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TRUCK DYNAMIC STABILITY

VOLUME I

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PARAMETRIC ANALYSIS OF HEAVY DUTY  
TRUCK DYNAMIC STABILITY

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16. Abstract The study sought to define the important parametric sensitivities which affect the directional performance limits of commercial vehicles. Rollover, unstable yaw response of the lead unit (spinout), and lightly damped yaw response of trailing units (rearward amplification) are identified as the three major response modes limiting directional performance. It is noted that both yaw response modes may precipitate rollover. The significant parametric sensitivities of commercial vehicles to each performance mode are identified by analytical means. Computer simulations of example vehicles, chosen for their peculiar susceptibility to one or more of the limiting performance modes, are used to demonstrate the parametric sensitivities.			
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## CHAPTER 1

### INTRODUCTION

This document constitutes the Final Technical Report by The University of Michigan Transportation Research Institute (UMTRI) on the research project entitled "Parametric Analysis of Heavy Truck Dynamic Stability." The project was sponsored by the National Highway Traffic Safety Administration (NHTSA) of the U.S. Department of Transportation under Contract Number DTNH22-80-C-07344.

The objective of this project is to characterize, by analytical means, the effects of vehicle design variations, and the limits within which drivers must operate their vehicles, as these considerations relate to the directional dynamic stability of large trucks. Computerized and theoretical analyses, serving to show the relationship of vehicle and operating parameters to the directional and roll stability of a particular sample of heavy vehicle configurations, are used to address this objective. In the broader sense, it is presumed that an improved understanding of these relationships will lead to a better basis for relating vehicle design and operating practices to safety.

Viewed in the context of the tremendous diversity of vehicle configurations which exist in the U.S. trucking fleet, the above-stated objective implies a study of considerable size. In order to limit the breadth of the undertaking and to focus the analytical effort near "the bounds of safe vehicle design," the study was defined to place emphasis on unusual, yet prevalent, vehicle configurations which might be hypothesized to require high levels of handling skills of their drivers. Accordingly, an early task of the project was a survey effort, aimed at canvassing the U.S. trucking community in order to identify vehicles that are difficult for drivers to control. From this activity a set of nine subject vehicle types was established, and the following analytical activity focused on these vehicles.

Commercial vehicles may be subject to two general classes of instability, namely, (1) divergent roll response, i.e., vehicle rollover, and (2) divergent yaw response, i.e., spinout or, in the case of articulated vehicles, jackknife or trailer swing. In the simplest sense, then, the directional response capability of a given commercial vehicle is generally limited by the instability (roll or yaw) which occurs at the lowest maneuvering level (as indicated by lateral acceleration).

A commercial vehicle's roll stability limit can be defined in terms of lateral acceleration. That is, below a given level of lateral acceleration, the vehicle remains roll stable; above that level, the vehicle is unstable in roll. In general, the roll stability limit is unaffected by velocity. Conversely, yaw stability of heavy vehicles may be a function of both velocity and lateral acceleration. Even commercial vehicles with the most severe yaw stability problems require a forward velocity of about 25 mph (40 k/hr) or greater, along with elevated lateral acceleration, in order to exhibit yaw instability.

In practice, roll and directional response cannot be divorced. If not the majority, then certainly a very large percentage of commercial vehicles will exhibit rollover at some level of attainable lateral acceleration (i.e., at a lateral acceleration level which is less than that required to produce saturation of tire side forces). As a result of this fact, a commercial vehicle (particularly a single-unit vehicle) whose limit performance is defined by yaw instability is likely, as a result of the occurrence of this instability, to proceed into a condition wherein lateral acceleration becomes sufficiently high to precipitate rollover. That is to say, while the limit of such vehicles is defined by a yaw instability, the safety-related consequence of exceeding the limit may be rollover. Situations wherein the vehicle is constrained to a curved path, but may be traveling at sufficient speed to aggravate yaw instability, such as may occur on a freeway exit ramp, have been hypothesized as producing rollover as a consequence of yaw instability.

In the case of articulated vehicles, yaw response of the vehicle may precipitate rollover in a somewhat different manner. The yaw response of

articulated units may be characterized by lightly damped dynamic modes dominated by yaw plane oscillations of the trailing units of the vehicle. Although such modes may be stable, in transient maneuvers (such as emergency lane changes) they can exacerbate the yaw response of the rearmost unit to such an extent that the roll stability limit of that unit is exceeded prematurely—prematurely in the sense that if the trailing unit had experienced the same level of excitation as the leading unit, rollover would not have occurred. This general phenomenon—the exaggerated or amplified lateral acceleration response of trailing units relative to the lead unit—has come to be known as "rearward amplification."

Accordingly, in this project, the sensitivities of the dynamic response of commercial vehicles are examined analytically. The validity of the general analytical findings is then demonstrated through the evaluation of parametric changes in a particular set of vehicles, chosen for study because they appear to be real-world examples of commercial vehicles that are particularly susceptible to one of the following classifications of limit performance:

- Divergent roll response, per se
- Divergent yaw response
- Lightly damped, oscillatory yaw response—rearward amplification

The remainder of this report is structured to provide an overview of the results of the study, followed by a more indepth discussion of the project findings. In accordance with this structure, Chapter 2 provides a narrative review of the findings of the study. Chapter 3 reviews the selection of sample vehicles for study. Chapters 4, 5, and 6 provide detailed technical discussions of the dynamic directional characteristics of heavy vehicles in general, and of selected vehicles in particular, relative to roll divergence, yaw divergence, and lightly damped yaw response, respectively.

## CHAPTER 2

### DISCUSSION OF RESULTS

This study has sought to define the important parametric sensitivities which significantly affect the limits of safe operation and design of commercial vehicles with respect to directional performance (that is, with respect to handling, but not braking performance). In the main, the directional performance of commercial vehicles is limited by one of the following:

- 1) Straight-forward vehicle rollover encountered in attempting turning maneuvers which directly exceed the roll stability of the vehicle.
- 2) Loss of directional control, particularly as a result of exceeding the vehicle's yaw stability limit and thereby initiating a "spinout" condition. For commercial vehicles, such unstable yaw response is likely to generate subsequent turning responses which exceed the vehicle's roll stability limit, thus precipitating rollover as a result of the spinout.
- 3) Exaggerated response of trailers, particularly of multiply-articulated vehicles. Articulated vehicles often have dynamic modes of behavior which are, technically, stable but which may be very lightly damped. For multiply-articulated vehicles, the result is a tendency for the rearward units of the vehicle to show exaggerated or amplified response, relative to the towing unit, in certain classes of turning maneuvers. "Rearward amplification" has important safety consequences when, in such maneuvers, the trailing units exceed their own roll stability limit, resulting in trailer rollover.



Accordingly, the parameter sensitivity analyses of this study have been conducted within the following three categories:

- 1) Parameter sensitivities affecting vehicle roll stability limits.
- 2) Parameter sensitivities affecting vehicle yaw stability limits.
- 3) Parameter sensitivities affecting rearward amplification.

The following three sections of this chapter will briefly review the findings of this project as they relate to these three topics, respectively. The findings presented refer to the mechanical (parametric) properties of vehicles and their components in contrast to a hardware-oriented approach in which the influences of specific pieces of hardware are compared. Designing satisfactory hardware is a matter of ingenuity and skill once appropriate mechanical specifications (goals) have been identified. A fundamental understanding of what is needed is the prerequisite to the design of hardware fulfilling the need. The findings of this study are intended to help provide that understanding.

## 2.1 Parameter Sensitivities Affecting Vehicle Roll Stability Limits

Figure 1 illustrates that as a vehicle undergoes a turn, it experiences a centrifugal force pulling outward from the center of the turn through the vehicle's center of gravity. This force tends to roll the vehicle outward from the turn, and if large enough, will cause the vehicle's inside tires to lift from the ground and roll the vehicle over.

The magnitude of this force is equal to the weight of the vehicle ( $W$ ) times the lateral acceleration ( $a_y$ ) generated by the turn. As the turn becomes more severe, lateral acceleration increases, causing an increase in the centrifugal force. Thus, the roll stability limit of the vehicle is generally identified by the maximum level of lateral acceleration which a vehicle can sustain without rolling over.

In addition to the centrifugal force, Figure 1 also shows that, as the vehicle rolls outward in a turn, its c.g. tends to shift outward

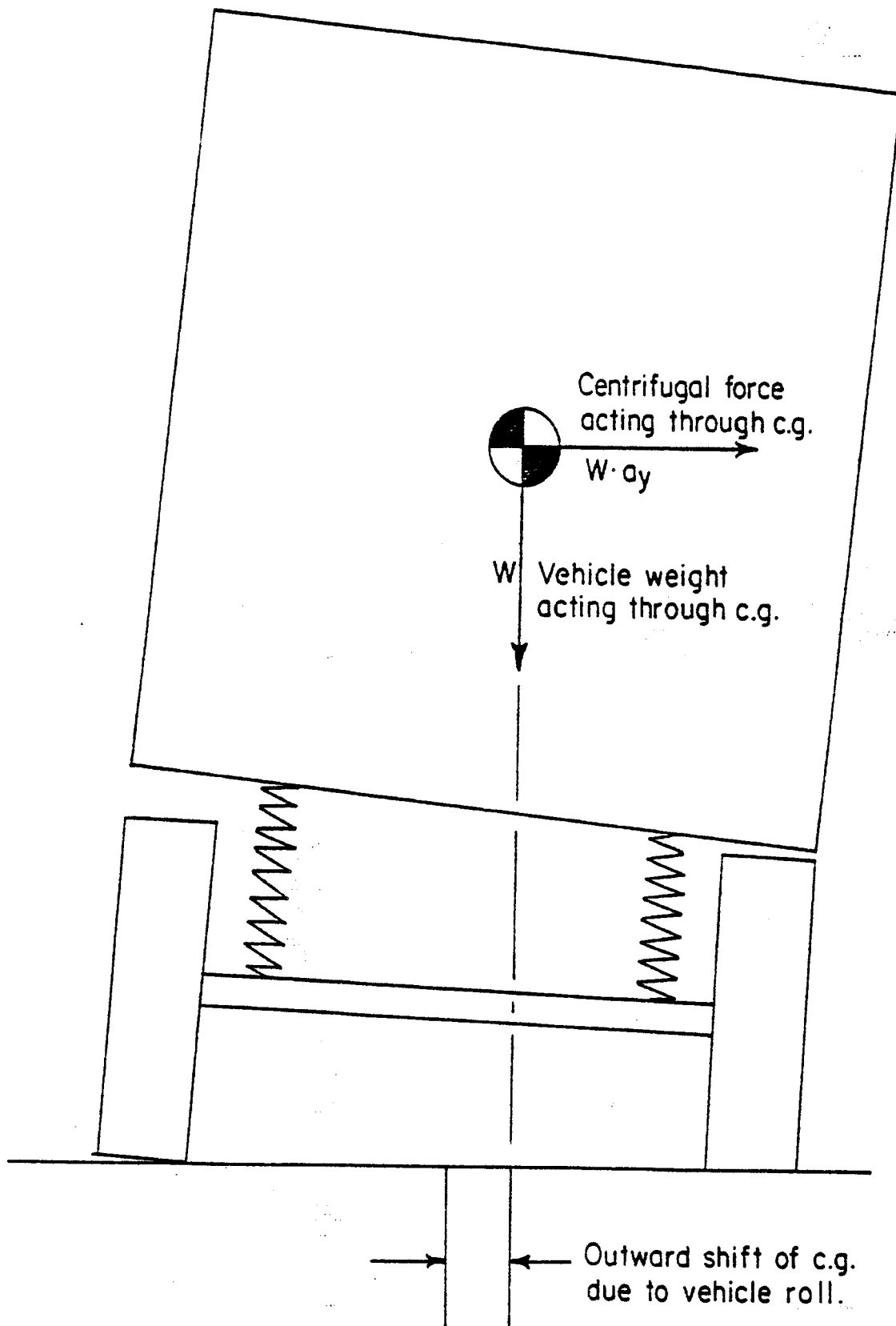


Figure 1. A schematic diagram of a commercial vehicle rolling in a turn.

relative to the center of the vehicle's track. This outward shift of the center of gravity also tends to promote rollover, serving to lower the roll stability limit.

A number of vehicle parameters can be identified which affect the roll stability limit of a vehicle. Generally, these parameters either (1) determine the direct effectiveness of the centrifugal force in generating rollover, or (2) contribute to determining the amount of outward shift of the center of gravity in a given turn.

The specific vehicle parameters which serve to determine the limit of roll stability are discussed below. The discussion is valid for any independently rolling vehicle unit. For example, a tractor-semitrailer represents a single unit since the fifth wheel does not allow independent rolling of the tractor and trailer. A truck-full trailer combination, however, would be two units, the truck and the trailer. That is, the pintle hook connection generally used allows independent rolling of the truck and the trailer.

The first parameter to be discussed, the ratio of track width to c.g. height, is the most basic parameter to determining the rollover limit. That is, it is the one parameter which determines "the direct effectiveness of the centrifugal force in generating rollover." All four parameters to be considered are involved in the secondary roll stability mechanism, i.e., the outward shift of the center of gravity during turning. To a large extent, the strength of the influence of each item is dependent on the values of the others. Thus, no infallible ranking of these four items can be given. Nonetheless, they are presented in an order generally indicative of their relative importance.

1) The ratio of track width to c.g. height. This ratio is the most fundamental parameter affecting the roll stability limit. As the ratio is increased, either by increasing track or decreasing c.g. height, the roll stability limit of the vehicle is improved. Commercial vehicles generally

have nearly identical track widths, but vary greatly in c.g. height.\* Accordingly, high c.g. vehicles can generally be suspected of being most prone to rollover.

2) The general level of tire and suspension roll stiffness. As tires and suspensions become stiffer, increasing the overall roll stiffness of the vehicle, the vehicle will roll less for a given level of lateral acceleration. As a result, the outward shift of the c.g. in a turn is reduced, causing an improvement in the roll stability limit of the vehicle.

Roll stiffness of typical steel spring suspensions can be increased by increasing the basic spring stiffness or increasing the lateral spacing between springs. The presence of freeplay, or lash, at the spring hangers of many suspensions decreases the overall roll stiffness of a suspension. Eliminating lash can, therefore, improve roll stability.

The suspension stiffness of concern is roll stiffness, not vertical stiffness. Many air suspensions, which have a very soft vertical rate, nonetheless are very stiff in roll due to substantial "auxiliary roll stiffness." (In some cases, this stiffness is provided by the rigid attachment of trailing arms to the axle.) Auxiliary roll stiffness mechanisms could be applied to many suspension types, thereby improving the roll stability limit. Some measured values of suspension roll stiffness are shown in Table 1. The roll stiffnesses of special suspensions can vary greatly from the values shown.

Overall roll stiffness is a function of both tire and suspension stiffness. Increasing tire stiffness (e.g., by employing maximum inflation pressures) can help improve roll stability. However, since tires typically contribute only 1/3 of the offending roll compliance, increasing tire

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\*For example, the typical five-axle, van, tractor-semitrailer combination may have a sprung mass c.g. height ranging from about 50 inches when empty, to in excess of 80 inches when fully loaded to a simultaneous "bulk-out" and "cube-out" condition. The typical five-axle MC306 tractor-semitrailer tanker has a c.g. height in the 75 to 80 inch range when fully loaded.

Table 1. Range of Measured Suspension Roll Stiffness

<u>Suspension Type</u>	<u>Roll Stiffness Range* (in-lb/deg)**</u>
Leaf-Spring Front Suspensions	25,000-36,000
Single-Axle Leaf-Spring Rear Suspensions (Tractor)	92,000
Four-Leaf-Spring Tandem Suspension (Tractor)	73,000-132,000
Four-Leaf-Spring Tandem Suspension (Trailer)	115,000-225,000
Leaf-Spring Walking-Beam Tandem Suspension (Tractor)	112,000
Rubber Block Walking-Beam Tandem Suspension (Trailer)	91,000
Tandem Air Suspensions	35,000-100,000
Tandem Torsion Bar Suspensions	62,000

\*For tandem suspensions, roll stiffnesses are on an average per axle basis, measured at 32,000 lb vertical suspension load. Front suspension rates were measured at a 12,000 lb axle load. Single-axle rear suspension rate was measured at 19,000 lb axle load.

\*\*1.0 Nm = 8.85 in-lb

1.0 N = 0.2248 lb

stiffness is not as powerful a mechanism for improving stability as that achieved through increasing suspension roll stiffness.

Additional roll stability can be provided by adding suspensions to the vehicle. More suspensions, of course, provide more total roll stiffness. However, the benefit of this additional stiffness is lost if the weight of the vehicle is increased proportionally (through added cargo).

3) The location of suspension roll centers. When a vehicle rolls in a turn, the body rotates in roll relative to the axles. The center of this rotation is defined by suspension geometry and is called the suspension roll center. (For typical leaf-spring suspensions, roll center height is near the height of the main leaf.) If the roll centers of a vehicle can be made to increase in height, the lateral shift of the c.g. for a given level of body roll is decreased. Thus, raising roll center heights generally improves the roll stability limit.

4) The distribution of roll stiffness among suspensions. Not only is the general level of roll stiffness important to roll stability, but the distribution of roll stiffness among suspensions is also important. In general terms, the optimum situation for roll stability is obtained when suspension roll stiffness is distributed in the same proportion as load. Accordingly, increasing the roll stiffness of the relatively softest suspension of a vehicle is most effective toward improving roll stiffness. Changes in the stiffest suspensions can be expected to be less effective. If the vehicle has more than two suspensions, changes in the stiffest suspension can be expected to be nearly insignificant.

This general rule is somewhat modified by the position of the suspensions along the length of the vehicle. Generally, suspensions near the center of the vehicle are less effective than those near either the front or rear. This implies that suspensions near the center need to be stiffer to do their "fair share." However, the importance of suspension location declines as speed increases. At highway speeds, there is very little effect. At "in-town" speeds, the effect can be significant.

Each of the four items discussed has an important influence in determining a commercial vehicle's rollover limit, but each is also subject

to practical design considerations which may limit the designer's ability to affect advantageous values of these parameters. In regard to the ratio of track width to c.g. height, c.g. height is often more under the influence of the vehicle user than the designer, and track width is generally limited by law to 96 inches (2.44 m) overall. Within this limit, nominal track width can be increased by the choice of wide-base singles to replace dual tires, but in general, this gain in track width is offset by reduced tire vertical stiffness. However, in those cases where legal width extends beyond 96 inches, full advantage should be taken by increasing track width accordingly.

The roll stiffness of commercially available suspensions varies greatly. Choosing suspensions which are stiff in roll and have minimal spring lash can have a significant effect on roll stability. Adding auxiliary roll stiffness to existing suspensions would also appear reasonable. Users, as well as designers, can play a role in assuring good distribution of roll stiffness among suspensions of a vehicle.

Roll center heights of commercial vehicle suspensions vary considerably. The more popular types of tandem suspensions, for example, may have roll center heights ranging from the low 20's (in inches) to the low 30's (1 m = 39.37 in). Moreover, current designs would indicate that the importance of roll center height is not broadly recognized. Reasonable modifications to suspension designs might be expected to move roll center heights into the upper 30's.

Roll stiffness distribution as well as the relative roll center height of the several suspensions of a vehicle affects the distribution of roll moment reactions and, thus, tire loading. Accordingly, these parameters influence yaw stability as well as roll stability. Yaw stability properties must, of course, be considered in the design process.

## 2.2 Parameter Sensitivities Affecting Yaw Stability Limits

The computer-based results discussed in Chapter 5 suggest that certain heavy trucks, characterized primarily by high centers of gravity, can develop yaw divergence instabilities (that is, a "spinout") during relatively moderate turning maneuvers. Increasing speed aggravates the problem,

but in some cases, the vehicle may become unstable in yaw at speeds as low as 25 mph (40 k/hr). The nature of yaw instability for heavy trucks can be described as a slow, continuous build-up of lateral acceleration by the vehicle for a fixed steering-wheel input. That is, the vehicle will not turn on a circle-like radius, but rather, turn on a tighter and tighter spiral. This tighter turning behavior leads to higher lateral acceleration levels and eventual rollover of the vehicle. A simplified analysis, directed toward identifying the sensitivity of yaw stability thresholds to typical vehicle parameter variations, was shown to predict reasonable results when compared to results of a more comprehensive computer simulation study. A more thorough examination of the dynamic behavior of these vehicles during steady turning was conducted with the use of comprehensive computer models used for simulating vehicle-driver-roadway interactions.

Specific conclusions, applicable to the class of high-center-of-gravity trucks examined here, and based upon the results presented in Chapter 5, are:

- 1) Yaw instability can occur with such vehicles during moderate turning maneuvers (0.2-0.3 g's) while operating at highway speeds. The significance of yaw divergence during steady turning is that it will lead to rollover in the absence of corrective steering action and/or reduced speed.

- 2) The principal mechanism responsible for yaw instability during moderate turning maneuvers at highway speeds is the nonlinear manner in which truck tires produce cornering forces for different vertical loads. As truck tires become more heavily loaded, their ability to produce lateral force, in proportion to the load they carry, is diminished. Hence, during steady-turning maneuvers, as load is transferred from the inside to the outside tires of a vehicle, a net loss of tire cornering force occurs on each axle. Because heavy trucks transfer much greater load across the rear suspension, the greatest losses in tire cornering forces occur at the rear axles. The net result is a tendency for the vehicle to lose its directional stability ("spin out") as greater and greater load transfer occurs.

- 3) Vehicle yaw divergence behavior may be stabilized by corrective steering actions of drivers. Whereas a vehicle may tend to become



directionally unstable as a result of its own properties, it may be stabilized to some extent by the corrective steering control of drivers. However, as vehicles exhibit greater levels of instability (vehicle-alone), the ability of drivers to stabilize them becomes an ever-increasing challenge—leading at some point to loss of control. An additional corrective action that can be used by drivers is simply to reduce speed. Since yaw stability is strongly dependent upon vehicle speed, slowing down during corrective steering should be the best countermeasure available to drivers.

4) The presence of superelevation in highway curves acts not only to contribute roll stabilization to such vehicles, but is also a particularly powerful means for reducing the likelihood of yaw divergence. Superelevation of highway curves produces reduced load transfer levels during steady turning, thereby leading to improved roll and yaw stability. The "ideal" level of superelevation for minimizing the occurrence of yaw divergence is that amount which produces no load transfer across the vehicle suspension. Since this level of superelevation, of course, depends upon the actual operating speed through the curve, no single value of "ideal" superelevation for a given curve is possible. In general, though, the amount of roadway superelevation typically specified by AASHTO highway curve design practice does greatly reduce the propensity of such trucks to develop yaw instabilities.

5) Reasonable vehicle-related modifications which could be performed to increase the yaw/roll stability of these vehicles are: (a) improvement in fore/aft roll stiffness distribution, (b) use of additional tires or axles (non-steering) at the rear of the vehicle, (c) lowering of the center of gravity, and (d) selection of rear tires with more linear-like variation of cornering stiffness with vertical load.

6) Vehicle parameters found to have the greatest influence upon the development of yaw divergence in the straight truck vehicle class examined here are:

a) Rear tire cornering stiffness variation with vertical load.

A tire which exhibits greater curvature, than a similar tire, in its cornering stiffness versus load plot (see Figure 38), will, in general, be a more likely contributor to vehicle yaw divergence.

- b) Center of gravity height. Vehicles possessing greater c.g. heights, in general, transfer more load side to side during cornering. Greater load transfer levels across an axle exaggerate the net loss of cornering stiffness (see Item (a) and Appendix A).
- c) Fore/aft roll stiffness distribution. The large differences in the roll stiffness of front and rear suspensions (front suspensions being softer) which are characteristic of the heavy truck, promote proportionately greater side-to-side load transfer across the rear suspension than the front. Correspondingly, greater opportunities to suffer cornering stiffness losses therefore exist at the rear axles—leading to vehicle oversteer and directional instability at elevated speeds.
- d) Number of axles. In general, addition of nonsteerable axles at the rear of the vehicle contributes to the directional and roll stability of such vehicles. (Steerable tag axles with freedom to steer through castering should be avoided since they produce no lateral force contributing to the yaw stability of the vehicle.) Additional rear axles: (i) increase total roll stiffness of the vehicle thereby reducing vehicle roll and rear-end (and total vehicle) load transfer, and (ii) increase immunity to yaw instability by producing less load transfer per rear tire as well.
- e) Wheelbase length. Wheelbase length,  $\ell$ , has a theoretical  $\sqrt{\ell}$  influence on the variation of critical velocity for an oversteer vehicle. That is, doubling a vehicle's wheelbase length for the same oversteer condition will raise the maximum, stable operating velocity by a factor of 1.4.

Generally, it is not possible to assign a fixed order of importance to these individual parameters. The nature of yaw divergence is such that a combination of various parameter conditions must exist simultaneously in order for it to develop. This synergistic quality of parametric interactions that surrounds the issue of "important parameters related to yaw divergence" is worth emphasizing and is discussed in some detail in Chapter 5.

### 2.3 Parameter Sensitivities Affecting Rearward Amplification

In sudden lateral displacement (obstacle-avoidance) maneuvers, the directional responses of certain articulated vehicles exhibit large amounts of rearward amplification; that is, the lateral acceleration of the last unit in the combination vehicle is a considerably amplified version of the lateral acceleration of the leading unit. Large amounts of rearward amplification are undesirable because (1) the last unit's path may extend well outside of the path of the first unit and (2) the lateral acceleration experienced by the last unit may be high enough to cause it to roll over, prematurely. If a vehicle has a large amount of rearward amplification, the driver may be able to steer the lead unit around an immediate obstacle without approaching the rollover limit of the lead unit, but the trailing units may (a) swing out of the path of the first unit thereby going off the road or striking other vehicles and/or (b) roll over due to the high lateral acceleration generated during the "correction phase" of the obstacle-avoidance maneuver (i.e., when the last unit is attempting to return to the original direction of travel after the obstacle has been avoided).

The rearward amplification phenomenon is at its worst when the following operating conditions prevail: (1) the vehicle is traveling at highway speeds (the faster the speed, the higher the amplification factor); (2) the vehicle is fully loaded (reasons pertaining to both rollover and directional response apply here); and (3) the steering activity required to avoid an obstacle or make a path correction contains a rapid reversal or rapid reversals of the steering-wheel angle.

Commercial vehicles that are likely to have high levels of rearward amplification possess some or all of the following parametric properties:

-The distance,  $x_{pc}$ , from the center of gravity (c.g.) of each towing unit to its pintle hitch, connecting the towing unit to the unit being towed, is large. (Towing units include not only tractors, but also straight trucks and, in doubles and triples combinations, semitrailers and full trailers. See Figure 2.)

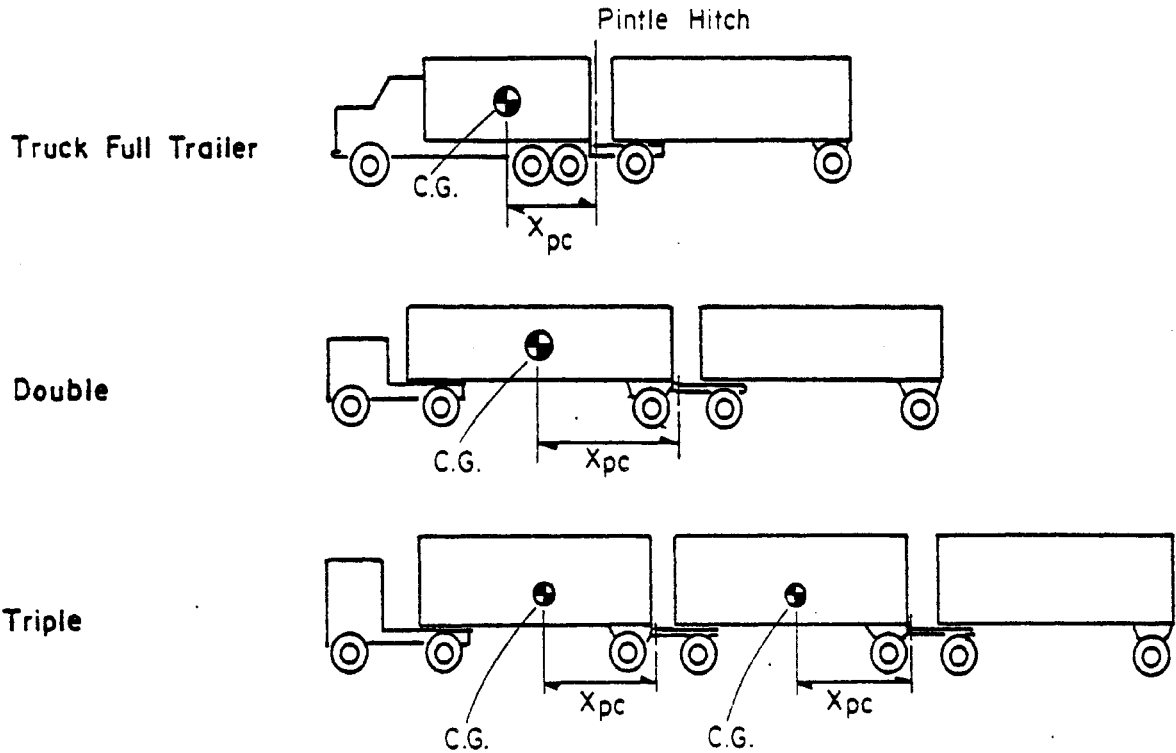


Figure 2. Diagram showing  $x_{pc}$ , the distances from the c.g.'s to the pintle hitches.

-The towing unit's design parameters are such that the vehicle will respond in yaw rotation to a much greater extent than it will translate laterally when performing a sudden lateral displacement maneuver. The specific combinations of parameters that contribute to this type of situation are:

- 1) The ratio of the total cornering stiffness of all the towing unit's tires to the weight (mass) of the towing unit is less than average. (That is, the "cornering coefficient" is less than that typically used. For example, the cornering stiffness of a conventional 10x20 bias-ply truck tire with rib tread is approximately 500 lbs/deg (2224 N/deg) when loaded to 4000 lbs (17,800 N), yielding a typical cornering coefficient equal to  $(500)/(4000) = 0.125 \text{ deg}^{-1}$ .) Presuming that standard tires are used, heavily loaded vehicles are worse off than lightly loaded vehicles.
- 2) The so-called "damping-in-yaw" (i.e., the influence of axle locations,  $x_i$ , and tire stiffnesses,  $C_{\alpha_i}$ , as expressed in an expression of the form  $\sum_i x_i^2 C_{\alpha_i}$  where  $x_i$  is the distance from the c.g. to the i-th axle) is small. Presuming again that standard tires are used, short-wheelbase towing vehicles will be worse off than long-wheelbase vehicles (e.g., 27-foot (8.2-m) trailers versus 45-foot (13.7-m) trailers as towing units in doubles combinations).
- 3) Although not as important as items (1) and (2) in this list, large amounts of overhang of the load beyond the wheelbase of the vehicle and relatively long distances from the c.g. location to the front axle also contribute to exaggerated yaw responses.

-The fundamental parameters having a first-order influence on the magnitude of the amplification factors pertaining to full trailers are:

$\frac{\Sigma C_{\alpha}}{W_T}$ , the total cornering coefficient for the trailer (i.e., the ratio of the sum of all of the cornering stiffnesses divided by the weight of the trailer), and

$x_{BA} + x_{BT}$ , where  $x_{BA}$  is the distance from the turntable to the pintle hitch on the dolly and  $x_{BT}$  is the distance from the c.g. of the full trailer to the turntable or fifth-wheel pivot.

Full trailers that are short, heavily loaded, and pulled by short drawbars or tongues (i.e.,  $x_{BA}$  is small) are likely to have amplification factors that make a significant contribution to the overall rearward amplification of a combination vehicle.

The overall rearward amplification factors for truck-full trailers, doubles, and triples combinations may be predicted (estimated) by multiplying the individual amplification factors corresponding to the properties of all of the individual units comprising these multiply-articulated vehicles. Hence, the parameter sensitivities, described herein before, for each unit, apply directly to the total combination vehicle. Nevertheless, towing and towed units that have similar design parameters can combine to make either very good overall combinations or very poor combinations. For example, if a triple is comprised of nearly identical trailers and all of these trailers have large amplification factors for both towing and being towed, the overall rearward amplification can be exceedingly large (on the order of 3 to 4). On the other hand, the theoretical results presented in Chapter 6 indicate that double drawbar arrangements in which (1)  $x_{BA}$  (the tongue length) is effectively increased and (2)  $x_{pc}$  (the distance from the c.g. of the towing unit to its pintle hitch) is virtually reduced to zero would be very effective for reducing the rearward amplification of conventional multiply-articulated vehicles.

## CHAPTER 3

### SAMPLE VEHICLE SELECTION

As indicated in Chapter 1, a selected set of commercial vehicles was to be used to demonstrate the parametric sensitivities of commercial vehicle performance with respect to (1) divergent roll response, (2) divergent yaw response, and (3) lightly damped, oscillatory yaw response. The basic premise for selecting the sample vehicles from the U.S. fleet was that each could reasonably be suspected of being unusually susceptible to one or more of these three performance limits.

In order to identify the sample vehicle set, the first task of the study included a survey effort in which a large number of individuals and organizations across the country who are associated with the U.S. trucking industry or the study of highway safety were contacted, and asked to identify heavy vehicles particularly subject to unstable performance. Their responses, an evaluation of formal accident data sets, and information gathered on a field trip, along with engineering judgment based on a fundamental understanding of heavy vehicle dynamics, were used to establish the sample vehicle set for the study [1].

Organizations contacted in this effort included:

- police, public safety, or transportation agencies of each of the 50 states
- 50 major U.S. trucking companies
- the Brotherhood of Teamsters
- member companies of the Truck Trailer Manufacturers Association
- the Motor Vehicle Manufacturers Association

Responses were received from 29 states, four trucking companies, and one trailer manufacturer. Two responses were received from the Teamsters.

By far, the largest area of concern indicated by responses received was vehicle rollover. A variety of high center of gravity vehicles were identified in this regard. High c.g. vehicles identified included (1) liquid and dry bulk tanker vehicles of tractor-semitrailer, truck-full trailer, and doubles configurations, (2) trucks and tractor-trailer combinations hauling shifting loads, including fluids, and swinging meat, (3) mobile homes in tow, and (4) special-purpose vehicles, including dump trucks and tractor-semitrailers, ready-mix cement trucks\* and garbage packers. The other major classification of vehicles identified was characterized by either multiple or unusual yaw articulation joints. Roll-over or yaw instabilities were associated with these vehicles. This group included doubles configurations, truck-full trailer configurations, dump trucks hauling pintle hitch semitrailers, tractor-semitrailer car haulers using "stinger" fifth wheels, and truck-semitrailer "dromedary" configurations.

BMCS accident file data were also examined to assist in the identification of sample vehicles for study. The data involved are all related to single-vehicle, heavy-vehicle accidents. BMCS files provide information on the type of accident and also indicate, in a limited way, the type of vehicle and cargo involved. Since interest lies in vehicle instability and handling problems, emphasis was placed on accident characterized as either jackknife or overturn and, within these categories, the purpose was to identify over-involvement of vehicles of any general vocational type.

The presentations of Figures 3 and 4 derive from the BMCS data files aggregated for the years 1976, 1977, and 1978. Figure 3 deals with rollover accident data for those years, and Figure 4 similarly deals with jackknife accident data.

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\*One cement truck of interest employed a rear-mounted, freely castering, air-tag axle.



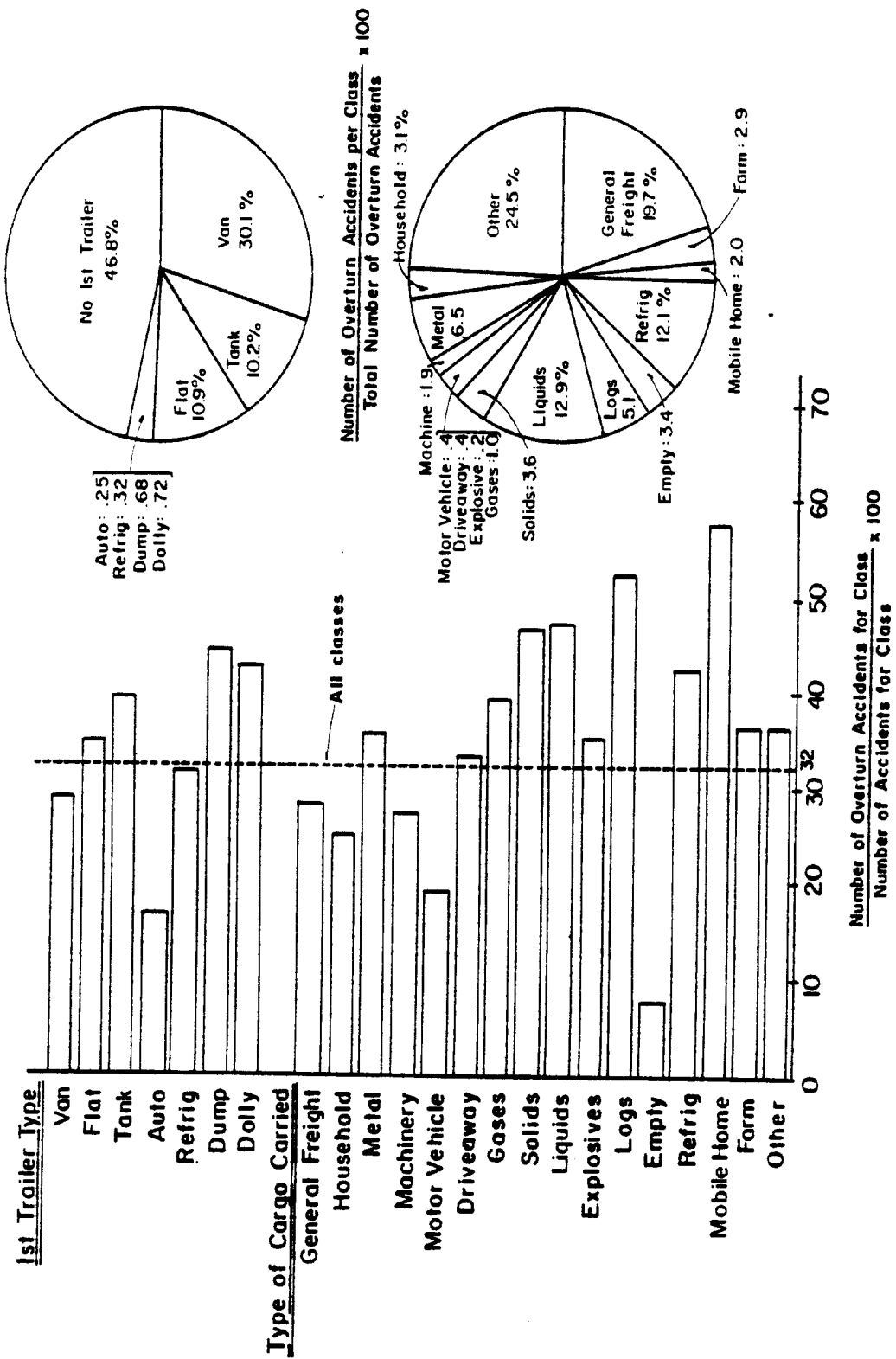


Figure 3. Distribution of overturn accidents.

