DATE UNCERTAINTY*

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This paper introduces an event-study method that incorporates the possibility of a random event date. Consistent with empirical evidence, we assume an event may affect not only the conditional mean of a security's return, but also its conditional variance. We compare the statistical power and efficiency of our maximum-likelihood method with the standard application of traditional event-study methods to multiday security returns. Assuming a two-day event period, our empirical results provide evidence that the multiday approach is robust. We use our maximum-likelihood method to investigate the valuation effects of stock splits and stock dividends.

1. Introduction

In an efficient financial market, security prices adjust instantaneously to reflect unanticipated information. Event studies focus on how firm-specific events affect the returns to the firm's securities. Often, an event's calendar date is uncertain. The date the *Wall Street Journal* announces an event need not correspond to the date the event affects security prices. The potential for event-date misspecification arises whenever price data are reported more precisely than information about the event date.

Brown and Warner's (1980) simulation analysis with monthly common stock return data establishes that if the time at which a specific event occurs is known, commonly used event-study methods detect abnormal performance

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adequately. When the event date is uncertain, however, these common methods often fail to reject the null hypothesis of no abnormal performance when abnormal performance is present. The use of daily common stock return data in event studies increases the possibility of event-date misspecification and, concommitantly, reduces the statistical power of commonly used methods.

This paper introduces and implements an event-study method that permits event-date uncertainty. The method's specification incorporates the possibility of a random event date. Further, we assume an event may affect not only the conditional mean of a security's return, but, consistent with empirical evidence, its conditional variance as well. Resulting likelihood-ratio tests provide a statistically powerful means of detecting the presence of these events and measuring their impact on underlying security returns efficiently. We also compare this new method with the event-study methods in common use.

The problem of event-date uncertainty is usually addressed by applying traditional event-study methods to multiday security returns. Information is lost by aggregating security returns, however, and efficiency and statistical power are consequently reduced. We establish that in the presence of event-date uncertainty, applying common event-study methods to multiday security returns provides method-of-moments estimates of security-price performance. A comparison of our method with this approach comes down to a comparison of maximum-likelihood estimation with method-of-moments estimation. In general, maximum-likelihood estimation provides the more efficient and powerful tool. For two-day returns, however, these gains are small, which is evidence that the multiday approach is robust.

Event-study methods often ignore the possibility of increases in security-return variance around events. We corroborate Christie's (1983) argument that errors of inference may result if a researcher assumes no increase in variance around an event when variance actually increases. Intuitively, the researcher may erroneously interpret actual changes in variance as evidence of security-price performance. By incorporating the possibility of increases in variance around events, our method provides a more accurate assessment of security-price performance.

We illustrate the applicability and advantages of our method by using it to investigate the response of common stock returns to announcements of stock splits and stock dividends. We use Grinblatt, Masulis, and Titman's (1984) sample of pure stock-split and stock-dividend announcements that are uncontaminated by other firm-specific information on the announcement dates. As noted by Grinblatt, Masulis, and Titman, on average, a firm's common stock price increases significantly when a stock split or a dividend is announced, even though these announcements do not directly affect the firm's future cash flows. Given uncertainty about the event date, Grinblatt, Masulis, and Titman apply standard event-study methods to multiday common stock returns. For comparison, we apply our maximum-likelihood method and investigate various hypotheses about the valuation effects of stock splits and stock dividends. Our results confirm Grinblatt, Masulis, and Titman's conclusions.

The plan of this paper is as follows. In section 2, we describe a security-return-generating process that incorporates the possibility of event-date uncertainty. We consider the maximum-likelihood estimation of this model in section 3. We use Dempster, Laird, and Rubin's (1977) EM algorithm to implement the proposed test procedures efficiently. We also discuss various implications of our specification, including the impact of event-date uncertainty on reported increases in daily security-return variance around certain events. For example, we find that event-date uncertainty accounts for approximately 24% of the estimated increase in the variance of standardized excess event-period returns in the Grinblatt, Masulis, and Titman sample of stock splits and stock dividends. In section 4, we use simulation techniques based on observed security-return data, as in Brown and Warner (1980, 1985), to examine the statistical properties of our method, as well as standard event-study methods applied to multiday security returns. We use our estimation procedures in section 5 to investigate the valuation effects of stock splits and stock dividends. Section 6 provides a summary and conclusions.

2. The model

We couch our analysis, without loss of generality, within a mean-adjusted returns framework [Masulis (1978)]. That is, we examine whether security returns in the event period are statistically different from returns in the estimation period, without taking into account marketwide movements or the systematic risk of the securities.

For each day t in the estimation period, we assume that the return to the ith security, r_{it}, is normally distributed with mean μ_i and variance σ_i^2:

$$r_{it} \sim \mathrm{N}\left(\mu_i, \sigma_i^2\right).$$

For the event-period, however, we model an alternative return-generating process that reflects the informational impact of the event. Our specification of this process permits the possibility of a random event date.

In characterizing event-period returns, we assume there is no possibility of an event occurring outside the event period. We standardize notation by assuming the event period is symmetric around the presumed event date, day 0. That is, we denote the event period as $(-c,\ldots,0,\ldots,+c)$. To set parameters for event-date uncertainty, for each day t in the event period, we define

$$\begin{aligned} \theta_t &= 1 \quad \text{if the event occurs on day } t,\\ &= 0 \quad \text{otherwise.} \end{aligned}$$

The random variable θ_t indicates whether the event occurs on day t of the event period. For example, standard event-study methods assume

$$\theta_t = 1 \quad \text{if} \quad t = 0,$$
$$= 0 \quad \text{otherwise.}$$

Assuming that exactly one event occurs during the event period, we define

$$p_t = \text{Prob}\left[\theta_t = 1, \theta_s = 0, s \in (-c, \dots, +c), s \neq t\right], \quad t = -c, \dots, +c.$$

Thus, p_t is the probability that the event occurs on day t, and only one event can occur during the event period. The standard event-study method can be embedded within this framework by assuming that $p_0 = 1$.

We specify the event-period security-return-generating process permitting the possibility of event-date uncertainty as follows. If an event does not occur on day t of the event period, we assume that

$$r_{it} | \theta_t = 0 \sim \text{N}(\mu_i, \sigma_i^2).$$

This is precisely the estimation-period security-return-generating process. Alternatively, if an event occurs on that day,

$$r_{it} | \theta_t = 1 \sim \text{N}(\mu_i + A\sigma_i, \delta^2\sigma_i^2),$$

where A is the abnormal performance introduced by the event. We assume that the abnormal performance is proportional to the security return's estimation-period standard deviation, which allows abnormal performance to differ across firms. Since A is a standardized return, it is not a measure of the mean effect of an event per se. We also allow the security return's event-period volatility to differ, $\delta^2 \neq 1$, from its estimation-period volatility. This is consistent with the empirical evidence of Beaver (1968), Christie (1983), Kalay and Lowenstein (1985), and others that the variance of a security's return can increase substantially around certain types of events. We assume A and δ^2 are cross-sectional constants.

We detect empirically the impact of the event under investigation by estimating the parameters A and δ^2. By estimating the parameters $(p_{-c}, \dots, p_{c-1})$, we estimate the probability of the event occurring on alternate days within the event period. As a result, this specification makes it possible to measure security-price performance in the presence of event-date uncertainty.

3. Estimation

We assume independence of the event's effect across firms. That is, there is no clustering in event dates. In addition, we assume that estimation-period security returns are independent over time, as are event-period security returns conditional upon the event occurring on a particular date. As we show later, however, unconditional returns within the event period will not be independent over time.

We illustrate our estimation procedure by considering initially an event study consisting of a single security, whose subscript we omit temporarily. We assume m security returns in the estimation period, and, as before, $2c+1$ returns in the event period.

The following two-stage procedure is used:

Stage 1. Using data from the estimation period only, compute the maximum-likelihood estimates of μ and σ^2:

$$\hat{\mu} = \frac{1}{m}\sum_t r_t \quad \text{and} \quad \hat{\sigma}^2 = \frac{1}{m-1}\sum_t (r_t - \hat{\mu})^2.$$

Stage 2. Proceed as if μ and σ^2 are estimated without sampling error. That is, assume that in the event period $\mu = \hat{\mu}$ and $\sigma^2 = \hat{\sigma}^2$. Using data from the event period, compute maximum-likelihood estimates of the $(2c+2)$-vector of parameters $\{A, \delta^2, p_{-c}, \ldots, p_{c-1}\}$.

This two-stage method can be compared with standard event-study procedures. Using estimation-period returns, we establish a benchmark. Empirically we then contrast the sample returns in the event period with this benchmark to ascertain any statistically significant deviations.

We use a hypothesis-testing framework to assess security-price performance. The null hypothesis, H_0, specifies no change in the return-generating process between estimation and event periods. By contrast, the alternative hypothesis, H_A, specifies that the posited change in the return-generating process occurs during the event period. Formally, the joint hypothesis is:

$$H_0: \quad A = 0, \delta^2 = 1, \qquad H_A: \quad \sim H_0.$$

We commit a type I error when we reject H_0 when in fact H_0 is true. This error corresponds to declaring that the event has an effect on security returns when in fact it does not. A likelihood-ratio test will limit the probability of this error to a prespecified level. In section 5, we investigate various extensions and modifications of this hypothesis-testing framework using the Grinblatt, Masulis, and Titman stock-split and stock-dividend data.

