ON THE LATTICE APPROACH TO THE MAXIMUM HIGGS BOSON MASS ☆

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If the triviality upper bound on the Higgs boson mass m_H occurs for strong self-coupling, inferring properties of the Higgs from the euclidean propagator is in principle theoretically difficult whether in coordinate or momentum space. In that case, common methods of identifying m_H in lattice field theory simulations may produce a value for which is at best distantly related to the true upper limit. We discuss some shortcomings and ambiguities of recent results suggesting that the maximum occurs for weak coupling and emphasize potential complications due to finite-size and non-Lorentz-invariant effects of the lattice. The situation is illustrated by reference to the behavior in an analytically soluble approximation based on a $1/N$ expansion.

In recent years, it has become widely believed that quartic scalar field theory, although renormalizable, is *trivial* in the technical sense that the scalar self-coupling λ can be non-zero only in the presence of a finite momentum cutoff [#1]. A manifestation of this triviality is that, as the normalization scale μ increases, the renormalized self-coupling $\lambda(\mu)$ tends to infinity at some finite scale Λ_c which may be regarded as the largest possible momentum scale at which the scalar effective field theory makes sense. The parameter Λ_c may be regarded as a physical parameter characterizing λ in much the same way that Λ_{QCD} characterizes the SU_3 gauge coupling in quantum chromodynamics.

Because the Higgs sector of the standard model (SM) of strong and electroweak interactions involves a self-coupled scalar field, it is interesting to ask whether it is also necessarily trivial. The answer seems to be yes, but, if the Higgs boson is not much more massive than the weak vector bosons $W^{\pm}$ and Z^0, the cutoff can be postponed well beyond the Planck scale [2]. Motivated in part by naturalness considerations [3] and the gauge hierarchy puzzle of grand unfication [4], there is a great deal of interest in the possibility that the true cutoff lies in fact within the TeV energy range and that this may be near if not coincident with λ_c. This suggests that the self-coupling may actually be considerably larger than the gauge couplings, that the associated Higgs boson may be extremely heavy (if it exists at all), and that the TeV range may be represent a new threshold for strong interactions associated with large λ [#2].

The smaller the value of Λ_c, the larger $\lambda(\mu)$ becomes (at some definite, fixed scale μ) and, correspondingly, the larger the Higgs mass m_H. Consequently, there is an upper limit to m_H [6] inasmuch as it is only sensible if it lies below the physical scale Λ_c beyond which the theory is physically modified in some manner to be determined by future experiments. At present, the primary effort to determine quantitatively this upper limit to m_H involves the methodology of lattice field theory (with the momentum cutoff replaced by the (inverse) lattice spacing) and there have been an increasing number of attempts to implement this program over the past several years (see footnote 1).

However, there are certain problems intrinsic to the lattice approach which have not been discussed in work thus far. It is the purpose of this letter to highlight their relevance generally and to illustrate their importance in the context of a certain, solvable ap-

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[#1] The voluminous original literature may be traced from ref. [1].

[#2] In fact, Veltman [5] suggests that this threshold may lie well below 1 TeV.

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proximation to the SM. We will work entirely in continuum language since it is easier analyticaly and since this is the situation the lattice approach hopes to reproduce. A basic point is that the upper limit being sought requires penetration of the structure of the theory at short distances, yet the measures that have been introduced to define or identify m_H either are unreliable or else correspond to long-distance or low-momentum (i.e., infrared) properties of the field theory. Wen commented upon at all, this methodology has been rationalized a posteriori by the claim that the upper limit on m_H lies within the perturbative domain, so that its difference from the physical mass is minor. This is to say the least a paradoxical situation in which it is claimed at one and the same time that the formulas developed based on the continuum perturbative analysis are valid but that the corrections to the continuum description are large. While we cannot claim that the $1/N$ approximation is quantitatively reliable, a very different qualilitative picture emerges from our solution which is at least intuitively plausible, viz., our upper bound on the Higgs mass occurs for strong coupling where perturbation theory cannot be trusted. In this regime, the correspondence between the infrared measures employed and the physical Higgs parameters is quite complicated. To relate those quantities easily determined numerically and these parameters would demand considerable theoretical effort.