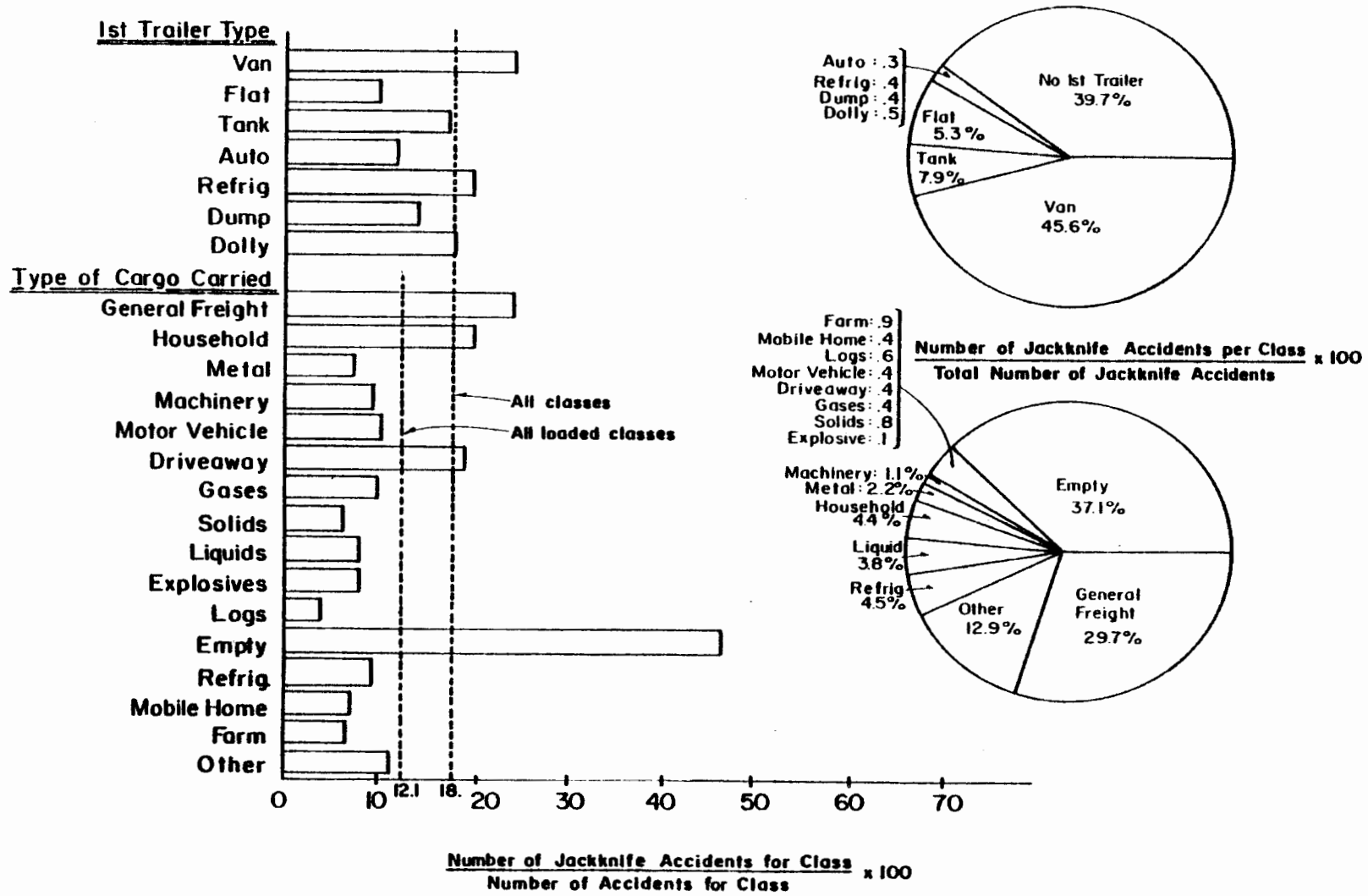


Figure 4. Distribution of jackknife accidents.

Each figure presents two bar graphs and two pie charts. The bar graphs indicate the number of subject accident types (rollover or jackknife) as a percentage of the total number of accidents for the specific vehicle or cargo class; that is, for example:

$$\frac{\text{Number of van rollovers}}{\text{Number of van accidents}} \times 100$$

Each bar graph has a vertical reference line indicating the same parameter (albeit for the entire population), that is,

$$\frac{\text{Total number of rollovers}}{\text{Total number of accidents}} \times 100$$

The pie charts indicate the number of accidents for all types for the specific class as a percentage of the accident population; for example,

$$\frac{\text{Number of van accidents}}{\text{Total number of accidents}} \times 100$$

Graphs are presented according to first trailer body type and according to type of cargo carried.

The BMCS data presented in these figures tends to confirm the various positions of the survey. Averaged over the three years examined, the data indicate that 32 percent\* of the heavy-vehicle accidents are classified as as rollovers, while 18 percent are jackknife, supporting the survey result

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\*Ongoing analysis of the BMCS data indicate that this figure might be substantially higher. The BMCS coding system demands that an accident be recorded in only one accident class (e.g., an accident which involved a jackknife and rollover must be classified as one or the other, but not both). Study of the more details accident reports shows that 1/2 to 3/4 of the single-vehicle accidents coded as "ran off road" also involve rollover. The "ran off road" category accounts for approximately 1/4 of the coded, single-vehicle accidents.

emphasizing rollover. (As a point of reference, similar rollover data reported for passenger cars range from 7 percent to 14 percent [2].)

Regarding rollover (Fig. 3), it can also be seen that the BMCS data tend to support the survey results with respect to specific vehicle properties. By body type, the graph shows that high c.g. tank and dump types and multiply-articulated dolly types are over-involved in rollovers. The majority of cargo types which are over-involved are easily identified as associated with high c.g. vehicles (metal coils, gases, liquids, solids, logs, and mobile homes).

The BMCS data of Figure 4 show jackknife to be, in large measure, an empty-vehicle problem. The mechanics of heavy vehicle systems would then imply that jackknife is largely a braking performance problem, not a handling problem (and consequently, beyond the scope of this study). Referring to the "1st Trailer" data of the figure, vans and refrigerated trailers are over-represented. The "Cargo" data are more revealing. Here, empty vehicles are very strongly over-represented relative to all other classes. In order to better evaluate the other cargo class, a second vertical reference line dealing only with loaded vehicles (i.e., neglecting the "empty" data) has been added to the graph. The line shows the value of:

$$\frac{\text{Total loaded vehicle jackknife}}{\text{Total loaded vehicle accidents}} \times 100\%$$

Relative to this reference, the categories of General Freight, Household, and Drive Away are over-represented. Presumably all of these categories are dominated by rather typical tractor/van-semitrailer combinations and therefore yield no indication that "special" vehicles are particularly prone to jackknife.

The information derived from the survey responses and accident data files becomes more meaningful when merged with a basic understanding of commercial vehicle dynamics. With respect to roll stability, per se, the basic determinant of the roll stability limit of any conventional highway vehicle is the ratio of the vehicle's c.g. height to track width. Since the c.g. height of commercial vehicles generally exceeds one half of the

track width, commercial vehicles can generally be considered susceptible to rollover. Further, as c.g. height increases, roll stability of these vehicles will generally decrease. Thus, the identification of high c.g. vehicles (of a variety of vocational types) as susceptible to rollover is not surprising.

Basic principles of vehicle dynamics also identify the commercial vehicles as being susceptible to monotonically unstable yaw response (i.e., spinout). This instability is sensitive to tire properties, to the manner in which the vehicle loads its tires, and in particular, to how the vehicle transfers tire load, side-to-side, during turning. High c.g. vehicles, in general, transfer more load during a given turning maneuver, and, thus, may be considered more susceptible to this instability.

Lightly damped, oscillatory motions of trailing units is the third dynamic response to which commercial vehicles are especially subject. The number and character of yaw articulation joints is known to play an important role in determining the nature of this dynamic performance regime. Thus, it is not surprising to find multiple and unusually articulated vehicles identified by the survey and accident analysis efforts.

Finally, both areas of concern regarding yaw performance (unstable response and lightly damped oscillatory response) can serve to generate vehicle motions which ultimately challenge basic roll stability. Accordingly, accidents which are initiated by yaw response properties may culminate in rollover, and the overwhelming nature of the rollover event may serve to obscure the causative role of yaw dynamics. Thus, where the survey and accident file data indicate concern for rollover, yaw performance must also be suspected, particularly where cited vehicle properties, combined with an understanding of vehicle dynamics, point toward yaw performance problems.

Through combination of the information obtained and an understanding of commercial vehicle dynamics, and in consultation with the Contract Technical Manager, the sample vehicles indicated in Table 2 were selected. The table also indicates the undesirable performance modes which each vehicle is suspected to exhibit and the primary physical parameter which leads to this suspicion. Photographs of individual examples of the selected

Table 2. The Subject Vehicles

	Dynamic Modes of Interest			Primary Generic Quality Leading to Selection			See Figures
	Rollover Per Se	Divergent Yaw Response	Lightly Damped Yaw Oscillation	High C.G.	Unusual Articulation	Multiple Articulation	
<b>Straight Truck</b>							
Cement mixers, 4 and 5 axle (including steerable tag)	X	X		X			5, 6, 7
Dump trucks, 3 and 5 axle	X	X		X			8, 9
Refuse packer, 3-axle front loader	X	X		X			10
<b>Semitrailer Combination Vehicles</b>							
Tractor-semi, dump trailer, 6 axles	X			X			11
Dump truck with pintle hook lowboy trailer			X		X		8 & 16
Car hauler with "stinger" 5th wheel			X		X		12
California Dromedary			X		X		13
<b>Full Trailer Combination Vehicles</b>							
California truck-full trailer, 5 axles			X			X	14
Michigan-style truck-full trailer, 11 axles	X		X	X		X	15

vehicle types appear in Figures 5 through 16. The dump truck-pintle hook trailer combination was composed of the three-axle dump truck (Fig. 8) and the low-boy equipment hauler (Fig. 16). Payload for the trailer was derived from measured inertial properties of a construction tractor equipped with a front-loader and backhoe.

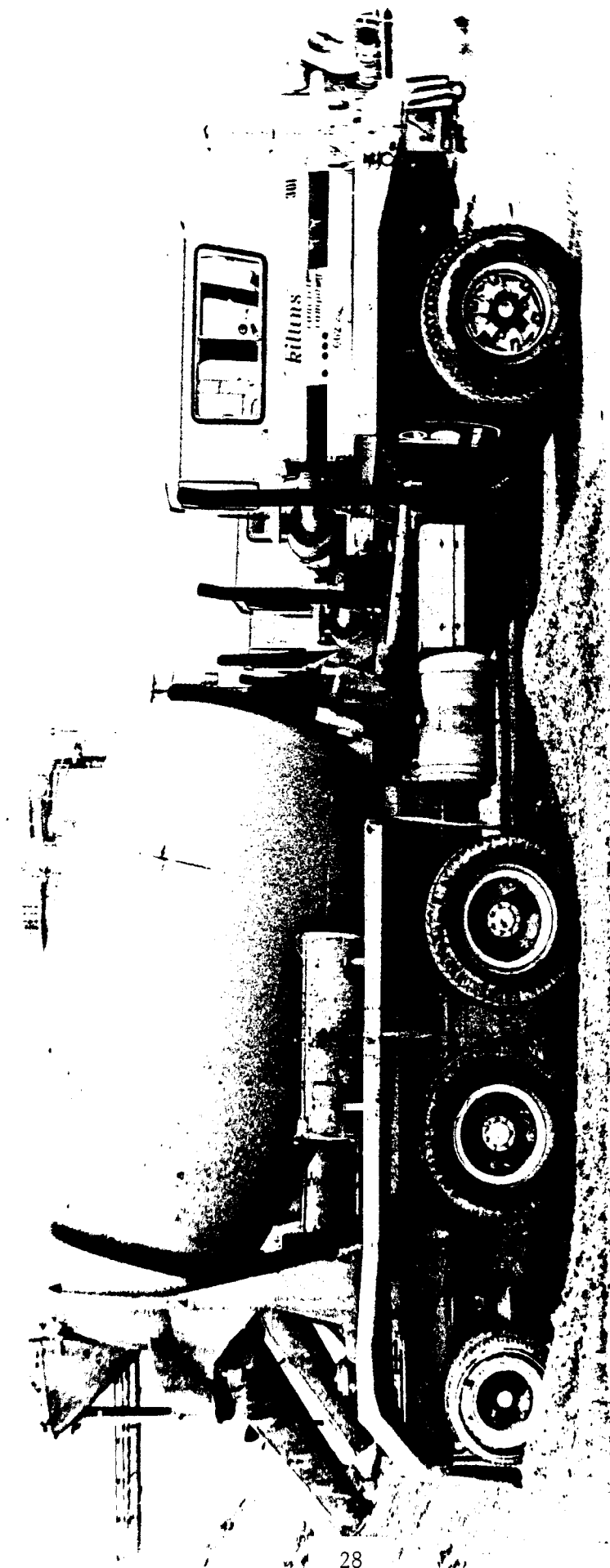


Figure 5. Four-axle ready-mix cement truck.



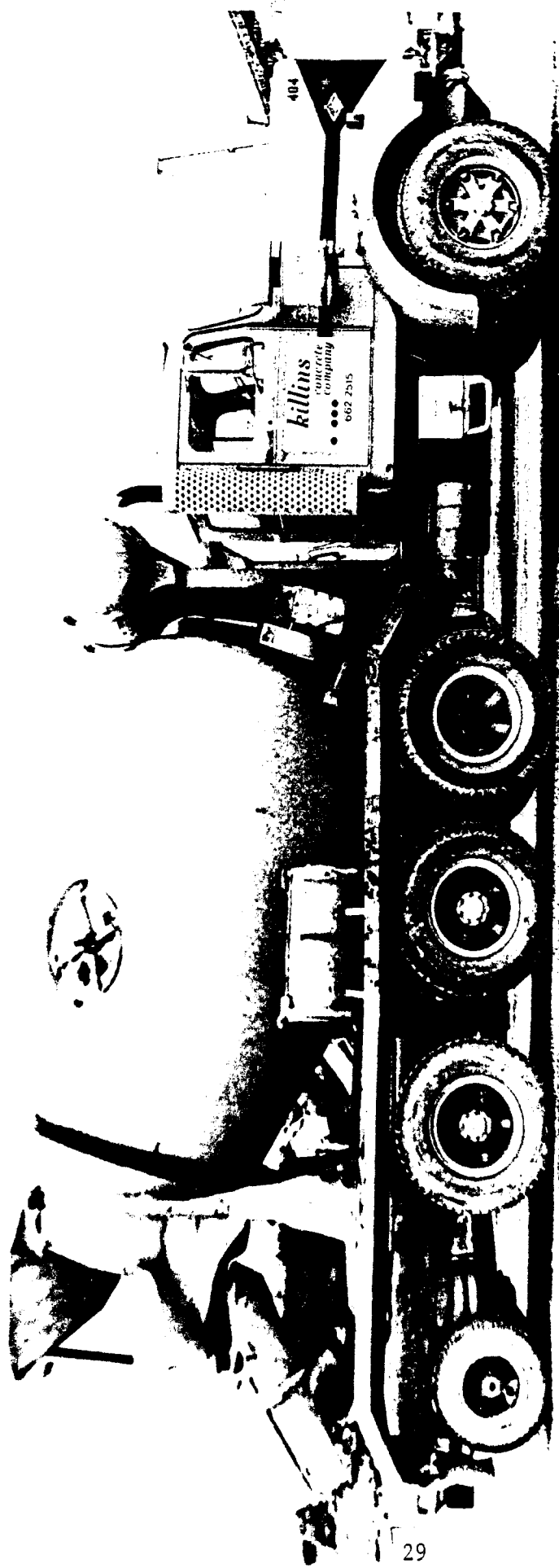


Figure 6. Five-axle ready-mix cement truck.

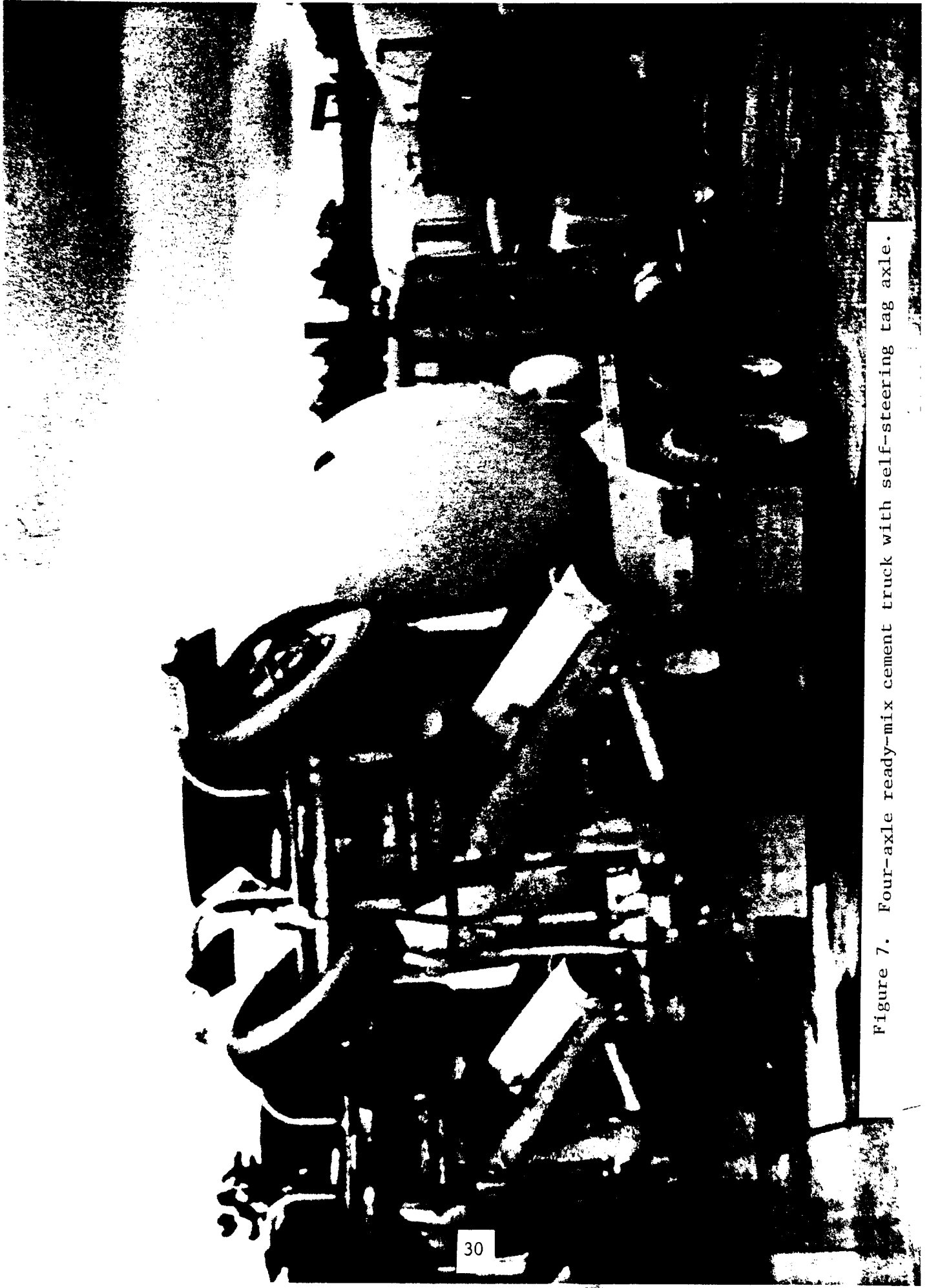


Figure 7. Four-axle ready-mix cement truck with self-steering tag axle.

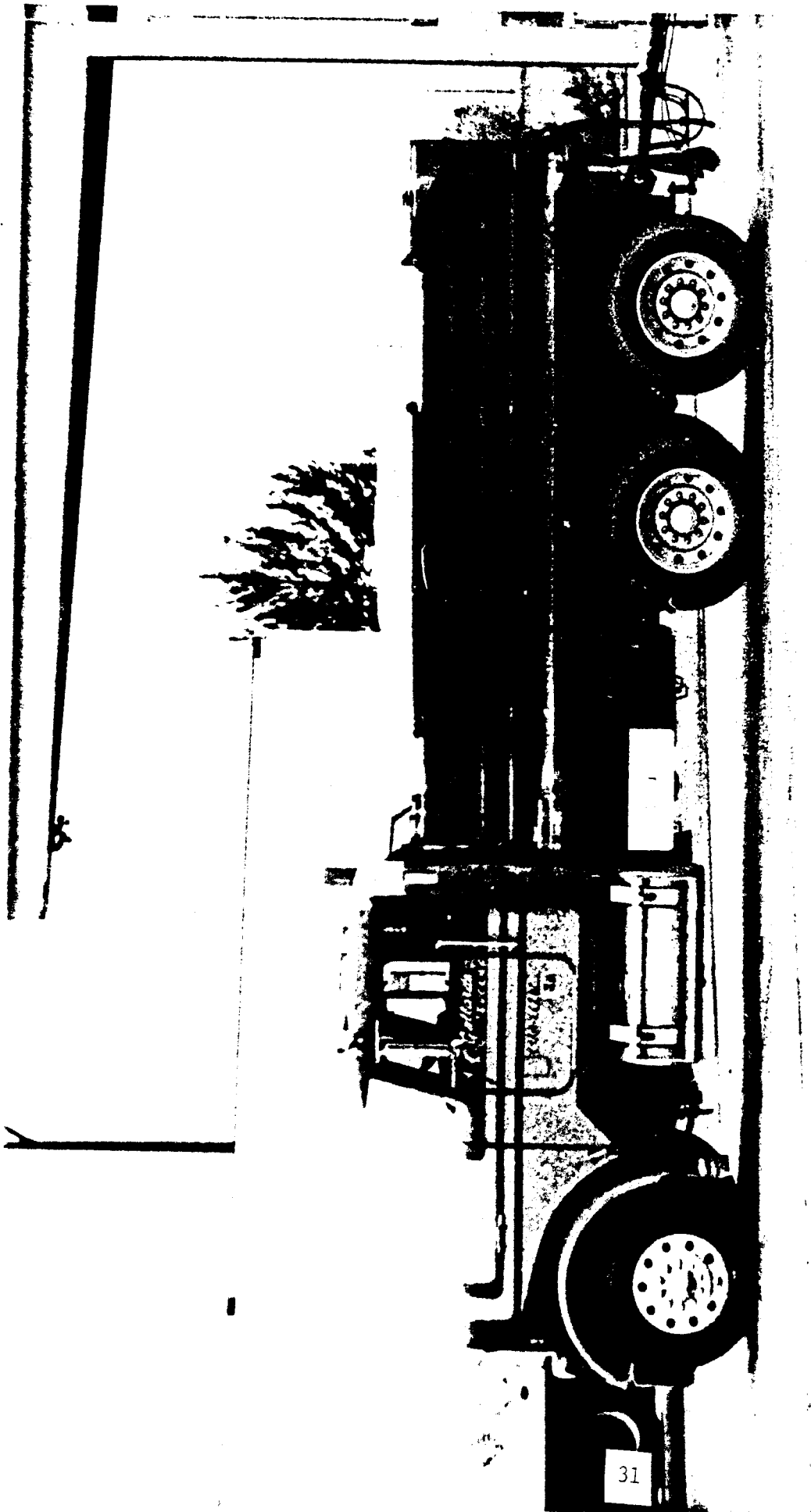


Figure 8. Three-axle dump truck.

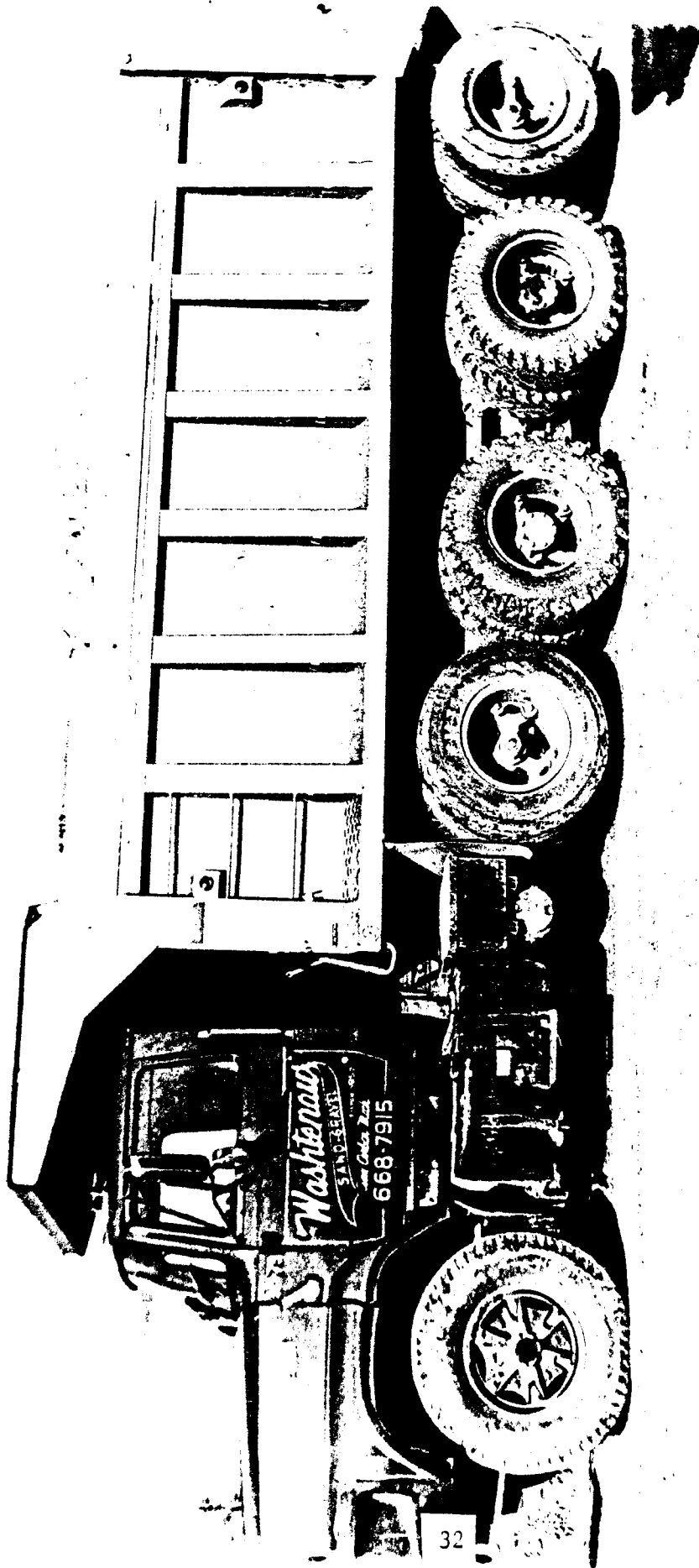


Figure 9. Five-axle dump truck.

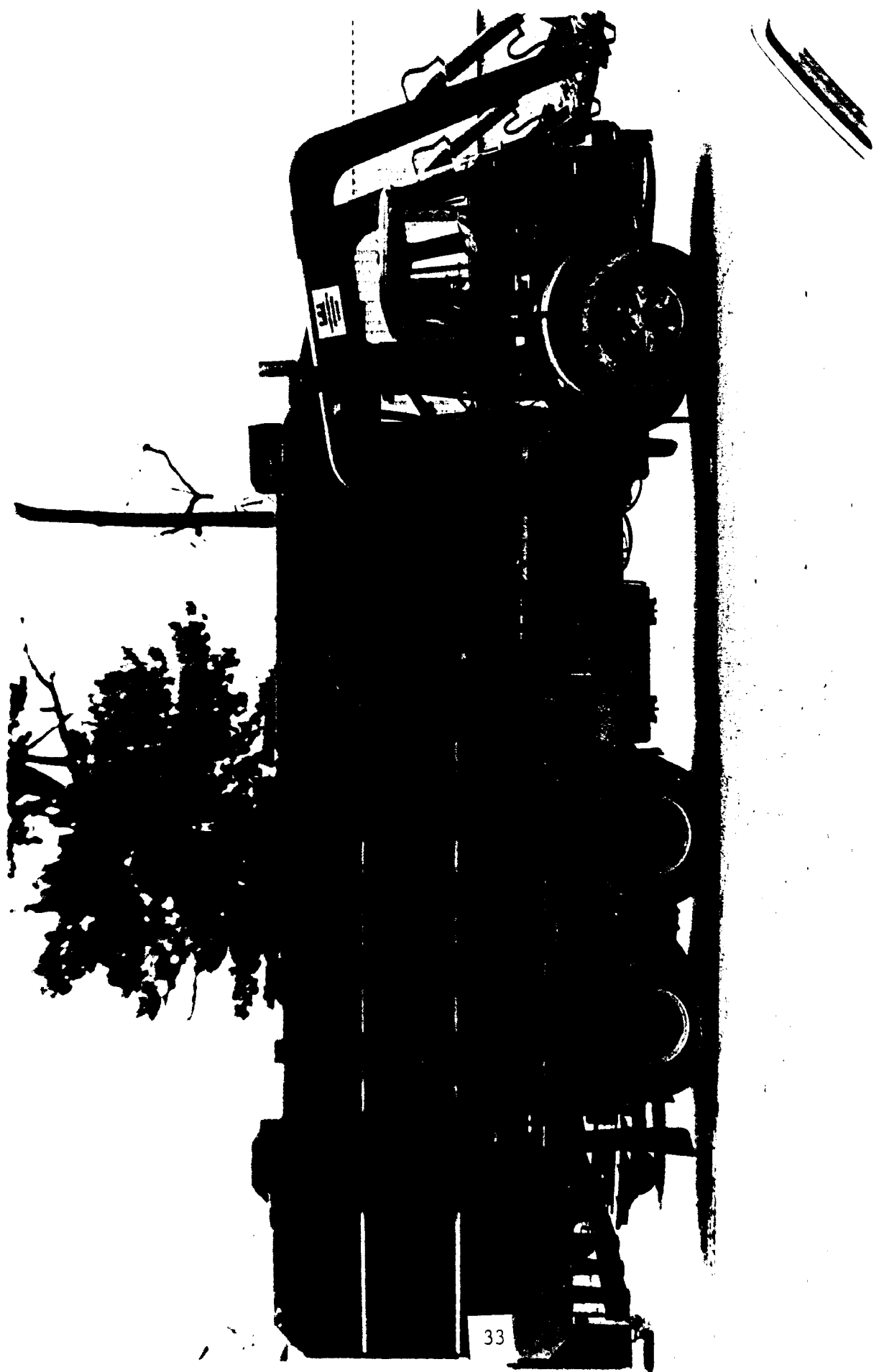


Figure 10. Three-axle refuse packer, front loader.

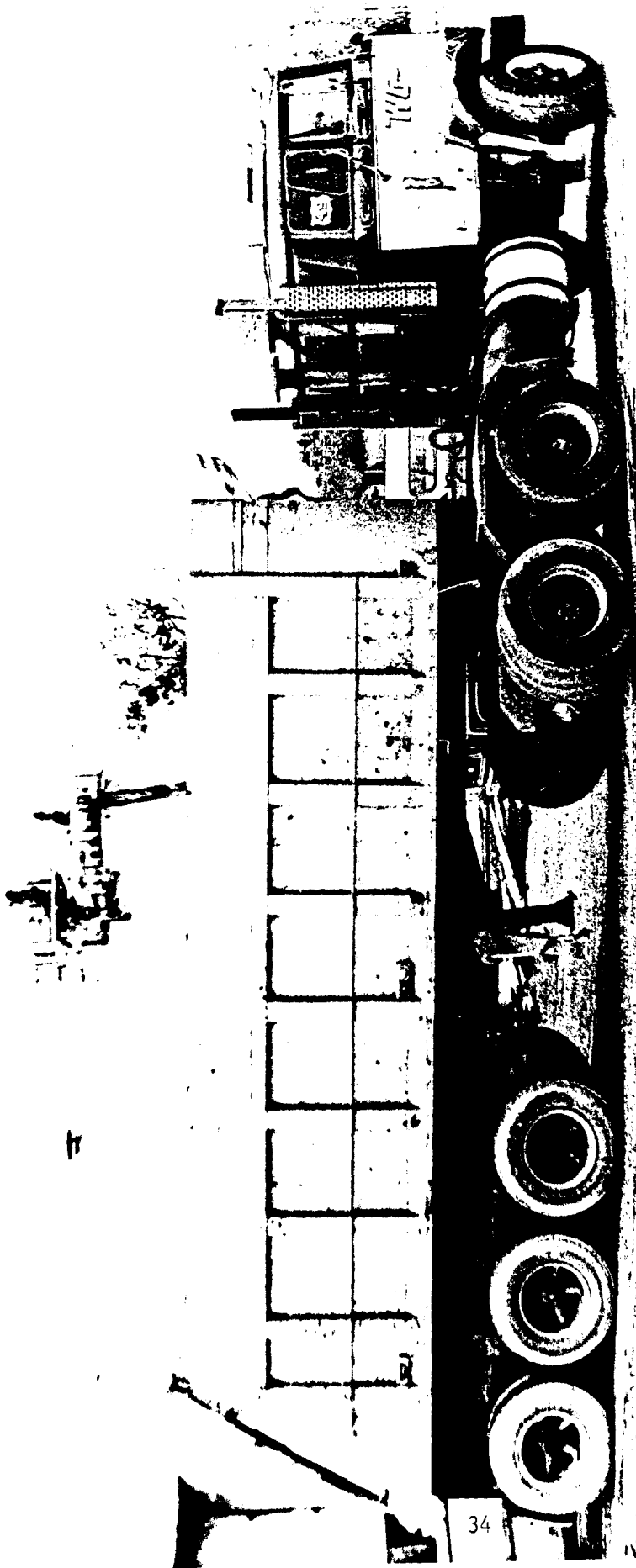
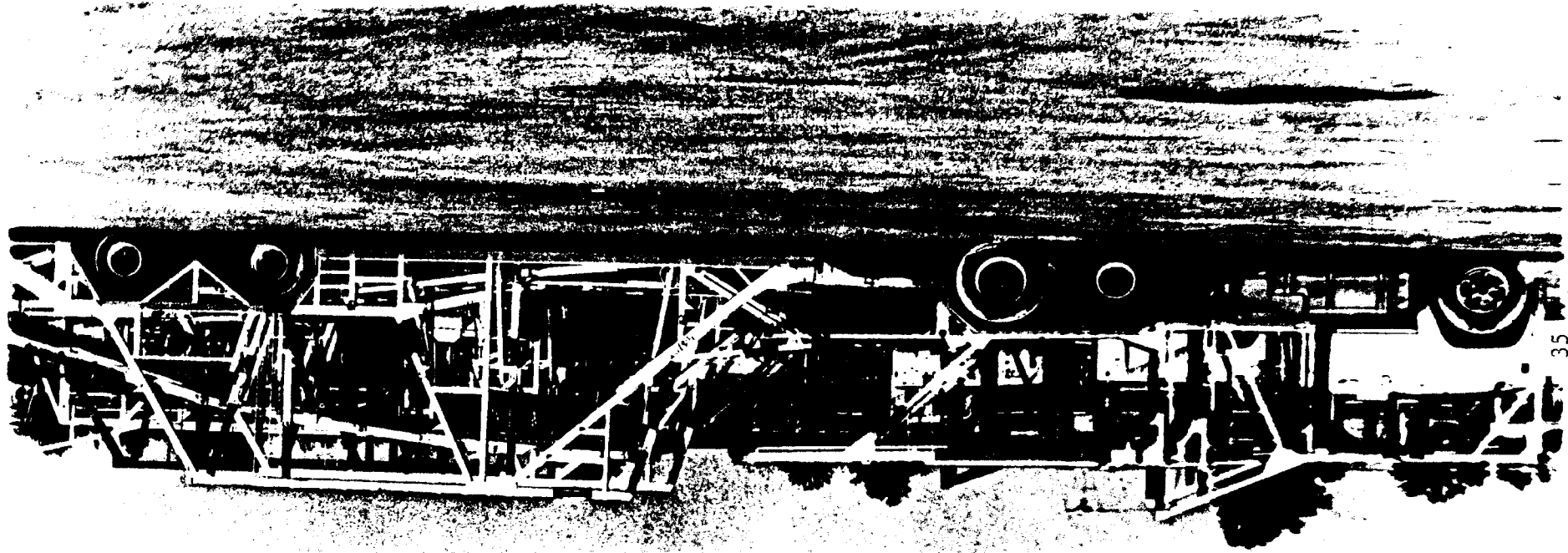


Figure 11. Six-axle tractor-semi, dump trailer.

Figure 12. Car hauler with "stinger" fifth wheel.



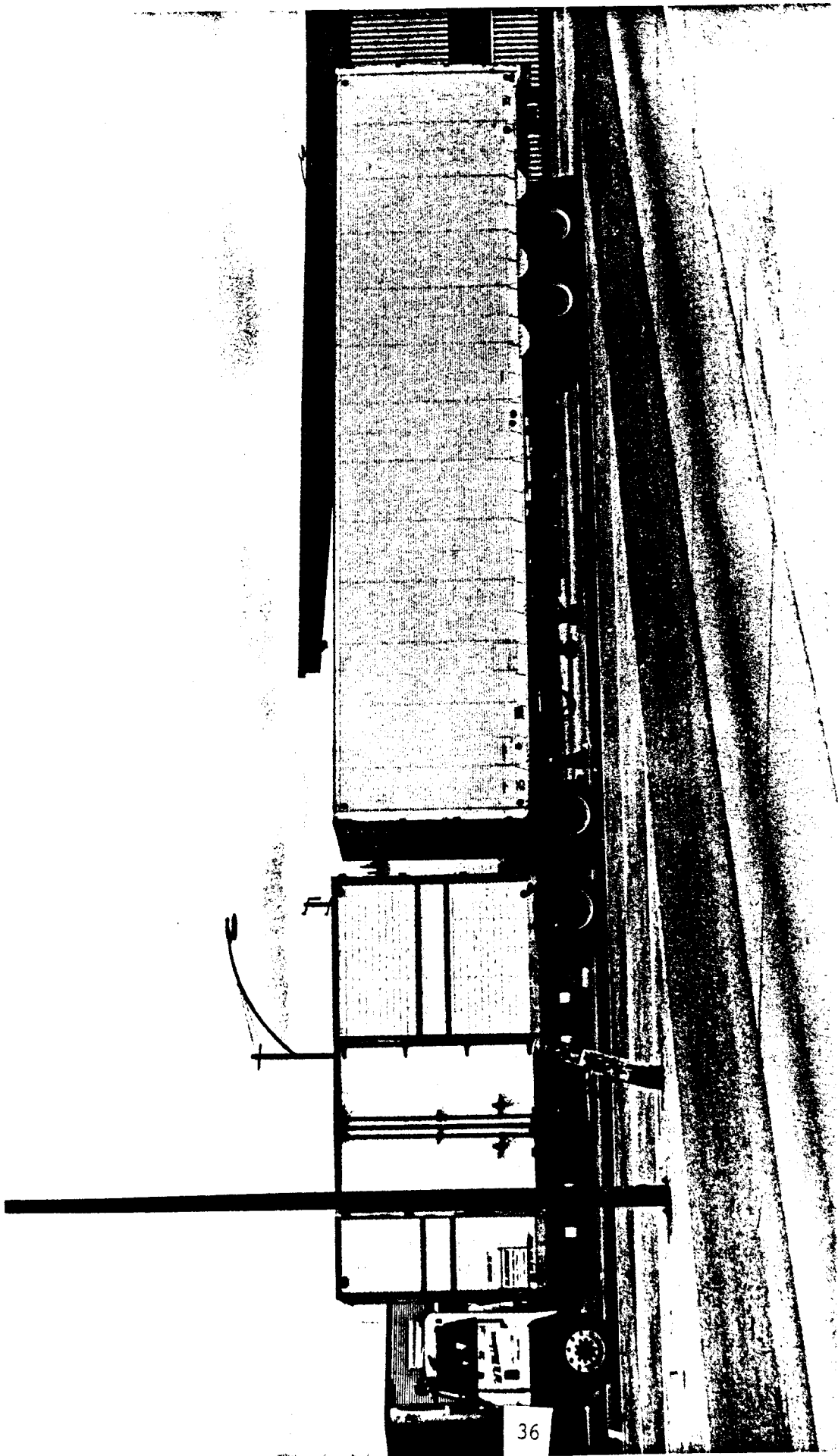


Figure 13. Dromedary truck-semitrailer.