3.1. Implications of the model

Let $\underline{x}$ be the $(2c+1)$-vector of standardized excess daily returns to the single security across the event period:

$$x_t = (r_t - \hat{\mu})/\hat{\sigma}, \qquad t = -c, \ldots, +c.$$

Under our assumptions,

$$x_t | \theta_t = 0 \sim \mathrm{N}(0,1), \qquad x_t | \theta_t = 1 \sim \mathrm{N}(A, \delta^2).$$

Let $f(\underline{x}|\underline{\theta})$ denote the conditional distribution of $\underline{x}$ given $\underline{\theta} \equiv (\theta_{-c}, \ldots, \theta_0, \ldots, \theta_{+c})$. That is, $f(\underline{x}|\underline{\theta})$ represents the distribution of the security's standardized excess daily event-period returns conditional on the event occurring on a particular date. This conditional distribution is multivariate normal. It can be shown that its mean vector is $A\underline{\theta}$, with corresponding covariance matrix $(\delta^2 - 1)\underline{\theta}\underline{\theta}^T + I_{2c+1}$. Therefore, the joint distribution of $\underline{x}$, $f(\underline{x})$, is a mixture of multivariate normal distributions

$$f(\underline{x}) = \int_{\underline{\theta}} f(\underline{x}|\underline{\theta})\,\mathrm{d}P(\underline{\theta}),$$

where $P(\cdot)$ represents the distribution function of $\underline{\theta}$.

By our earlier arguments, it follows that

$$f(\underline{x}) = \sum_{t=-c}^{+c} p_t g_t(\underline{x}),$$

where

$$g_t(\underline{x}) = f(\underline{x}|\theta_t = 1)$$
$$= \frac{1}{\sqrt{2\pi\delta^2}} \exp - \frac{(x_t - A)^2}{2\delta^2} \prod_{\substack{s=-c \\ s \neq t}}^{+c} \frac{1}{\sqrt{2\pi}} \exp - \frac{x_s^2}{2}.$$

Generalizing to n firms, the marginal distribution of x_{it}, the standardized excess event-period return to firm i on day t, is given by

$$f(x_{it}) = p_t \frac{1}{\sqrt{2\pi\delta^2}} \exp - \frac{(x_{it} - A)^2}{2\delta^2} + (1 - p_t)\frac{1}{\sqrt{2\pi}} \exp - \frac{x_{it}^2}{2},$$
$$t = -c, \ldots, +c.$$

The marginal distributions of the firms' standardized excess daily event-period returns are not normal, are not identically distributed, and are not independent.

Upon integration, we have

$$\mathrm{E}(x_{it}) = p_t A, \qquad s,t = -c,\dots,+c,$$

$$\operatorname{var}(x_{it}) = 1 + A^2(p_t - p_t^2) + p_t(\delta^2 - 1), \qquad s \neq t,$$

$$\operatorname{cov}(x_{is}, x_{it}) = -A^2 p_s p_t, \qquad i = 1,\dots,n.$$

Under this specification, we have a negative covariance in standardized excess daily event-period returns. Moreover, the variance of the standardized excess daily returns within the event period is no longer unity.

Evidence in many event studies is consistent with increases in a security's variance around events. For example, Christie (1983) presents evidence that the variance around certain types of events can increase by a factor of almost two.[1] Our results suggest that not only may the event itself induce the reported increase in variance, but given event-date uncertainty, this empirical regularity may be due to the mixing of heterogeneous populations. Later empirical analysis investigates these determinants of increased event-period return variance.

3.2. Maximum-likelihood estimation

Suppose we have standardized excess daily event-period return data on n securities corresponding to n firms:

$$\underline{\underline{x}} = \begin{bmatrix} \underline{x}_1 \\ \vdots \\ \underline{x}_n \end{bmatrix} = \begin{bmatrix} x_{1-c} & \cdots & x_{1+c} \\ \vdots & & \vdots \\ x_{n-c} & \cdots & x_{n+c} \end{bmatrix},$$

where x_{it} is the standardized excess return for the ith security on event day t, $i = 1,\dots,n$, $t = -c,\dots,+c$. The joint density of $\underline{\underline{x}}$ is given by

$$f(\underline{\underline{x}}) = \prod_{i=1}^{n} f(\underline{x}_i) \quad \text{where} \quad f(\underline{x}_i) = \sum_{t=-c}^{+c} p_t g_t(\underline{x}_i).$$

[1] For an analysis of the implications of variance increases for event-study methods, see Brown and Warner (1985), especially pp. 22–25.

The logarithm of the corresponding likelihood function is given by

$$\ln L(\underline{x}) = \sum_{i=1}^{n} \ln f(\underline{x}_i).$$

Maximizing $\ln L(\underline{x})$ as a function of the parameter vector $(A, \delta^2, p_{-c}, \ldots, p_{c-1})$ gives corresponding maximum-likelihood estimates. We use Dempster, Laird, and Rubin's (1977) EM algorithm.[2] Not only is the algorithm simple to apply, it provides further insights into measuring security-price performance in the presence of event-date uncertainty.[3]

Setting the partial derivatives of the logarithmic likelihood function for the parameters to be estimated equal to zero gives necessary conditions for the maximum-likelihood estimates. In the appendix, we establish that these likelihood equations may be rewritten as

$$p_t^{**} = (n)^{-1} \sum_{i=1}^{n} p^{**}[t|\underline{x}_i], \qquad t = -c, \ldots, +c, \tag{1}$$

$$A^{**} = (n)^{-1} \sum_{i=1}^{n} \sum_{t=-c}^{+c} p^{**}[t|\underline{x}_i] x_{it}, \tag{2}$$

$$\delta^{2**} = (n)^{-1} \sum_{i=1}^{n} \sum_{t=-c}^{+c} p^{**}[t|\underline{x}_i](x_{it} - A^{**})^2. \tag{3}$$

Here, $(A^{**}, \delta^{2**}, p_{-c}^{**}, \ldots, p_{c-1}^{**})$ denotes the vector of maximum-likelihood estimates, and $p^{**}[t|\underline{x}_i]$ represents the posterior probability of the event's occurring on day t given data for firm i at these maximum-likelihood estimates.

[2] The EM algorithm provides maximum-likelihood estimates without recourse to the numerical computation of first- and second-order partial derivatives. The algorithm is appropriate for estimation problems involving incomplete data. In our case, the event date is not certain. For further details on the EM algorithm see Everitt and Hand (1981) and Titterington, Smith, and Makov (1985).

[3] We performed experiments to investigate the convergence properties of the EM algorithm. The Newton–Raphson algorithm was considered for corroborative purposes. Given that the possibility of multiple maximums is always present, neither algorithm is guaranteed to converge to the global maximum. Using simulated data, which assumed that the event did not induce an increase in security-return variance, $\delta^2 = 1$, with 100 firms, $n = 100$, a five-day event period, that is, $c = 2$, three sets of parameter values, and ten sets of starting values, we compared the two methods. In every case the EM algorithm converged to a unique solution within 20 iterations and in many cases far fewer. For some starting values the Newton–Raphson algorithm did not converge; however, for plausible starting values the algorithm converged quickly, and in every case where convergence was obtained, it converged to the EM algorithm's solution.

This statement of the algorithm provides an intuitive interpretation. Eq. (1) states that p_t^{**} is the average posterior probability of the event occurring on event day t, $t=-c,\ldots,+c$. Eq. (2) estimates A with a weighted average of the returns where the weighting reflects the posterior probability of the event occurring, and eq. (3) estimates δ^2 with the corresponding weighted average of the sum of squared deviations from A^{**}.

Eqs. (1), (2), and (3) provide the basis of the EM algorithm. From initial parameter estimates, we compute an initial estimate of $p^{**}[t|\underline{x}_i]$ for $t=-c,\ldots,+c$, $i=1,2,\ldots,n$, and use this estimate in eqs. (1), (2), and (3) to update the parameter estimates. We repeat the procedure until we obtain satisfactory convergence. Since $0 \le p^{**}[\cdot|\cdot] \le 1$, it follows that $0 \le p_t^{**} \le 1$, $t=-c,\ldots,+c$, and $\sum_{t=-c}^{+c} p_t^{*}=1$. That is, at each step of the algorithm the estimates fall within the parameter space of the problem.

3.3. *Multiday estimation*

Applying standard event-study procedures to multiday security returns provides method-of-moments estimates. Our subsequent analysis will compare these multiday estimates to the maximum-likelihood estimates.

Recalling that

$$\mathrm{E}(x_{it}) = p_t A, \qquad t=-c,\ldots,+c,$$

it follows that

$$\mathrm{E}\left(\sum_{t=-c}^{+c} x_{it}\right) = A,$$

and it can be shown that[4]

$$\operatorname{var}\left(\sum_{t=-c}^{+c} x_{it}\right) = 2c + \delta^2.$$

[4] Intuitively, our model assumes that exactly one event occurs during the event period, implying that the random variable $\sum_{t=-c}^{+c} x_{it}$ is the sum of the $2c+1$ independent normal random variables, $2c$ of which have unit variance, whereas the remaining one has variance δ^2.