In the following, we begin with a review of a coordinate space approach. After summarizing the features of our solvable approximation, we then examine an alternative momentum space approach. Then we discuss other analytic methods and their results.

The first important point is that the heavier the Higgs boson, the greater its width is expected to be. The Higgs boson is normally associated with the pole s_σ determined by analytic continuation in $s \equiv p^2$ of the scalar propagator from the physical sheet to the so-called second sheet of the Riemann surface [7]. As is well known, in perturbation theory, the width grows proportional to m_H^3 and there is the expectation that, if the pole position in the SM could be determined for strong coupling, the imaginary part would be comparable to or larger than the real part. Thus, even in an ideal situation there is no such simple concept as the Higgs mass m_H, and the specification of the Higgs boson requires a determination of the complex number s_σ. With an imaginary part comparable to or larger than the real part, it will bear only a faint resemblance to the usual concept of a resonance. In fact, there may be an infinite number of "shadow" poles on other Riemann sheets associated with this pole in the lower half plane of the second sheet [7]; indeed, that is what we find in our solvable approximation below.

The most popular approach to lattice field theory generally deals with the analytic continuation from physical Minkowski space to the euclidean region. One common approach to determining m_H is to fit the asymptotic behavior of some correlation function, assuming the relevant correlation behaves as $\exp(-m_H r)$ for large separations r (see footnote 1; some recent representative references can be found in ref. [8]). While this asymptotic form is valid for a stable particle, it is not true for an unstable particle for which the ultimate asymptotic behavior is determined by the threshold for te continuum to which it is coupled. Nevertheless, a narrow resonance may in fact dominate the behavior for some intermediate range of r even if the true asymptotic behavior for large r is quite different. One would expect such pole dominance to deteriorate for a broad resonance. Moreover, the notion that a single Higgs resonance pole dominates the asymptotic behavior is obviously problematic, since the scalar propagator is real while the contribution from s_σ is complex. Therefore, it behooves those fitting simple exponentials to correlation functions to elaborate the theoretical basis for the interpretation of those fits.

One can discuss the asymptotic behavior of the Higgs field propagator $\Delta_F(p^2)$ quite generally starting from the Källén–Lehmann representation [9]. In coordinate space, the propagator may be written as

$$\Delta_F(x) = \int_0^{\sim\Lambda_c^2} ds\, \rho(s)\, \Delta_F(x; s) , \qquad (1)$$

where the spectral function $\rho(s)$ is necessarily non-negative, vanishing below the lightest threshold to which the Higgs field is coupled. The upper limit of integration must be cutoff at a scale of $O(\Lambda_c^2)$, as we assume that a local field theory approximation is good only for smaller energy scales. $\Delta_F(x; s)$ is the free particle propagator associated with a particle of mass $\sqrt{s}$. In the euclidean region ($x^2 \equiv -r^2 < 0$), this is simply

$$i\Delta_F(x;s) \equiv \Delta_E(r;s) = \frac{\sqrt{s}}{4\pi^2 r} K_1(\sqrt{s}\,r) . \quad (2)$$

For large r, $\Delta_E(r;s)$ behaves as $r^{-3/2}\exp(-\sqrt{s}\,r)$, so the true asymptotic behavior as $r\to\infty$ is determined by the behavior of $\rho(s)$ at the lightest threshold. In lowest order, the lower limit of integration is at $4m_e^2$ associated with the electron–positron state. In fact, for a very heavy Higgs, this contribution as well as those due to other known fermions [#3] is weak compared to the coupling to W^+W^- associated with a threshold at $4M_W^2$, and we shall neglect its contribution.