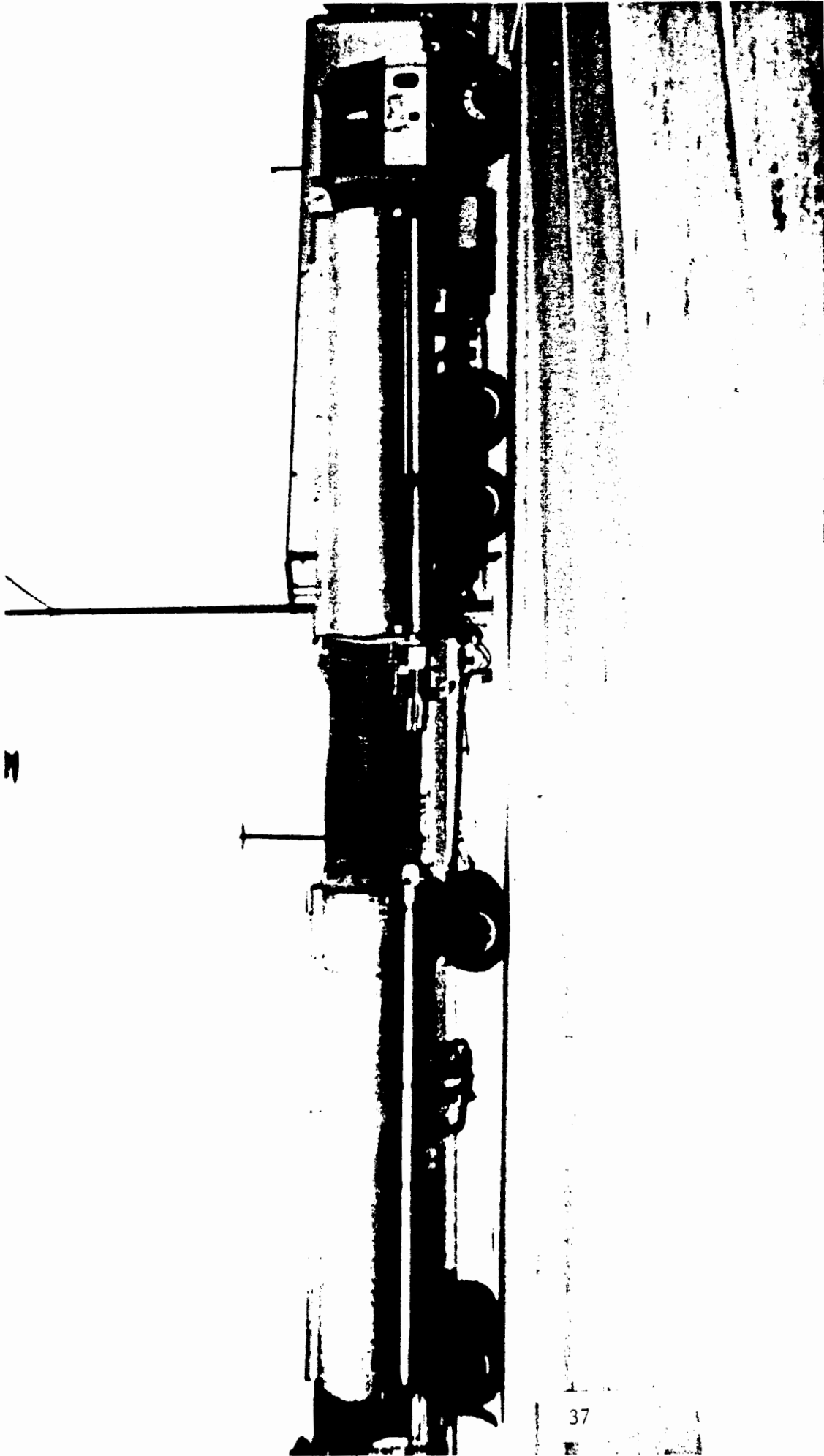


Figure 14. Five-axle truck-full trailer.



Figure 15. Eleven-axle truck-full trailer.

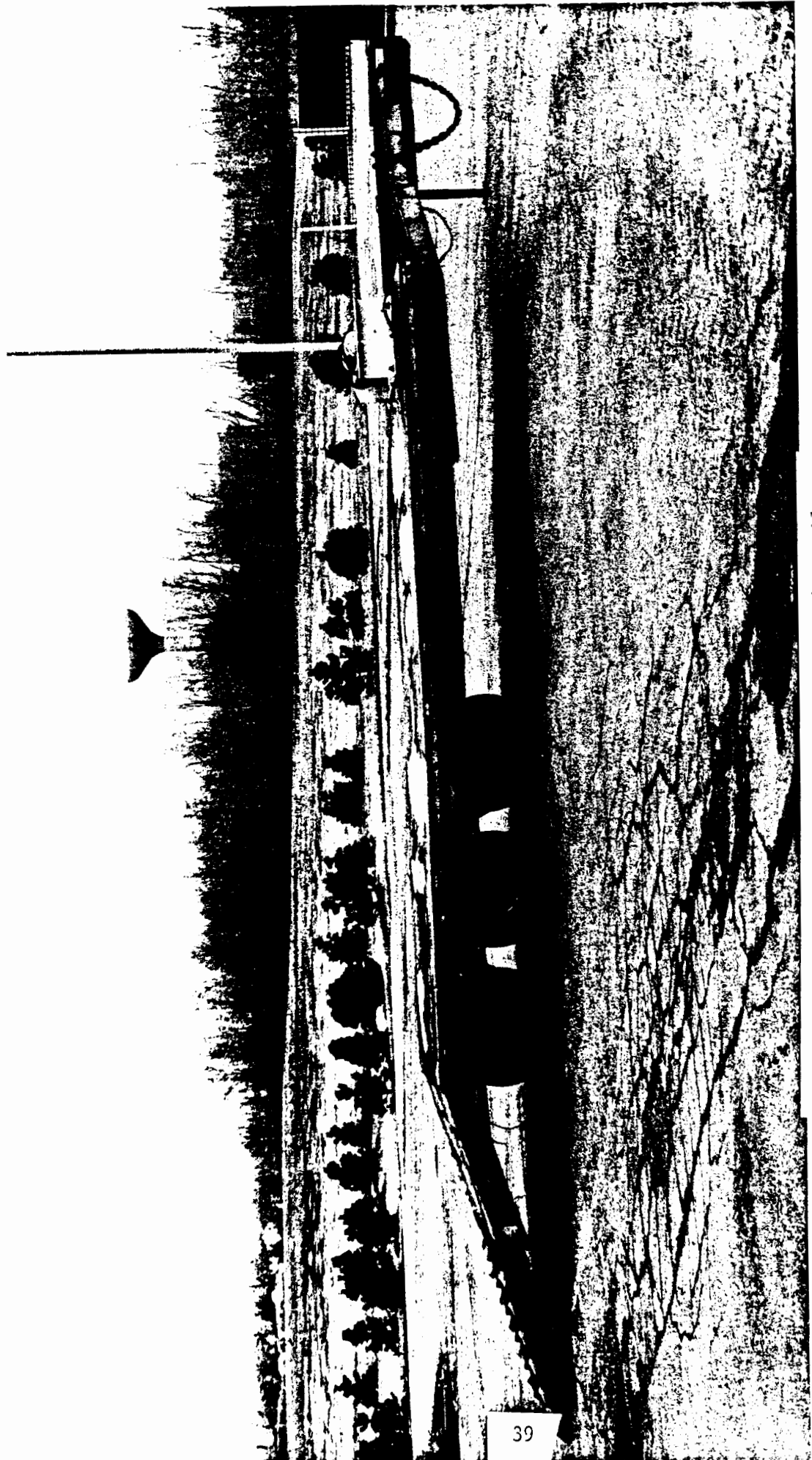


Figure 16. Low-boy, pintle hitch trailer.

## CHAPTER 4

### PARAMETRIC SENSITIVITY OF ROLLOVER LIMIT

It is the purpose of this section to discuss, in detail, the sensitivity of the rollover limit of commercial vehicles to the vehicle parameters pertinent to this limit. The interest, here, is in the rollover limit per se, i.e., in the maximum steady-state lateral acceleration which a given vehicle could sustain without becoming asymptotically unstable in roll. Conversely, there will be no consideration here of what level of lateral acceleration would actually be established in a given maneuver. This subject is in the realm of yaw plane dynamics, and will be discussed in Chapters 5 and 6. (This is not to say that yaw plane performance is not important to determining whether a vehicle will rollover in a given maneuver in practice. Indeed, yaw plane performance does establish the maximum level of lateral acceleration achieved by a vehicle in a given maneuver, and thus, helps determine whether or not rollover will take place.)

The discussion begins with a review of the physics of the rollover process, using simplified roll plane models as a basis. The presentation includes and expands on the work of Mallikarjunarao [3,4]. This review will serve to identify and explain the reasons for the parameter sensitivities of the rollover limit. Following this discussion, simulation study results demonstrating these sensitivities for the pertinent subject vehicle will be presented.

#### 4.1 The Physics of Commercial Vehicle Rollover

The most fundamental parameter affecting the rollover stability limit of commercial vehicles is the ratio of wheel track to c.g. height. Other vehicle parameters, including (1) tire and suspension roll compliances, (2) suspension freeplay, (3) suspension geometry, and (4) the distribution of compliance among the suspensions of the vehicle, contribute significantly to determining the roll stability limits of the vehicle. The remainder of Section 4.1 will be dedicated to a discussion of the physics of commercial

vehicle rollover, presented in a manner intended to explain the sensitivity of roll stability to these several vehicle parameters. The discussion is applicable to any vehicle unit with a single roll degree of freedom. For example, a tractor-semitrailer combination should be considered as one unit since the fifth-wheel coupling requires that the two vehicle elements roll as one.

4.1.1 The Basic Influence of the Ratio of Track Width to C.G.

Height. To begin at the primary level of importance, consider the roll plane model of Figure 17 in which the compliance of all suspension springs and tires is neglected. That is, tires and suspension are considered rigid. In the figure:

$W$  is the weight of the vehicle

$a_y$  is steady-state lateral acceleration

$T$  is 1/2 of the vehicle track

$h$  is the height of the c.g. above the ground

$\phi$  is the vehicle roll angle

(Note that since the vehicle is rigid,  $\phi = 0$  at all times until a tire lifts off of the ground.)

When the vehicle of Figure 17 is subject to a steady-state lateral acceleration, three moments act on the vehicle. Considering moments about point 0 in the figure, these three moments are (assuming small roll angles):

$-W \cdot a_y \cdot h$  the "overturning moment"

$(F_2 - F_1)T$  the "restoring moment"

$-W \cdot h \cdot \phi$  an additional overturning moment resulting from the lateral shift of the c.g. due to roll

For steady-state equilibrium, it is necessary that

$$W \cdot a_y \cdot h = (F_2 - F_1)T - W \cdot h \cdot \phi \quad (4.1)$$

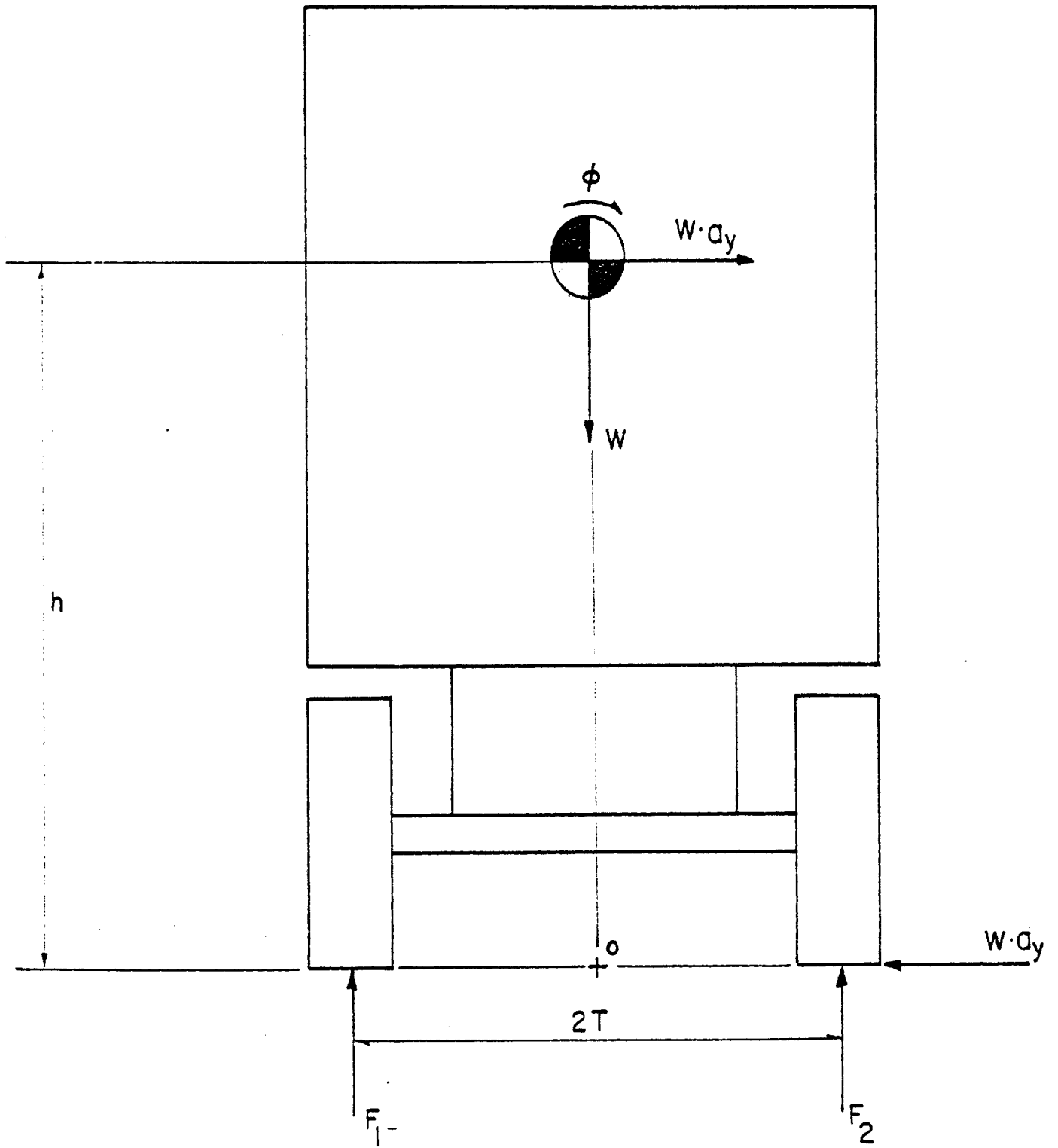


Figure 17, Rigid vehicle roll model.

Figure 18 presents a graphical representation of Equation (4.1). In the figure, the terms on the right side of the equation (as well as their sum) are represented as functions of  $\phi$  on the right side of the graph. The left side of the equation is represented as a function of  $a_y$  on the left side of the graph. As noted on the figure, the left-side moment can be thought of as the destabilizing moment due to lateral acceleration. The right-side moment may be thought of as the stabilizing moment provided by vehicle response. The vehicle will become unstable in roll at any acceleration level which causes the destabilizing moment (left side) to exceed the vehicle's ability to generate a stabilizing moment (right side).

Note that the term  $(F_2 - F_1)T$  has a maximum value of  $W \cdot T$  which is equivalent to the condition in which all of the vehicle weight has been transferred to the outboard tire. Since the vehicle is rigid, full load transfer occurs with zero roll angle. As roll angle increases beyond zero, the total moment on the right steadily decreases from this maximum ( $W \cdot T$ ) due to the influence of the  $W \cdot h \cdot \phi$  term.

For steady-state equilibrium in roll to exist, the left- and right-hand sides of the figure (Equation (4.1)) must produce equal moments. Thus, Figure 18 shows clearly that the maximum sustainable lateral acceleration for roll equilibrium is  $a_y = T/h$ . At this condition, a roll moment of  $W \cdot T$  is produced by both the right and left sides. At any higher level of acceleration, the right side cannot generate enough roll moment for equilibrium. The excess overturning moment ( $W \cdot a_y \cdot h$ ) will cause the vehicle to begin to roll to a larger angle (larger than zero for this rigid vehicle). As roll angle increases, the negative influence of the lateral shift of the c.g. actually decreases the net restoring moment causing an even greater imbalance, and so the rate of roll increases and the rollover process continues. That is to say, the system has become unstable in roll. In this and following graphical presentations, then, a negative slope of the net moment curve is the key indicator for an unstable roll condition. Or, equivalently, the maximum value of the net moment determines the roll stability limit of the vehicle. To express this limit in terms of lateral acceleration, the lateral acceleration equivalent to the peak net moment is determined from the left-hand portion of the graph.

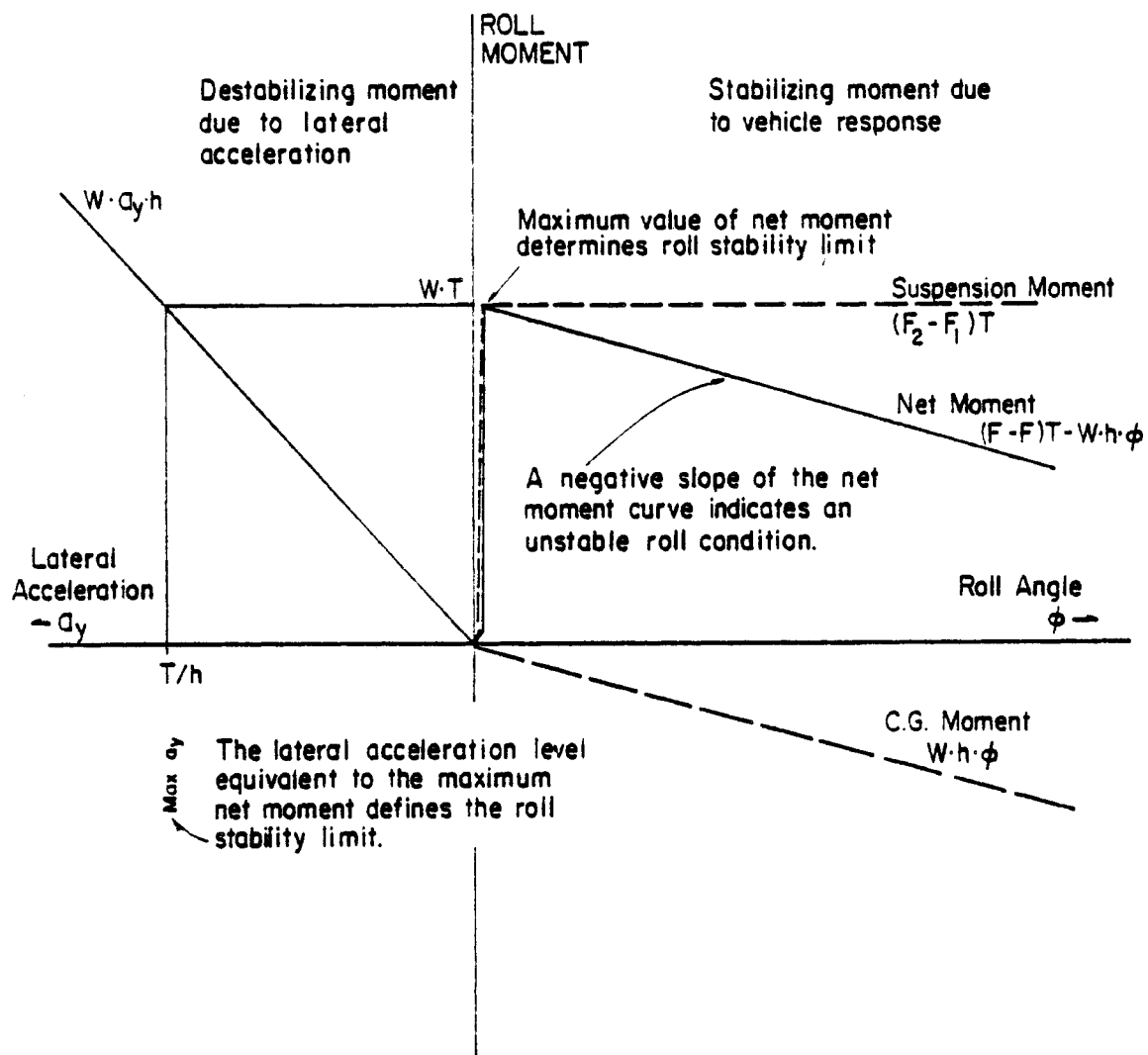


Figure 18. Roll response of rigid vehicle model.



Then, for this simple rigid model, the rollover limit of the vehicle (i.e., the maximum sustainable lateral acceleration) is identically  $T/h$ , the ratio of the  $1/2$  track to the c.g. height. In other words, in a "parameter sensitivity" context, we expect the roll stability limit to be most sensitive to this fundamental parameter.

4.1.2 The Basic Influence of Roll Displacement as Allowed by Tire and Suspension Compliance. Now consider the somewhat more complex roll model of Figure 19. This model includes both suspension and tire compliance, but, for the moment, we will include the simplifying assumption that the compliance of all the vehicle's tires and suspensions can be lumped into a single suspension model. Also, we will assume that the vehicle rolls around a point in the ground, i.e., that the suspension roll center is in the ground plane. These assumptions allow the simplest introduction of the degrading influence of roll compliance on the roll stability limit. In later sections, this influence will be examined in more detail.

For these assumptions, Equation (4.1) remains valid, but we require a new graphical representation to include the effects of compliance. The appropriate representation appears in Figure 20. In this figure, the representation of the  $(F_2 - F_1)T$  term now includes the composite effect of suspension compliance and tire compliance. That is, roll angle displacement is required in order to develop suspension restoring moment, and the maximum restoring moment ( $W \cdot T$ ) is not attained until the roll angle,  $\phi_\ell$ , is reached. At  $\phi_\ell$  wheel lift-off will occur. When this roll displacement effect is combined with the  $W \cdot h \cdot \phi$  term, the total effect is to lower the maximum available restoring moment from  $W \cdot T$  to  $W \cdot T - W \cdot h \cdot \phi_\ell$  and thereby lower the stability limit to a lateral acceleration that is less than  $T/h$ .

To put the influence of the  $W \cdot h \cdot \phi_\ell$  term in perspective, the example vehicle to be considered in Section 4.2 would have a roll stability slightly in excess of .5 g's if it were a rigid vehicle. In the baseline condition considered, however, the actual roll stability limit is .37 g's. In physical terms, then, the  $W \cdot h \cdot \phi_\ell$  effect (along with the more subtle

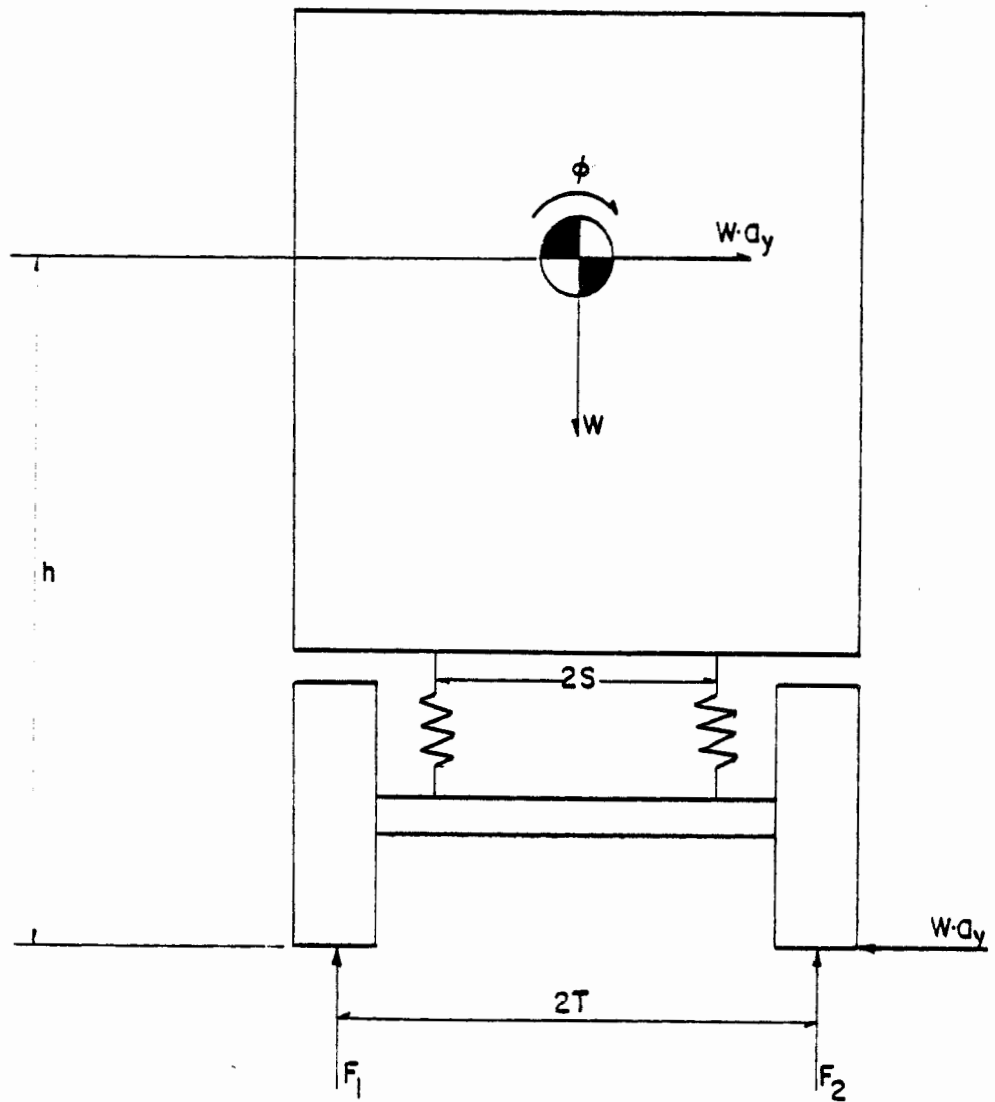


Figure 19. Vehicle roll model with lumped suspension compliance.

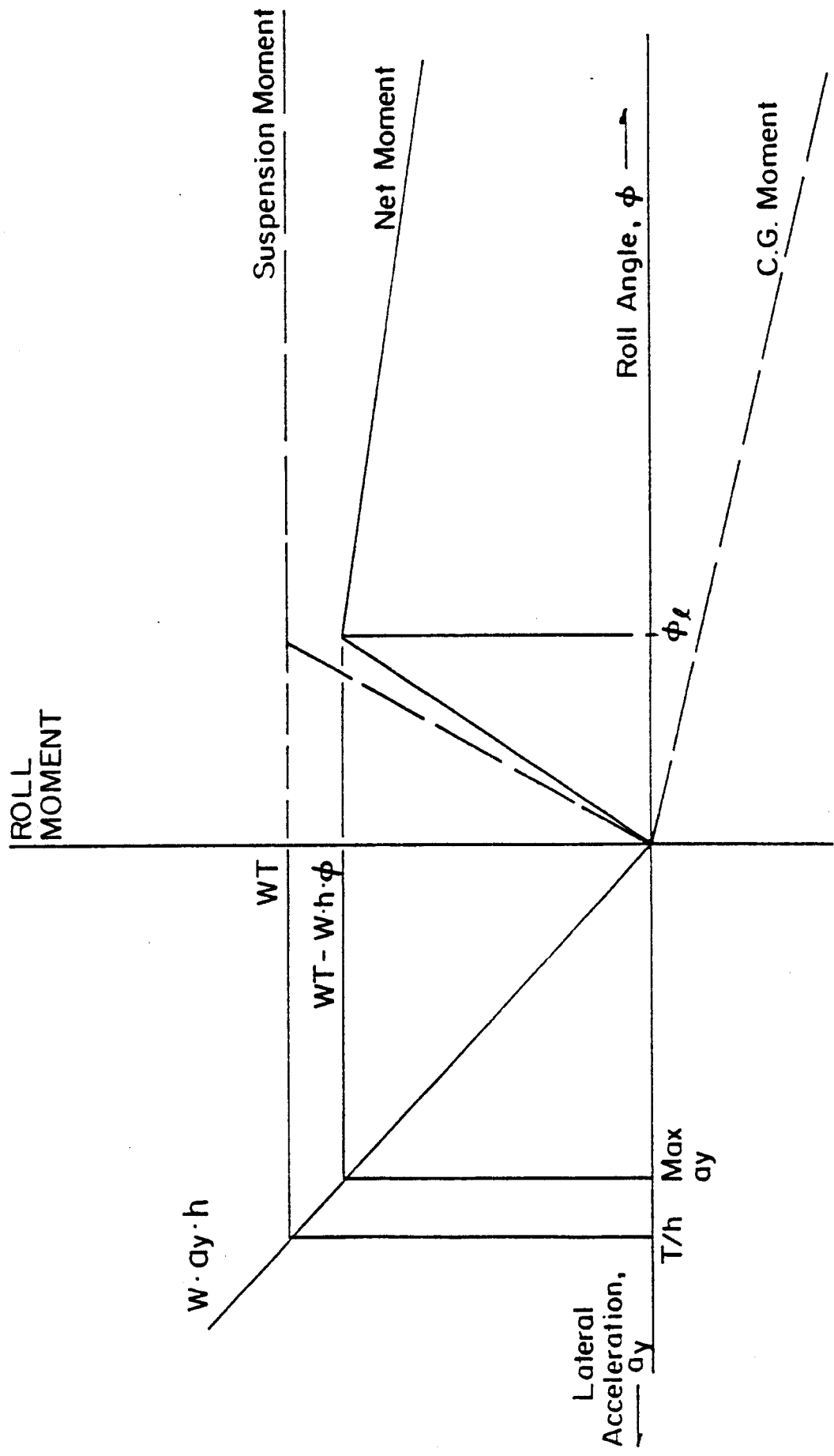


Figure 20. Roll response of vehicle with suspension compliance.

effects to be considered below) lowers the roll stability of this vehicle by 25%. Ervin [3] has shown that, in single-vehicle accidents, the likelihood of rollover increases from 15% to 40% for this degradation in roll stability limit.

In general, the more compliance, the lower the rollover limit. This can be seen graphically by imagining Figure 20 with a lower initial slope to the  $(F_2-F_1)T$  function, and therefore to the total function. The effect can be seen in equation form by examining the expression

$$WT - W \cdot h \cdot \phi_{\lambda} \quad (4.2)$$

More compliance implies a larger value of  $\phi_{\lambda}$  and, thus, a smaller value for the expression. Expression (4.2) also indicates a secondary influence of c.g. height. When compliance is present, increasing c.g. height not only reduces the reference,  $T/h$ , value, but increases the negative effect of the  $W \cdot h \cdot \phi_{\lambda}$  term, further reducing the stability limit.

4.1.3 The Influence of Suspension Spring Lash. Heavy vehicle suspensions, particularly four-leaf tandem suspensions, often exhibit spring lash as the lightly loaded spring passes from compression to tension on the way toward rollover. The amount of this lash can affect the rollover limit.

Consider Figure 21 which derives from the single-axle model with spring lash included. From the  $(F_2-F_1)T$  function, it can be seen that, as the lightly loaded spring passes through its lash, suspension roll displacement takes place without any increase in suspension restoring moment. (The magnitude of this roll displacement is  $\delta/2S$  where  $\delta$  is the amount of lash and  $2S$  is the spacing between the suspension springs.) The effect is to further increase the roll angle at which maximum total moment is obtained ( $\phi_{\lambda}$ ) and, again through the influence of the  $-W \cdot h \cdot \phi_{\lambda}$  term, to reduce this maximum moment and, thereby, the rollover limit.

It is of interest to note that the effect of lash is, in the end, similar to the effect of increased compliance. Figure 21 points this out by including plots of an "equivalent" suspension which is more compliant

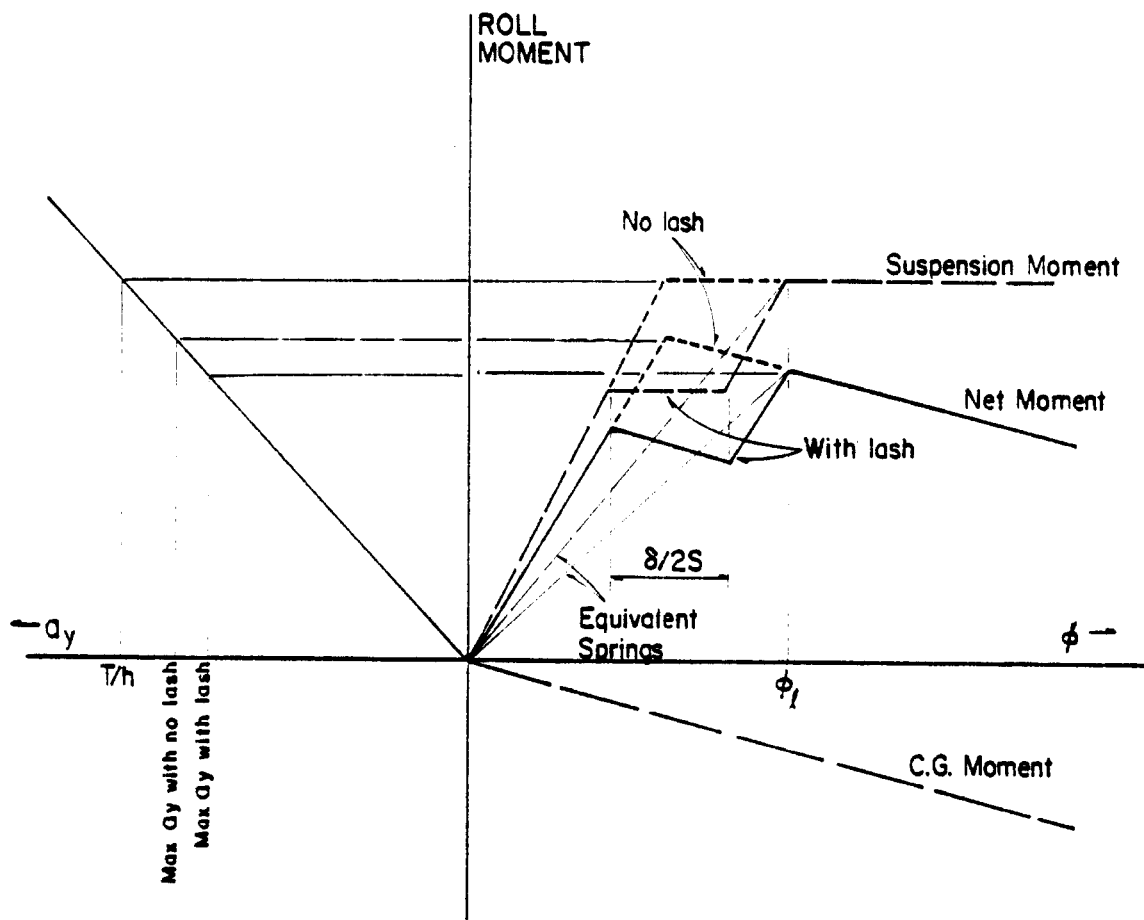


Figure 21. Roll response of vehicle including spring lash.

but has no lash. Since this suspension has a value of  $\phi_\ell$  that is identical to the suspension with lash, the resulting rollover limit is also identical. In effect, then, the equivalent compliance of a suspension is the average compliance exhibited up to the level of wheel lift-off. Vehicle roll stability will exhibit a parameter sensitivity to this effective compliance as it derives from both nominal roll rate and from suspension lash.

Recent recognition of the influence of spring lash has resulted in reduction of lash on the part of many manufacturers. Older suspensions, however, exhibited lash on the order of one inch. One inch of lash would contribute about 1.5 degrees of "free" roll out of a total of perhaps six degrees of roll required to reach the rollover limit for a relatively high c.g. vehicle. Accordingly, in a general sense, spring lash might account for nearly 25% of the roll stability limit degradation generally attributable to suspension compliance.

4.1.4 Effects of Suspension Roll Center Height. If we include suspension geometry, and in particular, roll center height in the vehicle model, we can discover an additional sensitivity.

Figure 22 illustrates the new model. The new parameters in this figure are

$h_1$  the height of the roll center above the ground

$h_2$  the height of the c.g. above the roll center

$\phi_1$  the roll angle of the unsprung mass

From the figure, it can be shown that, for small angles, the moment due to the lateral shift of the c.g. is

$$-W \cdot (h_1 \phi_1 + h_2 \phi) \quad (4.3)$$

In the previous model we assumed the roll center to be in the ground. In that case,  $h_1 = 0$  and  $h_2 = h$  and (4.3) simplified to  $-W \cdot h \cdot \phi$ .

For the moment, let us make the "opposite" assumption, viz., that the roll center is at the c.g. and that, therefore,  $h_1 = h = h_2 = 0$ .\*

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\*For heavy trucks, this condition never exists, but the assumption serves to make an important point.

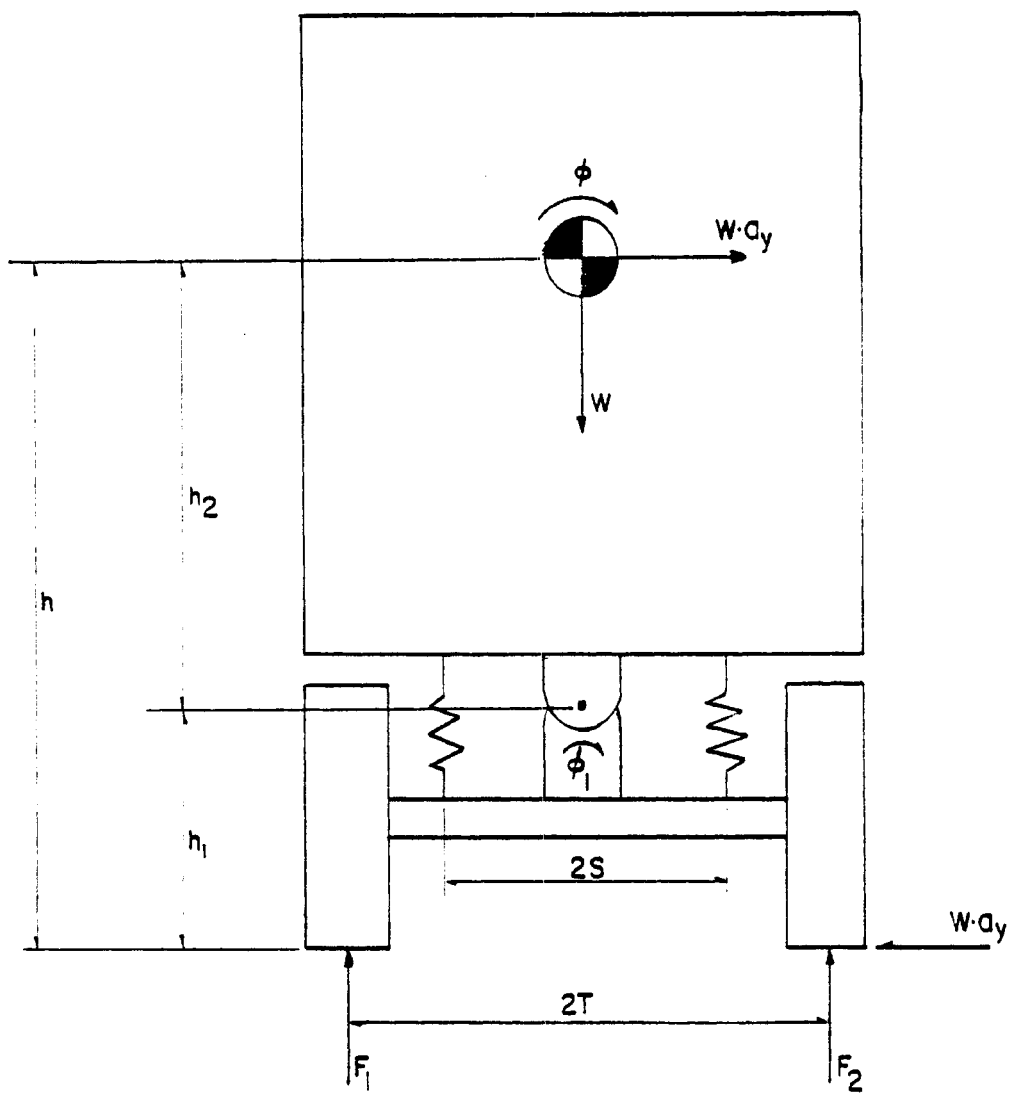


Figure 22. Vehicle roll model with roll center.