Defining

$$\bar{x}_t = (n)^{-1} \sum_{i=1}^{n} x_{it}, \qquad t = -c, \ldots, +c,$$

$$s^2 \equiv \text{sample variance}\left(\sum_{t=-c}^{+c} x_{it} \right),$$

$$\bar{\bar{x}} = \sum_{t=-c}^{+c} \bar{x}_t,$$

we have that

$$\mathrm{E}(\bar{x}_t) = p_t A, \qquad t = -c, \ldots, +c,$$

$$\mathrm{E}(\bar{\bar{x}}) = A,$$

$$\mathrm{E}(s^2) = 2c + \delta^2.$$

The multiday estimators $(\hat{A}, \hat{\delta}^2, \hat{p}_{-c}, \ldots, \hat{p}_{c-1})$ are given by

$$\hat{A} = \bar{\bar{x}},$$

$$\hat{p}_t = \bar{x}_t / \bar{\bar{x}}, \qquad t = -c, \ldots, +c-1,$$

$$\hat{\delta}^2 = s^2 - 2c.$$

In general, method-of-moments estimators are not efficient. For example, although $\sum_{t=-c}^{+c} \hat{p}_t = 1$, there is no assurance that a particular $\hat{p}_t$ is positive. However, the multiday estimates provide adequate starting values for the numerical maximization of the logarithmic likelihood function. In a majority of cases, we require only four or five iterations to achieve convergence.

4. Empirical properties of test procedures

Given event-date uncertainty, we contrast the empirical properties of our maximum-likelihood method with the standard multiday event-study method in detecting and measuring an event's abnormal performance. For illustrative purposes, we consider a two-day event period, (0, +1). This specification allows us to explore a number of the implications of event-date uncertainty.

A two-day event period is the shortest period that permits event-date uncertainty. We minimize the potential gains of our maximum-likelihood procedure by assuming this period. Longer event periods, consistent with very imprecise event dates, are found in regulatory studies and other accounting, tax, and industrial organization studies. The longer the event period, the less powerful and efficient the multiday approach. Intuitively, the longer the event period, the more information is lost when event-period security returns are aggregated. For a $(2c+1)$-day event study with n firms $\mathrm{SE}(\hat{A}) = \sqrt{(2c+1)/n}$, whereas without event-date uncertainty, the optimal estimator of A has standard deviation $\sqrt{1/n}$. Therefore, the maximum gain in efficiency of A^* over $\hat{A}$ is bounded by $2c \times 100\%$. Efficiency gains of the maximum-likelihood estimator may then be expected to grow linearly with the length of the event period. In unreported simulations, we contrast the competing methods for event periods of varying lengths up to 31 days. The limited empirical evidence is consistent with greater gains in the maximum-likelihood procedure's efficiency for longer event periods. With over 30 parameters to estimate, however, extensive computational effort is required to implement our method. Nevertheless, for a particular event study, our method permits a researcher to investigate security-price performance efficiently within an event period longer than two days.

When an event's abnormal performance, A, is small, the competing methods will have difficulty discerning its presence. For small levels of A, the relevant measure of comparison is a method's power. Therefore, we perform a power study to compare the methods for $A = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. Alternatively, when an event's abnormal performance is large, both methods should detect it. Under such circumstances, the power of both methods will be near unity. For large levels of A, the relevant measure of comparison is the relative efficiency of the corresponding parameter estimates. To compare the efficiency of the multiday estimates with our maximum-likelihood estimates, we perform a simulation analysis for $A = 1.0, 2.0, 3.0$.

4.1. Simulation methodology

To assess the performance of the competing methods, we use simulation techniques based on actual security-return data [Brown and Warner (1980, 1985)] as well as pure simulated data.[5] Our data source is the University of Chicago's Center for Research in Security Prices (CRSP) daily return file for the period 1962–1983. This provides common stock returns for 4,645 firms, with up to 5,652 daily records for each firm. Not all firms exist, however, for the entire 22-year period. Also, extensive periods of data are missing for some firms, and there are sporadic omissions for others. Our sampling scheme takes into account these missing data.

[5] The data are generated using pseudo random numbers [Wichman and Hill (1982)] with the appropriate mixture of multivariate normal distributions.

We assume that 250 days of security-return data are available for each event study. We designate the first 239 days as the estimation period and the remaining 11 days as the event period.[6] Our sampling plan gives an equal probability of selection to every group of 250 contiguous observations on the CRSP daily return file, provided no observations are missing from the event period and no more than two observations are missing from the estimation period. This screening minimizes problems associated with missing data, without biasing the results of our simulation experiments. We select sets of 250 daily returns to a particular firm according to a geometric random variable with parameter 0.0025 so that, on average, one set of 250 daily returns in every 400 available is selected. This sampling plan is equivalent to simple random sampling without replacement, and yet is easier to implement when using blocks of firm-specific data.

With this screening procedure, we make two independent passes through the entire CRSP daily return file and generate 59,669 sets of 250 daily returns. These returns data should not exhibit abnormal performance systematically. For each set of daily returns, we use estimation-period data to estimate corresponding means and standard deviations with which to standardize event-period returns. We select event periods at random from our population of 59,669 event periods to form the basis for our power and efficiency studies.

4.2. Power

A test's power indicates its ability to detect abnormal performance. Other things being equal, a more powerful test is preferred to a less powerful one. Brown and Warner (1980) establish that imprecise information about the timing of an event can result in a dramatic decrease in the power of standard event-study methods.

To concentrate on the effect of event-date uncertainty, we set $\delta^2 = 1$ and consider the hypothesis structure

$$H_0:\quad A = 0, \qquad H_A:\quad A \neq 0.$$

The power of a test, for a given significance level α, $0 < \alpha < 1$, is the probability of rejecting the null hypothesis when the alternative is true. That is, a method's power at a level of abnormal performance $A = a$ is given by

$$\Pi(a) = \text{Prob}[\text{rejecting } H_0 | A = a].$$

[6]Since we restrict attention to a two-day event period, we actually use only the first two days of this longer event period in the analysis that follows. Computational limitations prevent an extensive investigation of security-price performance given event-date uncertainty within an eleven-day event period.

Given our assumptions, we can derive the theoretical power function corresponding to applying standard event-study methods to multiday security returns. For a two-day event period, (0, +1), we have

$$x_0 + x_1 \sim \mathrm{N}(A,2).$$

For an event study consisting of n firms, it follows that

$$\bar{x}_0 + \bar{x}_1 \sim \mathrm{N}(A,2/n),$$

or equivalently,

$$\sqrt{n/2}\,(\bar{x}_0 + \bar{x}_1) \sim \mathrm{N}\left(\sqrt{n/2}\,A,1\right).$$

Using the test statistic

$$Z_1 = \sqrt{n/2}\,(\bar{x}_0 + \bar{x}_1),$$

the two-sided critical region at an $\alpha\%$ significance level is given by Z_1 such that

$$(Z_1)^2 > (Z_{\alpha/2})^2,$$

with $Z_{\alpha/2}$ implicitly defined by $1 - \Phi(Z_{\alpha/2}) = \alpha/2$ where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function. Therefore, the power of standard event-study methods in the presence of event-date uncertainty is given by

$$\begin{aligned}\Pi(a) &= \mathrm{Prob}\left[(Z_1)^2 > (Z_{\alpha/2})^2 \mid A = a\right] \\ &= \Phi\left(-Z_{\alpha/2} + a\sqrt{n/2}\right) + \Phi\left(-Z_{\alpha/2} - a\sqrt{n/2}\right).\end{aligned}$$

Assuming $n = 25$ firms, we tabulate this theoretical power function in table 1 for significance levels of 1% and 5% at various levels of abnormal performance. These results provide a benchmark for our empirical power study.

We cannot derive explicitly the power of the test corresponding to our maximum-likelihood method. The appropriate test statistic is given by

$$Z_2 = A^*/\mathrm{SE}(A^*),$$

where A^* is the maximum-likelihood estimate of abnormal performance and $\mathrm{SE}(A^*)$ is the standard error of A^*, subject to the restriction that $\delta^2 = 1$.

Table 1

Theoretical power function for multiday event-study method using Z_1 as the test statistic.

We assume a 25-firm event study with a two-day event period and report the theoretical power function for the multiday estimator $\hat{A}$ for levels of abnormal performance $A = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. The test statistic $Z_1 = \sqrt{n/2}\,(\bar{x}_0 + \bar{x}_1)$, where n is the number of firms in the event study and $\bar{x}_0$ and $\bar{x}_1$ denote, respectively, the security's mean standardized excess return on event days 0 and 1.

Level of abnormal performance	Significance level = 5%	Significance level = 1%
$A = 0.0$	5.0%	1.0%
$A = 0.2$	10.89%	3.13%
$A = 0.4$	29.30%	12.27%
$A = 0.6$	56.41%	32.48%
$A = 0.8$	80.74%	59.97%
$A = 1.0$	94.24%	83.14%

Asymptotically, under the null hypothesis H_0: $A = 0$, the test statistic Z_2 is standard normally distributed.

For comparison, we use simulated data to calculate empirical power functions corresponding to both test statistics Z_1 and Z_2. We assume an event study consisting of $n = 25$ firms. For each firm, we introduce abnormal performance of A standard deviations, $A = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$, with equal probability on one of the first two days of the event period,[7] and determine whether abnormal performance is detected by a particular test statistic. We consider both 1% and 5% levels of significance. For each level of abnormal performance, we repeat this experiment 500 times and record the percentage of times the null hypothesis is rejected.