From the positivity of the spectral function, it can be shown that the euclidean propagator $\Delta_E(r) \equiv i\Delta_F(x)$ is positive and monotonically decreasing, with successive derivatives alternating in sign. Consequently, no matter what the dynamics, $\Delta_E(r)$ decreases smoothly, without even so much as a wiggle!

As remarked earlier, it can happen that, because ρ is so much larger in a higher mass region, the behavior may be approximately described over an intermediate range of r by ignoring the lighter threshold. So it may be useful to record the contribution coming from a resonance. First, however, let us recall that, because the propagator is an analytic function real in the euclidean region, associated with a pole at position s_σ, there is a mirror pole at s_σ^*. Therefore, we may write this contribution to the spectral function as

$$\rho_\sigma(s) = \frac{1}{2\pi i}\left(\frac{Z_\sigma^*}{s-s_\sigma^*} - \frac{Z_\sigma}{s-s_\sigma}\right). \quad (3)$$

(This formula does not represent an analytically correct approximation to $\rho(s)$ but rather what we will mean by te pole approximation thereto. We suppose the 'wave function renormalization constant" Z_σ is such that $\rho_\sigma > 0$.) One may then ask what this contribution gives for the coordinate space behavior when inserted into eq. (1). As remarked earlier, the true asymptotic behavior will be determined by the threshold position, but we may isolate the behavior associated with the pole as follows: Since $\rho_\sigma(s)$ and $\Delta_E(r;s)$ are analytic in s in a neighborhood of the positive real axis, we may replace the integral in eq. (1) by

$$\Delta_E(r) = -\frac{1}{2\pi i}\int_C ds \ln(4M_W^2 - s)\,\rho_\sigma(s)\,\Delta_E(r;s) , \quad (4)$$

where the contour C wraps the positive real axis (plus a contour at $|s| = \Lambda_c^2$). Note that the integral is independent of the scale of the scale of the logarithm. Opening up the contour, one picks up the residues at the poles s_σ and s_σ^* plus a "background" contribution which we discard. Parameterizing the pole as $s_\sigma = |s_\sigma|\exp(-2i\theta)$ and assuming $|s_\sigma| \gg 4M_W^2$, we find

$$\Delta_E(r) \approx \left(1 - \frac{2\theta}{\pi}\right)\mathrm{Re}[Z_\sigma \Delta_E(r;s_\sigma)] , \quad (5)$$

where we have also dropped a term involving $\ln(|s_\sigma|)$ since its precise form depends on the threshold position (here at $s\approx 0$) and its value depends on the scale of the logarithm, i.e., the nature of the background. Defining the Higgs mass and width by $\sqrt{s_\sigma} \equiv m_H - i\Gamma_H/2$, the pole contribution for large r is

$$\Delta_E(r) \to \left(1 - \frac{2\theta}{\pi}\right)\frac{1}{2(2\pi r)^{3/2}}\,|Z_\sigma||s_\sigma|^{1/4}$$

$$\times \exp(-m_H r)\cos(\tfrac{1}{2}\Gamma_H r - \tfrac{1}{2}\theta + \zeta) , \quad (6)$$

where ζ is the phase of Z_σ. In principle, these formulas answer the question of how the Higgs pole parameters are related to the euclidean correlation function, demonstrating that a pole yields a behavior for the euclidean Green's function which is oscillatory with an exponentially decaying envelope. Although eq. (6) makes no assumption about the ratio Γ_H/m_H being small, a positive spectral function ρ_σ must, as noted above, produce a positive, monotonically decreasing propagator, so the approximations represented by eqs. (5) and (6) can apply at most to a fraction of the distance out to their first zero. There remains unanswered the question how the contribution from a resonance compares to the exact propagator, including the continuum "background".