With this assumption, (4.3) simplifies to  $-W \cdot h \cdot \phi_1$ . For a given lateral acceleration, however, we know that  $\phi_1$  is less than the value of  $\phi$  since  $\phi$  results from tire plus suspension compliance and  $\phi_1$  results only from tire compliance. Thus, the slope of the moment due to lateral c.g. shift is reduced, as indicated in Figure 23.

Further, with the roll center located at the c.g., the vehicle body will not roll with respect to the axle and the body roll angle,  $\phi$ , will equal  $\phi_1$ . In effect, the composite compliance of the suspension and tires is reduced to the compliance of the tires alone. The effect is to increase the initial slope of the  $(F_2 - F_1)T$  function, again shown in Figure 23. The figure also shows that the two effects combine to produce an increase in net moment, and, therefore, an improved roll stability limit.

It is probably safe to say that the importance of roll center height to the roll stability of commercial vehicles has not been generally recognized to date. Common commercial vehicle suspension designs do not show evidence of special efforts taken to control roll center height. As a general rule, roll center height is closely approximated by the point where side forces are transmitted between the vehicle frame and suspension. For most leaf-spring suspensions, then, the roll center height will be near to the height of the connection between the ends of the leaf springs and the frame. Trailing-arm air suspensions often have special lateral links which transmit lateral force between the suspension and frame, and would thus locate roll center height. Limited laboratory measurements of unloaded Class 8 commercial vehicles have indicated roll center heights above ground as follows: (1 m = 39.37 in)

Leaf-spring front suspension:	about 25 inches
Single-axle leaf-spring rear suspension:	about 30 inches
Four-spring tandem suspension:	about 30 inches
Walking-beam suspension with leaf springs:	about 22 inches

In the future, raising roll center height by specific design intent would appear to have potential as a practical and effective means of improving commercial vehicle roll stability. It would appear that roll



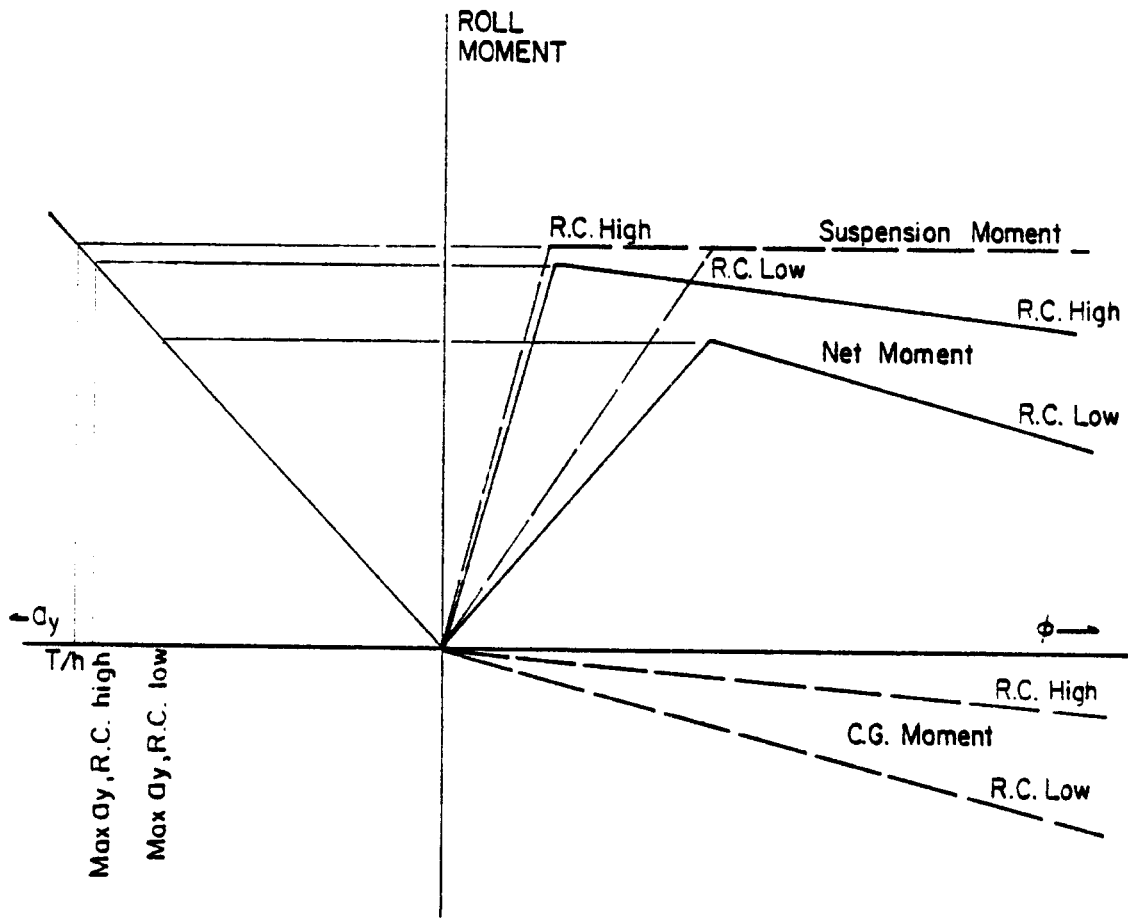


Figure 23. Effect of roll center height on roll response.

center heights in the range of 35-40 inches (1 m) are practically obtainable. It should be noted, however, that differences in roll center heights among the several suspensions of a vehicle affects the distribution of roll moment among those suspensions. Thus, as was the case for roll stiffness distribution, roll center height "distribution" can affect the vehicle's yaw stability as well.

4.1.5 The Influence of Distributed Suspension Roll Compliance. The single-axle model used above ignores the influence of the distribution of roll stiffness among the several suspensions of the vehicle. If the roll stiffness of the various suspensions are not proportional to the loads carried by the suspension, then the single-axle representation will generally predict a rollover limit which is higher than the true limit.

For example, consider the conventional tractor-semitrailer. Such vehicles are typically equipped with very soft front suspensions, a considerably stiffer rear tractor suspension, and a still stiffer trailer suspension. Figure 24 presents the graphical representation of the roll moments for such a three-suspension vehicle. The trailer suspension is shown as the stiffest, while the trailer and tractor rear suspension carry nearly equal load (i.e., nearly equal  $W \cdot T$  values). The tractor front axle is both softer and carries a much lower load.

The roll angles necessary for wheel lift at each of the three suspensions are indicated by the angles  $\phi_{\ell 1}$ ,  $\phi_{\ell 2}$ , and  $\phi_{\ell 3}$ , respectively. (If the stiffness of each suspension was proportional to its load, then these angles would all be equal and the model would converge to the equivalent of the lumped suspension model used earlier.) From the plot of the net moment function, we see that the maximum roll resistant moment occurs at the tire lift point for the tractor rear axle. At higher roll angles, even while the tractor front tires remain on the ground, net moment is decreasing. This implies that the front axle stiffness is so low that it does not compensate for the overturning moment generated by the continuing lateral shift of the c.g. This point, then, defines the limit lateral acceleration with respect to roll stability.

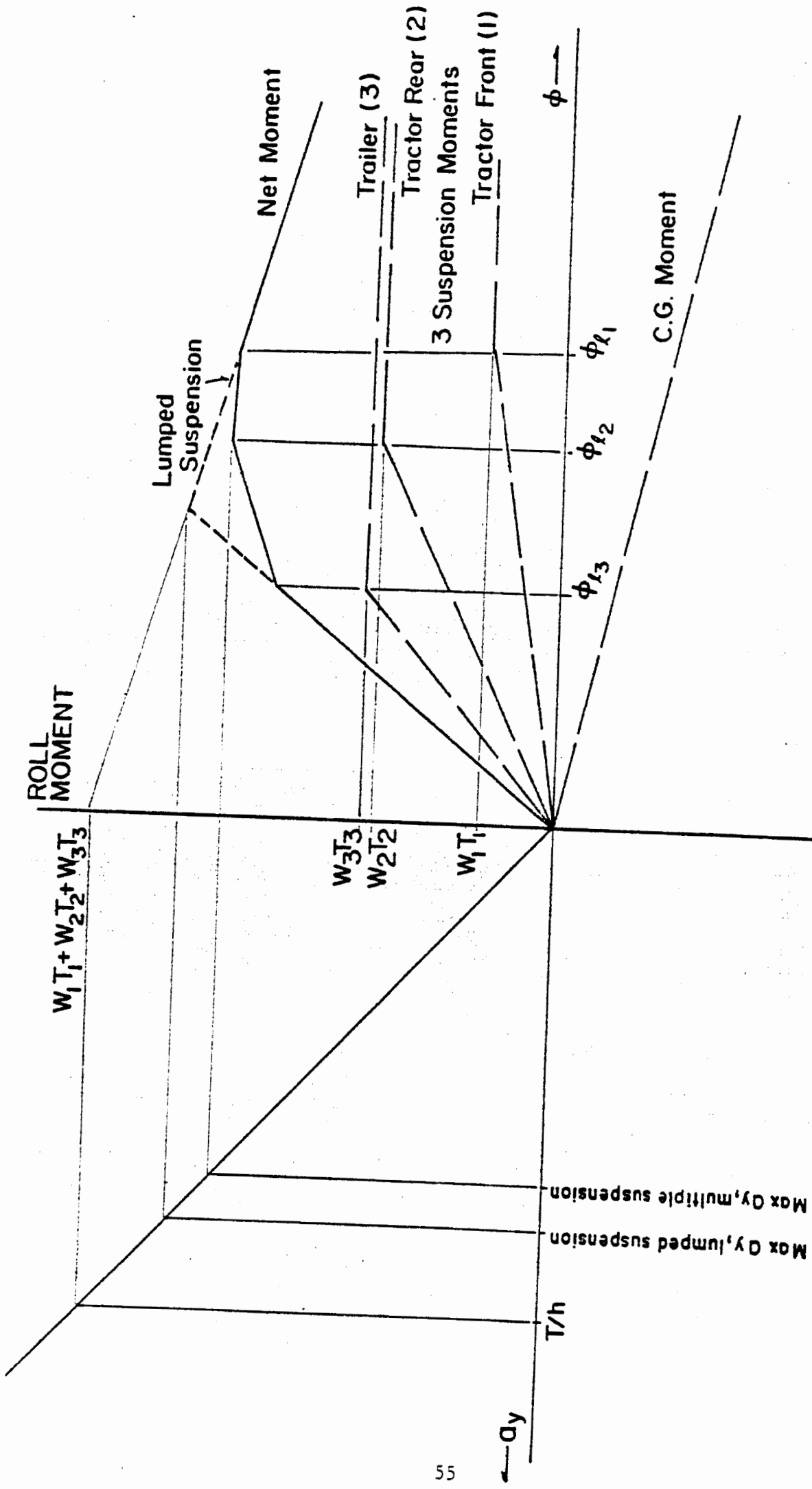


Figure 24. Roll response of combination vehicle with three compliant suspensions.

Figure 24 also includes a dashed line indicating the prediction of roll limit that would result if the lumped suspension model was used for this vehicle. Note that the lumped suspension model predicts a somewhat more roll-stable vehicle.

The Influence of Individual Suspension Stiffnesses - We will now consider the parameter sensitivity effect of changes in the individual suspension stiffness of Figure 24. For this purpose, we will define two classes of suspensions, viz., (1) "stiff" suspensions which are suspensions that exhibit tire lift at a roll angle less than the roll angle at which maximum net moment is obtained and (2) "soft" suspensions which are suspensions that exhibit tire lift at roll angles which are equal to or greater than the roll angle of maximum net moment. Our example vehicle has one "stiff" suspension, the trailer suspension, and two "soft" suspensions. This is typical of tractor-semitrailer vehicles. It is possible to have other mixtures. The only invariable rule is that every vehicle must have at least one "soft" suspension. That is, the two extreme possibilities are (1) maximum moment occurs with the last axle lift giving one "soft" suspension and all other suspensions "stiff" and (2) maximum moment occurs with the first axle lift, yielding all "soft" suspensions.

"Stiff" Suspensions - Figure 25 illustrates the effects of varying the stiffness of the trailer suspension (the only "stiff" suspension) of our example vehicle. Two variations from the baseline are shown: (1) the suspension is made stiffer and (2) the suspension is made softer to the extent that it becomes a "softer" type. The figure demonstrates that stiffening this "stiff" suspension (variation 1) reshapes the initial portion of the net moment curve, but does not affect the maximum value of the net moment. Thus, there is no effect on roll stability.\* On the other hand, softening this "stiff" suspension to the extent that it becomes a "soft" suspension (variation 2) lowers the maximum value of the net moment and therefore degrades the roll stability limit.

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\*Softening this suspension slightly, so that it remains a "stiff" suspension would, similarly, have no effect on the roll stability limit.

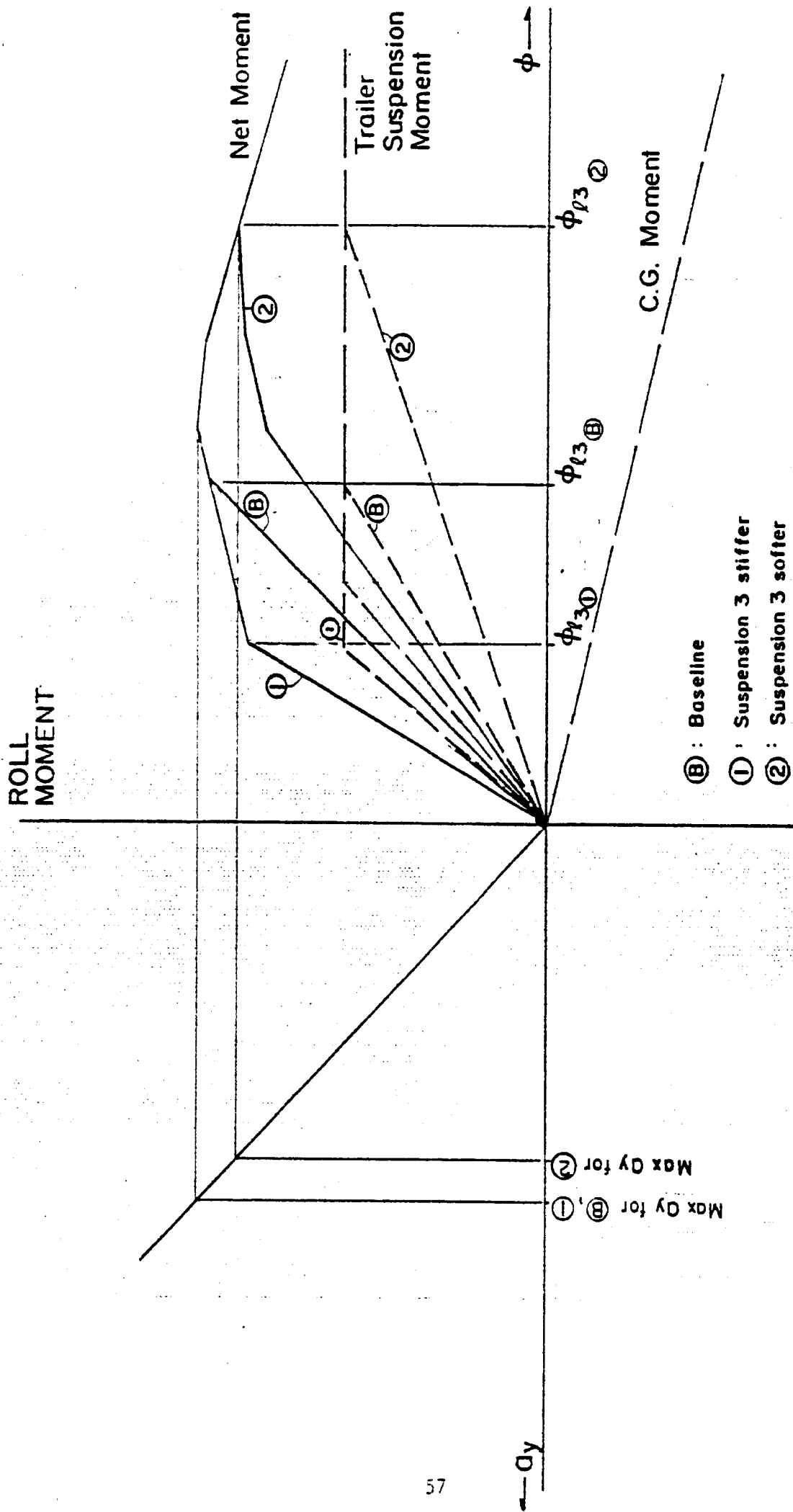


Figure 25. Effect of changes in compliance of the "stiff" trailer suspension.

"Soft" Suspensions - Figures 26 and 27 illustrate the influences of changes in stiffness of the two "soft" suspensions—the tractor rear and tractor front suspensions, respectively. These two figures illustrate that stiffening any "soft" suspension improves roll stability and, conversely, softening such suspensions degrades roll stability. This is so, since any change in a "soft" suspension affects the maximum net moment.

The maximum advantage to be gained by stiffening any soft suspension is, of course, limited by the point where the suspension eventually makes the transition to a "stiff" suspension type.

It should be noted that roll stiffness distribution is very significant in determining yaw stability, as well as roll stability, properties of commercial vehicles (Chapters 5 and 6). In the context of complete vehicle performance, optimizing roll stiffness distribution for roll stability alone may not be wise if this serves to unacceptably degrade yaw stability.

Influence of Suspension Lash - As pointed out earlier in the discussion on suspension lash based on the single suspension model, lash can be viewed simply as a mechanism which reduces the overall effective stiffness of a suspension up to tire lift. Accordingly, all the comments of the immediately preceding discussion are appropriate to the effects of lash, if we simply view lash as a mechanism which reduces suspension stiffness.

4.1.6 Suspension Location. In the previous section, we discussed the influence of the distribution of roll stiffness among the various suspensions of the vehicle. There is an additional, more subtle effect of multiple suspensions on roll stability which is related to the longitudinal position of the various suspensions on the vehicle.

Consider the free-body diagram of Figure 28. The figure shows the forces which act on an unsprung mass in steady-state, namely,

$F_1$ and $F_2$	the left and right side vertical tire forces
$F_{s1}$ and $F_{s2}$	the left and right side spring forces
$F_y$	the total tire side force

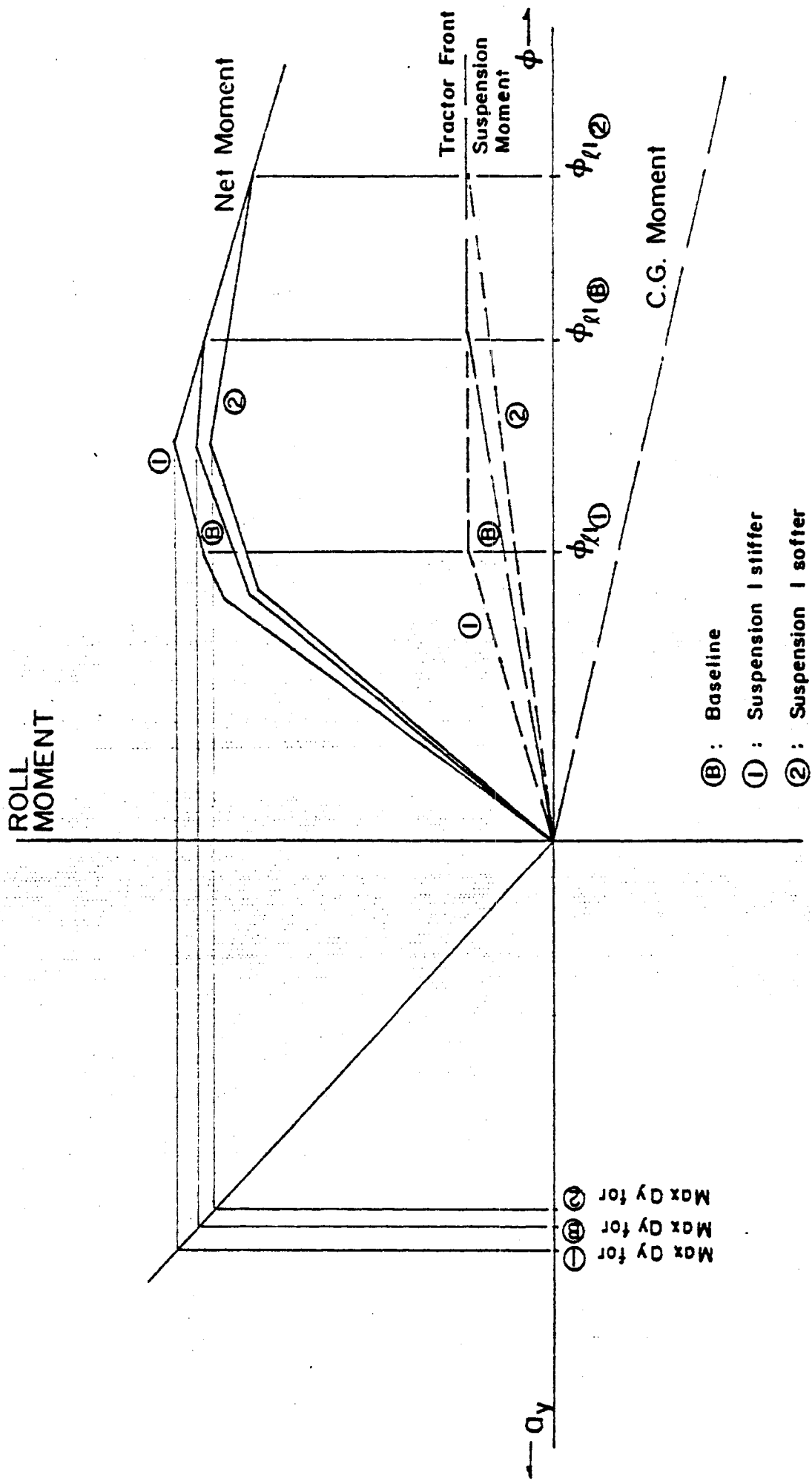


Figure 26. Effect of changes in compliance of the "soft" front suspension

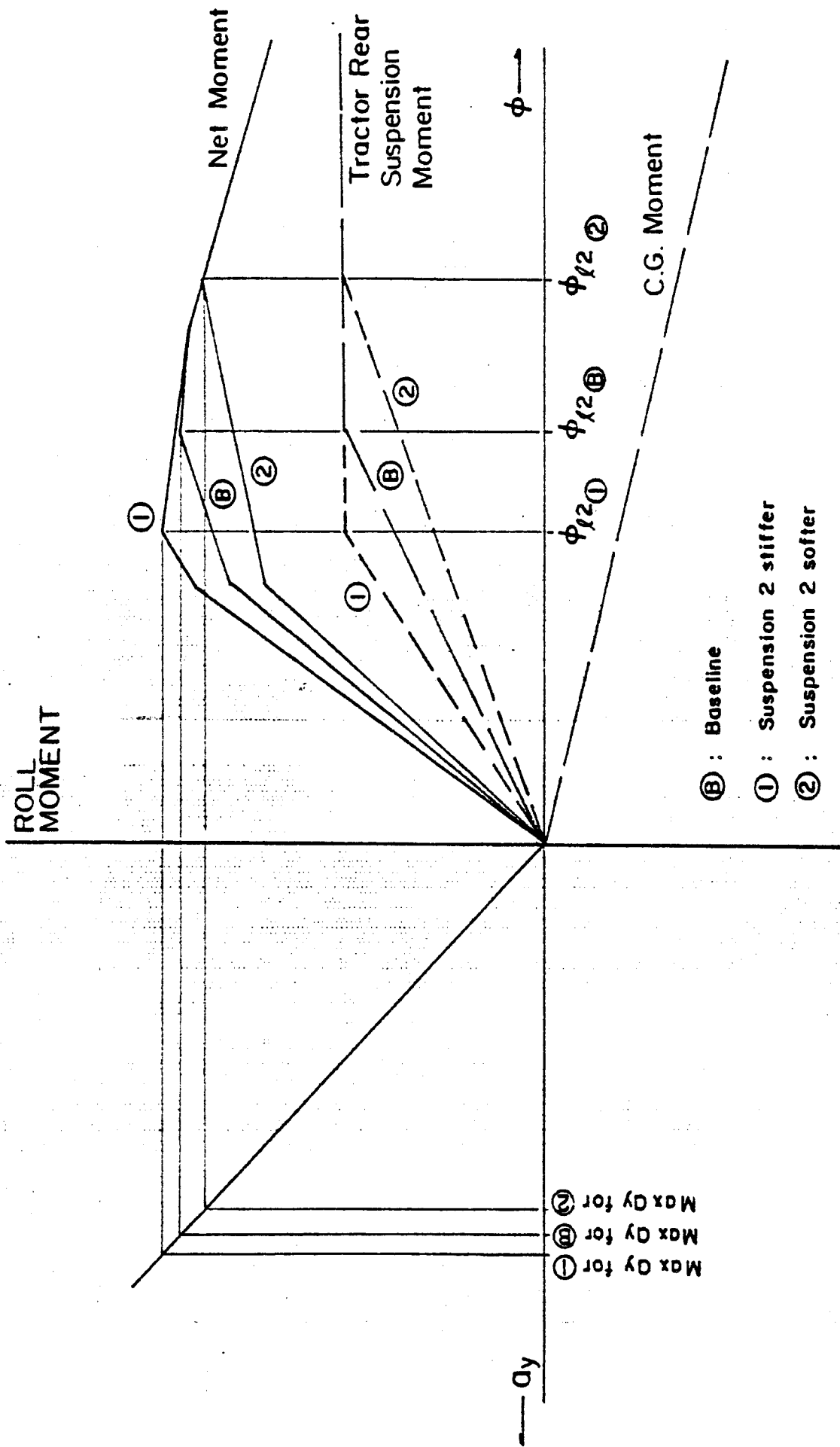


Figure 27. Effect of changes in compliance of the "soft" tractor rear suspension.



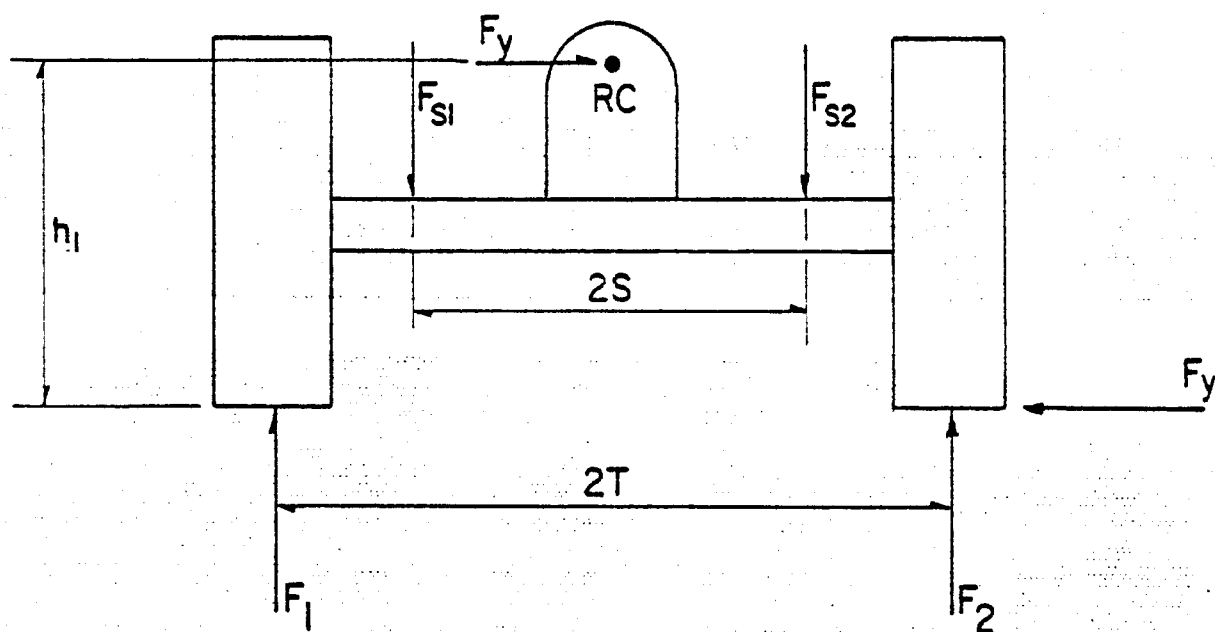


Figure 28. Free-body diagram of an unsprung mass.

The tire side force is reacted by an equal and opposite force at the roll center (RC). The spring spacing is  $2S$  and the track is  $2T$ . The roll center height is  $h_1$ .

Summing moments about the roll center yields

$$F_y h_1 - (F_2 - F_1)T + (F_{s2} - F_{s1})S = 0 \quad (4.3)$$

Now, define  $\phi$  and  $\phi_1$  as the roll angles of the sprung and unsprung masses, respectively, and define  $K_S$  and  $K_T$  as the equivalent torsional springs of the suspension and tires, respectively, such that

$$K_T \phi_1 = (F_2 - F_1)T \quad (4.4)$$

$$K_S (\phi - \phi_1) = (F_{s2} - F_{s1})S \quad (4.5)$$

Equations (4.3) through (4.5) may be combined and solved for  $\phi$ , yielding

$$\phi = (F_2 - F_1)T \left( \frac{1}{K_T} + \frac{1}{K_S} \right) - \frac{F_y h_1}{K_S} \quad (4.6)$$

Now, define  $W_S$  as the total vertical load on this suspension. Then wheel lift takes place for the suspension when  $F_2 = W_S$  and  $F_1 = 0$ . Then for this suspension

$$\phi_\ell = W_S T \left( \frac{1}{K_T} + \frac{1}{K_S} \right) - \frac{F_y h_1}{K_S} \quad (4.7)$$

where  $\phi_\ell$ , again, is the body roll angle at which wheel lift occurs.

The second term in Equation (4.7) shows that:

As the value of  $F_y h_1$  increases, the body roll angle at which tire lift occurs becomes smaller. That is, as the  $F_y h_1$  term becomes larger, the suspension becomes effectively "stiffer" per our previous definition of "stiff" and "soft" suspensions.

Accordingly, Equation (4.7) is another way of expressing the importance of roll center height. As the roll center height increases, the suspension appears "stiffer" as was determined in the previous discussion on roll center height.

Interpreting Equation (4.7) in another light, however, we see that the effective stiffness of a suspension is related to the amount of side force ( $F_y$ ) to which the suspension is subjected. If the side force is large, then tire lift occurs at a smaller body roll angle and the suspension is, in effect, stiffer.

The distribution of  $F_y$  among suspensions is related to yaw plane behavior. Sufficient for this discussion, it can be said that, as a general rule, for higher level (of lateral acceleration) steady-state turning, axles near the center of the vehicle unit\* are subjected to smaller slip angles than those closer to front or rear. Therefore, they will, in general, experience smaller levels of side force. Thus, axles placed near the center of the vehicle can be expected to appear "softer" than those placed far forward or aft, all other parameters being equal.

The strength of this effect is dependent on speed. For a fixed lateral acceleration, the difference between slip angles among axles generally will grow as speed decreases. Thus, axle placement is of greater importance in low-speed turning than in high-speed turning.

Equation (4.7) leads to one more interesting conclusion, viz., that self-steering axles can, in general, be expected to be effectively "softer" than they would otherwise be. There has recently been increased interest in the use of self-steering axles on heavy vehicles to improve low-speed maneuverability and to lessen tire wear. Since the general nature of self-steering axles reduces tire side force on that axle during turning, the effective stiffness of a self-steering axle can be expected to be lower than it would be for a similar, non-steering axle.

It should be noted that the issues considered in this section (axle location and self-steering axles) affect primarily stiffness distribution among axles as opposed to total stiffness. For a given steady-state lateral acceleration, a specific total tire side force is required. Accordingly,

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\*"Unit," here, refers to a single vehicle unit in the yaw plane.

when tire side force is found to be low on one axle due to location or a self-steering function, side forces on other axles will be larger, thus adding to their effective stiffness. Depending on the relative height of the roll center of the suspension's "tracking" side force, total effective stiffness may either increase or decrease somewhat, or remain constant.

4.1.7 Summary. The preceding discussion has served to highlight the significant parametric sensitivities of commercial vehicles with respect to the roll stability limit. Strictly speaking, the relative importance of these sensitivities can only be evaluated for a given vehicle system. Nevertheless, an effort has been made to order the following summary according to relative importance, given current general practice. The significant sensitivities are:

1) Sensitivity to track width and c.g. height. The ratio of track width to c.g. height is the fundamental determinant of the lateral acceleration level at which roll instability will occur. Lowering c.g. height and/or increasing track width have a stabilizing influence.

2) Sensitivity to the total (lumped) roll compliance of the vehicle's suspensions and tires. In general, body roll compliance that derives from suspensions and tire compliances degrades the roll stability limit of the vehicle from the reference level defined by the track width to c.g. ratio. This degradation derives from the lateral shift of the c.g. which occurs as the vehicle rolls on compliant suspensions.

3) Sensitivity to suspension lash. The lash which is present in many heavy vehicle suspensions may contribute to the effective roll compliance of the suspension as the vehicle approaches rollover. Accordingly, suspension lash is seen as a portion of the more general compliance affect, but it can contribute significantly to the degradation of the roll stability limit.

4) Sensitivity to suspension geometry: Roll center height. Roll center height has an influence on the effective roll compliance of a suspension and on the amount of lateral c.g. shift which occurs per unit of roll. Accordingly, the roll stability limit is sensitive to roll center heights. In general, higher roll centers increase the roll stability limit.

Current practice suggests that the influence of roll center height on roll stability is not widely recognized and that significant gains in roll stability might be made through advantageous suspension design changes.

5) Sensitivity to roll compliance distribution among suspensions. The distribution of compliance among the various axles of the suspension can affect the roll stability limit. Given that the suspensions, in total, exhibit some specific level of roll stiffness, the optimum distribution of that stiffness among the suspensions is in proportion to the vertical load carried by each suspension. Variations from this distribution degrade the roll stability limit. Further, stiffening or softening suspensions which are proportionately too stiff is ineffectual toward altering the roll stability limit. For suspensions that are proportionately too soft, stiffening will increase the limit and softening will degrade the limit.

6) Sensitivity to axle location. Particularly at lower speeds, the effective stiffness of a given axle is sensitive to its longitudinal placement. Axles nearer the center of the vehicle appear softer; those close to either the front or rear appear stiffer. Thus, the issue of roll compliance distribution (item 5) is affected by longitudinal placement of axles. By a very similar mechanism, self-steering axles also appear to be effectively softer in roll than they would if they were non-steering axles. This effect is not speed sensitive, however, so that self-steering axles always have a special influence on roll stiffness distribution.

#### 4.2 Demonstration of Parameter Sensitivities of Vehicle Roll Stability

4.2.1 Introduction. In Section 4.1, simplified analytical techniques were used to explain the physics of the vehicle rollover process. In this way, rational explanations for the parameter sensitivities of the vehicle roll stability limit were developed. In this section, these parameter sensitivities will be "demonstrated" quantitatively using computer simulation.

This section uses the five-axle dump truck of Figure 9 as the "demonstration" vehicle. Major descriptive parameters for this vehicle as simulated are:

-Total weight	70,000 lbs (311,400 N)
-Sprung mass c.g. height	78.3 in. (1.99 m)
-Five axles	
-Axle loads	
-steering	18,000 lbs (80,000 N)
-each of four non-steering	13,000 lbs (57,000 N)
-Suspension types	
-leaf spring steering axle	
-Axles 2 and 5, tag axles	
-Axles 3 and 4, tandem-axle pair	

Axles 3 and 4 are a rather typical tandem suspension pair. The air-suspended tag axles (2 and 5) have a rather low vertical stiffness, but substantial auxiliary roll stiffness. These two axles have identical parameters in the simulation, as do axles 3 and 4. A full parameter set for the baseline vehicle can be found in Appendix E.

The model used for the "demonstration" is the yaw/roll model described in Appendix D. The simulated maneuver is a slow ramp steer. This maneuver begins with the vehicle traveling straight ahead. As the maneuver progresses, steering is slowly increased from an initial value of zero until a level of steering is reached at which the vehicle finally rolls over. The maneuver is not directly related to any one "real-world" maneuver, but rather provides a practical and efficient method to span a broad range of lateral acceleration conditions. The majority of simulation runs were conducted at a velocity of 20 mph (32 k/hr). This relatively low speed was chosen to avoid yaw instability whose occurrence can serve to reduce the fidelity of the analysis of roll plane performance. (In general, roll plane performance is not sensitive to speed, but rather to lateral acceleration. Only the effect of longitudinal axle position is speed sensitive. An additional run at 55 mph (88 k/hr) was made to examine this influence.

4.2.2 Roll Stability of the Baseline Vehicle. The baseline vehicle demonstrates a roll stability limit of  $143 \text{ in/sec}^2$  (.37 g's) of lateral

acceleration. At this level of lateral acceleration, the last of the four, non-steering axles completes side-to-side load transfer and lifts its inside tire. At this point, the only axle with all tires on the ground is the front axle which is not sufficiently stiff to provide roll stability. Thus, as shown in Figure 29, at a lateral acceleration of  $143 \text{ in/sec}^2$ , the vehicle roll angle begins to increase rapidly with no further increase in lateral acceleration. This condition indicates that the vehicle is unstable in roll.

Figure 30 is a plot of the light-side wheel loads as a function of vehicle roll angle for each of the five axles of the vehicle. (Axles are numbered 1 through 5, beginning with the front axle and moving rearward.) Assuming constant total axle loads, these plots are equivalent in form to the roll moment versus roll angle plots used in Section 4.1. (The vertical load scales of the plot have been inverted and shifted so that the plots take on the same appearance as those of Section 4.1.) As such, they indicate the relative stiffness of the axles in the same manner as did the plots of Section 4.1.

Figure 30 indicates that axle 1 is clearly the softest axle and that axle 5 is clearly the stiffest. Axles 2, 3, and 4 are rather close in relative stiffness—2 and 4 being slightly "stiffer" than 3. According to our definition of "stiff" and "soft" suspension types, then, axles 5, 2, and 4 are "stiff" and axles 1 and 3 are "soft." Axles 2 and 4 are, however, rather close to the boundary. Note also that the spring lash in suspensions 3 and 4 is clearly apparent in the figure. The other three suspensions have no lash.