We present the results of our empirical power study in table 2. By comparing table 1 with the 'multiday method' columns of table 2, we see that the empirical power function of Z_1 corresponds well to its theoretical counterpart at both the 1% and 5% significance levels. For example, for $A = 0.4$, Z_1 theoretically rejects the null hypothesis of no abnormal performance 29.30% of the time at the 5% significance level, whereas for the pure simulated data the null hypothesis is rejected 28.20% of the time, and for the CRSP simulated data it is rejected 28.00% of the time. Also, the maximum-likelihood test is more powerful than the test based on multiday security returns, particularly at the 1% significance level. For the CRSP simulated data, the percentage gain in

[7]By assuming that the event occurs with equal probability on each of the event dates we maximize the unconditional variance of the individual returns. Specifically, $\text{var}(x_0) = \text{var}(x_1) = 1 + A^2(p - p^2)$ where p is the probability that the event occurs on day zero. Also, $\text{cov}(x_0, x_1) = -A^2 p(1 - p)$. The relative power gains and efficiency of the maximum-likelihood procedure are expected to be weakest when $p = 0.5$.

Table 2

Empirical power functions for multiday and maximum-likelihood methods using both pure simulated and CRSP simulated data.

This power study assumes a 25-firm event study with a two-day event period and abnormal performance of $A = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. We consider both 1% and 5% significance levels. For each value of A, we repeat the experiment 500 times and record the percentage of times the null hypothesis, H_0: $A = 0$, is rejected using the multiday and maximum-likelihood estimation procedures for both pure simulated data and CRSP simulated data. In addition, we report the percentage gain in efficiency of the maximum-likelihood estimation procedure over the multiday approach. For this study $p = 0.5$ and $\delta^2 = 1.0$

Level of abnormal performance	Significance level = 5% Multiday method	Maximum-likelihood method	Percentage increase in power	Significance level = 1% Multiday method	Maximum-likelihood method	Percentage increase in power
			Pure simulated data			
$A = 0.0$	4.6	6.8	—	0.6	1.0	—
$A = 0.2$	11.2	13.4	19.6	2.6	3.8	46.2
$A = 0.4$	28.2	32.4	14.9	12.4	18.2	46.8
$A = 0.6$	57.8	60.2	4.2	35.6	43.6	22.5
$A = 0.8$	82.0	86.0	4.9	61.2	74.8	22.2
$A = 1.0$	93.6	95.2	1.7	82.8	91.0	9.9
			CRSP simulated data			
$A = 0.0$	5.8	7.2	—	1.4	2.0	—
$A = 0.2$	11.8	12.2	3.4	2.6	4.0	53.8
$A = 0.4$	28.0	29.8	6.4	9.6	16.4	70.8
$A = 0.6$	57.4	60.6	5.6	32.2	40.8	26.7
$A = 0.8$	81.0	85.8	5.9	60.0	72.4	20.7
$A = 1.0$	94.8	96.8	2.1	82.8	92.6	11.8

the maximum-likelihood method's power at $A = 0.4$ is 6.4% at the 5% significance level and 70.8% at the 1% significance level. Although the percentage gain increases as the assumed level of abnormal performance decreases, the absolute gain in power is less pronounced. When an event's abnormal performance is minimal, the power gain from being able to isolate the event date is less significant. Further, for the assumed two-day event period, both the maximum-likelihood and multiday approaches tend to detect abnormal performance adequately as its assumed level increases. For $A = 1.00$, both methods reject the null hypothesis of no abnormal performance at the 5% significance level approximately 95% of time with either the pure or CRSP simulated data.

4.3. *Efficiency*

When an event is unanticipated, the magnitude of abnormal performance measures the event's wealth effects. The more efficiently an event-study method estimates abnormal performance, the more accurately we can measure these wealth effects. We use simulated data to compare the relative efficiency of the multiday estimates of abnormal performance with our maximum-likelihood estimates in the presence of event-date uncertainty. To concentrate on efficiency, we consider highly abnormal performance, which both methods should detect.

To emphasize event-date uncertainty, we assume initially that $\delta^2 = 1$. We introduce abnormal performance of $A = 1.0, 2.0, 3.0$ standard deviations with equal probability on one of the first two days of the event period. Event studies consisting of 20, 40, 60, 80, and 100 firms are considered. For each event study, we apply standard event-study methods to the two-day event-period returns to provide multiday estimates of abnormal performance, $\hat{A}$. We also apply our maximum-likelihood method to provide maximum-likelihood estimates of abnormal performance, A^*, and the probability of the event occurring on day 0, p^*. We repeat this procedure 500 times to derive the sampling distributions of $\hat{A}$, A^*, and p^*.

We tabulate our results for both the pure and the CRSP simulated data in tables 3 and 4, respectively. The results in the two tables are quantitatively very close. The maximum-likelihood estimators A^* and p^* are unbiased in all cases, as is the multiday estimator $\hat{A}$. For example, assuming CRSP simulated data, for an event study consisting of $n = 60$ firms with $A = 2.0$ and $p = 0.5$, from table 4 we note that the mean of A^* is 2.0160, the mean of p^* is 0.4976, and the mean of $\hat{A}$ is 2.0161. The standard deviation of A^* is markedly less than the standard deviation of $\hat{A}$ in all cases, however, with greater gains in efficiency for higher levels of abnormal performance. For example, from table 4, for an event study consisting of $n = 60$ firms, maximum-likelihood estimation is approximately 41.4% more efficient than multiday estimation when $A = 1.00$, but approximately 89.8% more efficient when $A = 3.00$. Intuitively, the greater the abnormal performance, the greater the loss of information when security returns are aggregated.

To investigate the performance of the EM algorithm when security-return variance increases around an event, we perform a simulation using CRSP data assuming $A = 2.0$ for $\delta^2 = 1.0, 1.5, 2.0$. The results are shown in table 5. Multiday estimates $\hat{A}$, $\hat{p}$, and $\hat{\delta}^2$ and maximum-likelihood estimates allowing a changing conditional variance A^{**}, p^{**}, and δ^{**} as well as maximum-likelihood estimates restricting $\delta^2 = 1$, A^*, and p^* are considered. The multiday estimates and the unrestricted maximum-likelihood estimates both appear to be unbiased throughout. For example, for $\delta^2 = 1.5$, the mean of A^{**} is 2.0023, the mean of p^{**} is 0.5007, and the mean of $\hat{A}$ is 2.0090.

The standard deviations of the maximum-likelihood estimate of abnormal performance are smaller than the standard deviations of the multiday estimate in all cases, however. For $\delta^2 = 1.5$, the standard deviation of A^{**} is 0.1240 and the standard deviation of $\hat{A}$ is 0.1565.

Table 3

Efficiency tests with pure simulated data assuming no increase in conditional event-period variance.

This efficiency study provides summary statistics on the maximum-likelihood estimators A^* and p^* assuming $\delta^2 = 1$ as well as the multiday estimator $\hat{A}$ and the test statistic $Z_2 = A^*/\text{SE}(A^*)$, where SE(A^*) is obtained from the inverse Hessian matrix at the maximum-likelihood estimates. We report information on event studies consisting of 20, 40, 60, 80, and 100 firms with a two-day event period whose returns are subject to abnormal performance of $A = 1.0, 2.0, 3.0$ standard deviations with $p = 0.5$. In all cases the number of simulations is 500. The data are generated using pseudo-random numbers with the appropriate mixture of multivariate normal distributions.

Statistic	Minimum	Maximum	Mean	Std. dev.	Skewness	Kurtosis
			Case 1: $A = 3.0$, $p = 0.5$, Number of simulations = 500			
			Number of firms = 20			
$\hat{A}$	2.1163	3.9124	3.0150	0.3108	−0.118	−0.140
A^*	2.3933	3.6690	3.0044	0.2166	0.002	−0.096
p^*	0.1497	0.8370	0.4878	0.1126	−0.076	−0.135
Z_2	10.5400	16.4010	13.3810	0.9900	−0.022	−0.072
			Number of firms = 40			
$\hat{A}$	2.3031	3.6892	3.0011	0.2226	−0.204	−0.143
A^*	2.5110	3.3929	2.9975	0.1619	−0.112	−0.211
p^*	0.2972	0.7718	0.4973	0.0757	0.205	0.001
Z_2	15.7180	21.4170	18.8840	1.0458	−0.125	−0.198
			Number of firms = 60			
$\hat{A}$	2.3498	3.6368	3.0116	0.1809	−0.150	0.159
A^*	2.6659	3.3780	3.0030	0.1283	0.027	−0.038
p^*	0.3277	0.7182	0.4978	0.0682	0.131	−0.201
Z_2	20.5240	26.1360	23.1720	1.0144	0.021	−0.038
			Number of firms = 80			
$\hat{A}$	2.5337	3.4000	2.9970	0.1503	0.033	0.059
A^*	2.6816	3.3188	2.9975	0.1128	−0.028	−0.182
p^*	0.3508	0.6627	0.5045	0.0542	0.099	−0.381
Z_2	23.8060	29.6380	26.7070	1.0313	−0.038	−0.180
			Number of firms = 100			
$\hat{A}$	2.6047	3.3644	2.9947	0.1397	−0.013	−0.071
A^*	2.7081	3.2608	2.9916	0.0978	0.071	−0.228
p^*	0.3133	0.6719	0.4950	0.0554	−0.020	0.256
Z_2	26.8700	32.5030	29.7990	0.9992	0.061	−0.234
			Case 2: $A = 2.0$, $p = 0.5$, Number of simulations = 500			
			Number of firms = 20			
$\hat{A}$	1.1612	2.8672	1.9554	0.3124	0.022	−0.270
A^*	1.1109	2.5627	1.9765	0.2332	−0.098	−0.000
p^*	0.0757	0.9125	0.4967	0.1335	0.150	0.362
Z_2	4.5555	11.3650	8.5922	1.1153	−0.119	−0.020
			Number of firms = 40			
$\hat{A}$	1.3426	2.7276	2.0177	0.2273	−0.180	0.032
A^*	1.4286	2.5489	2.0046	0.1617	0.067	0.287
p^*	0.2172	0.7566	0.5028	0.0890	−0.002	−0.024
Z_2	8.4529	15.9490	12.3410	1.0898	0.044	0.243