We shall discuss below similar difficulties associ-

[#3] The inclusion of a very heavy fermion (as the top quark may very well be) requires a new treatment which we would expect to alter the results below quantitatively but not qualitatively.

ated with isolating the Higgs parameters for euclidean momentum space. First, however, it is useful to analyze these complications in an approximation which was introduced some years ago [10] [#4,5]. For λ large compared to the gauge couplings and for energy scales large compared to the $W^\pm$ and Z masses, the gauge couplings (and vector boson masses) can be neglected in first approximation. In addition, with the possible exception of the top quark, fermions can also be neglected as a first approximation. Then the $SU_2 \otimes U_1$ Higgs sector of the SM becomes the O_4 Higgs–Goldstone model with a scalar field $\boldsymbol{\pi}$ in the fundamental representation of O_4. In this previous work, we suggested that, to understand the behavior qualitatively, O_4 be replaced by O_{2N} and, in the now-familiar manner, expanded in $1/N$ for a fixed value of the self-coupling λN. The results to leading order may be derived either by summing the leading Feynman diagrams [10,11] or by manipulating the path integral [12]. The solution in leading order may be summarized as follows: In the broken symmetry phase, the field acquires a vacuum expectation value, say, $\langle \pi_N \rangle = v$. This corresponds in the SM to the weak scale. The renormalized coupling λN may be related to the bare coupling $\lambda_0 N$ according to

$$\frac{1}{\lambda(M)N} = \frac{1}{\lambda_0 N} + \frac{1}{8\pi^2} \ln\left(\frac{\Lambda^2}{M^2}\right), \tag{7}$$

where Λ is the momentum cutoff and M is an arbitrary normalization mass. It is of course redundant to speak of varying both the bare coupling and the cutoff, and we replace them by a single physical parameter Λ_c defined by

$$\frac{1}{\lambda N(M)} \equiv \frac{1}{8\pi^2} \ln\left(\frac{\Lambda_c^2}{M^2}\right). \tag{8}$$

Given the value of the coupling on any scale, one may think of Λ_c as the maximum possible momentum scale which can be discussed in this effective field theory.

To leading order in $1/N$, the weak scale v is a renormalization group invariant which we may imagine to be fixed by the Fermi constant G_F or, after reintroducing the gauge couplings, by the vector boson masses. (Recall that v is formally of $O(\sqrt{N})$.) As usual, the Higgs field σ is associated with the shifted field $\pi_N = v + \sigma$. The none may determine the inverse Higgs propagator exactly to O(1) to be

$$D_{\sigma\sigma}(p^2)^{-1} = p^2 - \frac{16\pi^2 v^2/N}{\ln[e^2\Lambda_c^2/(-p^2)]}, \tag{9}$$

where $\ln e \equiv 1$. The branch of the logarithm is chosen so that the propagator is real on the physical sheet for spacelike momenta ($p^2 < 0$). The different branches of the logarithm (for which the arguments differ by $2\pi i$) correspond to the different Riemann sheets for the propagator. The position of the poles on each sheet is determined by those momenta s_n at which $D_{\sigma\sigma}(s_n)^{-1} = 0$. The pole s_σ which is nearest the physical region for timelike $p^2 + i\epsilon$ is generally what is meant by "the Higgs boson", for it is this pole which evolves continuously as $\lambda N \to 0$ to the free stable particle pole. The behavior of this pole position was worked out in ref. [10] and is reproduced here in fig. 1. (To compare with the SM, we simply set $v/\sqrt{N} = 174$ GeV, its value for the SM ($N=2$).) As expected quite generally, Im s_σ generally increases with λN but Re s_σ increases only up to a point and then