At this point, it is convenient to introduce Figure 31. This figure is a bar graph showing the roll stability limit, expressed in  $\text{in/sec}^2$  of lateral acceleration, as a function of a variety of vehicle parameter changes. The baseline vehicle is shown as the bar on the far left. For reference, the roll stability limit of this vehicle is indicated by the solid line extending across the entire graph. In the following discussion, this figure will be referred to numerous times.

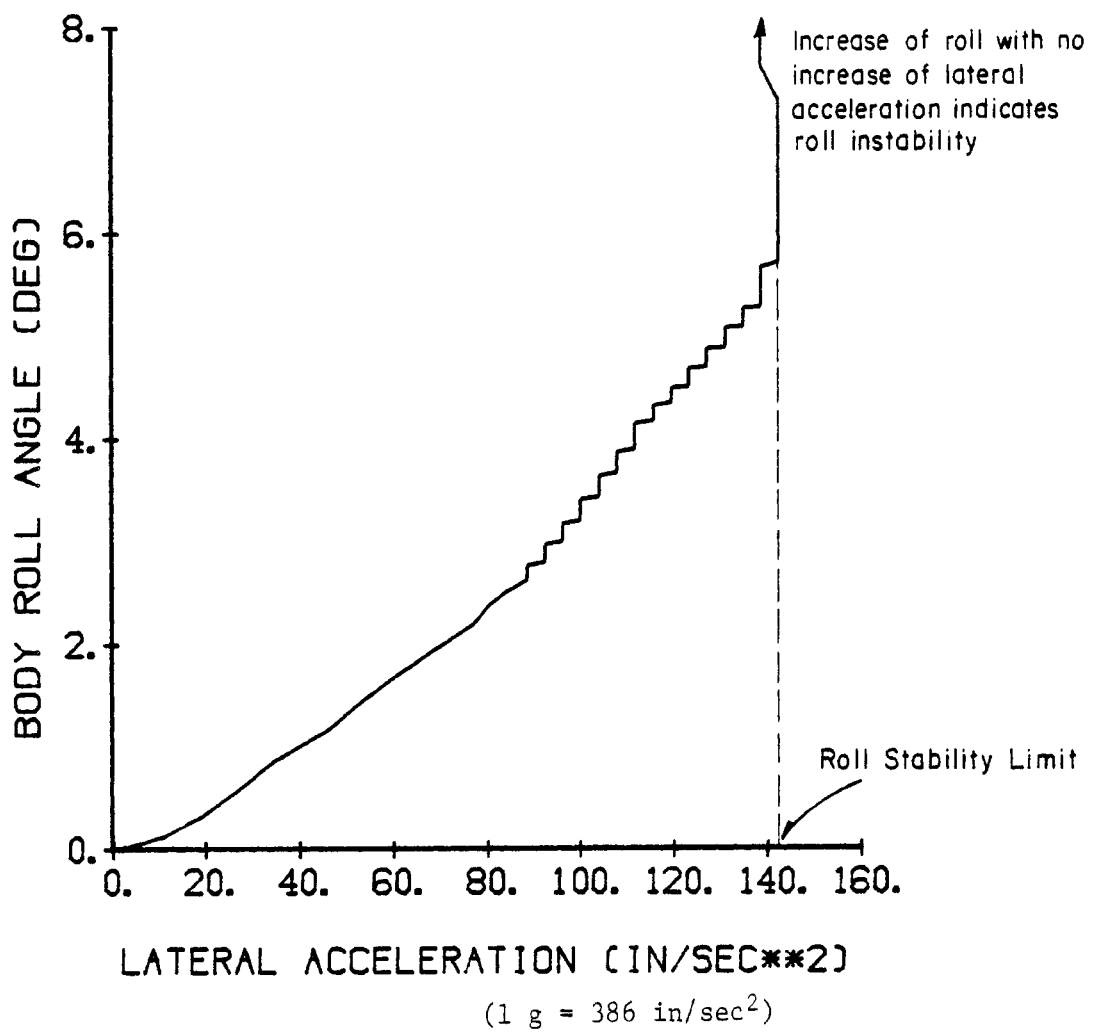


Figure 29. Roll response to lateral acceleration of simulated five-axle dump truck.



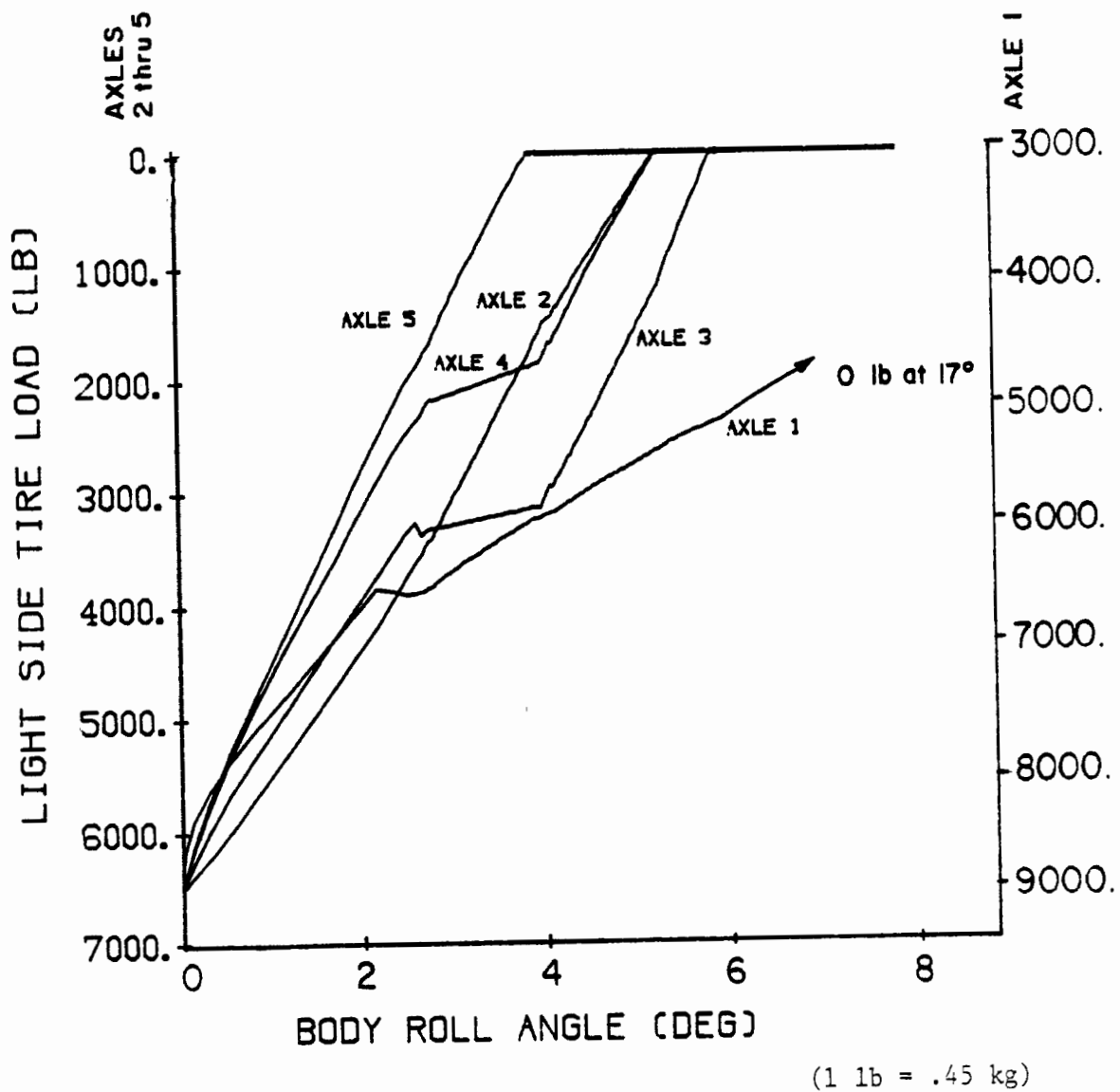
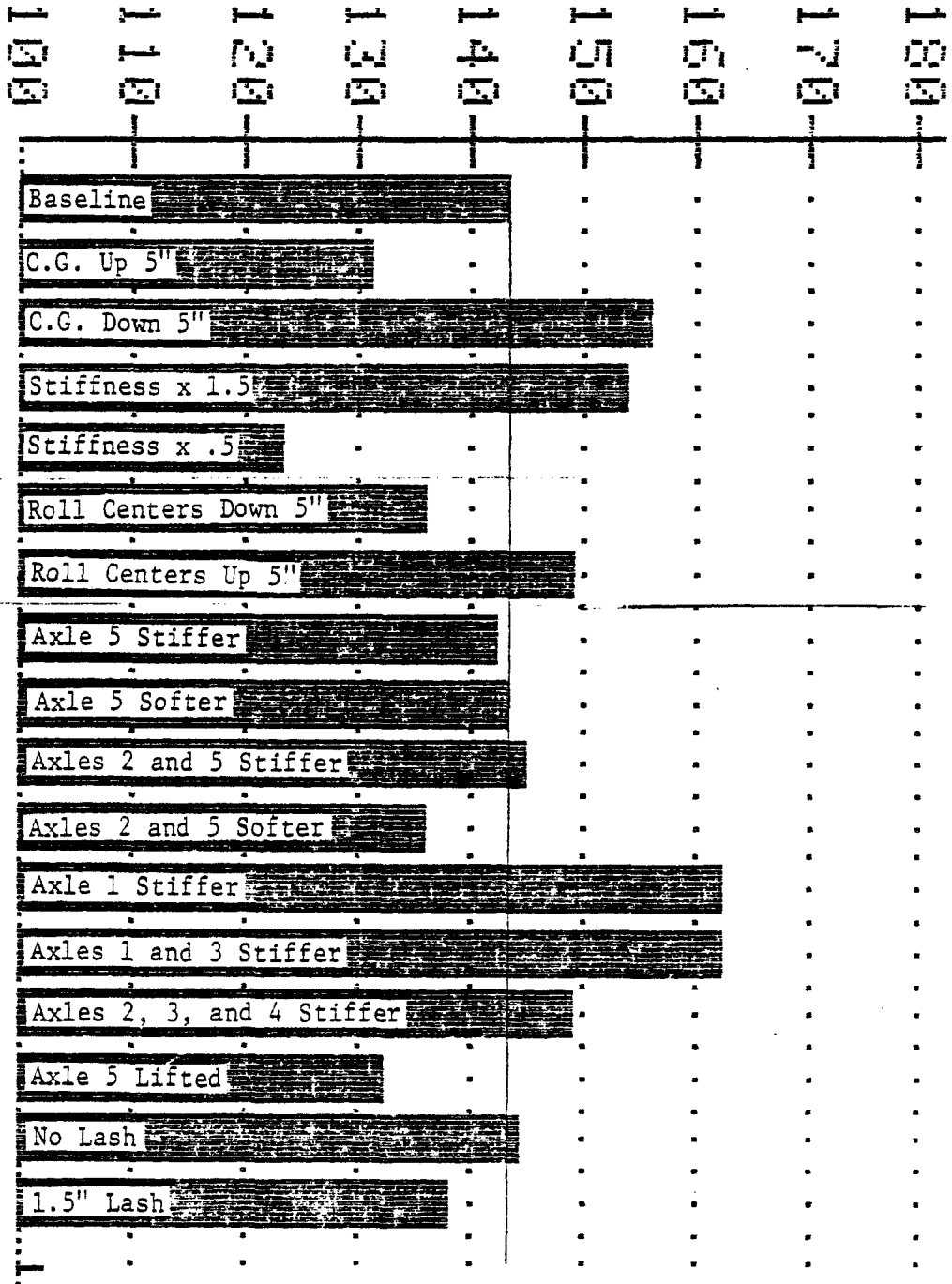


Figure 30. Light-side tire load response to roll of simulated five-axle dump truck at 20 mph (32 k/hr).

# LATERAL ACCEL

in/sec<sup>2</sup> (1 g = 386 in/sec<sup>2</sup>)



## PARAMETER CHANGES

Figure 31. Parameter sensitivity of roll stability limit of simulated five-axle dump truck.

4.2.3 The Effect of Changes of Track Width to C.G. Height. Section 4.1 indicated that the most fundamental parameter which affects the roll stability limit is the ratio of 1/2 track width to c.g. height. To demonstrate the effect of this parameter, two runs, one with the sprung mass c.g. raised five inches (.127 m) and one with the c.g. lowered five inches (.127 m), were made. This represents changes of about  $\pm 6\%$  in the c.g. height of the sprung mass.

Figure 31 indicates that raising the c.g. lowered the rollover limit about 8% and lowering the c.g. raised the rollover limit about 9%. The sensitivity to the track width to c.g. height parameter accounts for  $\pm 6\%$  (resulting directly from the percentage change of the parameter as it would affect a rigid vehicle). The excesses indicate the secondary effect of changes in c.g. height as discussed in Section 4.1.2 and predicted by Equation (4.2).

4.2.4 The Effect of Changes in Total Suspension and Tire Stiffness. The next effect to be examined is that of the total suspension and tire roll stiffness. The lumped suspension model of Section 4.1 indicates that increases in this parameter should increase the roll stability limit. This effect is shown clearly in Figure 31 by the bars marked "Stiffness x 1.5" and "Stiffness x .5." These bars indicate the results from increasing and decreasing, respectively, all tire and suspension stiffness by 50%. These are rather large changes in stiffness, but they produce substantial changes in the roll stability limit, namely, + 7% and - 13%.

4.2.5 The Effect of Changes in Roll Center Height. Two bars of Figure 31 show the influence of changes in suspension roll center height. They illustrate the effect of raising and lowering all suspension roll center heights by five inches (.127 m) (probably within attainable limits).

The effect of these changes is remarkably strong. For this example vehicle, note that a five-inch (0.127-m) increase in the roll center heights

is about 50% as effective in increasing the roll stability limit as is a five-inch (.127-m) decrease in the very basic c.g. parameter.

It should be noted that increasing roll center height improves roll stability by decreasing the amount of body roll prior to rollover. Thus, the strength of this effect will be dependent on the nature of other parameters affecting roll. For example, a vehicle equipped with very rigid suspensions would not roll much prior to the limit. Thus, changes in roll center height of such a vehicle would not have as strong an effect. On the other hand, vehicles with a low rollover threshold generally will have substantial roll prior to the limit. Thus, roll center height adjustments can generally be expected to be effective for "problem" vehicles.

#### 4.2.6 The Effect of Changes in Selected Suspension Stiffnesses.

This general topic will be considered under several headings dealing with "stiff" and "soft" axles, suspension lash, and suspension location.

"Stiff" and "Soft" Axles - As noted above, axle 5 is the only axle of this vehicle which is well within the "stiff" axle classification. Two runs are shown in Figure 31 to illustrate the effects of changing the stiffness of this axle. In one run, labeled "Axle 5 Stiffer," the auxiliary roll stiffness of axle 5 was increased threefold, from 100,000 to 300,000 in-lb/deg (11,300 to 33,900 Nm/deg). As indicated by Figure 31, since this axle was already "stiff," virtually no change in rollover limit was obtained.

In the second run, labeled "Axle 5 Softer," the auxiliary roll stiffness of the axle was decreased to 50,000 in-lb/deg (5,650 Nm/deg). This is about the maximum decrease which still leaves axle 5 in the "stiff" classification. Again, Figure 31 shows that the change has no effect on roll stability limit.

Axle 2 is also a "stiff axle," but is relatively close to the "stiff"/"soft" boundary. In two additional runs, the auxiliary roll stiffness of axles 2 and 5 were each increased and decreased by 50,000 in-lb/deg (5,650 Nm/deg), respectively. As expected, when these "stiff" axles are stiffened further, virtually no gain is found. But when they are softened, axle 2 crosses the boundary and becomes a "soft" axle. The loss of stiffness in this "soft" axle then leads to a decreased roll stability limit.

Axle 1 is the only axle of the vehicle which is "soft" by quite a margin. To show the effect of changing the stiffness of a "soft" axle, an auxiliary roll stiffness of 50,000 in-lb/deg (5,650 Nm/deg) was added to this axle. As discussed in Section 4.2, this stiffening of a "soft" axle improves the vehicle roll stability.

On the baseline vehicle, axle 3 is also a "soft" axle, but is quite close in stiffness to axle 4, a "stiff" axle. In another run, the stiffness of both axle 1 and axle 3 was increased by adding 50,000 in-lb/deg (5,650 Nm/deg) of auxiliary roll stiffness to both axles. Figure 31 indicates that this has nearly the identical effect as stiffening axle 1 only. The reason, of course, is that with added stiffness, axle 3 moves past axle 4 into the "stiff" classification almost immediately. Further stiffening of this now "stiff" axle has almost no effect, as predicted.

Unfortunately, in practice, substantial increases in the effective roll stiffness of the front axle are difficult. Front-axle spring rates are, of course, influential with respect to ride quality and stiffer rates may be undesirable for this reason. Also, the torsional compliance of commercial vehicle frames makes it difficult to effectively utilize front suspension roll stiffness [5].

As another example, the stiffness of axles 2, 3, and 4 was increased simultaneously. These three axles are the three which are closely grouped in stiffness and define the boundary between the "stiff" and "soft" class. Increasing the stiffness of all of them shifts the boundary and allows each increase in stiffness to be effective. Figure 31 shows the resulting influence on the roll stability limit.

Finally, in one run, the stiffness of axle 5 was "reduced to the limit," i.e., to zero, by "lifting" this tag axle. This obviously moves axle 5 into the "soft" axle class. Of course, it also causes other effects, particularly the redistribution of vertical load, but it does represent a real-world option which is not uncommon for such vehicles when operated under "in-town" conditions. The bar marked "Axle 5 Lifted" indicates the serious consequences with respect to roll stability of radically lowering the stiffness of a "stiff" axle.

Suspension Lash - Axles 3 and 4 of the baseline vehicle each have .75 inches (1.9 cm) of suspension spring lash. In one simulation run, this lash was eliminated; in another it was doubled to values of 1.5 inches (3.8 cm). Figure 31 indicates the results with the bars labeled "No Lash" and "1.5 (3.8 cm) Lash."

As noted in Section 4.2, lowering lash is equivalent to increasing stiffness and vice versa. Since axle 4 is a "stiff" axle and axle 3 lies near the "soft"/"stiff" boundary, stiffening these suspensions by eliminating lash is rather ineffectual. On the other hand, softening these suspensions by increasing lash does degrade the rollover limit. Axle 3 is a "soft" axle, and so its reduction in stiffness is a degrading factor. To the extent that axle 4 moves below the previous effective stiffness of axle 3, its reduction is also a degrading factor.

Note that if all four of the rear suspensions had lash, eliminating their lash (at least for 2, 3, and 4) would be effective since the combined change would drive the "soft"/"stiff" boundary upward. Also, in the higher speed example to follow, axles 3 and 4 both lie near the "soft"/"stiff" boundary. For the higher speed condition of this vehicle, then, eliminating lash would also be more effective.

The Effect of Axle Location - Figure 30 indicates that in "effective stiffness," axle 5 is significantly stiffer than axle 2. As mentioned above, however, these two axles are identical air-suspended tag axles, each carrying 13,000 lbs (57,800 N). The significant difference between the two is their longitudinal position on the vehicle. Axle 2 is quite centrally located while axle 5 is located very rearward. Thus, in the low-speed turn which is simulated, axle 2 is subjected to a much lower side force than is axle 5. (For example, at 100 in/sec (.26 g) lateral acceleration, axle 2 generates 1,644 lbs (7,300 N) lateral tire force, while axle 5 generates 5,729 lbs (25,500 N).) As predicted by Equation (4.7), axle 2 is then seen as having a much lower "effective stiffness."

To see that increased speed reduces this effect, consider Figure 32. This figure is like Figure 30 in that it shows light-side wheel load for

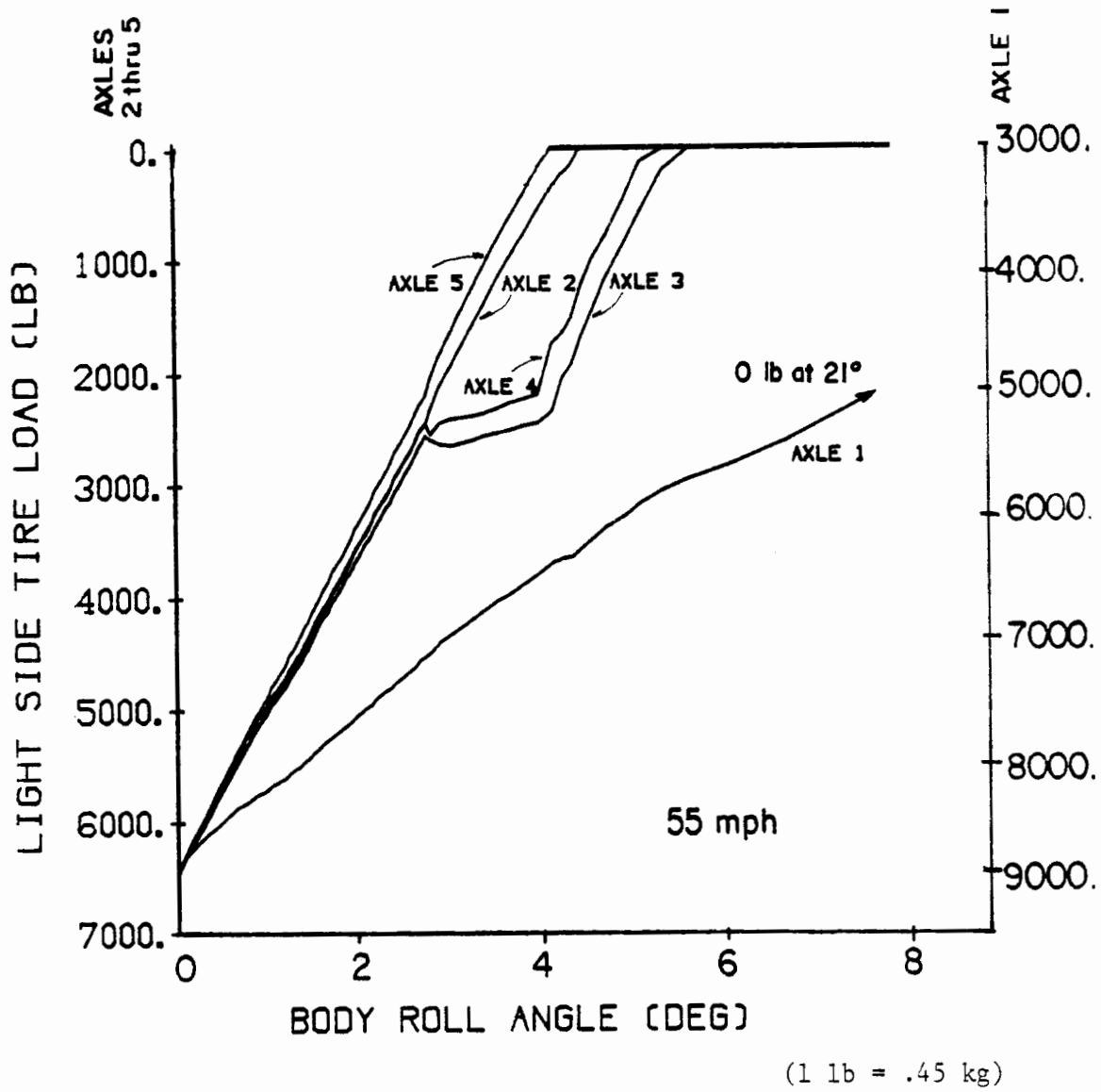


Figure 32. Light-side tire load response to roll of simulated five-axle dump truck at 55 mph (88.5 k/hr).

each axle of the baseline vehicle as a function of body roll, but the forward velocity of this run is 81 ft/sec (55 mph) (88.5 k/hr). In this case, we see that axle 2 has moved strongly into the "stiff" axle class and appears nearly identical to axle 5. The reason for this is that all rear axles are operating at more nearly equal tire lateral force. (At 100 in/sec<sup>2</sup> (.26 g), axle 5 generates the largest side force of 3,553 lbs (15,800 N) and axle 2 generates the smallest at 2,483 lbs (11,045 N). Thus, the influence of longitudinal axle position is sharply reduced.

In summary, the baseline vehicle considered here is a relatively unstable vehicle in roll, having a roll stability limit of .37 g's. The fundamental reason for this low limit is the vehicle's relatively high c.g. The vehicle's ratio of track width to c.g. height establishes its roll stability limit at a theoretical limit of .5 g's (which would result if the vehicle were rigid in roll). Practical methods of recouping the stability loss (from .5 to .37 g's, suffered due to the vehicle's ability to roll) include:

- 1) Stiffening suspensions in roll
  - particularly suspensions 2, 3, and 4
  - through eliminating lash in suspensions 3 and 4
  - through stiffening springs or adding auxiliary roll stiffness
- 2) Raising the suspension roll center

Effective use of these alterations could be expected to regain 1/3 of the stability loss below .5 g. Of course, the most effective route to gaining roll stability would be to widen the vehicle's track or reduce its c.g. height, presumably requiring changes in chassis and/or body design.



## CHAPTER 5

### YAW/ROLL STABILITY OF STRAIGHT TRUCKS

This chapter is concerned with the issue of yaw divergence or directional instability as it relates specifically to the heavy truck. The example, or "problem" class of vehicles seen as being most susceptible to the directional instability discussed here is that particular group of straight trucks characterized primarily by high centers of gravity and listed under the Straight Truck category of Table 2. Examples of specific types of vehicles lying within this vehicle class would include certain types of cement mixers, dump trucks, and trash haulers—all having relatively high centers of gravity when compared to the "average" straight truck population.

The principal aim of this chapter is to explain and illustrate, through use of simplified analysis and computer simulation, the relationship between directional (yaw) stability and roll stability of heavy trucks with high centers of gravity. The emphasis here, as will become clear, is not directed toward roll-related dynamics and identification of roll thresholds for such vehicles, but instead, toward an explanation of yaw divergence, per se, and its potential for precipitating roll instability (rollover) at highway speeds and low levels of lateral acceleration. That is, yaw divergence which precedes and subsequently causes rollover is the phenomenon being examined here.

Yaw instability, or yaw divergence as it is frequently referred to, manifests itself as the tendency of a vehicle's heading to diverge, or increasingly point away, from the direction of travel. Terms such as "spinout" for motor cars, and "jackknife" for articulated vehicles, are commonly used to describe this general behavior. Although yaw divergence phenomena are frequently encountered with vehicles during braking maneuvers, the concern and attention here is directed only at the occurrence of yaw divergence during steady turning at relatively high forward speeds. As will be shown, the principal mechanism responsible for the onset of yaw divergence in heavy trucks is the combination of: (1) cornering stiffness

sensitivity of truck tires to vertical load, (2) fore/aft roll stiffness distribution, (3) high center of gravity heights, and (4) elevated speeds. Moderate reductions in items (1), (3), or (4) or increases in (2) can have a significant effect in improving the directional stability of such vehicles at elevated speeds.

### 5.1 Background and Related Work

The previous experimental and computer simulation work of Ervin, et al. [5,6] in studying the yaw stability of tractor-semitrailer vehicles during cornering has shown that tractor yaw instability (jackknifing) can occur well below the rollover threshold for certain vehicles. Modification of such vehicles' fore/aft roll stiffness distribution (through torsional frame stiffening and employment of a front-axle roll bar) was shown experimentally, and supported by computer simulation results, to eliminate the occurrence of tractor yaw divergence. In addition, other vehicle parameters were systematically examined to evaluate their influence in increasing or decreasing the potential for tractor yaw instability during steady turning.

A significant analytical by-product of the work of Ervin, et al., was the introduction of a graphical plot herein termed the "non-linear handling diagram." The prior analytical work of Pacjeka [7] in defining a so-called "handling diagram" and its subsequent adaptation by Ervin, et al., in the above study to the non-linear steady turning response of tractor-semitrailers, has led to the use of the analogous "non-linear handling diagram" for analyzing the directional stability of such vehicles.

A recent paper by Segel and Ervin [8] concerned with the influence of tire factors on truck stability, and based largely upon the results of a previous research project [9], concluded that: (1) certain heavy truck vehicles were quite capable of exhibiting yaw divergent behavior during steady turning (high speed) at a severity level far below that needed to achieve limit response of passenger cars and (2) that a marked degradation in directional controllability can accrue well in advance of the maneuver severity required to roll over the vehicle. Furthermore, Segel and Ervin

concluded that the typically low values of fore/aft roll stiffness distribution\* was the primary mechanism serving to aggravate the yaw stability of the heavy truck, and that the commercial vehicle tire was seen to exhibit certain shear-force-related properties which cause trucks to respond differently to parametric variations than typical passenger cars.

A strong relationship exists between the above work and that discussed in this chapter. The principal differences between the two lie with: (1) examination of two different and distinct heavy truck tire properties, and (2) the level of severity of the steady-turning maneuver. The referenced study examined the development of yaw divergent behavior as the cornering limit of the vehicle was approached and demonstrated dependence of such behavior upon the non-linear shear force properties of the tires. In contrast, this study examines the variation of tire cornering stiffness with vertical load—a low level, linear regime tire property—and demonstrates how it, acting in concert with load transfer mechanisms, also can precipitate yaw divergence, but at levels of lateral acceleration considerably below those identified in the above study. As will be seen, the prerequisites for the latter behavior to occur are: (1) as in the above study, significant fore/aft roll distribution margins must be present, and (2) in contrast with the referenced study, the vehicle must possess a relatively high center of gravity.

A number of investigators have studied the static and steady-turning roll stability of articulated vehicles [10,11,12,13]. More recently, the work of Mallikarjunarao and Ervin [4,14] in comprehensively examining the individual mechanical elements that contribute to the rollover event has led to a good understanding and explanation of the articulated vehicle rollover process. Since most of these findings are seen as equally applicable to the straight truck vehicle during steady turning, this chapter simply accepts, and elects not to reiterate herein, those same results for the straight truck vehicle. Chapter 4 of this report has addressed the

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\*That is, commercial vehicles typically have low values of front-axle roll stiffness and higher values of rear-axle roll stiffness.

mechanics of the roll process in greater detail. Therefore, the view adopted here with regard to heavy truck roll stability under steady-turning conditions, is that a rollover threshold exists and is expressible in terms of a lateral acceleration level (e.g., g-units), above which the vehicle rolls over; below which it does not. The rollover event, whether precipitated by yaw divergence at higher speeds or encountered without yaw divergence at lower speeds, is viewed as defining the absolute or maximum possible maneuvering regime. The results of computer simulation runs encountering rollover serve to define the absolute maneuvering range, or rollover threshold.

To summarize, previous experimental and analytical research that has addressed the yaw stability issue in commercial vehicles has focused largely upon the development of yaw divergence within the subrollover-limit maneuvering range. Furthermore, the vehicles examined possessed center of gravity heights that would be classified as moderate. In contrast, the computer-based results discussed in this chapter focus on the development of yaw divergence and/or roll instability for trucks possessing high centers of gravity, operating at low levels of lateral acceleration. The principal mechanism responsible for such behavior is shown to be the non-linear variation of truck tire cornering stiffness with vertical load, acting in conjunction with typical, heavy truck fore/aft roll stiffness distributions.

Chapter 5 begins with a brief discussion of yaw/roll stability concepts, followed by results of a simplified analysis that assumes typical truck tire cornering stiffness variation with vertical load. Results from a comprehensive computer simulation, which represents the vehicle dynamic behavior in greater detail, are then presented, showing essential qualitative agreement with the results of the preceding simplified analysis. The straight truck vehicles of Table 2 serve as the specific subjects for most of these studies. Finally, the topic of closed-loop driver control of yaw-divergent vehicles, as represented by a computer model, is introduced and discussed.

## 5.2 Yaw/Roll Stability

One of the purposes of this chapter is to provide a better understanding of how yaw divergence can occur for heavy trucks and its related role as a precipitant of roll instability. That is, the "de-coupling" or separate treatment of roll stability and yaw stability that is often appropriate for motor cars and many commercial vehicles is not particularly appropriate with the class of high c.g. heavy truck vehicles being examined here (Table 2). Yaw divergence in such heavy trucks inevitably leads to rollover, unless the tire/road friction coupling is less than the relatively low rollover threshold of these vehicles. The general nature of the yaw instability can be described as an ever tightening spiral of path response for a fixed steer input. The resulting build-up of lateral acceleration then causes rollover. Even though passenger cars and certain tractor-semitrailers can frequently exhibit yaw divergent behavior ("spinouts" and "jackknives") on high friction surfaces without experiencing rollover as well, the high c.g. truck vehicle, in general, cannot. The development of ever increasing sideslip during yaw instability precipitates a corresponding increase in lateral acceleration which can very quickly lead to an unstable roll response. In fact, many of the same physical mechanisms that define, to first order, the roll stability of such vehicles (e.g., c.g. height, track, fore/aft roll stiffness distribution) also play an important role in defining, with tire force vertical load sensitivity, the yaw stability of these vehicles.

One way of illustrating the yaw and roll stability relationship is to plot, for a given vehicle, its yaw divergent or "critical" velocity as a function of lateral acceleration (see Figure 33, line A). Also shown on the same plot is a vertical line, B, defining the rollover threshold for the given vehicle. The yaw/roll stability regime for this vehicle could then be defined as that velocity/lateral acceleration area lying to the left of the combined curves. Any combination of vehicle velocity and acceleration lying to the right of the combined curve will produce an unstable roll and/or yaw response.

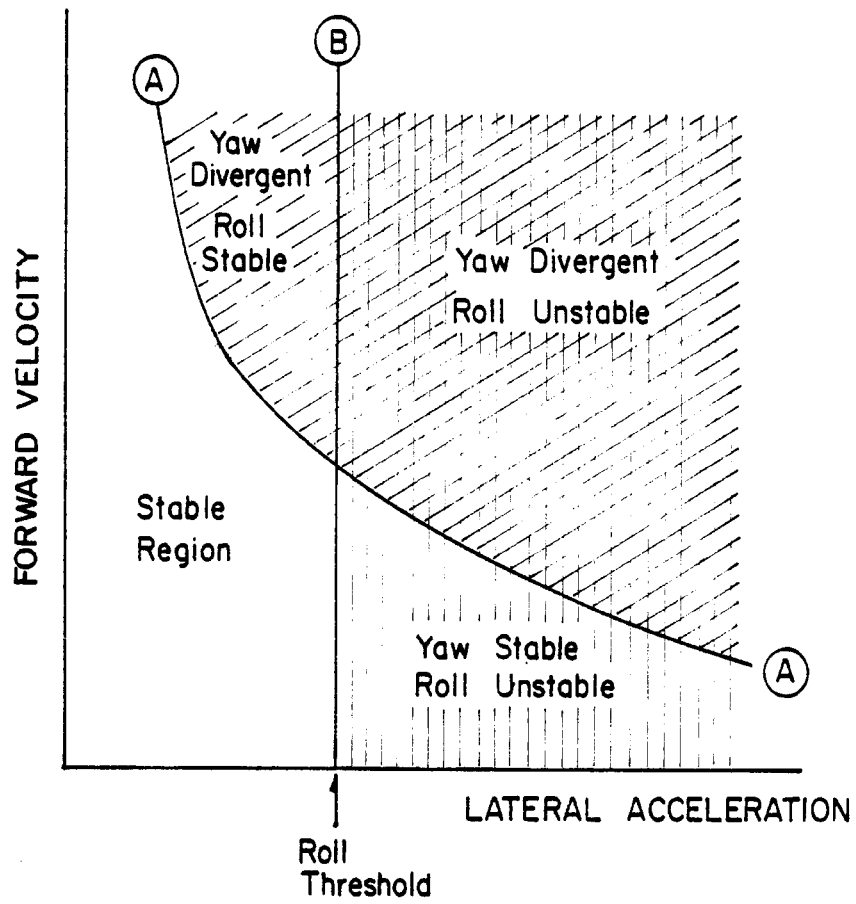


Figure 33. Yaw/roll stability plot.

The yaw/roll stability plot illustrated in Figure 33 is comprised of four distinct regions: (1) the stable region, (2) the yaw-stable/roll-unstable region, (3) the yaw-divergent/roll-stable region, and (4) the yaw-divergent/roll unstable region. We see that at low speeds and increasing levels of lateral acceleration (tighter and tighter low-speed turning), the principal stability concern is that of rollover. However, at elevated speeds, as lateral acceleration increases, the principal stability concern is yaw divergence prior to reaching the rollover threshold. The danger of yaw divergence, if not attended to by corrective driver steering control and/or reduced speed, is that it will lead to a further increase in a vehicle's lateral acceleration and thereby precipitate rollover.

The yaw/roll stability plot is seen as a convenient means for displaying absolute stability levels for a given vehicle, as well as a means for readily illustrating sensitivities of these stability boundaries to variations in typical vehicle parameters. That is, it provides a method for graphically describing regions of stability/instability in terms of easily understandable quantities—speed and lateral acceleration. The next section demonstrates how such a plot can be used, in combination with simplified analyses, to study yaw divergence sensitivity to variations in vehicle parameters. The subsequent section, concerned with more realistic computer simulation predictions, employs such a plot in an absolute sense, to summarize results of computer-based studies illustrating the interaction/relationship between yaw stability and roll stability for the particular straight truck vehicles listed in Table 2.

### 5.3 Simplified Analysis of Yaw Stability

This section presents results of simplified calculations of yaw stability based on the analysis presented in Appendices A and B. An "average" vehicle representation for the class of high c.g. trucks under study was selected to serve as the baseline calculation. The "average" high c.g. vehicle is defined here as having the average c.g. height, average suspension loading, average roll compliance, etc., for those straight truck vehicles listed in Table 2. Variations were then made in several of the vehicle parameters in order to reveal sensitivities of yaw stability

boundaries to changes in each of the examined parameters. The calculations shown below make use of Equation (B-5) and illustrate the relationship between critical velocity and lateral acceleration operating level. Note that this analysis is conservative in its calculation of yaw divergence, and that its principal aim is to identify sensitivities of yaw stability boundaries to variations in vehicle parameters.

The "average" high c.g. straight truck vehicle was defined as having the following baseline properties for the purpose of these simplified calculations:

185-inch wheelbase (4.699 m)

18,000-lb front axle load (80,071 N)

48,000-lb rear suspension load shared by three  
axles, (213,523 N) dual tires, 50-inch axle  
spacing (1.27 m)

10:1 front-to-rear roll compliance

80-inch front track (2.032 m)

72-inch rear track (1.829 m)

16.5x22.5 front tire

10Fx20 rear tire

70-inch total c.g. height above ground (1.778 m)

and average cornering stiffness sensitivity to vertical load for the tires identified above.