Table 3 (continued)

Statistic	Minimum	Maximum	Mean	Std. dev.	Skewness	Kurtosis
Case 2: A = 2.0, p = 0.5, Number of simulations = 500						
Number of firms = 60						
$\hat{A}$	1.4699	2.6587	2.0027	0.1820	0.256	0.232
A^*	1.5503	2.4546	2.0003	0.1355	0.004	0.033
p^*	0.2617	0.6820	0.5022	0.0756	−0.209	−0.062
Z_2	11.4090	18.7250	15.0820	1.1165	−0.017	−0.005
Number of firms = 80						
$\hat{A}$	1.4730	2.5257	1.9963	0.1648	−0.014	0.205
A^*	1.4720	2.3128	1.9958	0.1191	−0.191	0.573
p^*	0.2924	0.7044	0.0543	0.0659	0.061	−0.282
Z_2	12.3640	20.3840	17.3740	1.1361	−0.213	0.588
Number of firms = 100						
$\hat{A}$	1.5941	2.3409	1.9988	0.1405	0.049	−0.187
A^*	1.6067	2.2877	1.9963	0.1046	−0.073	0.235
p^*	0.3521	0.6915	0.5001	0.0560	0.121	−0.134
Z_2	15.2140	22.5460	19.4330	1.1164	−0.089	0.239
Case 3: A = 1.0, p = 0.5, Number of simulations = 500						
Number of firms = 20						
$\hat{A}$	−0.0466	1.9316	0.9767	0.3066	0.110	0.236
A^*	−0.2398	1.7340	0.9718	0.2565	−0.186	0.777
p^*	0.0000	1.0000	0.5005	0.2232	0.022	−0.353
Z_2	−0.8694	7.3942	3.8152	1.1600	0.025	0.290
Number of firms = 40						
$\hat{A}$	0.3254	1.7180	0.9860	0.2299	0.173	0.287
A^*	0.3522	1.5554	0.9911	0.1862	−0.011	−0.185
p^*	0.0000	0.9240	0.4898	0.1408	−0.145	0.075
Z_2	1.7393	9.3113	5.4937	1.2249	0.064	−0.216
Number of firms = 60						
$\hat{A}$	0.5036	1.4733	0.9922	0.1754	−0.017	−0.303
A^*	0.5404	1.4328	0.9900	0.1502	−0.065	−0.189
p^*	0.1240	0.9600	0.5033	0.1159	0.142	0.277
Z_2	3.5006	10.3720	6.7205	1.1923	0.030	−0.225
Number of firms = 80						
$\hat{A}$	0.5199	1.5122	1.0030	0.1576	0.113	−0.098
A^*	0.5536	1.4203	0.9982	0.1272	0.123	0.246
p^*	0.2029	0.8460	0.5021	0.1041	−0.000	−0.019
Z_2	3.8702	11.7130	7.8307	1.1827	0.192	0.213
Number of firms = 100						
$\hat{A}$	0.5376	1.4845	0.9988	0.1467	0.027	0.127
A^*	0.6279	1.3152	0.9958	0.1164	−0.232	0.185
p^*	0.1707	0.7484	0.4963	0.0877	−0.365	0.373
Z_2	4.9022	12.0540	8.7242	1.2036	−0.161	0.074

Table 4

Efficiency tests with CRSP simulated data assuming no increase in conditional event-period variance.

This efficiency study provides summary statistics on the maximum-likelihood estimators A^* and p^* assuming $\delta^2 = 1$ as well as the multiday estimator $\hat{A}$ and the test statistic $Z_2 = A^*/\mathrm{SE}(A^*)$, where $\mathrm{SE}(A^*)$ is obtained from the inverse Hessian matrix at the maximum-likelihood estimates. We report information on event studies consisting of 20, 40, 60, 80, and 100 firms with a two-day event period whose returns are subject to abnormal performance of $A = 1.0, 2.0, 3.0$ standard deviations with $p = 0.5$. In all cases the number of simulations is 500. The data are generated from the CRSP daily return file.

Statistic	Minimum	Maximum	Mean	Std. dev.	Skewness	Kurtosis
			Case 1: $A = 3.0$, $p = 0.5$, Number of simulations = 500			
			Number of firms = 20			
$\hat{A}$	1.9995	4.2453	3.0096	0.2963	0.078	0.637
A^*	2.0807	3.9333	3.0085	0.2139	0.190	1.113
p^*	0.1757	0.8409	0.5030	0.1144	0.004	−0.192
Z_2	9.0993	17.5640	13.4120	0.9930	0.155	1.149
			Number of firms = 40			
$\hat{A}$	2.2283	3.8763	3.0031	0.2260	0.097	0.536
A^*	2.5622	3.5013	3.0112	0.1496	0.296	0.382
p^*	0.2651	0.7753	0.4992	0.0779	−0.021	0.082
Z_2	16.0950	22.1100	18.9870	0.9623	0.283	0.360
			Number of firms = 60			
$\hat{A}$	2.4761	3.6231	3.0136	0.1736	0.094	0.067
A^*	2.6590	3.3812	3.0174	0.1259	0.018	−0.062
p^*	0.3125	0.7493	0.5006	0.0689	0.165	0.055
Z_2	20.4520	26.1590	23.3020	0.9924	0.005	−0.051
			Number of firms = 80			
$\hat{A}$	2.4537	3.5472	3.0185	0.1655	−0.065	0.401
A^*	2.6700	3.3696	3.0219	0.1152	0.029	0.182
p^*	0.3194	0.6548	0.4977	0.0553	−0.117	0.013
Z_2	23.7490	30.1050	26.9480	1.0483	0.020	0.185
			Number of firms = 100			
$\hat{A}$	2.6455	3.4480	3.0194	0.1424	0.117	−0.149
A^*	2.7198	3.3675	3.0226	0.1102	0.059	0.190
p^*	0.3587	0.6634	0.5012	0.0520	−0.067	−0.254
Z_2	27.0510	33.6230	30.1360	1.0438	0.044	0.191
			Case 2: $A = 2.0$, $p = 0.5$, Number of simulations = 500			
			Number of firms = 20			
$\hat{A}$	1.2037	3.3178	2.0163	0.3313	0.406	0.377
A^*	1.3614	2.8142	2.0366	0.2453	0.365	0.324
p^*	0.0865	0.8666	0.4916	0.1226	0.002	−0.035
Z_2	5.4791	12.5320	8.8539	1.1768	0.308	0.276
			Number of firms = 40			
$\hat{A}$	1.3994	2.7134	2.0285	0.2267	0.195	0.035
A^*	1.4841	2.6490	2.0205	0.1733	0.223	0.230
p^*	0.2290	0.7540	0.5042	0.0834	−0.103	0.144
Z_2	8.6410	16.6320	12.4150	1.1808	0.179	0.199

Table 4 (continued)