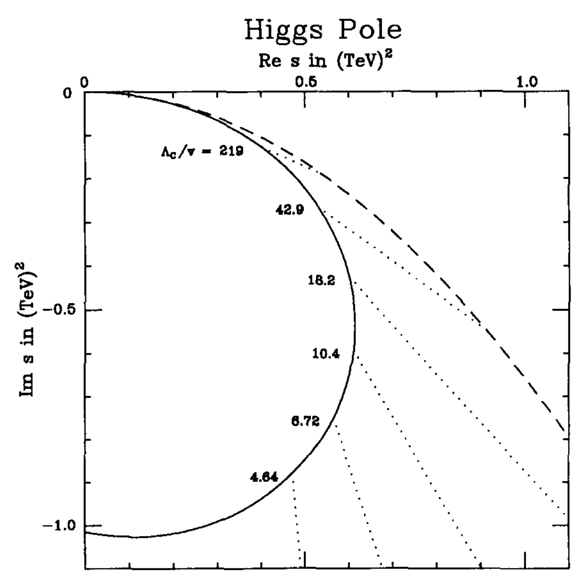

Fig. 1. Solid line: Higgs pole position s_σ; dashed line: perturbative Higgs pole position, normalized at $|s_\sigma|$. Dotted lines connect curves at points having a common value of Λ_c/v.

[#4] For the most part, we shall follow the notation and conventions introduced there.

[#5] While this was the first application of the O_{2N} model to the SM, the solution for this model had been worked out previously in other contexts. See ref. [11].

turns around [#6]. For comparison, we have also plotted the perturbative (tree) results for the Higgs mass and width, with the coupling normalized (arbitrarily) at the scale $|\sqrt{s_\sigma}|$. With this convention the perturbative Higgs mass tends to infinity at the crossover point where $|s_\sigma| = \Lambda_c^2$. Large deviations from the perturbative value are evident for a perturbative mass above about 800 GeV.

In fig. 2, the relation between the momentum cutoff and the magnitude of the Higgs pole $|\sqrt{s_\sigma}|$ is displayed. We have chosen the abscissa to be proportional to $\sqrt{\lambda N}$, where we arbitrarily take the scale of λ to be the weak scale $\sqrt{2/N}\,v$. Thus, the abscissa characterizes theories with different coupling strengths but at a scale well below all relevant values of the cutoff. This choice is further motivated by the observation that, for weak coupling, this scale is directly proportional to the Higgs mass. The Higgs modulus $|\sqrt{s_\sigma}|$ saturates to an approximately constant value around 1 TeV. As depicted in fig. 2, the crossover point at which the momentum scale $|s_\sigma|$ associated with the Higgs pole becomes equal to the cutoff Λ_c^2 turns out to be $\Lambda_c \approx 1.8\pi v/\sqrt{N}$, corresponding to an upper limit on the Higgs modulus of about 1.0 TeV. From fig. 2, it is easy to see that this upper limit is rather insensitive to weakening the crossover criterion. For example, if it should be that significant deviations from the SM already appear when $|s_\sigma| = \Lambda_c^2/4$, then the crossover is at $|\sqrt{s_\sigma}| = 0.86$ TeV instead. To be more precise requires a careful examination of the higher dimensional operators entering the effective lagrangian and consideration of their effects in particular processes. It is perhaps fortuitous but nevertheless striking that the absolute upper limit determined here for the Higgs modulus turns out to be numerically very nearly the same as the perturbative upper bound on the Higgs mass [13]!

[#6] It is important to observe that, in physical scattering processes, for example, in W^+W^- scattering, the center-of-mass energy E_{cm} at which the Higgs pole is closest corresponds to $E_{cm}^2 = \mathrm{Re}\, s_\sigma = m_H^2 - \Gamma_H^2/4$. As a result, this "peak" position *never* becomes larger than about 780 GeV. However, since $\mathrm{Im}\, s_\sigma$ becomes large in the strong coupling regime, the pole is so far removed that it would be impossible to detect its presence as an ordinary resonance.