Figure 34 shows a typical result from the simplified analysis for rear axle and c.g. height variations. It describes the sensitivity of the "average" baseline vehicle's yaw stability boundary to the removal or addition of the rearmost axle and to increases and decreases in c.g. height. Figures 35 and 36 show similar parameter sensitivity results from the simplified analysis. In a more complete portrayal of the vehicle dynamics, as for example in the computer simulation study of the next section, these boundaries will shift further to the left and display a more restrictive operating range of yaw stability. The results of Figures 34-36 indicate that, with respect to yaw stability, this class of vehicle benefits (and



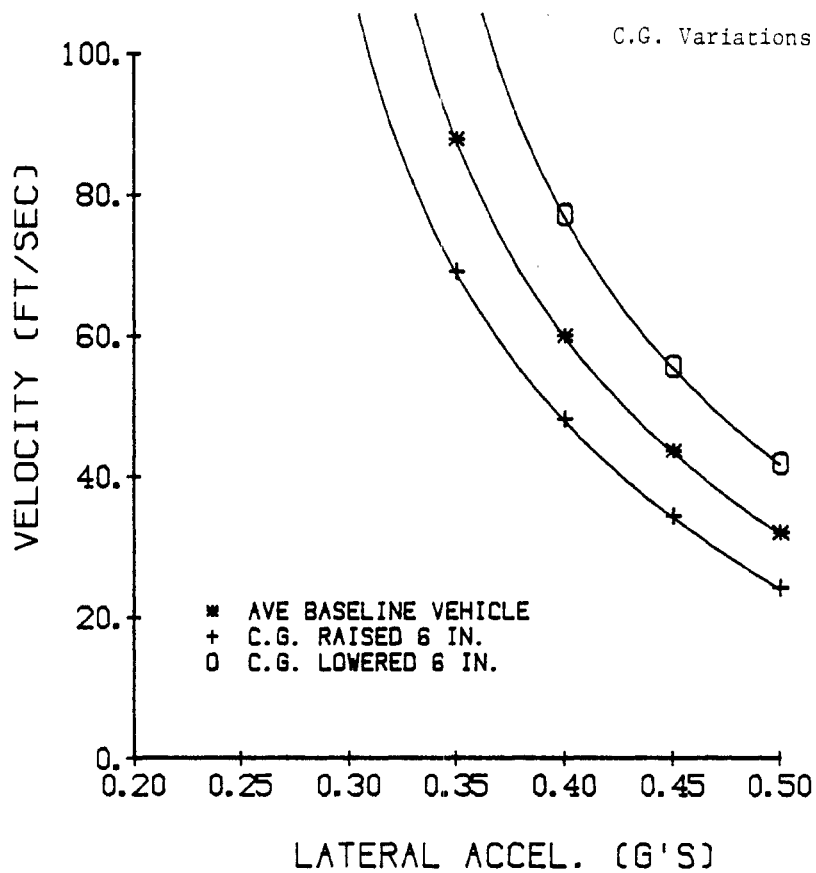
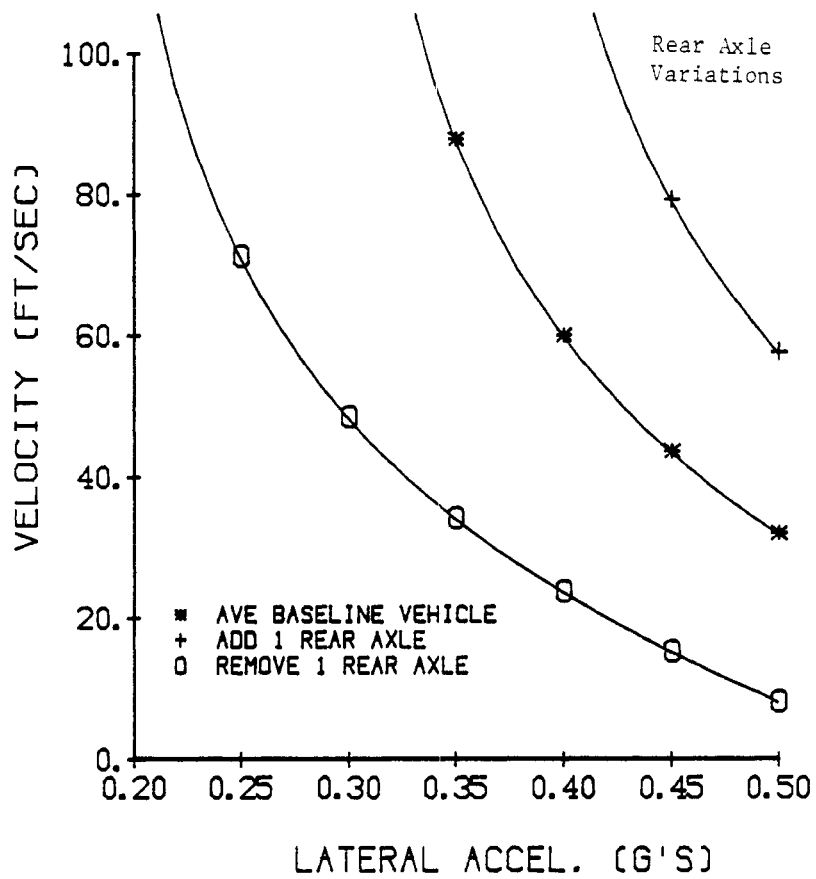


Figure 34. Sensitivity of yaw stability boundaries predicted by the simplified analysis (rear axle and c.g. height variations).

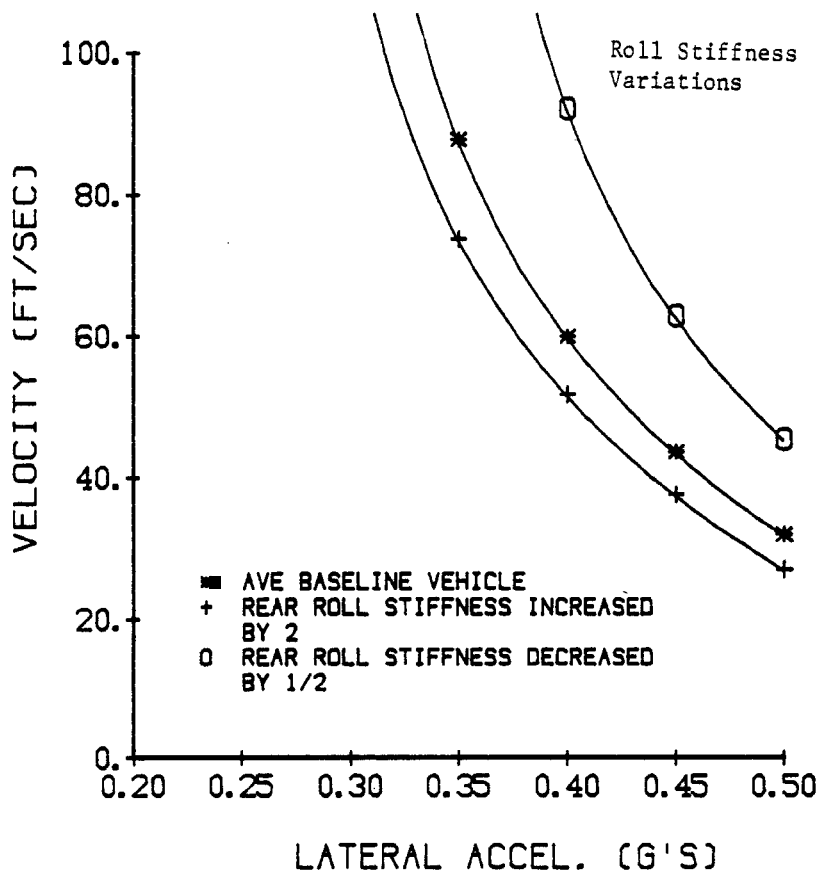
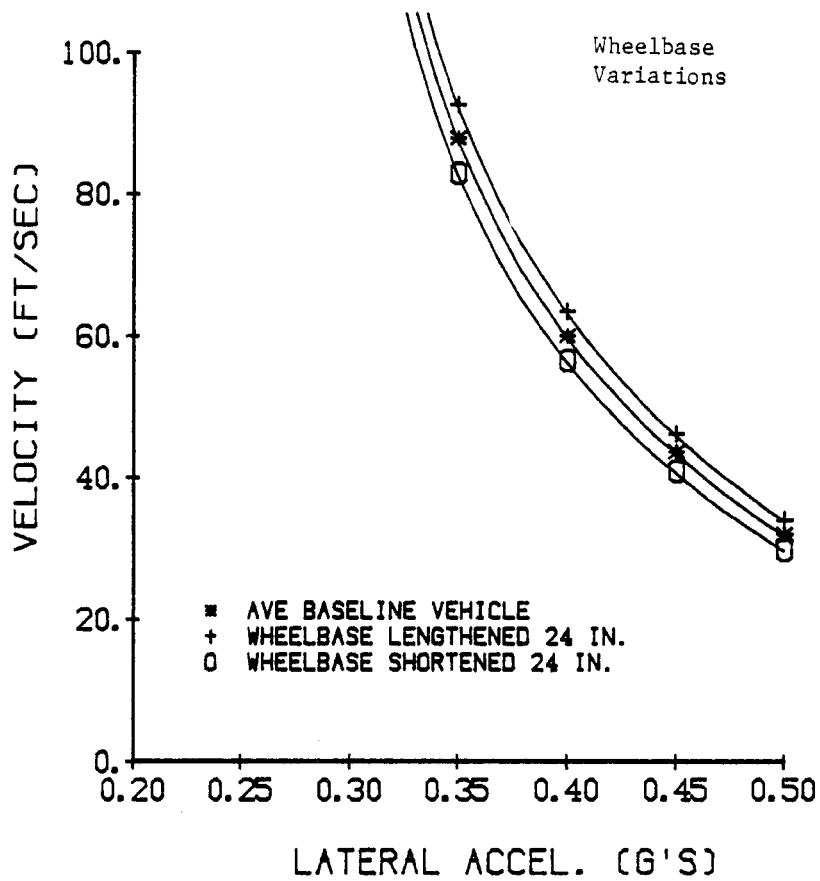


Figure 35. Sensitivity of yaw stability boundaries predicted by the simplified analysis (wheelbase and roll stiffness variations).

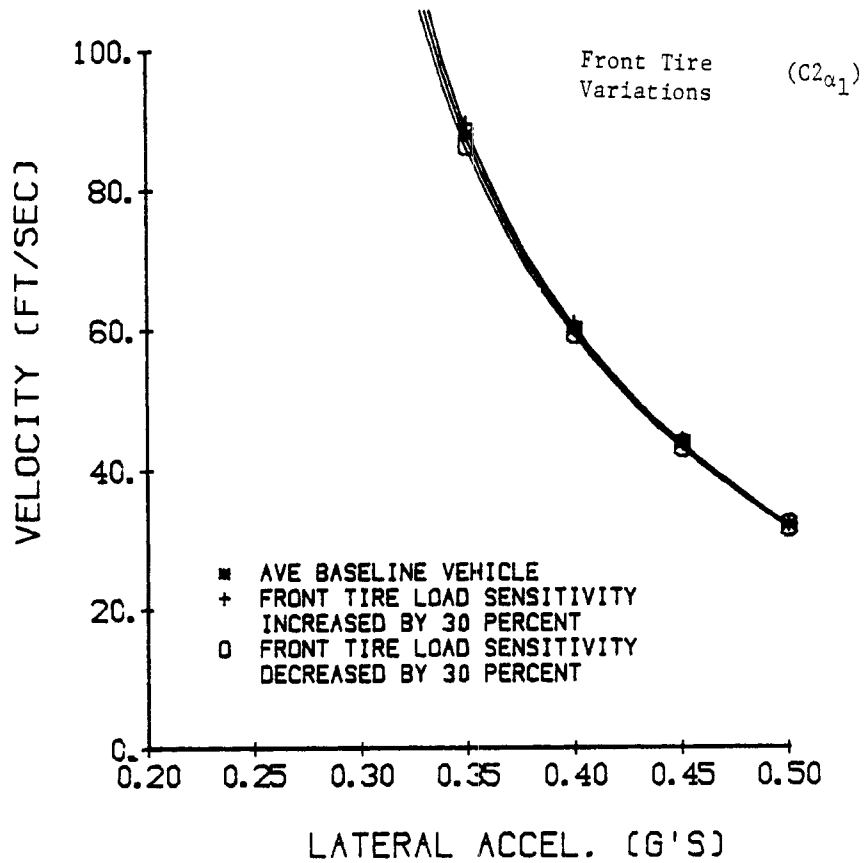
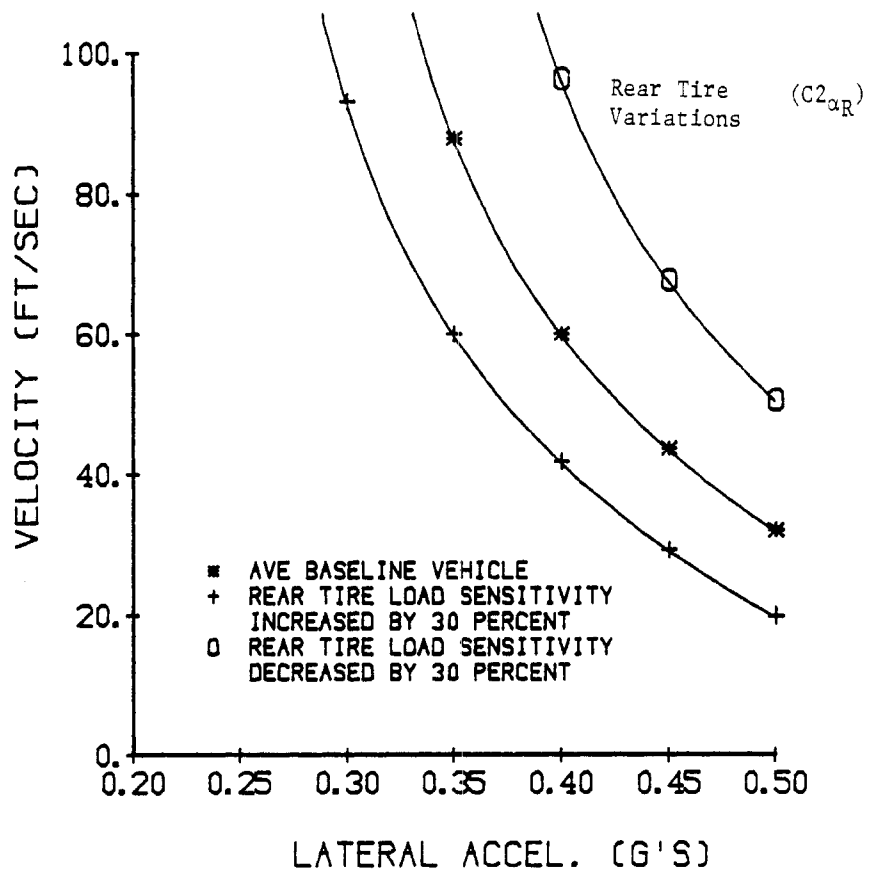


Figure 36. Sensitivity of yaw stability boundaries predicted by the simplified analysis (rear/front tire property variations,  $C2_{\alpha R}$ ,  $C2_{\alpha 1}$ ).

suffers) the most from: (a) addition (removal) of rear axles, (b) changes in c.g. height, and (c) variations in rear tire load sensitivity to vertical load. Greater sharing of lateral load transfer by the front axle also indicates significant improvement in yaw stability.

As indicated, the parameter sensitivity results shown in Figures 34-36 are based upon the simplified analysis contained in Appendices A and B. This analysis extends the classical steady-turning equation to include the effect of side-to-side load transfer upon the cornering stiffness of each tire. The net result is a more rapid loss of cornering stiffness at the rear of the vehicle, relative to the front, due to typical fore/aft roll stiffness distributions of heavy trucks, as steady turn lateral acceleration levels are increased.

The classical steady-turning equation can be written, with the primed quantities denoting dependence upon lateral acceleration, as:

$$\delta = \frac{\lambda'_e}{R} + K' a_y \quad (5.1)$$

where

$\delta$  is front wheel angle

R is path radius

$a_y$  is lateral acceleration

$\lambda'_e$  is the so-called effective wheelbase for a vehicle with tandem rear suspension, but here, dependent upon lateral acceleration

$K'$  is the classical understeer gradient, K, evaluated with lateral acceleration-dependent cornering stiffnesses  
(see Appendix A)

The calculation of critical velocity,  $U_c$ , or yaw stability boundaries appearing in Figures 34-36 is based upon Equation (B.5), or

$$U_c = \left[ \frac{-\lambda'_e - \frac{\partial \lambda'_e}{\partial a_y} \cdot a_y}{K' + \frac{\partial K'}{\partial a_y} \cdot a_y} \right]^{1/2} \quad (5.2)$$

where  $\lambda'_e$  and  $K'$  are defined as above;  $\partial \lambda'_e / \partial a_y$  and  $\partial K' / \partial a_y$  represent the variations of  $\lambda'_e$  and  $K'$  with respect to lateral acceleration (see Appendix B). Noting that the denominator of the critical velocity expression in Equation (5.2) is simply the lateral acceleration-dependent understeer gradient,  $S$ , Equation (5.2) can be expressed as:

$$U_c = \left[ \frac{-\lambda'_e - \frac{\partial \lambda'_e}{\partial a_y} \cdot a_y}{S} \right]^{1/2} \quad (5.3)$$

If the quantity,  $S$ , is plotted as a function of lateral acceleration for some representative high c.g. heavy truck, we would obtain a graph of the form shown in Figure 37. We see that at low levels of lateral acceleration the quantity  $S$  has a positive value and falls off in a rapid, quadratic-like fashion as lateral acceleration increases above 0.2-0.3 g. (The transition of this quantity from positive to negative values as lateral acceleration increases is analogous to the polarity of the slope of the non-linear handling diagram in reflecting the transition from understeer to oversteer.) Also shown in this figure are three additional lines representing various elements comprising the quantity  $S$ . Two of these elements, "Steering System Compliance" and "Roll Steer," appear as constant quantities on this plot; the one labeled "Tires" is the element contributing to the quadratic-like reduction in  $S$  as lateral acceleration is increased. (A value of 3 deg/g was assumed for these constant quantities in the calculations appearing in Figures 34-36.) The thrust of this figure illustrates, quite convincingly, that the constant-like understeer mechanisms of steering system compliance and roll steer are very quickly overwhelmed by the quadratic-like oversteer contributions deriving from the net loss of rear tire cornering stiffness as lateral acceleration begins exceeding 0.2-0.3 g for such vehicles. (As lateral acceleration increases further, additional lateral tire force losses deriving from shear-force-related properties of

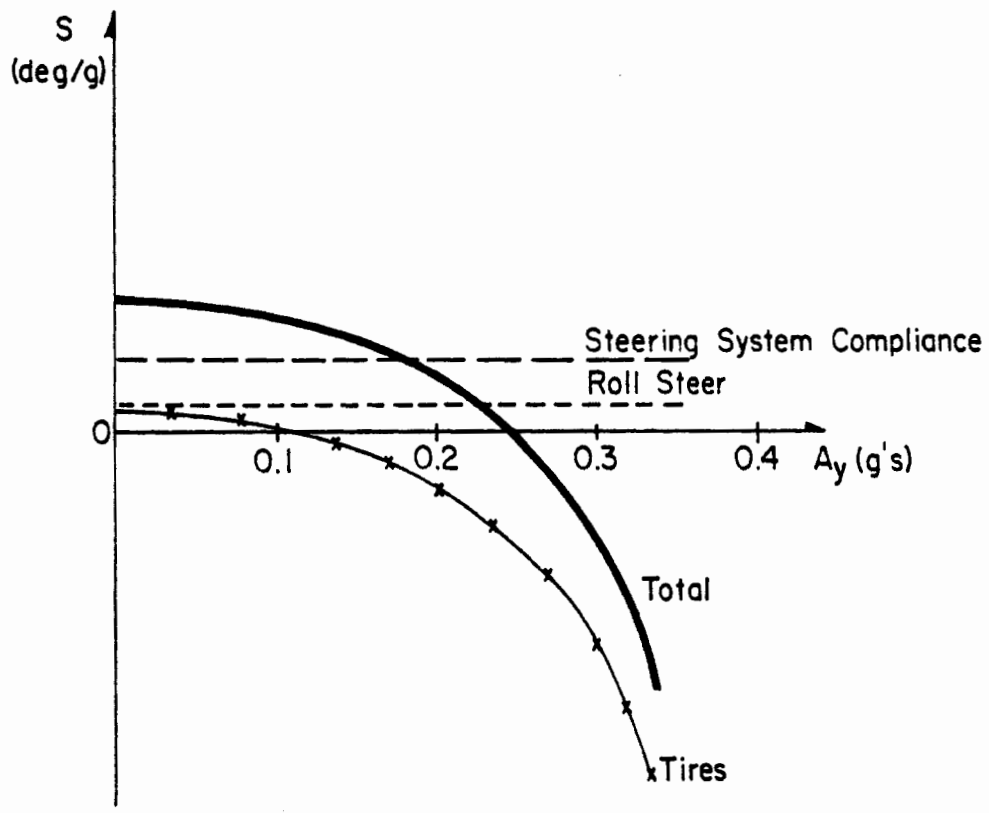


Figure 37. The influence of lateral acceleration upon various understeer/oversteer-contributing mechanisms.

the truck tire, and not included here, would gradually supplement the cornering stiffness losses discussed above.) Without having similar quadratic-like contributions from other understeer mechanisms, the vehicle designer is left with the following means for improving the directional stability of such vehicles: (1) increasing fore/aft roll stiffness distributions through increased front suspension and/or frame roll-torsional stiffness, or (2) selection of tires whose cornering stiffness varies more linearly than others with increasing load (i.e., less curvature in the  $C_a$  versus load plot), or (3) addition of more tires or axles at the rear of the vehicle. Items (1) and (3) would also serve to improve the roll stability of the vehicle.

#### 5.4 Computer Simulation Study

The following section describes the results of the computer simulation study which examined the turning response of three similar, though distinct, heavy truck vehicles (see Table 2), as represented by a comprehensive computer model. The principal difference between this study and the simplified analysis of the previous section is in the level of detail defining the mechanics of the vehicle and its dynamic behavior. The computer model used here is the same model employed in Section 4.2 and is described in Appendix D.

5.4.1 Estimation of Vehicle Parameters. The computer model was used to simulate the turning response of the three basic straight truck vehicles described in Table. Each was assumed to possess mass distribution and geometric properties considered characteristic of this general class of high c.g. trucks (e.g., certain types of dump trucks, cement mixers, and front-end loading trash haulers, some of which are peculiar to certain states or regional areas). The parameter estimates used to represent this class of vehicles in the computer model were based upon simple field measurements and photographs of these types of vehicles. In addition, UMTRI's resource of vehicle parameters, which include representative suspension and tire data, were also used to estimate parameter values considered characteristic of various vehicle components.

Table 3 lists the general description of three trucks identified as Vehicle A, Vehicle B, and Vehicle C, which represent here the three straight truck vehicle configurations identified in Table 2. Figure 38 shows the cornering stiffness variation with vertical load assumed for both the front and rear tires. The tire data was taken from a previous study [9] in which various tire measurements were conducted on tires similar in size and construction to those commonly observed on vehicles of this class. The suspension rates shown in Table 3 are for the entire axle (left and right springs). Tag axles equipped with air suspensions were represented by low vertical spring rates (2K) and high auxiliary roll stiffnesses. Careful attention was given to assembling computer model data sets, upon which the vehicle descriptions in Table 3 are based, so as to avoid portraying unrealistic ranges of geometric and mass distribution properties. Hence, the following computer simulation predictions, corresponding to vehicles A, B, and C, should be viewed as representing a certain class of high c.g. trucks, and not applicable to trucks possessing moderate or low center of gravity heights.

5.4.2 Computer Simulation Results. The methodology, or computer-based technique, used for conducting the computer simulation study is described as follows: For a particular forward speed of the vehicle, a sequence of steady turns were conducted. Each steady turn during this sequence was slightly more severe than the one before due to an incremented steering angle. This process was continued until an instability developed, either in the form of rollover or yaw divergence. The forward velocity and lateral acceleration level at which the instability first occurred during each sequence was then recorded and used to define one point on the yaw/roll stability plot, similar to Figure 33. This process would then be repeated for several forward velocities until an adequate "mapping" of the yaw/roll stability boundary was achieved for each vehicle.

Figure 39 shows a representative time history result for Vehicle C, illustrating the yaw divergence phenomenon and how it leads to roll instability if the steering angle remains fixed. This simulation run was conducted at a forward speed of 55 mph (88.5 k/hr) and represents a velocity-lateral acceleration point, at about one second, which is well within the



Table 3. Straight Truck Parameters - Computer Simulation Study.

Vehicle	Sprung Mass C.G. Height	No. of Axles	Wheelbase*	Static Axle Loads (1b/1000)	Axle Suspension Rate (1b/in/1000)	Fore/Aft Roll** Compliance Ratio
A (Mixer)	72" (1.829 m)	5	196" (4.978 m)	18/9.6/18/18/6	4/2/15/15/2	12
B (Dump)	78" (1.981 m)	5	189" (4.801 m)	18/13/13/13/13	4/2/15/15/2	16
C (Refuse Packer)	82" (2.083 m)	3	173" (4.394 m)	18/20/20	4/15/15	7.5

\*Front axle to centerline of rear suspension

\*\*Including auxiliary axle roll stiffness

1 meter = 39.37 inches

1 Newton = .2248 lb

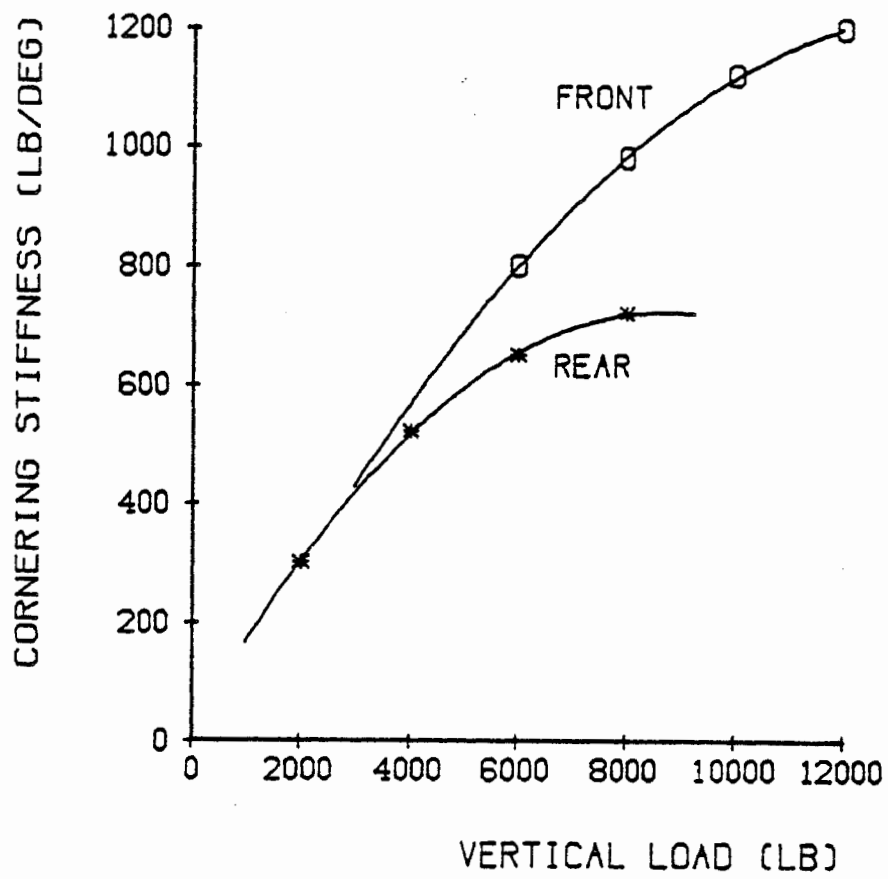


Figure 38. The variation of front and rear tire cornering stiffnesses with vertical load.

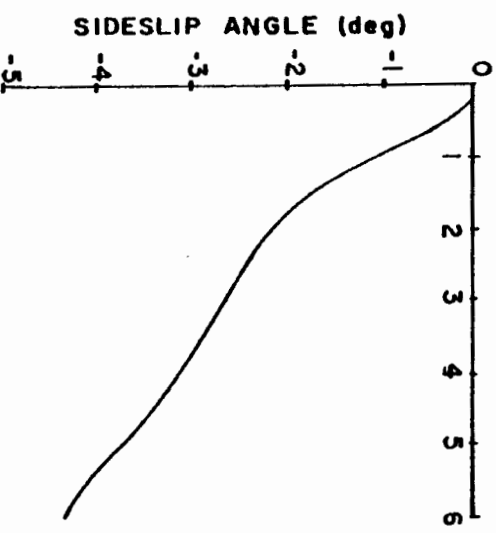
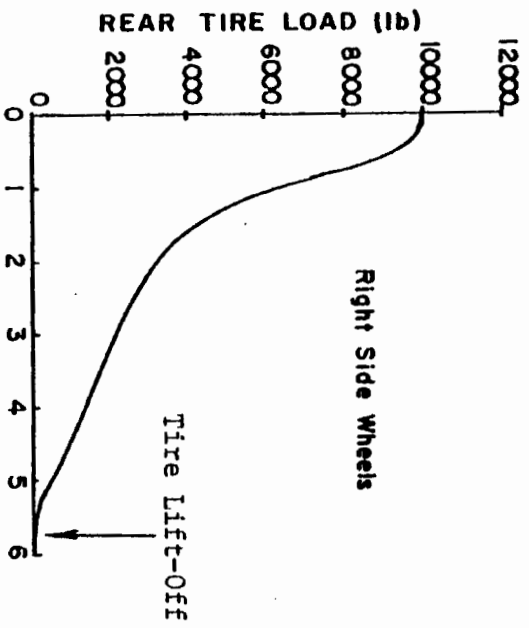
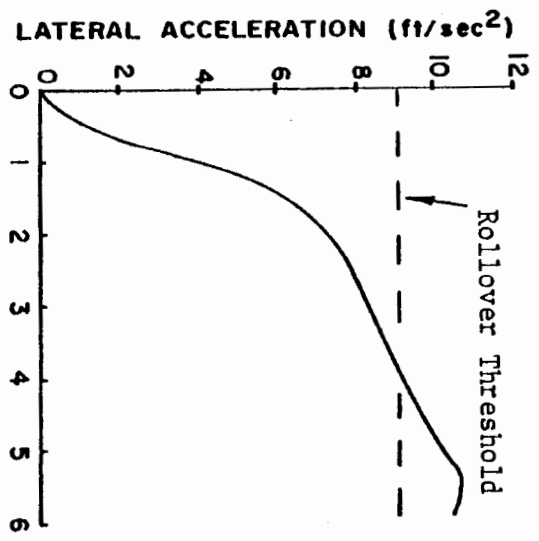
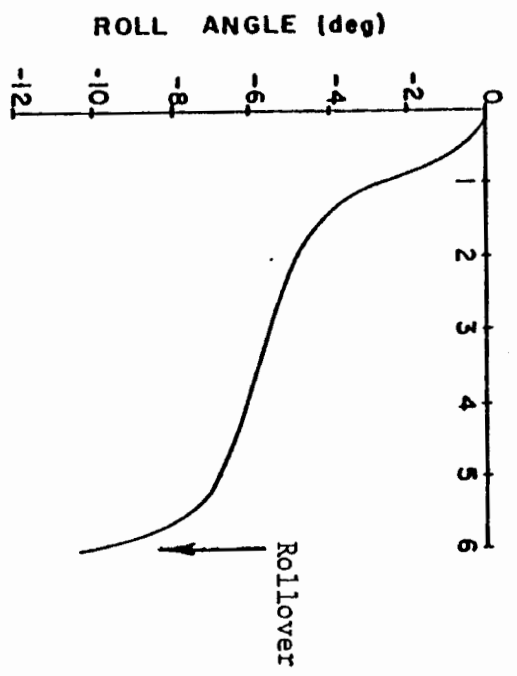
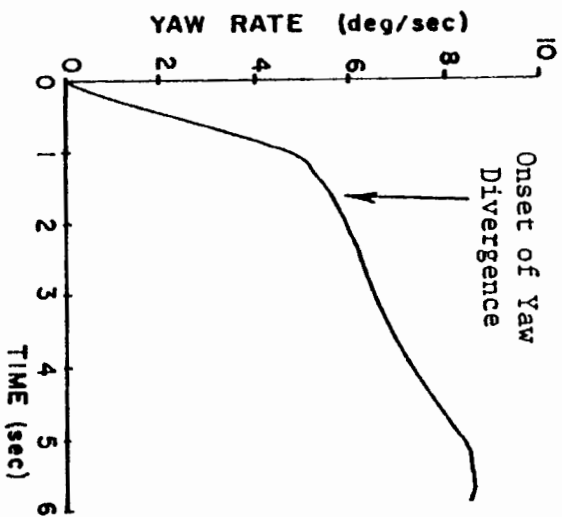
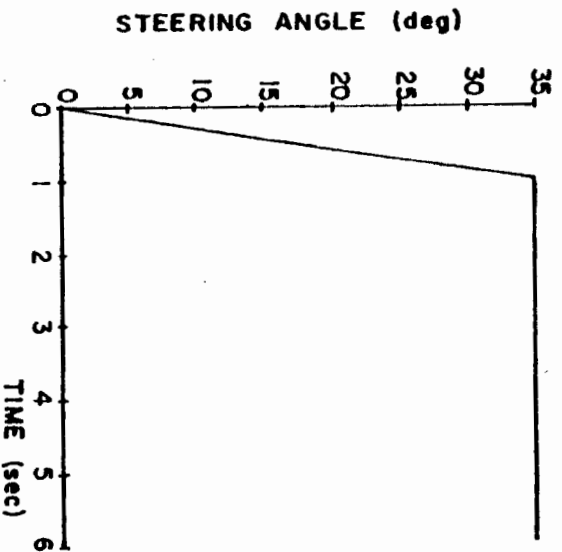


Figure 39. Illustration of yaw divergence/rollover for Vehicle C.

yaw-divergent/roll-stable region of Figure 33 and moves, for the remaining time, toward the roll stability threshold. Shortly after one second, the continuously increasing yaw rate response, characteristic of yaw divergence, has begun. We observe that after five seconds the roll angle is exceeding seven degrees and the rear inside tires are approaching lift-off. Note that the start of yaw divergence is clearly apparent within 1.5 seconds for this run at a relatively low lateral acceleration level of  $6 \text{ ft/sec}^2$  (0.2 g). In fact, turns as low in severity as 0.18 g's were shown to produce yaw divergence (requiring more time to develop) in this vehicle at a forward speed of 55 mph (88.5 k/hr).

Figure 40 summarizes, in the same manner as Figure 33, the results of the computer simulation study for Vehicles A, B, and C. The vertical lines represent the roll thresholds predicted for each vehicle; the lines curving upward to the left represent the yaw stability boundaries predicted for each vehicle. Also shown in this figure are two dashed lines which show the movement, leftward, of the yaw stability boundaries for Vehicles A and B as a result of having removed the rearmost axle from each vehicle.

The general range of yaw stability boundaries predicted by the computer simulation model in Figure 40 are observed to be somewhat more restrictive (located more to the left) than the similar range of yaw stability boundaries predicted by the simplified analysis of the previous section. This difference is due to: (1) the static side-to-side load transfer assumption used in the simplified analysis is especially conservative, and (2) additional, mild non-linearities present in the computer simulation tire representation at low lateral acceleration levels further contribute to the destabilization process. Recall, however, that the primary purpose of the simplified sensitivity analysis was to aid in understanding the development of yaw divergence, and also, to provide a simple means of examining the sensitivity of yaw stability boundaries to vehicle parameter variations. Furthermore, if Figure 40 is viewed as a parameter variation exercise (i.e., removal of one or more axles, variations in c.g. heights and fore/aft roll stiffness), some of which are occurring simultaneously, many of the sensitivities suggested by the simplified analysis of Section 5.3 are strongly reflected in the computer simulation results of this section.

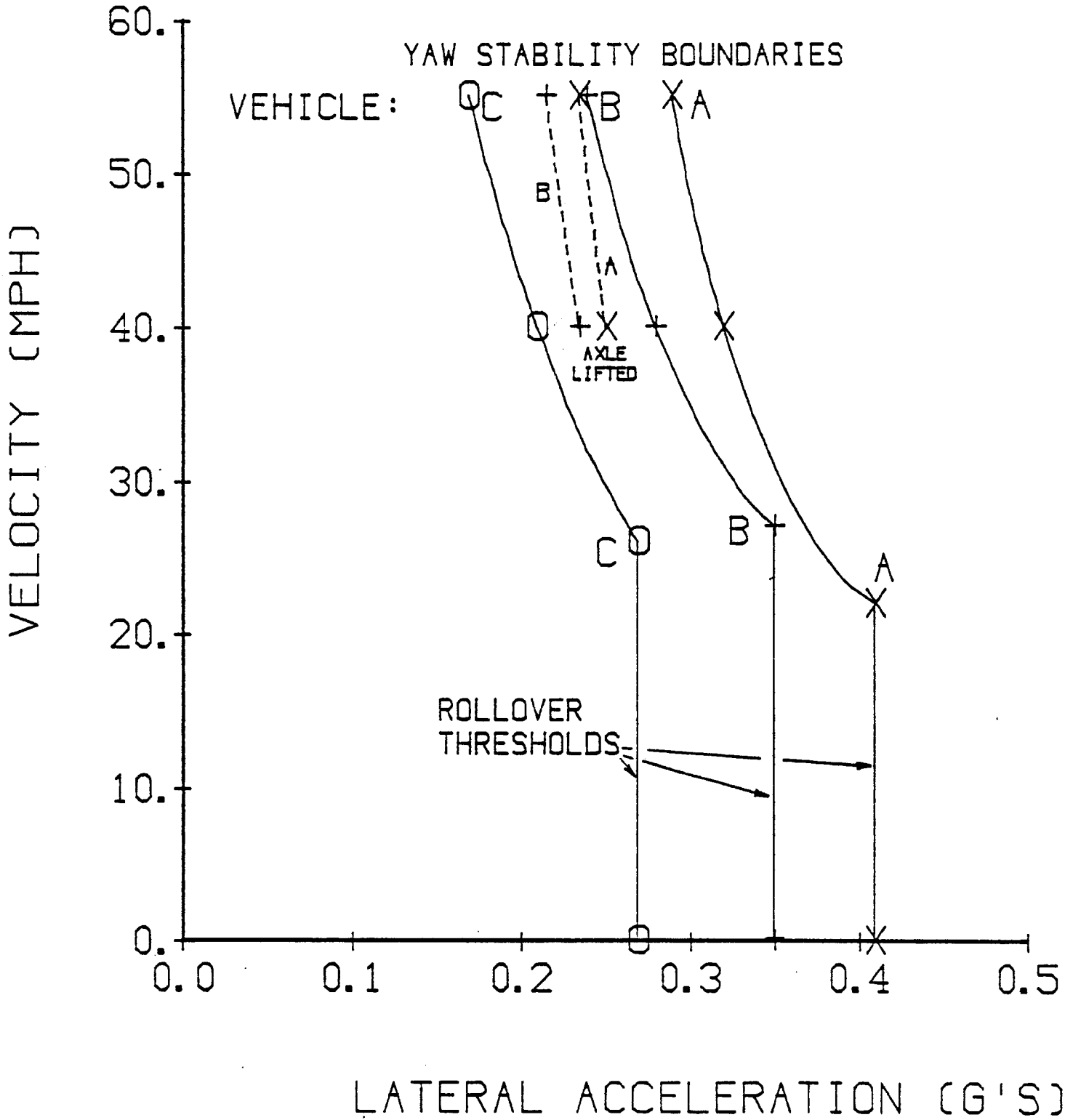


Figure 40. Summary of the yaw/roll stability findings from the computer simulation study.

The results of the computer simulation study presented in Figure 40 suggest that under steady-turning conditions:

1) The maximum lateral acceleration operating range for these vehicles is greatest at low speeds, becoming increasingly restricted as speed increases.

2) Increasing the number of rear axles and/or lowering the center of gravity height are effective methods for reducing the likelihood of yaw instabilities occurring at highway speeds in vehicles of this general class. (An approximate measure of the relative importance of performing one or the other of these changes independently can be seen in the simplified analysis calculations of Figure 34.)