Case 2: A = 2.0, p = 0.5, Number of simulations = 500						
Number of firms = 60						
$\hat{A}$	1.5398	2.5221	2.0161	0.1817	0.235	0.031
A^*	1.6278	2.4643	2.0160	0.1297	0.237	0.079
p^*	0.2866	0.6986	0.4976	0.0691	−0.023	−0.200
Z_2	11.9160	18.8340	15.1700	1.0780	0.217	0.038
Number of firms = 80						
$\hat{A}$	1.5519	2.4283	2.0135	0.1576	−0.053	−0.149
A^*	1.6823	2.4361	2.0170	0.1139	0.091	0.109
p^*	0.3039	0.6880	0.5014	0.0622	−0.089	0.137
Z_2	14.3510	21.5500	17.5290	1.0970	0.069	0.098
Number of firms = 100						
$\hat{A}$	1.6106	2.4520	2.0149	0.1386	−0.138	−0.100
A^*	1.7253	2.3455	2.0210	0.0101	0.013	−0.045
p^*	0.3307	0.6475	0.5025	0.0564	−0.263	−0.113
Z_2	16.4390	23.0790	19.6440	1.0919	−0.011	−0.052
Case 3: A = 1.0, p = 0.5, Number of simulations = 500						
Number of firms = 20						
$\hat{A}$	0.1584	1.9846	1.0367	0.2983	−0.008	0.042
A^*	0.1574	1.7500	0.9813	0.2544	0.086	0.143
p^*	0.0000	1.0000	0.5071	0.2185	−0.091	−0.335
Z_2	0.5409	7.4890	3.8800	1.1520	0.246	0.012
Number of firms = 40						
$\hat{A}$	0.5422	1.7788	1.0171	0.2138	0.229	−0.251
A^*	0.5179	1.6153	0.9535	0.1822	0.290	−0.030
p^*	0.0681	0.9909	0.5069	0.1520	0.089	0.170
Z_2	2.6507	9.6383	5.2892	1.1767	0.369	0.010
Number of firms = 60						
$\hat{A}$	0.4198	1.5190	1.0254	0.1861	−0.172	0.147
A^*	0.5119	1.5351	0.9650	0.1564	0.002	0.178
p^*	0.0488	0.9042	0.5058	0.1271	0.114	0.333
Z_2	3.2199	11.1580	6.5546	1.2359	0.091	0.150
Number of firms = 80						
$\hat{A}$	0.5966	1.6311	1.0210	0.1634	0.256	0.080
A^*	0.5666	1.4472	0.9647	0.1387	0.230	0.199
p^*	0.1632	0.8485	0.5052	0.1066	−0.019	0.176
Z_2	4.0320	12.1370	7.5657	1.2725	0.319	0.278
Number of firms = 100						
$\hat{A}$	0.5605	1.3595	1.0101	0.1353	−0.073	−0.104
A^*	0.5704	1.2942	0.9575	0.1218	0.145	−0.055
p^*	0.1981	0.7524	0.5103	0.0919	−0.023	0.001
Z_2	4.6843	11.7960	8.3825	1.2424	0.189	−0.085

Table 5

Efficiency tests with CRSP simulated data assuming an increase in conditional event-period variance.

Summary statistics are given for the multiday estimators $\hat{A}$, $\hat{p}$, and $\hat{\delta}^2$, the unrestricted maximum-likelihood estimators A^{**}, p^{**}, and δ^{2**}, and the maximum-likelihood estimators A^* and p^* subject to the constraint $\delta^2 = 1$. We report information on event studies consisting of 100 firms with a two-day event period whose returns are subject to abnormal performance of $A = 2.0$ standard deviations with $p = 0.5$. Three levels of variance, $\delta^2 = 2.0, 1.5, 1.0$ are examined. In all cases the number of simulations is 500. The data are generated from the CRSP daily return file.

Statistic	Minimum	Maximum	Mean	Std. dev.	Skewness	Kurtosis
			$\delta^2 = 2.0$			
A^{**}	1.5492	2.4218	1.9942	0.1424	−0.015	−0.072
A^*	1.6355	2.6044	2.0637	0.1410	0.251	0.484
$\hat{A}$	1.5229	2.6477	2.0122	0.1713	0.283	0.126
δ^{2**}	0.8931	9.6838	2.0872	0.9308	3.521	20.254
$\hat{\delta}^2$	0.5233	9.0544	1.9962	1.0097	2.878	14.498
p^{**}	0.3199	0.7012	0.4969	0.0603	0.015	−0.029
p^*	0.3256	0.6928	0.4978	0.0549	0.165	0.108
$\hat{p}$	0.3347	0.7565	0.4975	0.0652	0.269	0.181
			$\delta^2 = 1.5$			
A^{**}	1.5656	2.3898	2.0023	0.1240	0.028	0.331
A^*	1.6731	2.4040	2.0370	0.1248	0.150	0.131
$\hat{A}$	1.5278	2.4662	2.0090	0.1565	0.164	0.215
δ^{2**}	0.6228	6.3708	1.5280	0.6870	3.143	14.761
$\hat{\delta}^2$	0.2574	6.4605	1.4958	0.7801	2.103	8.745
p^{**}	0.3548	0.6540	0.5007	0.0598	0.019	−0.421
p^*	0.3599	0.6560	0.5009	0.0574	−0.001	−0.399
$\hat{p}$	0.3377	0.7194	0.5008	0.0641	−0.008	−0.403
			$\delta^2 = 1.0$			
A^{**}	1.7332	2.2613	2.0156	0.0994	−0.048	−0.238
A^*	1.7346	2.2774	2.0171	0.1037	0.022	−0.266
$\hat{A}$	1.6297	2.3755	2.0139	0.1369	0.067	−0.177
δ^{2**}	0.3415	3.9509	1.0016	0.3526	2.617	13.838
$\hat{\delta}^2$	0.0390	4.8187	0.9993	0.5633	1.902	7.459
p^{**}	0.3215	0.6838	0.4967	0.0557	0.037	0.341
p^*	0.3281	0.6817	0.4968	0.0556	0.039	0.282
$\hat{p}$	0.2881	0.6589	0.4947	0.0605	0.007	−0.006

The relative efficiency of the maximum-likelihood estimates tends to decrease with increasing δ^2. For example, whereas maximum-likelihood estimation is approximately 59.3% relatively more efficient when $\delta^2 = 1.5$, it is approximately 44.7% relatively more efficient when $\delta^2 = 2.0$. Intuitively, the more variable event-day returns, the more difficult it is to measure abnormal performance accurately.

For a two-day event study with n firms, $\mathrm{SE}(\hat{A}) = \sqrt{(1+\delta^2)/n}$. Without event-date uncertainty the optimal estimator of A has standard deviation

$\sqrt{\delta^2/n}$. Therefore, the maximum gain in efficiency of A^{**} over $\hat{A}$ is bounded by $(1/\delta^2) \times 100\%$. As δ^2 increases this bound decreases to zero. In particular, for $\delta^2 = 2$ the bound is 50% and for $\delta^2 = 1.5$ the bound is 66.7%. For the parameter values selected in this simulation the efficiency of the maximum-likelihood estimator A^{**} is very close to these bounds.

As expected, for $\delta^2 = 1.0$, the maximum-likelihood estimator allowing a changing conditional variance and the maximum-likelihood estimator assuming $\delta^2 = 1.0$ provide comparable performance. For example, in this case the mean of A^{**} is 2.0156 with a standard deviation of 0.0994 and the mean of A^* is 2.0171 with a standard deviation of 0.1037. For $\delta^2 > 1$, however, the maximum-likelihood estimates of A (but not p) assuming erroneously that $\delta^2 = 1.0$ appear upward biased. For example, for $\delta^2 = 2.0$, the mean of A^* is 2.0637 and the mean of p^* is 0.4978. Increases in security-return variance around an event may be interpreted erroneously as evidence of abnormal performance.

5. Valuation effects of stock splits and stock dividends

We use our maximum-likelihood method to investigate empirically the valuation effects of stock splits and stock dividends in the presence of event-date uncertainty. Grinblatt, Masulis, and Titman's (1984) empirical evidence is consistent with positive stock-price reactions, on average, to announcements of stock splits and stock dividends that are uncontaminated by other contemporaneous firm-specific announcements.

The valuation effects of stock splits and stock dividends obtain even though these announcements do not directly affect the future cash flows of the firm. As Brennan and Copeland (1987) argue, however, a firm declaring a stock split increases the number of its shares outstanding, thereby imposing additional transaction costs on its shareholders. Since low share prices are costly, high-value firms can signal credibly by declaring a stock split. Furthermore, Grinblatt, Masulis, and Titman observe larger announcement effects for stock dividends than stock splits. These differences are consistent with the so-called retained earnings hypothesis. That is, given legal restrictions on retained earnings and since for stock dividends the value of the newly issued shares is subtracted from retained earnings, firms that expect poor earnings will find it costly to mimic the stock-dividend signals of firms that expect good earnings.

5.1. Empirical results

In contrasting the multiday estimates of security price performance with maximum-likelihood estimates, we restrict our attention to Grinblatt, Masulis, and Titman's pure sample of 84 stock dividends and 244 stock splits that are uncontaminated by other contemporaneous firm-specific announcements. Be-

Table 6

Descriptive statistics on daily stock returns in the Grinblatt, Masulis, and Titman (1984) pure sample of stock-split and stock-dividend announcements

	Mean	Std. dev.
Panel A: Entire sample (n = 328)		
Day 0 return	0.8342	1.6570
Day 1 return	0.6916	1.5789
Two-day return	1.5257	2.2517
Standard error of the mean two-day return = 0.1243		
Panel B: Stock splits (n = 244)		
Day 0 return	0.7445	1.5381
Day 1 return	0.5463	1.5229
Two-day return	1.2908	2.1679
Standard error of the mean two-day return = 0.1355		
Panel C: Stock dividends (n = 84)		
Day 0 return	1.0945	1.9493
Day 1 return	1.1136	1.6702
Two-day return	2.2081	2.3624
Standard error of the mean two-day return = 0.2436		

cause of event-date uncertainty, Grinblatt, Masulis, and Titman apply the mean-adjusted returns method to a two-day (day 0 and day +1) announcement return. Day 0 is defined as the earlier of the trading day before the issue date of the *Wall Street Journal* in which the event is announced or the declaration date of the event on the CRSP daily master file. The corresponding estimation period is taken to be days 4 through 43 following day 0.