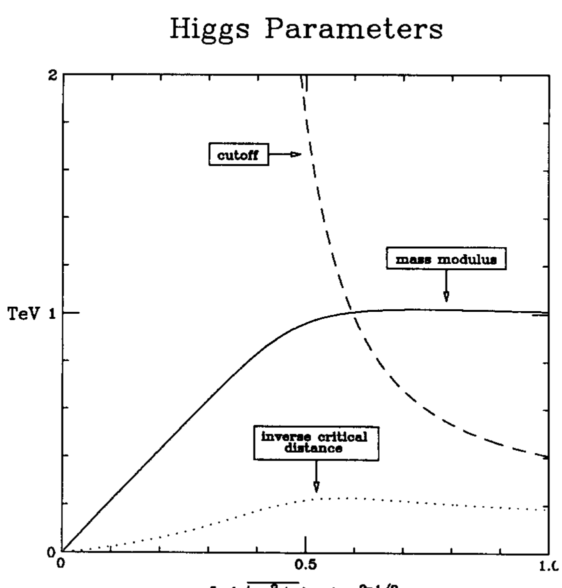

Fig. 2. The Higgs mass modulus $|s_\sigma|$, cutoff Λ_c, and inverse critical distance r_c^{-1} as functions of the coupling strength.

The spectral function $\rho(s)$ may be written as

$$\rho(s) = \frac{16\pi^2 v^2/N}{[s\ln(e^2\Lambda^2/s) - 16\pi^2 v^2/N]^2 + \pi^2 s^2}\,. \tag{10}$$

The true asymptotic behavior is determined by the threshold behavior

$$\lim_{s\to 0} \rho(s) = \frac{1}{16\pi^2 v^2/N}\,. \tag{11}$$

The asymptotic behavior in coordinate space of the propagator in the euclidean region may then be determined to be

$$\Delta_E(r) \to \frac{1}{\pi^2}\frac{\pi(0)}{r^4}\,. \tag{12}$$

Notice that this leading behavior is completely independent of the Higgs mass parameters [#7]! In the real world where the W-boson has non-zero mass, this simple power behavior would be tempered by a factor of $\exp(-2M_W r)$. Nevertheless, it will decrease far less rapidly than might be expected on the basis of the Higgs mass scale.

As mentioned earlier, it is uncertain whether a pole approximation to the propagator will be valid for a

[#7] It is tempting to speculate that this is a general consequence of the infrared freedom of the scalar sector, but it remains to be proved.

strongly coupled Higgs. To understand the transition from weak to strong coupling and to compare with the true asymptotic behavior, it may be useful to record the behavior for this model for the pole approximation, eq. (3). Z_σ may be written as [#8]

$$Z_\sigma^{-1} = 1 - \frac{\sin 2\theta\, e^{-2i\theta}}{\pi + 2\theta} . \quad (13)$$

Since the magnitude of $2\theta < \pi/2$, Z_σ differs from 1 by less than 20% regardless of the coupling strength. In this approximation, the crossover point for maximum Higgs "mass" at $|s_\sigma| \approx 1.0\ (\text{TeV})^2$ corresponds to $\theta \approx 32.5°$ so that the ratio of width to mass Γ_H/m_H is $\tan\theta \approx 0.64$. In this O_{2N} model, it would be interesting to know the range of r, if any, over which the pole contribution is a good approximation to the exact behavior, especially in the strong coupling regime near the crossover point. This requires detailed numerical investigation to which we shall return in a subsequent paper. To get a rough indication of the possibility, we define r_c to be that value of r at which the envelope in eq. (6) is equal to the true asymptotic behavior, eq. (12). Thus, for all $r > r_c$, the pole approximation certainly does not predominate. This critical distance r_c is plotted in fig. 2, from which one observes that in the region near where $|s_\sigma|$ reaches the crossover point, the pole approximation does in fact dominate the asymptotic behavior (by about a factor of two, as it turns out) on distance scales out to about 5 or 10 times the cutoff $1/\Lambda_c$, which in lattice calculations would presumably correspond to a/π, with a the lattice spacing. This provides some grounds for optimism for lattice simulations.