3) Yaw instabilities are not likely to be encountered at speeds less than 25 mph (40.2 k/hr) for this class of vehicles.

And more specifically,

4) Moderate wheelbase trucks which (a) possess sprung mass center of gravity heights greater than 80 inches (2.03 m) (payload + truck body weight), (b) are equipped with a rear tandem suspension of average stiffness, (c) possess tires of average traction which exhibit cornering stiffness variation with vertical load similar to that shown in Figure 38, and (d) are loaded in a manner similar to Vehicle C of Table 3, are capable of developing yaw instabilities at lateral acceleration levels less than 0.2 g while operating on horizontal surfaces at speeds above 40 mph (64.4 k/hr).

5) A representative range of rollover thresholds is 0.28 to 0.41 g's for the group of vehicles examined in this study.

#### 5.5 Closed-Loop Stabilization of Yaw-Divergent Vehicles

The results of the previous sections suggest that certain, high-center-of-gravity trucks, operating at highway speeds can exhibit yaw divergence behavior under relatively low lateral acceleration turning conditions. If so, a natural question which arises is, "Can drivers, through normal compensating or corrective steering action, stabilize the vehicle

and maintain adequate maneuverability under these conditions?" To begin to address this question, a small study was conducted to examine the question of closed-loop stabilization of yaw-divergent vehicles within the yaw-divergent/roll-stable region of Figure 33. That is, referring to Figure 33, operation within a region to the right of the vehicle's yaw stability boundary and to the left of its rollover threshold.

Results of several computer simulation runs performed with the aid of a driver model [17], indicated that stabilization of the closed-loop (driver-controlled) system was possible while retaining maneuverability or path control in this regime. However, both stability and maneuverability degrade rapidly as lateral acceleration approaches the rollover threshold.

The comprehensive computer model [16] and associated driver model [17] were employed in this activity to simulate the closed-loop driver/vehicle system. Average driver model parameters reflecting drivers' "look-ahead" time and response lag were used to simulate the driver control characteristics. Figures 41 and 42 show computer simulation results for a three-axle vehicle, similar to Vehicle C (but with reduced axle loadings of 16K/16K/16K (71KN/71KN/71KN) and 75-inch (1.9-m) c.g. height), performing a 50-mph (80.5-k/hr) closed-loop circular turn at 0.19 g's and 0.26 g's, respectively. Figure 41 corresponds to a lateral acceleration level which is below this vehicle's yaw stability boundary, while Figure 42 corresponds to a point above the vehicle's yaw stability boundary but below its rollover threshold. Both driver/vehicle systems track the circular paths with little difficulty. However, as can be seen in these two figures, the nature of the closed-loop steering functions are very dissimilar. Figure 41 shows a relatively constant steer control after 3-4 seconds (as one would expect), while Figure 42 shows a slow oscillatory steering control.

The time-varying, closed-loop steering control characteristics for the yaw-divergent vehicle in Figure 42 can be understood by noting that a yaw-divergent vehicle (without driver control) will respond to any disturbance by diverging away from an otherwise straight-line path. Clearly, a fixed level, closed-loop steering control, as is appropriate for following a circular turn in the case of a yaw-stable vehicle, is not appropriate

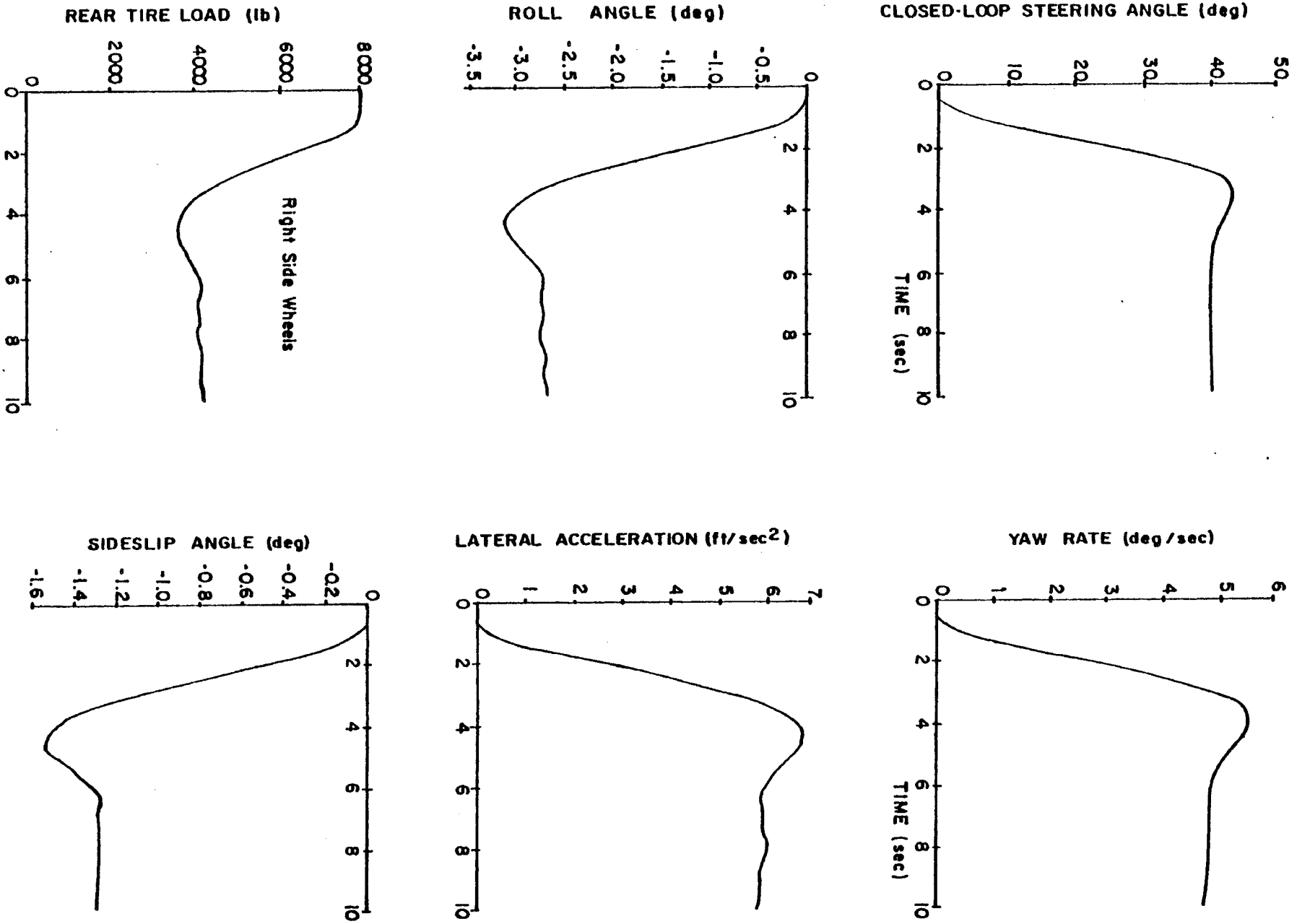


Figure 41. Closed-loop (driver-controlled) circular turning maneuver, operating within the vehicle yaw-stable boundary.



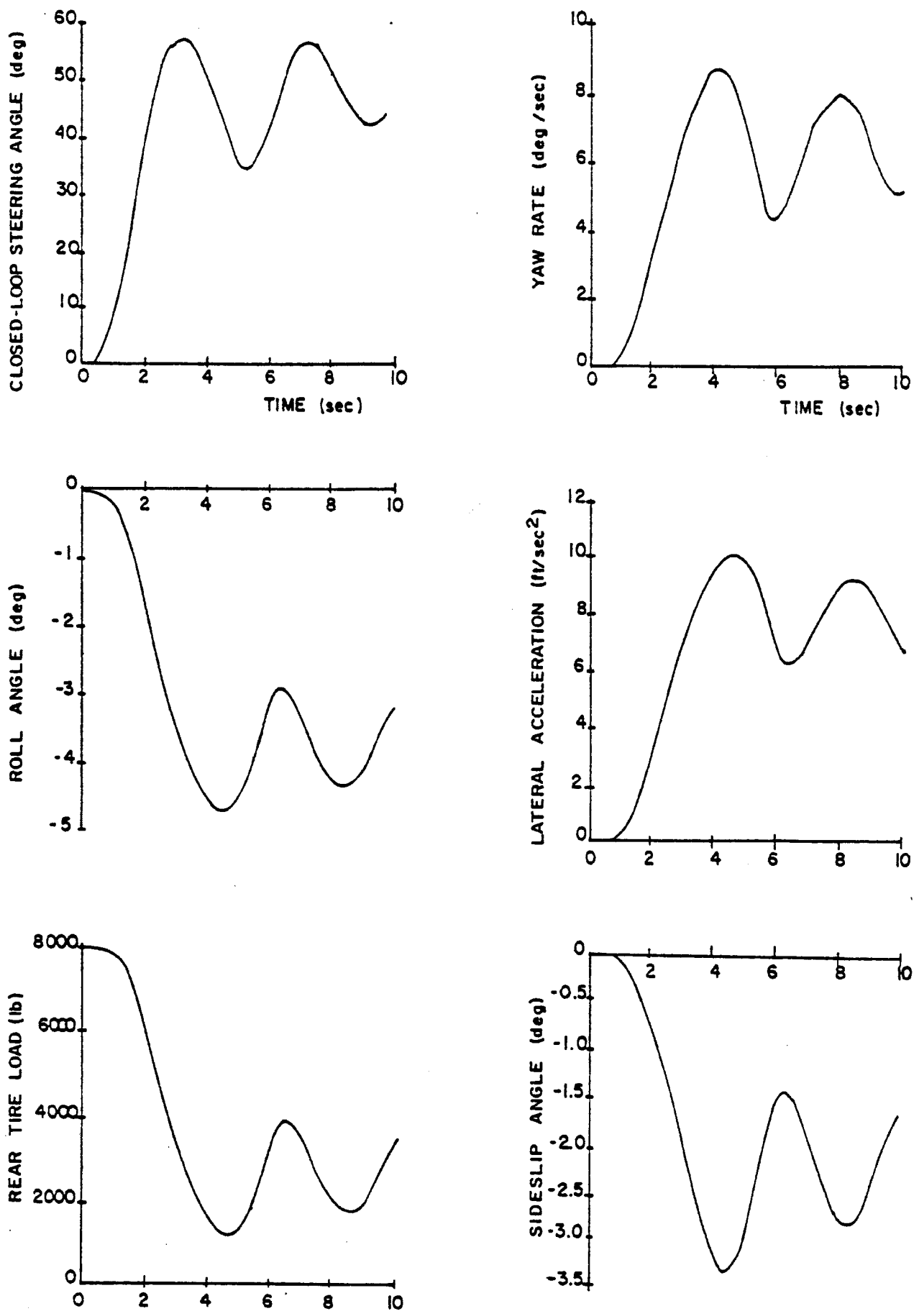


Figure 42. Closed-loop (driver-controlled) circular turning maneuver, operating outside the vehicle yaw-stable boundary.

for a yaw-divergent vehicle. Instead, a continually changing steering control, which constantly corrects the path-divergence tendencies of the more dynamically varying yaw-divergent vehicle, is the required steering control strategy. (If the maneuver, as described here, lasted for a long enough period of time, the oscillatory steering control would eventually die out, settling on a quasi-like fixed value.) Needless to say, such vehicle characteristics require very attentive drivers under these circumstances and are hardly desirable.

The purpose of presenting and discussing such results is not to sanction or encourage closed-loop operation within the yaw-divergent/roll-stable regime, but instead, to try to address certain questions and prevailing notions that entering this regime will precipitate a suddenly unstable or uncontrolled driver/vehicle system. Based on these results, there appears to be nothing "magical" about the yaw stability boundary for closed-loop operation except for the requirement of drivers to use a continuously varying (and presumably undesirable and more demanding) steering control to stabilize the directional vehicle dynamics at lateral acceleration levels above this boundary. In addition, it was observed that the directional stability of the closed-loop driver/vehicle system is gradually degraded as lateral acceleration operating conditions are increased from low levels to the rollover threshold.

Another example, using the same model, demonstrates the yaw stabilizing benefits derived from roadway superelevation. Figure 43 shows a limit-level 0.3 g, 55 mph (88.5 k/hr), closed-loop circular turn by the same vehicle (Figs. 41, 42) on a flat horizontal road and operating above its yaw stability boundary. Figure 44 shows the same maneuver repeated, but now on a roadway which gradually increases in superelevation to a fixed value of eight percent. While the horizontal-plane lateral acceleration is still 0.3 g's in negotiating this turn, the presence of superelevation reduces the lateral tire force requirements to maintain the same circular path and also decreases the side-to-side load transfer. Since rear-end load transfer/tire load sensitivity is the principal mechanism responsible for yaw-divergent vehicle dynamic behavior, a stabilizing effect should accrue from the presence of superelevation. Figure 44 clearly reflects

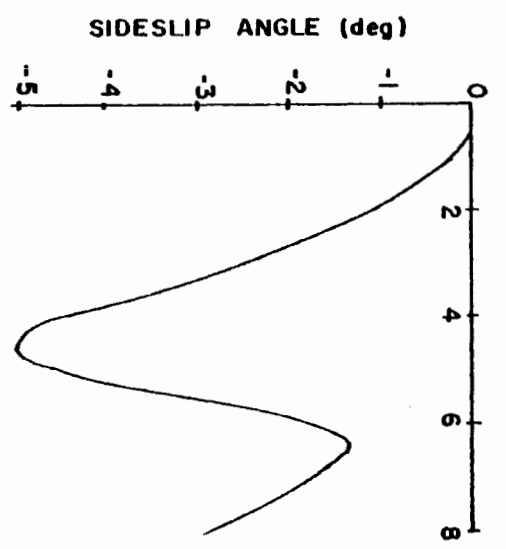
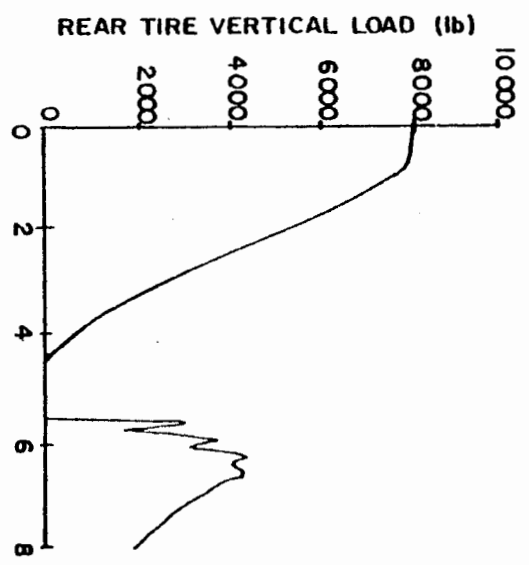
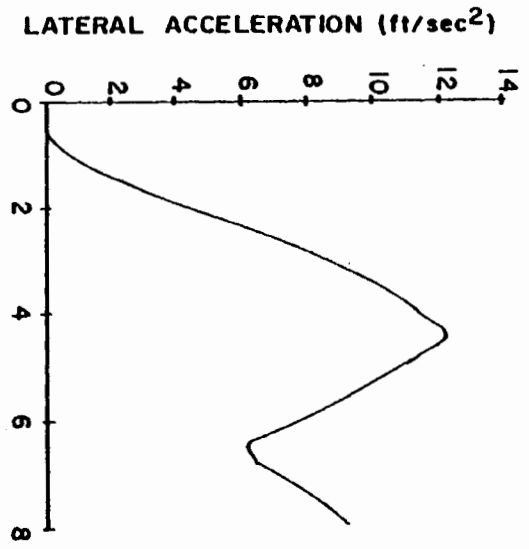
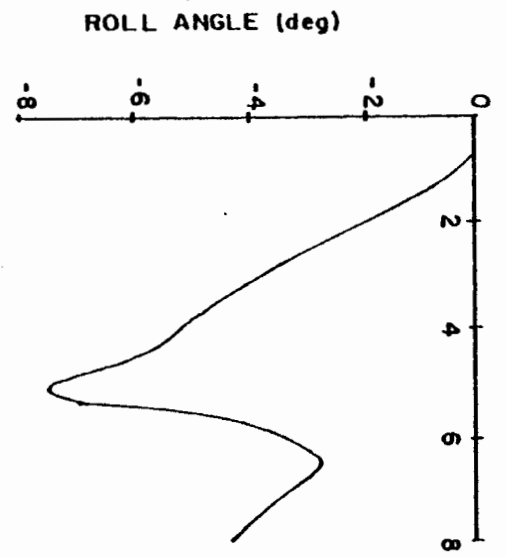
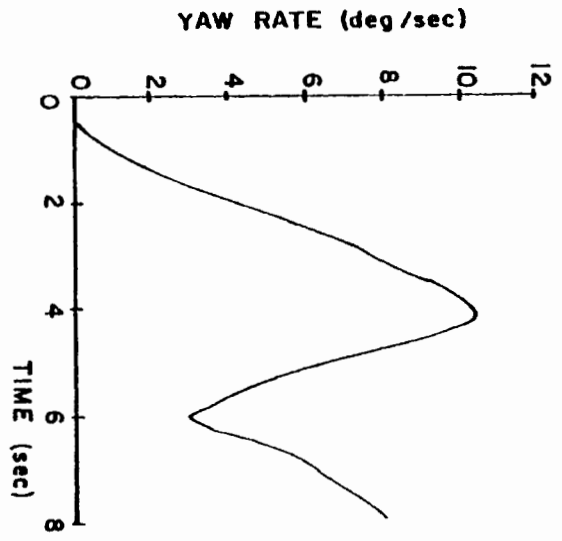
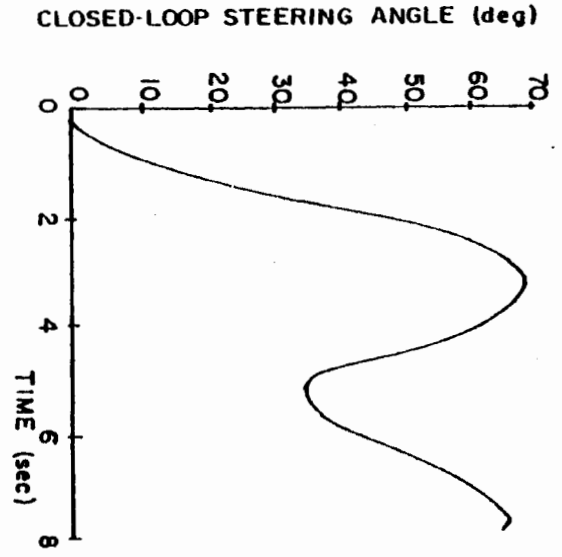


Figure 43. Closed-loop (driver-controlled) limit-level turning maneuver on a flat, horizontal road surface.

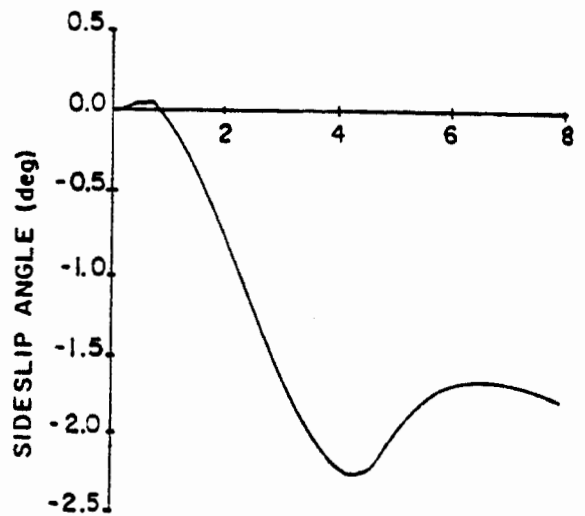
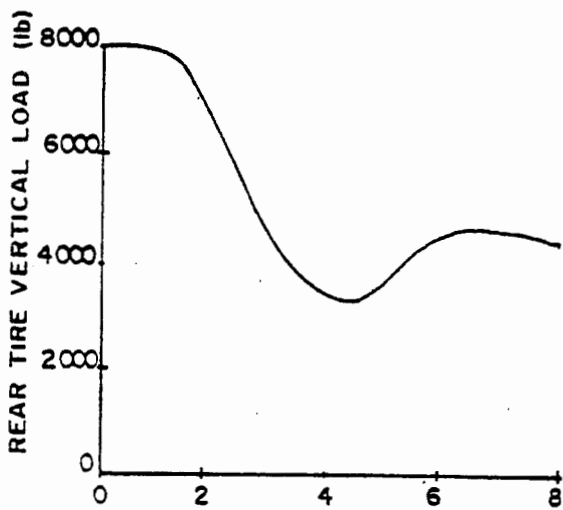
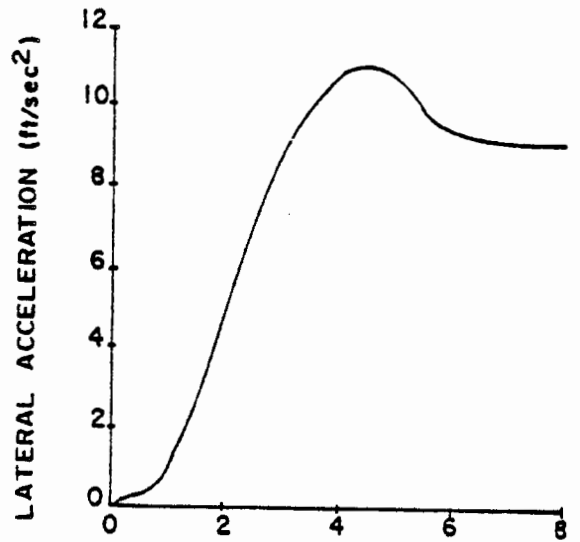
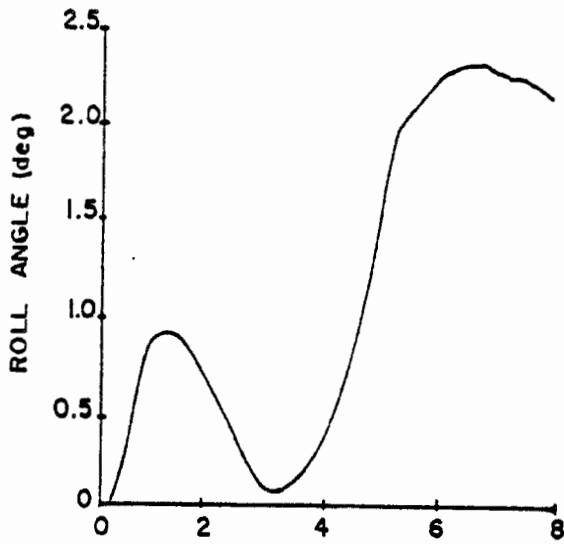
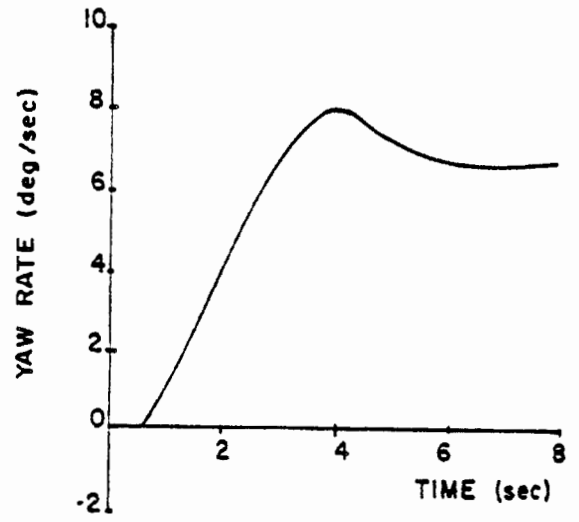
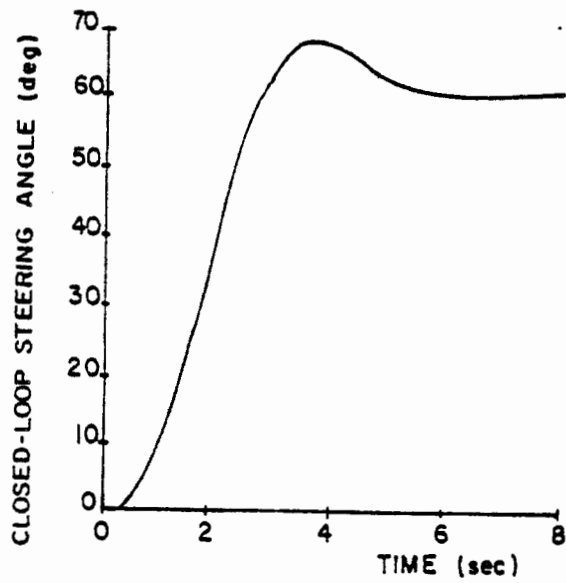


Figure 44. Repeat of the maneuver shown in Fig. 43 on an 8% super-elevated curve.

such an improvement with a yaw-stable, closed-loop steering control similar to that shown in Figure 41, and greatly reduced side-to-side load transfer.

These last examples raise the interesting question of whether the yaw divergence phenomena, as suggested by these computer-based findings, and so effectively countered by superelevated curves and to some extent by driver steering control, should be, in a very practical sense, a real concern. That is, since the development of yaw divergence requires high-speed turning on primarily horizontal-like surfaces, do ample opportunities or scenarios actually exist within the highway system for vehicles, such as those studied here, to develop yaw instabilities? Does the level of superelevation in expressway curves, exit ramps, and rural highway curves, as determined by common design practice, effectively negate the likelihood of yaw instabilities occurring in such vehicles? To what extent are actual drivers capable of detecting, controlling, and overcoming yaw-divergent behavior of certain vehicles? While such questions are clearly pertinent and should be pursued, it is also true that the design of heavy truck vehicles, so as to greatly diminish or eliminate the possibility of yaw instability occurrences, is undoubtedly the most effective means for counteracting this potential issue.

#### 5.6 Steerable Tag Axles

A trend toward equipping certain types of commercial vehicles with steerable tag axles has become apparent in recent years. The principal reason for using steerable tag axles (in addition to being able to carry greater load) is to retain maneuverability and avoid tire wear (scrubbing) under low-speed turning conditions. Apparently, a variety of such arrangements exist in the commercial vehicle population. However, one vehicle which UMTRI had an opportunity to study in some detail was an otherwise conventional four-axle cement mixer, but which included a tag axle located approximately 12 feet (3.66 m) behind the rear tandem suspension. The rear tag axle was typically loaded to 12,000 lb (53,380 N) (62,000-72,000 lb (275,800-320,300 N) gross vehicle weight) and was equipped with steerable wheels which castered. By "dialing in" different yardages of cement at the side of the vehicle, hydraulic rams load the rear tag axle to prescribed

or preset vertical loads. "Booster Mixer" ("booster" load on the tag axle) is the most common term used for describing such vehicles.

Initial hypotheses about this particular configuration of a steerable tag axle was that it would promote poor handling qualities. The rearward positioning and loading to 12,000 lbs (53,380 N) of a non-steerable tag axle would normally produce a (destabilizing) rearward shift in c.g. which would be approximately offset by a (stabilizing) lengthened wheelbase effect. However, steering of the tag axle wheels does not lengthen the effective wheelbase (no lateral tire forces produced) and thereby results in a destabilizing rearward c.g. shift alone. Since such vehicles are highly rear biased in load to begin with, the large aft positioning of the steerable tag axle could significantly shift the c.g. position rearward, thereby degrading the already precarious yaw stability levels for this kind of vehicle.

Results of a computer simulation study which examined this type of vehicle confirmed the initial suspicion that steerable tag axles, as employed in the above-described manner, produce a rapid degradation of high-speed directional stability. Figures 45 and 46 show example results for a simulated "Booster Mixer" with axle loads of 18K/19K/19K/12K (80KN/84.5KN/84.5KN/53.4KN) performing a 20-degree fixed-steer turning maneuver at 55 mph (88.5 k/hr). Figure 45 is the same run as shown in Figure 46, but without tag axle steering. As seen, the effect of permitting the rear tag axle to caster, rather than remain fixed, produces an unstable yaw response resulting in rollover of the vehicle. The particularly interesting feature of this instability is that it is present at very low levels of lateral acceleration. Whereas the vehicles examined earlier in this chapter required that a certain level of lateral acceleration be achieved before precipitating directional instability at highway speeds, this vehicle appears to exhibit little or no lateral acceleration stability margin at 55 mph (88.5 k/hr).

To examine what steering control challenges such a vehicle might present to drivers during high-speed highway maneuvers, a brief series of closed-loop (driver-controlled) simulations were performed. A 12-foot (3.66 m) lane change served as the nominal maneuver. Figure 47 shows an

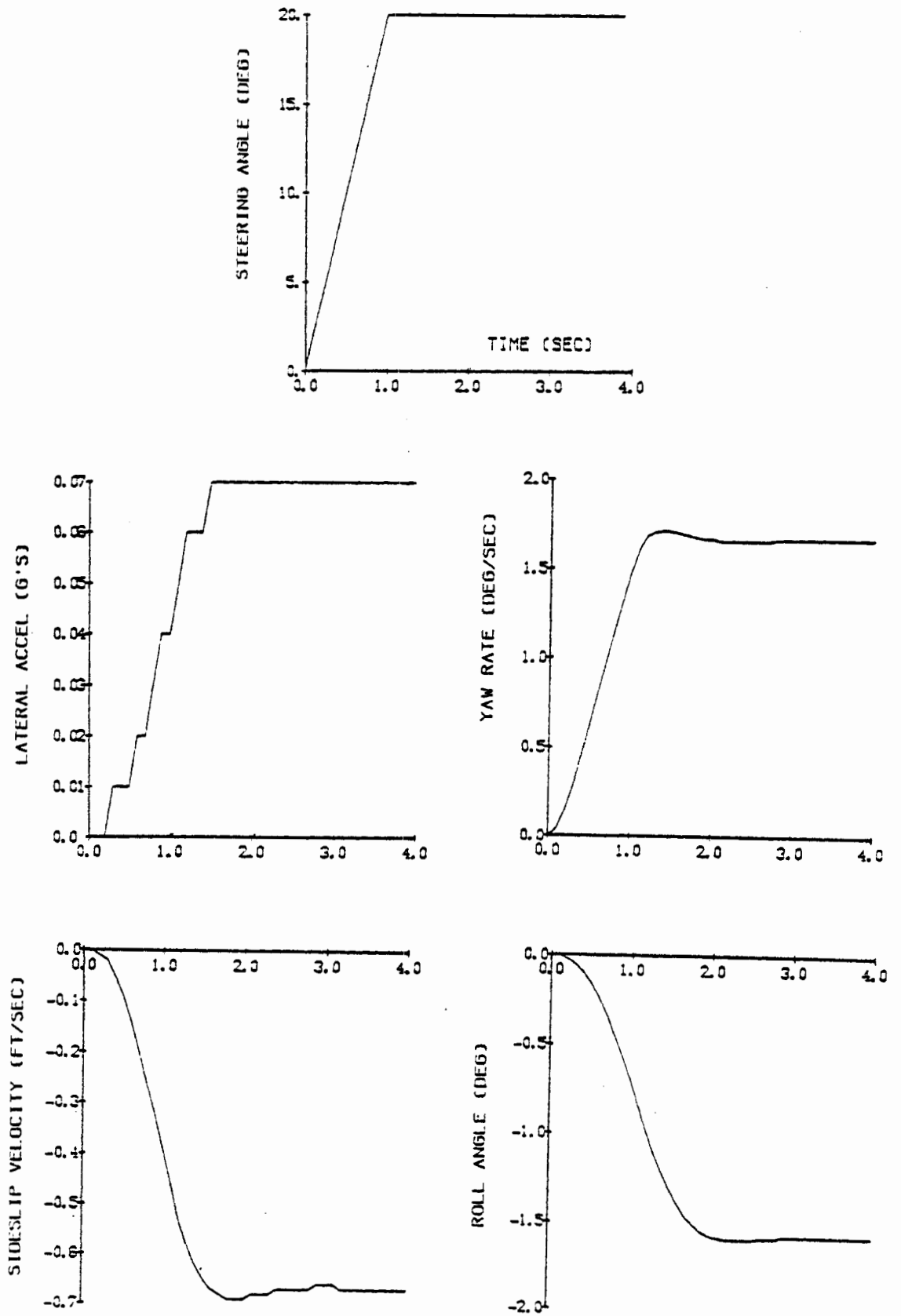


Figure 45. Simulated "Booster Mixer" - no tag axle steering - 55 mph (88.5 k/hr), 20° fixed steer turning maneuver.

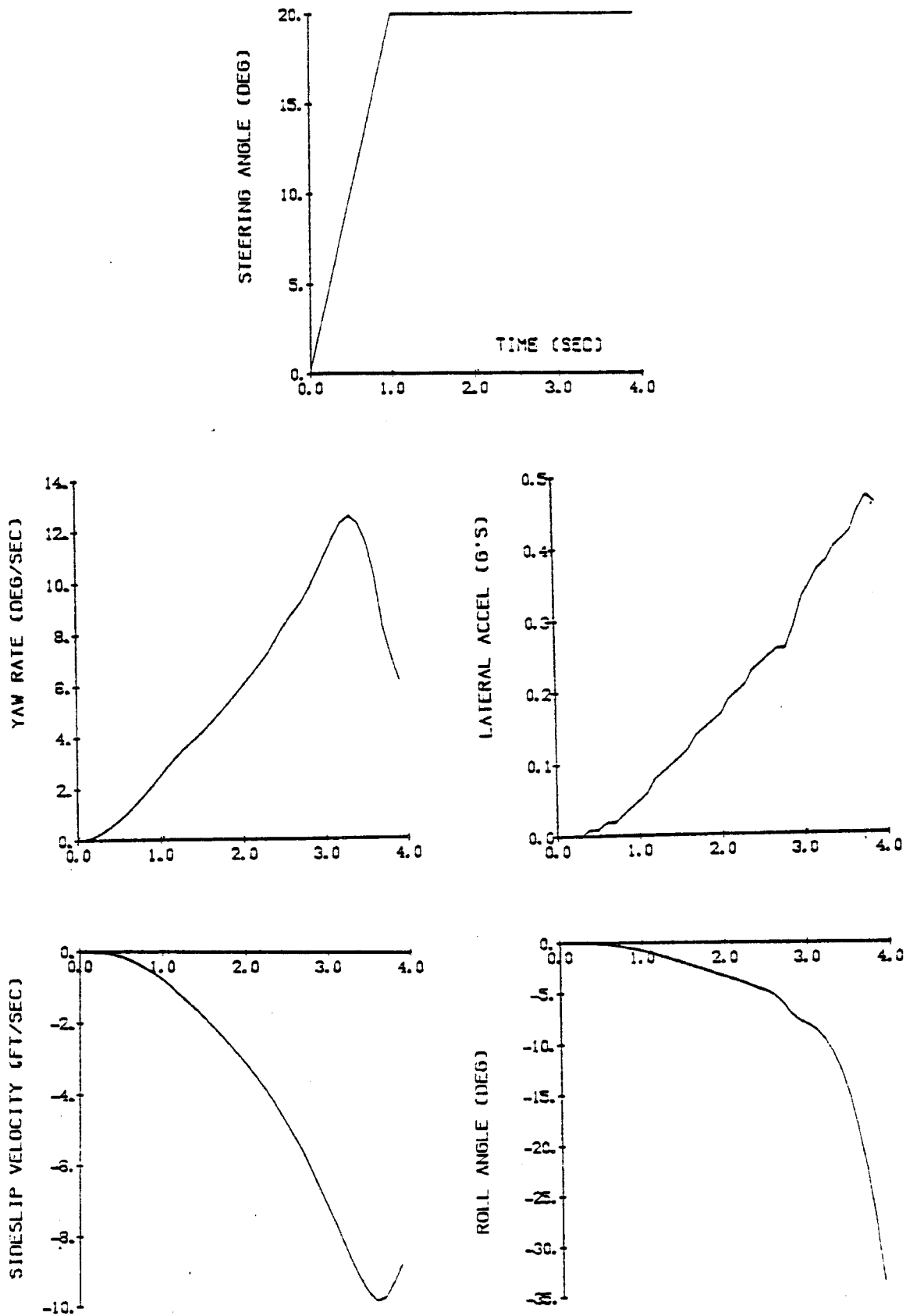


Figure 46. Simulated "Booster Mixer" - with tag axle steering - 55 mph (88.5 k/hr), 20° fixed steer turning maneuver.



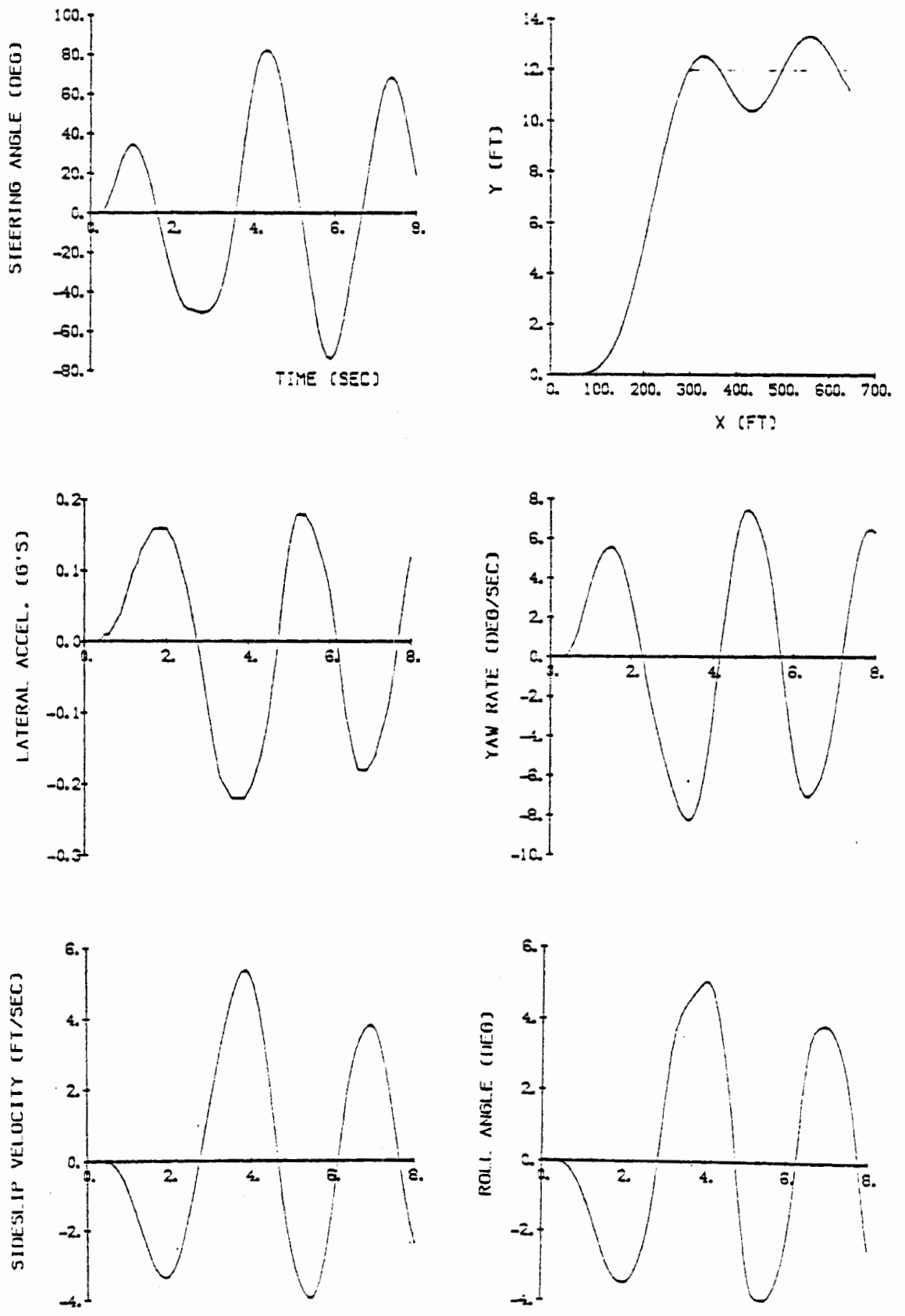


Figure 47. Simulated "Booster Mixer" - 55 mph (88.5 k/hr), driver-controlled lane-change maneuver.

example result performed at a speed of 55 mph (88.5 k/hr) suggesting a marginally stable driver/vehicle system. Repeating the same maneuver at 40 mph (64.4 k/hr) (Figure 48) demonstrated considerably improved path and directional responses, but still required a relatively "active" steering control by the simulated driver. Variation in the basic driver model parameters (driver preview time and transport lag) did not have significant influence in altering the basic results shown in Figures 47 and 48. This could suggest that new drivers of such vehicles would find limited means of compensating their normal control strategy to overcome the inherent deficiencies present in the vehicle directional dynamics. Presumably more experienced drivers of booster mixers have developed a set of learned steering responses which are capable of stabilizing the closed-loop system to significantly greater degree than that depicted in Figure 47 under similar circumstances.