For comparison, we apply the mean-adjusted returns method to standardized excess returns within the two-day event period (0, +1). Table 6 presents the results for both individual-day and multiday returns. Note that this table reports mean returns, not mean standardized returns, so the results are not comparable with those in tables 7 and 8. Panel A reports results for the entire sample, and panels B and C give results for stock splits and stock dividends, respectively. The empirical results are consistent with significant positive excess standardized returns to both stock splits and stock dividends, the valuation effects being larger for dividends. The variance of excess standardized event-period returns also appears to be larger for dividends.

Our maximum-likelihood procedures provide a more powerful and efficient means to investigate valuation effects in the presence of event-date uncertainty. We initially restrict $\delta^2 = 1$. In table 7 we present maximum-likelihood estimates of abnormal performance, A^*, maximum-likelihood estimates of the

Table 7

Maximum-likelihood estimates of abnormal performance for the Grinblatt, Masulis, and Titman (1984) pure sample of stock-split and stock-dividend announcements assuming no increase in conditional event-period variance.

We report information on the maximum-likelihood estimators A^* and p^* subject to the constraint $\delta^2 = 1$. We employ the Grinblatt, Masulis, and Titman (1984) daily return data for firms declaring stock dividends and stock splits. We examine the entire sample of stock splits and stock dividends, the sample of stock splits only, and the sample of stock dividends only. The likelihood-ratio statistic χ^2(stat) tests the null hypothesis H_0: $A = 0$ against the alternative H_A: $A \neq 0$. Under the null hypothesis this statistic is distributed χ^2 with one degree of freedom. In addition, $Z^* = A^*/SE(A^*)$.

Statistic	Entire sample $n = 328$	Stock splits $n = 244$	Stock dividends $n = 84$
A^*	1.4773	1.2643	2.0673
$SE(A^*)$	0.0583	0.0688	0.1117
p^*	0.5378	0.5688	0.4738
$SE(p^*)$	0.0390	0.0494	0.0654
Z^*	25.34	18.38	18.51
$\ln L(A^*, p^*)$	−1394.0866	−986.5141	−388.4368
χ^2(stat)	515.6417	270.3216	283.5916

probability of the event occurring on day 0, p^*, their corresponding asymptotic standard errors, and the corresponding maximized value of the logarithmic likelihood function. We also give values of the likelihood-ratio statistic for testing

$$H_0: \quad A = 0, \qquad H_A: \quad A \neq 0,$$

assuming $\delta^2 = 1$. The appropriate likelihood ratio test statistic is twice the difference of the maximum value of the logarithmic likelihood function under H_A and its maximum value under H_0. Since we are restricting only one parameter, under the null hypothesis of no abnormal performance, the likelihood-ratio statistic is asymptotically distributed χ^2 with one degree of freedom. At the 1% significance level, we can reject the null hypothesis of no abnormal performance due to stock splits and stock dividends. For the entire sample, the sample of stock splits only, and the sample of dividends only, the corresponding observed values of the likelihood-ratio test statistic exceed 6.63, the critical value of a χ^2 random variable with one degree of freedom at the 1% significance level.

Table 8 shows the results of applying our maximum-likelihood procedures with no restrictions on the conditional variance of excess standardized event-period returns. In addition to maximum-likelihood estimates of abnormal performance, A^{**}, and the probability of the event occurring on day 0, p^{**}, we present maximum-likelihood estimates of the variance measure, δ^{2**}. The

Table 8

Maximum-likelihood estimates of abnormal performance for the Grinblatt, Masulis, and Titman (1984) pure sample of stock-split and stock-dividend announcements allowing an increase in conditional event-period variance.

We report information on the unrestricted maximum-likelihood estimators A^{**}, p^{**}, and δ^{2**}, as well as the multiday estimators $\hat{A}$, $\hat{p}$, and $\hat{\delta}^2$. We employ the Grinblatt, Masulis, and Titman (1984) daily return data for firms declaring stock dividends and stock splits. We examine the entire sample of stock splits and stock dividends, the sample of stock splits only, and the sample of stock dividends only. Two likelihood-ratio statistics are reported: $\chi^2(\text{stat})^a$ tests H_0: $\delta^2 = 1$ against H_A: $\delta^2 \neq 1$, while $\chi^2(\text{stat})^b$ tests H_0: $A = 0$, $\delta^2 = 1$ against H_A: $\sim H_0$. Under their respective null hypotheses, $\chi^2(\text{stat})^a$ is distributed χ^2 with one degree of freedom, while $\chi^2(\text{stat})^b$ is distributed χ^2 with two degrees of freedom. In addition, $Z^{**} = A^{**}/\text{SE}(A^{**})$.

Statistic	Entire sample $n = 328$	Stock splits $n = 244$	Stock dividends $n = 84$
$\hat{A}$	1.5257	1.2908	2.2081
A^{**}	1.2846	1.0854	1.8815
SE(A^{**})	0.1089	0.1218	0.2231
$\hat{p}$	0.5467	0.5768	0.4957
p^{**}	0.5066	0.5309	0.4620
SE(p^{**})	0.0481	0.0604	0.0789
$\hat{\delta}^2$	4.0699	3.6999	4.5810
δ^{2**}	3.5797	3.3200	3.7993
SE(δ^{2**})	0.2923	0.3124	0.6402
Z^{**}	11.80	8.91	8.43
ln $L(A^{**}, p^{**}, \delta^{2**})$	−1233.9638	882.9261	−345.1344
$\chi^2(\text{stat})^a$	320.2456	207.1350	86.6048
$\chi^2(\text{stat})^b$	835.8873	477.4466	370.1964

[a] χ^2 for testing H_0: $\delta^2 = 1$ against H_a: $\delta^2 \neq 1$.
[b] χ^2 for testing H_0: $A = 0$, $\delta^2 = 1$ against H_a: $\sim H_0$.

asymptotic standard errors of the maximum-likelihood estimates, the corresponding maximized value of the logarithmic likelihood function, and the multiday estimates used as starting values are also presented. Given our previous results, we can test statistically for changes in the variance of excess standardized event-period returns due to the announcement of stock splits and stock dividends:

$$H_0: \quad \delta^2 = 1, \qquad H_A: \quad \delta^2 \neq 1,$$

when the level of abnormal performance, A, is unrestricted. The resulting likelihood-ratio statistics, which are asymptotically distributed χ^2 with one degree of freedom under the null hypothesis, are tabulated in table 8. For the entire sample, the sample of stock splits only, and the sample of stock dividends only, we can reject at the 1% significance level the null hypothesis of no change in the conditional variance of excess standardized event-period returns. We also assess the valuation effects of stock splits and stock dividends by examining

$$H_0: \quad A=0, \delta^2=1, \qquad H_A: \quad \sim H_0.$$

Since we are restricting two parameters, under the null hypothesis the likelihood-ratio statistic is asymptotically distributed χ^2 with two degrees of freedom. The corresponding likelihood-ratio statistics are also tabulated in table 8. For the entire sample, the sample of stock splits only, and the sample of stock dividends only, the corresponding observed values of the likelihood-ratio statistic exceed 9.21, the critical value of a χ^2 random variable with two degrees of freedom at the 1% significance level. Therefore, at the 1% significance level we reject the null hypothesis, providing empirical evidence consistent with either abnormal performance or increases in the conditional variance of excess standardized event-period returns brought about by the announcement of stock splits and stock dividends.

Event-date uncertainty contributes to the increase in the variance of standardized excess event-period daily returns. From our previous analysis, we have that

$$\operatorname{var}(x_{i0}) = 1 + A^2(p_0 - p_0^2) + p_0(\delta^2 - 1).$$

This increase in variance is factored into two components: $A^2(p_0 - p_0^2)$, reflecting uncertainty in event dates, and $p_0(\delta^2 - 1)$, reflecting the actual increase in the conditional variance. Using the entire sample of stock splits and dividends and corresponding maximum-likelihood estimates, we estimate that event-date uncertainty accounts for 23.99% of the increase in the variance of standardized excess event-period returns, whereas the announcement accounts for 76.01% of the increase. In this particular case, event-date uncertainty contributes significantly to the observed increase in the variance of returns.

By comparing the results of tables 7 and 8, we investigate Christie's (1983) conditional hypothesis inherent in event studies. Christie argues that the use of the wrong standard deviation, estimation period rather than event period, leads to serious errors of inference. It is important to include estimation of δ^2 when estimating A. Assuming that an event conveys information, we may then investigate whether the information has a significant effect on security prices. Our empirical results reinforce Christie's arguments. Table 7 reports the test statistic $Z^* = A^*/\mathrm{SE}(A^*)$, the ratio of A^* to its standard error, and table 8 reports the test statistic $Z^{**} = A^{**}/\mathrm{SE}(A^{**})$, the ratio of A^{**} to its standard error. The statistic Z^{**} tests for the presence of abnormal performance without restricting δ^2. Under the null hypothesis of no abnormal performance, that is, $A=0$, Z^{**} is asymptotically distributed standard normal. By contrast, the statistic Z^* tests for the presence of abnormal performance subject to the restriction that $\delta^2 = 1$. If $\delta^2 \neq 1$, then under the null hypothesis of no abnormal performance, $A = 0$, Z^* is not asymptotically distributed standard normal.

Since we reject statistically the restriction that stock splits and stock dividends do not affect the conditional variance of standardized excess event-period returns, inferences about the valuation of stock splits and stock dividends assuming the validity of this restriction are misleading. For example, using the entire sample of stock splits and stock dividends, from panel A of table 7 we have $Z^* = 25.34$, whereas from panel A of table 8 we have $Z^{**} = 11.80$. The value of the Z^* statistic is much larger, suggesting erroneously much more statistically significant valuation effects.