Another more recent approach to this problem involves a study of the lattice propagator in (euclidean) momentum space [14]. A Monte Carlo evaluation of the inverse propagator is fit to the form of a free propagator which, in the continuum limit, corresponds to $(p^2 + m_R^2)/Z$ and from the slope and intercept, the wave function renormalization constant Z and renormalized "Higgs mass" m_R are extracted. For reasons already presented, it is very dubious that the behavior for $p^2 \to 0$ reflects much about the Higgs properties, and these doubts are reinforced by comparison with the result of the $1/N$ approximation, which, for euclidean momenta $p_E^2 = -p^2 > 0$, may be simply written as

$$D_E(p_E^2)^{-1} \equiv -D_{\sigma\sigma}(p_E^2)^{-1} = p_E^2 + 2\lambda(\sqrt{p_E^2}/e)v^2 , \quad (14)$$

that is, it looks just like the tree aproximation with the scale of the running coupling proportional to the momentum scale of interest. Because the coupling varies only logarithmically, this function may *appear* to be linear over some range of p_E^2. Nevertheless, the intercept at $p_E^2 = 0$ is precisely zero (as one might have guessed from the fact that the coupling is infrared free) but the slope at $p_E^2 = 0$ is infinite. This is a situation in which a linear fit to the function produces an apparent non-zero intercept extremely sensitive to the points nearest to $p_E^2 = 0$, which in lattice calculations is presumably constrained by the finite lattice size [#9]. This suggests that the method of ref. [14] is suspect, although it may be that this deficiency can be satisfactorily dealt with [16]. Assuming that this does not significantly alter their results, these authors claim a small upper bound for the Higgs mass, around $m_H = 550$ GeV, assuming non-scaling effects are evident at a correlation length of 5 lattice spacings. In fact, the data presented do not appear to show significant deviations for correlation lengths larger than about 1.2 lattice spacings, corresponding to $m_H/v \approx 2.8$ or $m_H \approx 700$ GeV.

Another approach combining both numerical and analytical methods involves an attempt to exploit an expansion in the Higgs field kinetic energy (high temperature expansion) to solve for the relation between the Higgs mass and the cutoff [17] [#10]. As yet this technique has only been applied to the single component ϕ^4 theory, where there are no Goldstone bosons to cause infrared problems. The renormalized Higgs mass, defined as in ref. [14] at zero momentum, is compared with the ultraviolet cutoff. Requiring $m_H < \Lambda_c/2$, the upper limit on the self-coupling is only about 2/3 the unitarity bound. With appropriate modifications of the definition of the Higgs mass to avoid infrared singularities, the authors anticipate

[#8] This corrects a factor of 2 error in ref. [10].

[#9] Neuberger has suggested that such finite-size sensitivity might be usefully exploited [15].

[#10] This line of development has been reviewed recently by Lüscher [18].

that the method can be extended to the O_4 theory [19]. The role of the lattice enters only through their use of the coefficients for the high temperature expansion, and within the perturbative domain, their technique would appear to be reliable.

In each of these approaches [14,17,18], the technique relies on perturbation theory to determine the relation of m_H as defined to the physical Higgs mass. This is valid only so long as the upper limit on $m_{\rm H}$ corresponds to "small" coupling. If the perturbative formulas can be trusted, using the perturbative beta function $\beta_\lambda(\mu)$ would suggest that, even at the cutoff, the coupling is still within the perturbative regime. Why is the continuum perturbation expansion so misleading as to its range of validity?

One must be cautious about conclusions drawn from non-scaling effects on the lattice. If the modification of the SM required by the underlying physical theory does not break Lorentz invariance, for example, then one must distinguish those non-scaling effects associated with the triviality cutoff and those lattice artifacts associated with non-Lorentz-invariant corrections. Neuberger has argued similarly and suggested an F_4 lattice may be better in this regard [15,20]. One would expect, with a Lorentz-invariant cutoff, scale-breaking effects to be of order $m_{\rm H}^2/\Lambda_{\rm c}^2$. Thus, an upper limit of $m_H \approx 550$ GeV, corresponding to $m_{\rm H}/\Lambda_{\rm c} < 1/5\pi$ [14], would imply an intrinsic scale-breaking of about 0.4%.