A further comparison of tables 7 and 8 reveals that abnormal performance estimates assuming no increase in the conditional variance of standardized excess event-period returns are upward biased. For example, for the entire sample of stock splits and stock dividends, $A^* = 1.4773$ and $A^{**} = 1.2846$. Furthermore, the corresponding standard errors are suspect. Given that the conditional variance increases, asymptotic standard errors assuming no increase in the conditional variance are inappropriate. For the entire sample of stock splits and stock dividends, the standard error of A^* is 0.0583 and the standard error of A^{**} is 0.1089. Again, ignoring increases in the conditional variance of standardized excess event-period returns suggests erroneously much more statistically significant valuation effects.

5.2. A comparison of stock-split and stock-dividend valuation effects

In comparing valuation effects of stock splits and stock dividends, we take into account throughout any increases in the conditional variance of standardized excess event-period returns caused by the announcements. To examine the differences in abnormal performance caused by the announcements of stock splits and stock dividends, we consider

$$H_0: \quad A_{\text{split}} = A_{\text{div}}, \qquad H_A: \quad A_{\text{split}} \neq A_{\text{div}}.$$

To derive the appropriate test statistic, we appeal to asymptotic maximum-likelihood theory and note that

$$A^{**}_{\text{split}} - A^{**}_{\text{div}} \sim \mathrm{N}\left(A_{\text{split}} - A_{\text{div}}, \mathrm{SE}(A^{**}_{\text{split}})^2 + \mathrm{SE}(A^{**}_{\text{div}})^2\right).$$

where $\mathrm{SE}(A^{**}_{\text{split}})$ and $\mathrm{SE}(A^{**}_{\text{div}})$ are obtained from the appropriate diagonal elements of the corresponding inverse of the negative Hessian matrix evaluated at the maximum-likelihood estimates. Hence, the appropriate test statistic is

$$Z = \frac{A^{**}_{\text{split}} - A^{**}_{\text{div}}}{\sqrt{\text{SE}(A^{**}_{\text{split}})^2 + \text{SE}(A^{**}_{\text{div}})^2}} \sim \text{N}(A_{\text{split}} - A_{\text{div}}, 1).$$

We reject the null hypothesis at the $\alpha\%$ significance level if

$$|Z| > Z_{\alpha/2}.$$

The observed value of the test statistic is -3.1320 and we reject the null hypothesis at the 1% significance level. Therefore, the announcement of a stock dividend induces greater abnormal performance than the announcement of a stock split.

We test for differences in the conditional variance of excess standardized event-period returns by examining

$$\text{H}_0: \quad \delta^2_{\text{split}} = \delta^2_{\text{div}}, \qquad \text{H}_A: \quad \delta^2_{\text{split}} \neq \delta^2_{\text{div}}.$$

Again, appealing to asymptotic maximum-likelihood theory, the appropriate test statistic is

$$Z = \frac{\delta^{2**}_{\text{split}} - \delta^{2**}_{\text{div}}}{\sqrt{\text{SE}(\delta^{2**}_{\text{split}})^2 + \text{SE}(\delta^{2**}_{\text{div}})^2}} \sim \text{N}(\delta^2_{\text{split}} - \delta^2_{\text{div}}, 1).$$

The observed value of the test statistic is 0.6934 and we cannot reject the null hypothesis. The empirical evidence is consistent with the increase in security-return variance around the announcement of a stock split not differing from the increase around the announcement of a stock dividend.

To investigate differences in the timing of the announcements, we consider

$$\text{H}_0: \quad p_{\text{split}} = p_{\text{div}}, \qquad \text{H}_A: \quad p_{\text{split}} \neq p_{\text{div}}.$$

The appropriate test statistic is now given by

$$Z = \frac{p^{**}_{\text{split}} - p^{**}_{\text{div}}}{\text{SE}(p^{**}_{\text{split}})^2 + \text{SE}(p^{**}_{\text{div}})^2} \sim \text{N}(p_{\text{split}} - p_{\text{div}}, 1).$$

The observed value of this test statistic is -0.6678 and so the null hypothesis of no difference in the timing of stock-split and stock-dividend announcement cannot be rejected.

6. Summary and conclusions

Standard event-study methods are vulnerable to errors in the specification of event time. We model the effect on security returns of the arrival of unanticipated information, incorporating explicitly the possibility of random event dates. The EM algorithm provides an efficient technique for implementing the model's maximum-likelihood estimation. Given event-date misspecification, this method is statistically more efficient than traditional event-study methods. Simulation studies using both artificial and actual return data confirm the power of our proposed estimation procedure when the event date is uncertain and demonstrate its ability to detect the effects of unanticipated information efficiently.

Appendix

The corresponding likelihood equations are given by

$$\frac{\partial \ln L(\underline{x})}{\partial p_t} = \sum_{i=1}^{n} (f(\underline{x}_i))^{-1}(g_t(\underline{x}_i) - g_c(\underline{x}_i)) = 0, \tag{A.1}$$

$$t = -c, \ldots, c-1,$$

$$\frac{\partial \ln L(\underline{x})}{\partial A} = \sum_{i=1}^{n} (f(\underline{x}_i))^{-1}\left[\sum_{t=-c}^{+c} p_t g_t(\underline{x}_i)(x_{it} - A)\delta^{-2}\right] = 0, \tag{A.2}$$

$$\frac{\partial \ln L(\underline{x})}{\partial \delta^2} = \sum_{i=1}^{n} (f(\underline{x}_i))^{-1}$$

$$\times \left[\sum_{t=-c}^{+c} p_t g_t(\underline{x}_i)\left((x_{it} - A)^2(2\delta^4)^{-1} - (2\delta^2)^{-1}\right)\right]$$

$$= 0. \tag{A.3}$$

By Bayes theorem

$$P[\theta_t = 1 | \underline{x}_i] = (f(\underline{x}_i))^{-1}(p_t g_t(\underline{x}_i)).$$

This expression represents the posterior probability that an event took place on day t given the data for the ith firm. Denote the maximum-likelihood estimates by $(A^{**}, \delta^{2**}, p_{-c}^{**}, \ldots, p_{c-1}^{**})$ and set $p^{**}[t|\underline{x}_i] = P[\theta_t = 1 | \underline{x}_i]$ at the maximum-likelihood estimates.

Since

$$\sum_{t=-c}^{+c} p_t^{**} \frac{\partial \ln L(\underline{x})}{\partial p_t} = 0,$$

appealing to eq. (A.1) gives that

$$n = \sum_{i=1}^{n} (f(\underline{x}_i))^{-1} g_c(\underline{x}_i).$$

From eq. (A.1) we have that

$$p_t^{**} \frac{\partial \ln L(\underline{x})}{\partial p_t} = 0,$$

or

$$\sum_{i=1}^{n} (f(\underline{x}_i))^{-1} (p_t^{**} g_t(\underline{x}_i)) = \sum_{i=1}^{n} (f(\underline{x}_i))^{-1} (p_t^{**} g_c(\underline{x}_i)),$$

$$\sum_{i=1}^{n} p^{**}[t|\underline{x}_i] = p_t^{**} \sum_{i=1}^{n} (f(\underline{x}_i))^{-1} g_c(\underline{x}_i).$$

giving

$$(1) \qquad p_t^{**} = (n)^{-1} \sum_{i=1}^{n} p^{**}[t|\underline{x}_i], \qquad t = -c, \ldots, c,$$

where implicitly

$$\sum_{t=-c}^{+c} p_t^{**} = 1.$$

From eq. (A.2) it follows that

$$\sum_{i=1}^{n} (f(\underline{x}_i))^{-1} \left[\sum_{t=-c}^{+c} p_t^{**} g_t(\underline{x}_i) x_{it} \right]$$

$$= \sum_{i=1}^{n} (f(\underline{x}_i))^{-1} \left[\sum_{t=-c}^{+c} p_t^{**} g_t(\underline{x}_i) A^* \right],$$

or

$$\sum_{i=1}^{n} \sum_{t=-c}^{+c} p^{**}[t|\underline{x}_i] x_{it} = nA^{**},$$

and hence

$$A^{**} = (n)^{-1} \sum_{i=1}^{n} \sum_{t=-c}^{+c} p^{**}[t|\underline{x}_i] x_{it}. \tag{2}$$

From eq. (3) we have that

$$\sum_{i=1}^{n} (f(\underline{x}_i))^{-1} \left[\sum_{t=-c}^{+c} p_t^{**} g_t(\underline{x}_i)(x_{it} - A^{**})^2 \right]$$

$$= \sum_{i=1}^{n} (f(\underline{x}_i))^{-1} \left[\sum_{t=-c}^{+c} p_t^{**} g_t(\underline{x}_i) \delta^{2**} \right],$$

or

$$\sum_{i=1}^{n} \sum_{t=-c}^{+c} p^{**}[t|\underline{x}_i](x_{it} - A^{**})^2 = n\delta^{2**},$$

and

$$\delta^{2**} = (n)^{-1} \sum_{i=1}^{n} \sum_{j=-c}^{+c} p^{**}[t|\underline{x}_i](x_{it} - A^{**})^2. \tag{3}$$

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