Finally, two different approximate but analytic renormalization methods have been employed to obtain the relation between $\Lambda_{\rm c}$ and $m_{\rm H}$, the latter quantity being defined by the curvature of the effective potential at its minimum [21]. This quantity will be simply related to the physical Higgs parameters only so long as perturbation theory is valid. Requiring $m_{\rm H}/\Lambda_{\rm c} < 1/3$, these authors conclude that $m_{\rm H} < 800$ GeV. Had they, like the preceding authors [17,18], required $m_{\rm H}/\Lambda_{\rm c} < 1/2$, their upper limit would have been nearly 900 GeV. So we conclude that these approximate results are inconclusive as to whether the upper bound lies within the weak coupling or strong coupling domain.

If a Higgs particle does not exist below the triviality bound on $m_{\rm H}$, the SM will begin to fail at energies above this scale in its description of vector boson interactions. Thus, even though it may be that new physical thresholds do not appear until energies on the order of $\Lambda_{\rm c}$, it may be near $\Lambda_{\rm c}/2$ rather than around $\Lambda_{\rm c}$ at which significant deviations from the continuum field theory would appear. This can be seen by reference to composite models such as minimal technicolor, in which the cutoff $\Lambda_{\rm c}$ would correspond to the compositeness scale around 1 TeV with perhaps no Higgs particle at all below this scale. Yet one would expect that the interactions among longitudinal vector bosons would be strong even below 1 TeV. A triviality bound on $m_{\rm H}$ well below 1 TeV, as has been suggested by most of the investigations cited herein, implies that in the range above the bound but below 1 TeV, the SM lagrangian would not adequately describe this strong interaction. Since the SM lagrangian contains all terms of dimension 4 and below consistent with the gauge symmetries and particle multiplet assignments, the only possibility would be that there would be non-negligible terms of higher dimension which could come into play above an energy $m_{\rm H}$. This conclusion is clearly not peculiar to the technicolor model; in general, if there is no Higgs particle below its triviality bound, the implication would be that the SM requires modification at energies not above $\Lambda_{\rm c}$ but already above that bound.

As emphasized in refs. [17,18], an uper bound on $m_{\rm H}$ well below the unitarity bound means that there is no self-consistent, strongly coupled ϕ^4 theory. Despite the evidence to the contrary, we believe that when finite size and other lattice artifacts are treated more precisely and when a careful assessment of the size of the Lorentz-invariant, non-scaling corrections is made, the upper limit to the Higgs mass wil, as in the $1/N$ model, be determined to occur in the strong coupling regime where perturbation theory cannot be trusted. For the theoretical reasons discussed, at this time it is unclear to us how in principle to deduce properties of the Higgs boson in the strongly coupled regime from lattice simulations.

In conclusion, we wish to point out how extremely important it is to achieve quantitative accuracy on the implications of triviality. The difficulties observing longitudinal W^+W^- scattering, even with te design parameters of the SSC, may make it difficult to identify deviations from the SM unless they are rather large (say, 50%). As a second example, consider calculations of the usual kinds of radiative corrections to the ρ-parameter relating the Fermi constant $G_{\rm F}$ to the vector boson masses, frequently discussed as a precision test of standard model [#11]. Typically, the dependence on m_H or on fermion mass splittings (such as the difference between the top and bottom quark masses) are at the level of less than 1% even when these parameters are so large as to reach the limits of perturbation theory [#12]. If there are additional higher dimensional operators in the effective lagrangian with coefficients of order, say, $1/\Lambda_c^2$, at what point will these begin to compete with the usual radiative corrections?

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[#11] For a recent review of data see ref. [22]. For original theoretical work see ref. [23].

[#12] Elsewhere, it has been argued that Yukawa couplings may also be trivial [24].

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