It should be emphasized that the kind of booster mixer being discussed above is distinct in having tag axle wheels which are essentially free to caster. That is, unlike some booster mixers which have hydraulically-controlled steerable wheels (linked by a servo-controlled device to the driver-steered front wheels), the vehicles being examined here employ no active steering controller at the steerable tag wheels. It is very likely that the majority of hydraulically-controlled booster mixers do not suffer the same degradation in handling qualities as the type examined here.

## 5.7 Chapter Summary and Conclusions

The computer-based results discussed in this chapter suggest that certain heavy trucks, characterized primarily by high centers of gravity, can develop yaw divergence instabilities during high-speed steady turns at relatively low levels of lateral acceleration. A simplified analysis, directed toward identifying the sensitivity of yaw stability thresholds to typical vehicle parameter variations, was shown to predict reasonable results when compared to results of a more comprehensive computer simulation study. A more thorough examination of the dynamic behavior of these vehicles during steady turning was conducted with the use of comprehensive computer models used for simulating vehicle-driver-roadway interactions.

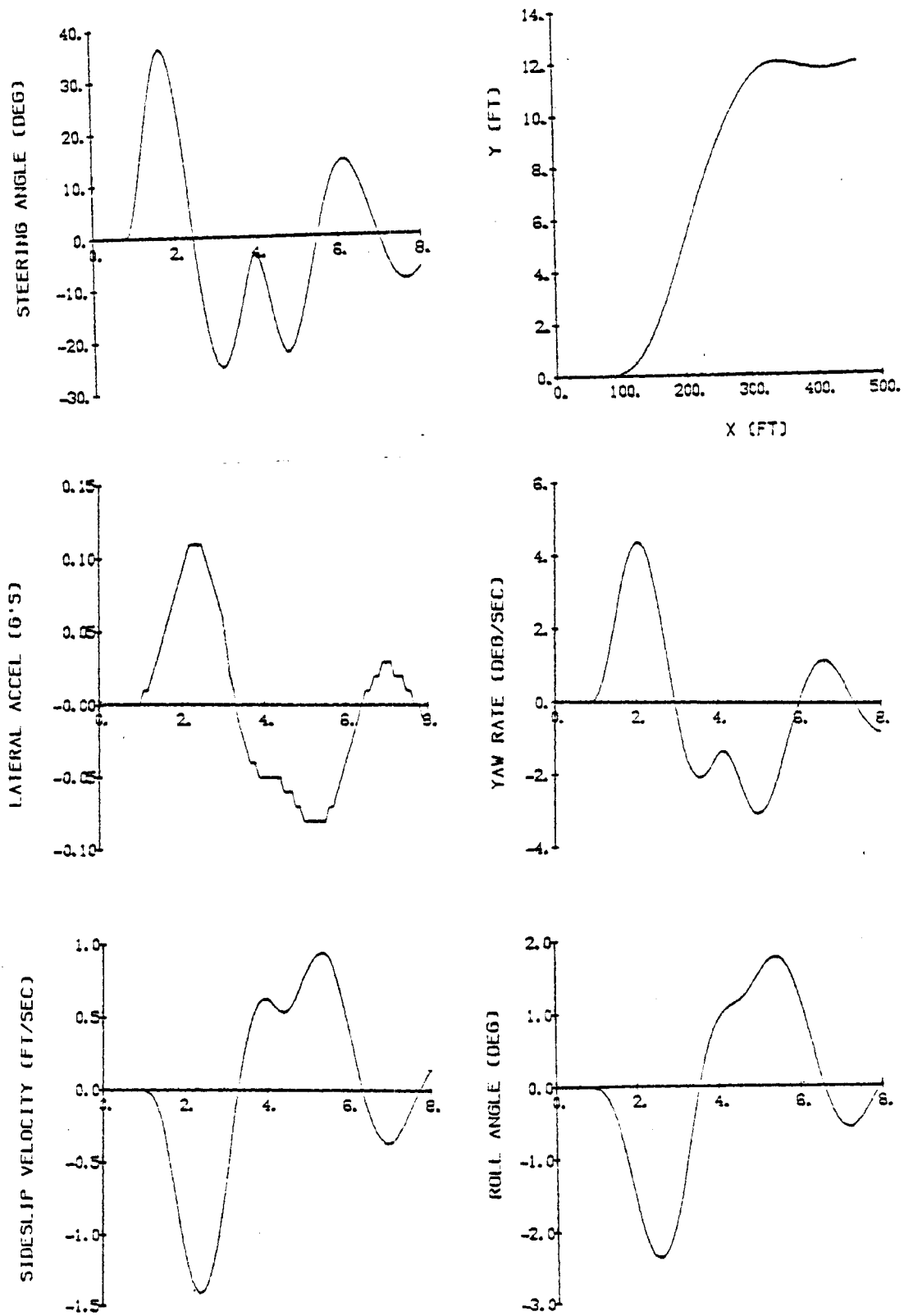


Figure 48. Simulated "Booster Mixer" - 40 mph (64.4 k/hr), driver-controlled lane-change maneuver.

Specific conclusions, applicable to the class of high center of gravity trucks examined here, and based upon the results of the computer simulation study are:

A) Yaw divergence behavior during steady turning is possible in such vehicles operating at elevated speeds and relatively low lateral acceleration levels on flat horizontal surfaces.

B) The occurrence of yaw divergence during steady-turning conditions will lead to rollover in the absence of corrective steering action and/or reduced speed.

C) The principal mechanism responsible for the production of yaw divergent behavior at low lateral acceleration levels and elevated speeds is the non-linear sensitivity of truck tire cornering stiffness to vertical load (i.e., the degree of curvature in a tire's cornering stiffness versus vertical load plot).

D) Although vehicle yaw divergence behavior may be stabilized by corrective steering actions of drivers, the margin of stability of such driver-controlled systems is significantly less than similar driver/vehicle systems possessing stable yaw dynamics.

E) The presence of superelevation in highway curves acts not only to contribute roll stabilization to such vehicles, but is also a particularly powerful means for reducing the likelihood of yaw divergence.

F) Reasonable vehicle-related modifications which could be performed to increase the yaw/roll stability of these vehicles are: (1) increases in fore/aft roll stiffness distribution, (2) use of additional tires or axles (non-steering) at the rear of the vehicle, (3) lowering of the center of gravity, and (4) selection of rear tires with more linear-like variation of cornering stiffness with vertical load.

G) Vehicle parameters found to have the greatest influence upon the development of yaw divergence in the straight truck vehicle class examined here are:

- 1) Rear tire cornering stiffness variation with vertical load. A tire which exhibits greater curvature, than a similar tire, in its cornering stiffness versus load plot (see Fig. 38) will, in general, be a more likely contributor to vehicle yaw divergence.
- 2) Center of gravity height. Vehicles possessing greater c.g. heights, in general, transfer more load side to side during cornering. Greater load transfer levels across an axle exaggerate the net loss of cornering stiffness (see item (1) and Appendix A).
- 3) Fore/aft roll stiffness distribution. Low values of fore/aft roll stiffness distributions, characteristic of the heavy truck vehicle, promote proportionately greater side-to-side load transfer across the rear suspension than the front. Correspondingly, greater opportunities to suffer cornering stiffness losses therefore exist at the rear axles—leading to vehicle oversteer and directional instability at elevated speeds.
- 4) Number of axles. In general, addition of non-steerable axles at the rear of the vehicle contributes to the directional and roll stability of such vehicles. (Steerable tag axles with freedom to steer through castering should be avoided.) Additional rear axles: (a) increase total roll stiffness of the vehicle thereby reducing vehicle roll and rear-end (and total vehicle) load transfer at a given lateral acceleration, and (b) increase immunity to yaw instability by producing less load transfer per rear tire as well.
- 5) Wheelbase length. Wheelbase length,  $\ell$ , has a theoretical  $\sqrt{\ell}$  influence on the variation of critical velocity for an oversteer vehicle. That is, doubling a vehicle's wheelbase length for the same oversteer condition will raise the maximum, stable operating velocity by a factor of 1.4.

## CHAPTER 6

### AMPLIFIED LATERAL RESPONSES IN COMBINATION VEHICLES

#### 6.1 Introduction

The phenomenon addressed in this chapter is referred to as "rearward amplification." It is typically characterized by the ratio of (a) the lateral acceleration of the trailing unit in a combination vehicle divided by (b) the lateral acceleration of the tractor or towing unit [19].

Some articulated vehicles have very large amounts of lateral acceleration gain (i.e., rearward amplification) during the transient phases of sudden lateral displacement maneuvers, others do not [20]. Large amounts of rearward amplification are dangerous in an obstacle-avoidance maneuver, not only because the rear unit in a combination vehicle tends to oscillate well out of the path of the lead unit but, also, because the rear unit may experience a level of lateral acceleration sufficient to cause it to roll over. The primary purpose of this chapter is to identify combinations of vehicle parameters that may significantly influence the tendency for articulated vehicles to "crack the whip" in obstacle-avoidance maneuvers at highway speeds.

The scope of this discussion is limited to articulated vehicles. Within the category of articulated vehicles, typical tractor-semitrailers do not have a problem with rearward amplification. The lateral acceleration of the center of gravity of a typical semitrailer differs from that of the center of gravity of the tractor by a multiplicative factor ranging from 0.8 to 1.2 in obstacle-avoidance maneuvers [20]. Nevertheless, the properties of semitrailers will be examined in connection with their use in doubles and triples combinations employing full trailers.

The study of rearward amplification is most important with regard to commercial vehicles incorporating full trailers where a full trailer consists of a steerable dolly, an articulation joint, and a semitrailer supported by the dolly.

In various States, full trailers are currently in common use in doubles and triples, as well as in truck-full trailer combinations. The trend towards larger vehicles to enhance fuel and other operational economies is likely to lead to an increased use of these multiply-articulated vehicles. Multiple articulation joints are popular because they can be arranged to provide good low-speed maneuverability and little off-tracking even for long vehicle trains. Nevertheless, multiply-articulated vehicles can have large amounts of rearward amplification if care is not exercised in arranging combinations with low levels of lateral acceleration gain. In some cases, a tradeoff between low-speed maneuverability and highway-speed obstacle-avoidance capability may be necessary if the potential for avoiding rollover accidents in situations involving high-speed maneuvering is to be increased.

## 6.2 Summary of Simplified Analytical Techniques for Studying Rearward Amplification

The problems associated with rearward amplification had been discovered previously in connection with rollover events following avoidance maneuvers performed by double-tanker gasoline trucks operating in Michigan [21]. Although progress has been made in studying this particular type of vehicle [20,21,14], generalized findings have not been previously forthcoming. This report provides a generic treatment of the rearward amplification phenomena associated with combination vehicles in order to provide a basis for (1) readily identifying vehicles that may have rearward amplification problems, (2) indicating the vehicle parameters that contribute to these problems, and (3) suggesting changes that can be made to reduce rearward amplification.

To fill the need for a generic treatment of the amplification properties of combination vehicles, linearized equations of motion applying to these vehicles have been studied in detail. Derivations of the results summarized in this section are presented in Appendix C of this report and also in SAE Paper No. 821259 entitled "The Transient Directional Response of Full Trailers" [22]. The methodology underlying these results consists of using frequency response methods to develop transfer functions describing

how forward velocity, distances from pintle hitches to center-of-gravity locations, and cornering coefficients influence rearward amplification in commercial vehicle combinations employing full trailers.

The results of this linear analysis can be expressed in reasonably simple form for each unit of a combination vehicle (that is, for trucks, tractor-semitrailers, or full trailers) because (1) the lateral forces of constraint at pintle hitch connections to steerable dollies are small and (2) full trailers are typically loaded with approximately equal loads on all their tires, with these tires all having similar mechanical properties. (See Appendix C for discussion of the technical matters pertaining to these two simplifying approximations.) To the extent that the two items enumerated above are applicable to a particular vehicle, the following simplified results (see Table 4) can be used to make first-order estimates of the overall rearward amplification factor for the complete combination vehicle using individual amplification factors applying to each unit in the combination.

For example, given a combination vehicle consisting of a truck towing a full trailer, the rearward amplification (expressed as the ratio of the lateral acceleration of the center of gravity of the full trailer to the lateral acceleration of the center of gravity of the tractor) may be studied using the following equation.

$$\frac{A_{yc_2}}{A_{yc_1}} = \frac{A_{yp_1}}{A_{yc_1}} \frac{A_{yc_2}}{A_{yp_1}} \quad (6.1)$$

where

$\frac{A_{yp_1}}{A_{yc_1}}$  is the lateral acceleration gain (transfer function) between the motions of the truck's c.g. and the pintle hitch connection point (Item 1 in Table 4), and

$\frac{A_{yc_2}}{A_{yp_1}}$  is the transfer function between the acceleration of the pintle hitch and the acceleration of the c.g. of the full trailer (Item 4 in Table 4).



Table 4. Amplification Factors [22]

Note: For each towing unit, the symbol  $x_{PC}$  represents the distance from the c.g. to the pintle hitch. The other symbols are defined in Table 5.

1. Towing Unit: Straight Truck

Rearward amplification between the c.g. of a straight truck and its pintle hitch

$$\frac{A_{yp}}{A_{yc}} = (1 + \Delta A)$$

where

$$\Delta A = \left[ \frac{\frac{-x_{PC}}{u} j\omega \left( \frac{m_1 u}{\Sigma C_\alpha} j\omega + 1 \right)}{1 - \frac{I_1}{x_{11} \Sigma C_\alpha} \omega^2 + \frac{j\omega \Sigma x^2 C_\alpha}{x_{11} u \Sigma C_\alpha}} \right]$$

2. Towing Unit: Tractor-Semitrailer

a. Rearward amplification between the c.g. of a semitrailer and its pintle hitch connection to the unit being towed

$$\frac{A_{yp}}{A_{yc}} = (1 + \Delta A)$$

where

$$\Delta A = \left[ \frac{\frac{-x_{PC} j\omega}{u} \left( \frac{u m_2 j\omega x_{2A}}{\Sigma(x_{2i} + x_{2A}) C_{\alpha 2i}} + 1 \right)}{1 - \left( \frac{I_2 \omega^2}{\Sigma(x_{2i} + x_{2A}) C_{\alpha 2i}} \right) + j\omega \left( \frac{\Sigma x_{2i} (x_{2i} + x_{2A}) C_{\alpha 2i}}{u \Sigma(x_{2i} + x_{2A}) C_{\alpha 2i}} \right)} \right]$$

b. Note that for typical tractor-semitrailers [20], the rearward amplification between the c.g. of the tractor and the c.g. of the semitrailer may range from a maximum of approximately 1.2 to a minimum of approximately 0.8 in the frequency range from 0 to 3.5 rad/sec. Vehicles with short semitrailers tend to have maximum amplification factors greater than 1.0 at frequencies in the range from 1 to 4 rad/sec. Vehicles with longer semitrailers tend to have amplification factors of 1.0 at low frequencies with their amplification factors falling off to approximately 0.8 in the neighborhood of 3 rad/sec. For first-order estimates of overall rearward amplification, a reasonable compromise is to assign an amplification factor of 1.0 between the c.g. of the tractor and the c.g. of the semitrailer if this amplification factor is not known from prior work.

3. Towing Unit: Full Trailer

Rearward amplification between the c.g. of a full trailer and the pintle hitch connection to the unit it is towing

$$\frac{A_{yp}}{A_{yc}} = (1 + \Delta A)$$

where

$$\Delta A = \left[ \frac{\frac{-x_{PC}}{u} j\omega \left( \frac{m_T u}{\Sigma C_\alpha} j\omega + 1 \right)}{\left( 1 - \frac{I_T}{x_{BT} \Sigma C_\alpha} \omega^2 \right) + \frac{j\omega \Sigma x^2 C_\alpha}{x_{BT} u \Sigma C_\alpha}} \right]$$

(The amplification factor for a towed full trailer is given next in Item 4.)

4. Towed Unit: Full Trailer

Rearward amplification between the pintle hitch connection to the towing unit and the c.g. of the full trailer

$$\frac{A_{yc}}{A_{yp}} (j\omega) = \frac{1}{1 - \left( \frac{\omega}{\omega_{nc}} \right)^2 + j 2\zeta_c \frac{\omega}{\omega_{nc}}}$$

where

$$\omega_{nc} = \sqrt{\frac{\Sigma C_\alpha}{m_T} \frac{1}{x_{BT} + x_{BA}}}$$

$$\zeta_c = \frac{1}{2u} \sqrt{\frac{\Sigma C_\alpha}{m_T} (x_{BT} + x_{BA})}$$

See Equations (12) and (13) for determining the maximum value of  $A_{yc}/A_{yp}$ .

Table 5. Symbols, Subscripts, and Definitions.

Force and Moment Coefficients Used in Linear Equations of Motion

$F_v$	the rate of change of lateral force with respect to $v$
$F_r$	the rate of change of lateral force with respect to $r$
$F_\psi$	the rate of change of lateral force with respect to $\psi$
$F_y$	the rate of change of lateral force with respect to $(y_A - y_m)$
$F_\delta$	the rate of change of lateral force with respect to $\delta$
$T_v$	the rate of change of yaw moment with respect to $v$
$T_r$	the rate of change of yaw moment with respect to $r$
$T_\psi$	the rate of change of yaw moment with respect to $\psi$
$T_y$	the rate of change of yaw moment with respect to $(y_A - y_m)$
$T_\delta$	the rate of change of yaw moment with respect to $\delta$

Operators and Frequency Response Quantities

$p$ or $(\cdot)$	indicates differentiation with respect to time
$\omega$	frequency, rad/sec
$j$	complex number equal to $\sqrt{-1}$
$\phi$	phase
$K$	amplitude
$A_{y2/Ay1}$	lateral acceleration transfer function between points 1 and 2
$A$	amplification factor for a towing unit
$Y_o$	open-loop transfer function
$Y_c$	closed-loop transfer function
$N$	numerator
$D$	denominator
$Y_z$	quantity pertaining to complex conjugate zeros
$Y_p, Y_{po}$	quantity pertaining to complex conjugate poles
$\omega_n$	natural frequency
$\zeta$	damping ratio
$\omega_{max}$	frequency at maximum gain
$G_{max}$	maximum gain for a full trailer

Special Points Used in Subscripts

Points	Location
3	turntable of a full trailer
T	c.g. of a full trailer

A	pinch hitch of a full trailer, also fifth wheel of a tractor-semitrailer, generally the articulation joint closest to the front of the vehicle
P	pinch hitch of any towing unit
C	c.g. of any towing unit
1,2,3 etc.	rear axles of a full trailer starting from the rear axle closest to the front of the trailer; also, these numbers are used in a double subscript notation (4) to denote the $j^{\text{th}}$ axle on the $i^{\text{th}}$ unit of a train. For example, $x_{13}$ is the distance from the center of gravity of a 3-axle tractor to its rearmost axle

Special Summations

$\sum C_i$	the sum of the cornering stiffnesses of all the tires on a full trailer or straight truck
$\sum_i C_{oi}$	the sum of the cornering stiffnesses of the tires on the front axle(s) of a full trailer or straight truck
$\sum_i x_{i3} C_{oi}$	the sum of the products of the distance from the c.g. to each rear axle with its cornering stiffness for full trailers
$\sum x_i^2 C_i$	the sum of the products of the square of the distance from the c.g. to each axle times the cornering stiffness for that axle (see $T_i$ in Fig. 2 for full trailer situations)

Motion Variables - Definitions

$v$	lateral velocity
$r$	yaw rate
$\psi$	heading angle
$y$	lateral displacement
$A_y$	lateral acceleration
$\gamma$	articulation angle
$\alpha$	tire slip angle
$\delta$	steering angle of front wheels

Parameters - Definitions

$u$ or $\dot{x}_A$	forward velocity
$C_i$	the sum of the cornering stiffnesses of all the tires mounted on a designated axle
$x$	longitudinal distance between points indicated by subscripts, e.g., $x_{3A}$ is the distance from point 3 to point A
$m$	mass
$I$	yaw moment of inertia
$m_1, m_2, m_3$	masses of straight trucks, semitrailers, and full trailers, respectively
$I_1, I_2, I_3$	moments of inertia of straight trucks, semitrailers and full trailers, respectively

The two terms on the right side of Equation 6.1 are independent of each other in that the transfer function  $A_{yp_1} / A_{yc_1}$  depends only upon the design parameters of the towing unit, e.g., the truck, and  $A_{yc_2} / A_{yp_1}$  depends only upon the design parameters of the towed unit, the full trailer. This independence feature is very useful because it allows towing units or towed units to be analyzed individually thereby (1) reducing the number of important combinations of parameters needed to evaluate the performance of any one unit and (2) allowing a variety of combination vehicles to be studied once the performance characteristics of several basic towing and towed units have been determined.

The amplification factors presented in Table 4 are expressed in terms of (a) basic design parameters (such as masses, inertias, hitch and c.g. locations, and tire cornering stiffnesses), (b) forward velocity,  $u$ , and (c) the frequency,  $\omega$ , employed in describing each transfer function. All of these quantities (i.e., design parameters, forward velocity, and frequency of excitation) are important in analyzing the rearward amplification phenomenon. The next section will discuss the sensitivity of rearward amplification to values of design parameters for vehicles traveling at highway speeds and at maneuvering rates ranging from low frequencies up into frequencies challenging the limits of driver capability in rotating the steering wheel. Since the simplified analysis has produced symbolic expressions (as contrasted to numerical results), the analysis of parametric sensitivities can proceed from (a) basic generalizations pertaining to the various types of heavy commercial vehicle combinations currently in use to (b) detailed numerical analyses (simulations) of particular vehicles identified in Table 2.

### 6.3 Parametric Sensitivities: The Influences of Design Parameters on Rearward Amplification

Examination of Table 4 indicates that the full trailer is the only type of towed unit to be considered in analyzing heavy vehicle combinations. The semitrailer part of a tractor-semitrailer vehicle is treated as a towing unit for a full trailer and not as a towed unit because no method is available for analyzing tractors and semitrailers separately. Furthermore, the pintle hitch connection at the back of the semitrailer is a logical

choice for separating a doubles or triples combination into constituent parts because the force at the pintle hitch is so small that the motion of the unit that the driver is steering (i.e., the tractor-semitrailer portion) is not noticeably influenced by the characteristics of the following trailer(s). Accordingly, the types of towing units considered include trucks, tractor-semitrailers, and, in the case of triples combinations, the full trailer that pulls the last full trailer in the triple.

Clearly, the overall rearward amplification of combination vehicles is determined by the product of the transfer functions applicable to the towing and towed units comprising the vehicle. Nevertheless, each transfer function may be examined separately to relate vehicle design parameters to overall rearward amplification. The following discussion starts with the full trailer—the only unit common to the basic types of multiply-articulated vehicles typically employed in the U.S.

6.3.1 The Influence of Full Trailer Parameters (When the Full Trailer is a Towed Unit). The simplified transfer function for a full trailer is characterized by a second-order system with natural frequency,  $\omega_{nc}$ , and damping ratio,  $\zeta_c$ , as specified in Item 4 in Table 4. This transfer function will have a maximum value, corresponding to maximum rearward amplification of the motion of the pintle hitch, at a frequency,  $\omega_{max}$ , given by the following equation:

$$\omega_{max} = \omega_{nc} \sqrt{1-2\zeta_c^2} \quad \text{for} \quad \zeta_c < .707 \quad (6.2)$$

(where  $\omega_{nc}$  and  $\zeta_c$  are expressed in terms of design parameters in Item 4 of Table 4).

From a practical standpoint, it is important to know whether  $\omega_{max}$  is within the frequency range that a driver is likely to excite. If it is, the trailer may be susceptible to rolling over (if the rearward amplification is large enough). Example results for three currently employed full trailers (see Table 6) indicate that  $\omega_{max} = 3.9, 3.4,$  and  $2.7$  rad/sec for the pup trailer of a "Michigan" double tanker, a 27-foot (8.23-m) trailer

Table 6. Full Trailer Examples.

Name: Pup Trailer of a Michigan Double Tanker

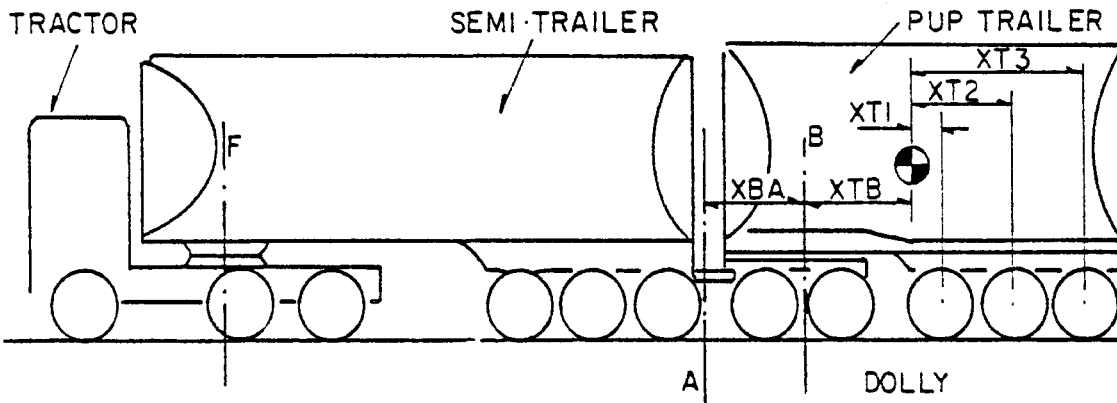
Parametric Values:

$$\begin{aligned} \dot{x}_A &= 968 \text{ in/sec (55 mph)} \\ m_T &= 167.1 \text{ lb-sec}^2/\text{in} \\ x_{BT} &= 75.32 \text{ in} \\ x_{BA} &= 70 \text{ in} \\ C_\alpha &= 95,860 \text{ lb/rad (all axles)} \\ x_{T1} &= 7.683 \text{ in} \\ x_{T2} &= 49.683 \text{ in} \\ x_{T3} &= 91.683 \text{ in} \\ I_T &= 875,220 \text{ in-lb-sec}^2 \end{aligned}$$

Numerical Results:

$$\begin{aligned} \frac{\Sigma C}{m_T} &= 2868 \text{ in/rad-sec}^2 \\ (x_{BT} + x_{BA}) &= 145.3 \text{ in} \\ \zeta_c &= 0.333 \\ G_{\max} &= 1.59 \\ \omega_{nc} &= 4.445 \text{ rad/sec} \\ \omega_{\max} &= \omega_{nc} \sqrt{1-2\zeta_c^2} = 3.9 \text{ rad/sec} \end{aligned}$$

1 m = 39.37 in = 3.281 ft  
 1 mph = 1.609 k/hr  
 1 N = 0.2248 lb



Profile view of 55-foot, 11 axle, double-bottom tanker.  
 F- Conventional fifth wheel; A- Pintle-hook connection,  
 B- Kingpin connection.

Table 6. (Cont.)

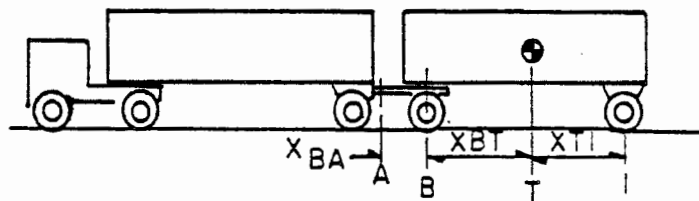
Name: 27-Foot Trailer (Conventional Five-Axle 65-Foot Double)

Parametric Values:

$$\begin{aligned} \dot{x}_A &= 968 \text{ in/sec (55 mph)} \\ m_T &= 90.67 \text{ in-sec}^2/\text{in} \\ x_{BT} &= 117 \text{ in} \\ x_{BA} &= 80 \text{ in} \\ C_\alpha &= 162,100 \text{ lb/rad (all axles)} \\ x_{T1} &= 135 \text{ in} \\ I_T &= 630,400 \text{ in-lb-sec}^2 \end{aligned}$$

Numerical Results:

$$\begin{aligned} \frac{\Sigma C}{m_T} &= 3578.9 \text{ in/rad-sec}^2 \\ (x_{BT} + x_{BA}) &= 197 \text{ in} \\ \zeta_c &= 0.434 \\ G_{\max} &= 1.28 \\ \omega_{nc} &= 4.26 \text{ rad/sec} \\ \omega_{\max} &= \omega_{nc} \sqrt{1-2\zeta^2} = 3.36 \text{ rad/sec} \end{aligned}$$



5 Axle Double

Table 6. (Cont.)

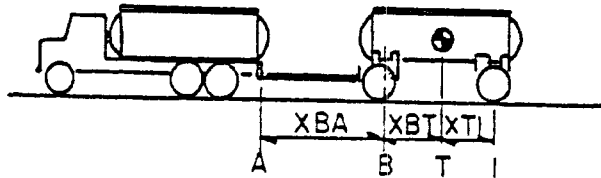
Name: Full Trailer from a California Truck-Full Trailer  
(See Figure 14)

Parametric Values:

$$\begin{aligned} \dot{x}_A &= 968 \text{ in/sec (55 mph)} \\ m_T &= 98.34 \text{ lb-sec}^2/\text{in} \\ x_{BT} &= 106.6 \text{ in} \\ x_{BA} &= 148 \text{ in} \\ C_\alpha &= 154,710 \text{ lb/rad} \\ x_{T1} &= 115.4 \text{ in} \\ I_T &= 683,600 \text{ in-lb-sec}^2 \end{aligned}$$

Numerical Results:

$$\begin{aligned} \frac{\Sigma C}{m_T} &= 3143 \text{ in/rad-sec}^2 \\ (x_{BT} + x_{BA}) &= 254.6 \text{ in} \\ \zeta_c &= 0.462 \\ G_{\max} &= 1.22 \\ \omega_{nc} &= 3.52 \text{ rad/sec} \\ \omega_{\max} &= \omega_{nc} \sqrt{1-2\zeta^2} = 2.67 \text{ rad/sec} \end{aligned}$$



California Truck Full Trailer

incorporated in a conventional 65-foot (19.8-m) double, and a full trailer from a "California" truck-full trailer combination, respectively. These frequencies (i.e., values of  $\omega_{\max}$ ) are within the range of frequencies that a driver could excite in an emergency situation when maneuvering to avoid an obstacle.

For second-order systems with damping ratios  $\zeta < .707$ , the maximum amplification,  $G_{\max}$ , which occurs at  $\omega_{\max}$ , may be expressed as a function of damping ratio, viz.,

$$G_{\max} = \frac{1}{2\zeta_c \sqrt{1-\zeta_c^2}} \quad (6.3)$$

When evaluated for various values of  $\zeta_c$  (see the graph and table in Figure 49), Equation (6.3) indicates that significant amounts of rearward amplification result for damping ratios less than 0.47.

(If  $\zeta_c > .707$ , the maximum gain is unity and it occurs at low (zero) frequency.)

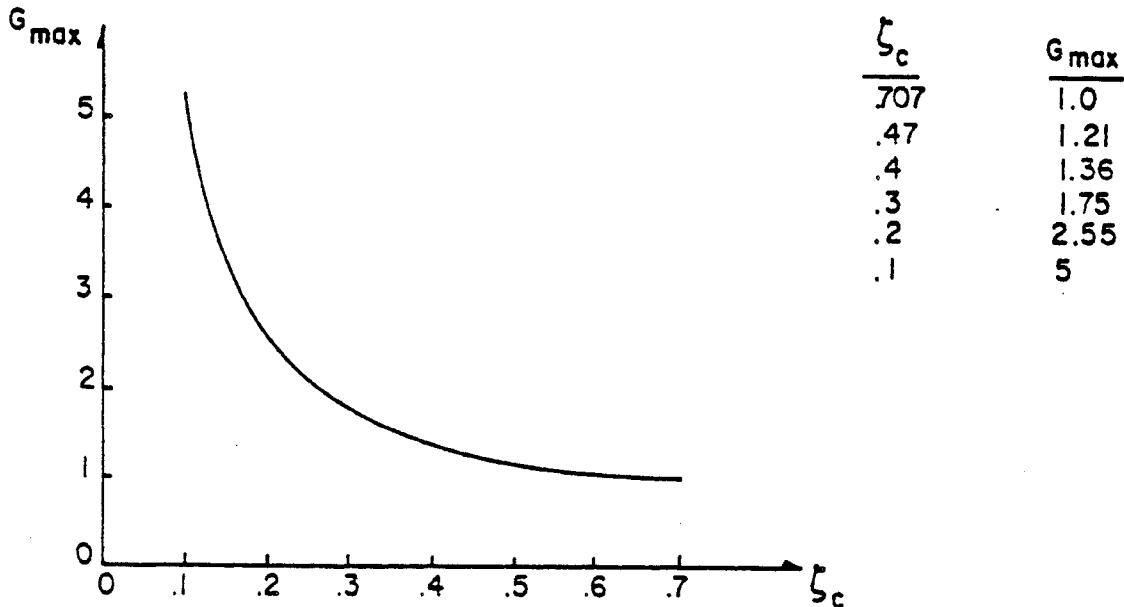


Figure 49.  $G_{\max}$  versus  $\zeta_c$ .



The basic results concerning the influence of full trailer parameters on rearward amplification are contained in the expression for  $\zeta_c$  which is repeated below:

$$\zeta_c = \frac{1}{2u} \left[ \left( \frac{\Sigma C_\alpha}{m_T} \right) (x_{BT} + x_{BA}) \right]^{1/2} \quad (6.4)$$

where

$u$  is the forward velocity

$\left( \frac{\Sigma C_\alpha}{m_T} \right)$  is the cornering coefficient for the trailer (i.e., the sum of the cornering stiffnesses of all the trailer's tires divided by the mass of the trailer)

and  $(x_{BT} + x_{BA})$  is the distance from the pintle hitch to the center of gravity of the full trailer

Equation (6.4) indicates that the damping ratio decreases (thereby causing the rearward amplification to increase) if (a) the forward velocity,  $u$ , is increased, (b) the total cornering coefficient,  $\Sigma C_\alpha / m_T$ , is decreased, or (c) the distance from the c.g. to the pintle hitch is decreased. Since damping ratio is inversely proportional to forward velocity, the magnitude of the velocity is a critical consideration when examining rearward amplification. The influences of the cornering coefficient and the distance from the pintle hitch to the c.g. both follow a square root characteristic. Hence, they are equally powerful (on a percentage basis) with regard to their influences on damping ratio and rearward amplification, but less powerful than the velocity factor.

In a qualitative sense, large amounts of rearward amplification are predicted at high velocities for heavily loaded short trailers with low values of cornering coefficients. Numerical examples for three full trailers that are employed in combination vehicles having large amounts of rearward amplification [20] are presented in Table 6. Even though the pup trailer of the double tanker has five axles (20 tires) as compared to the other two full trailers (see Table 6) which have two axles (eight tires), the overall cornering coefficient ( $\Sigma C_\alpha / m_T$ ) is significantly smaller for

the pup trailer than it is for the other two trailers (this is because the pup trailer employs small tires and carries a very heavy load). In addition, the pup trailer has a relatively short tongue length,  $x_{BA}$ , and a short overall length factor ( $x_{BA} + x_{BT}$ ). Consequently, the rearward amplification is much larger for the pup trailer than it is for the other two trailers.

Although the full trailer from the so-called "California truck-full trailer" has the lowest amplification factor,  $G_{\max}$ , it also has the lowest value of  $\omega_{\max}$  (see Table 6). This means that this full trailer has its maximum gain at a lower frequency which is easier, and probably more likely, for the driver to employ. Examination of the expressions for  $\omega_{nc}$  and  $\zeta_c$  (see Table 4) indicate that increasing the tongue length,  $x_{BA}$ , has both a major advantage, because it increases the damping ratio of the equivalent second-order system, and also a minor disadvantage connected with lowering the natural frequency of the equivalent second-order system, thereby producing additional amplification in less rapid maneuvers. Nevertheless, increasing the tongue length is an important countermeasure (design factor) to consider when attempting to reduce rearward amplification.

6.3.2 The Influence of Towing Unit Parameters. An appreciable fraction of the overall rearward amplification of multiply-articulated vehicles results from the fact that pintle hitches are not located at or near the centers of gravity of towing units. The lateral acceleration of the pintle hitch differs from that of the c.g. of a towing unit by a factor determined by the product of the yawing rotational acceleration of the towing unit times the distance,  $x_{pc}$ , from the c.g. back to the location of the pintle hitch. Since this fundamental fact applies to all types of towing units whether they be trucks, tractor-semitrailers, or full trailers, the expressions pertaining to the amplification factors for the three basic types of towing units given in Table 4 have several important similarities.

(Again, see Appendix C for a detailed technical discussion of the derivation of the expressions given in Table 4. The intention here is to use the items in Table 4 to develop an understanding of the sensitivity of rearward amplification to the values of design parameters.)

First observe (see Table 4) that for each towing unit a quantity,  $\Delta A$ , may be conveniently used to evaluate the amount that the amplification factor differs from 1.0, that is, from "zero amplification." This convenience is a direct result of the fundamental connection between the lateral acceleration of the c.g. of a towing unit and the lateral acceleration of the location of its pintle hitch. Furthermore, note that in all cases (Items 1, 2, and 3 in Table 4)  $\Delta A$  would be zero if  $x_{pc}$  were zero. Clearly,  $x_{pc}$ , the distance from the c.g. of the towing unit to the pintle hitch, is a crucial parameter in determining rearward amplification with large values of  $x_{pc}$  tending to promote increased amplification.

Pintle hitches are connected to the rear of towing units because other arrangements are usually not practical—e.g., rear axles are in the way of long drawbars and suitable attachment points with adequate clearance for rotations are not available near the centers of gravity of towing units. However, the possibility of using double drawbar arrangements has been considered recently by analysts, designers, experimentalists, inventors, and researchers in general [23,24,25,26,22]. Although double drawbars are not commonly used in commercial service, they have been successfully employed in special situations [7]. These double drawbar setups result in four-bar linkages that are approximately equivalent to providing a very long tongue that is effectively attached to the towing unit at a location well forward of the rear of the towing unit. For example, using a double drawbar arrangement, the effective value of  $x_{pc}$  could be reduced to zero, thereby eliminating any contribution from the towing unit to the overall rearward amplification of the entire vehicle [22].

For conventional towing units, for which  $x_{pc}$  is not readily altered by a significant amount, the influences of design parameters other than  $x_{pc}$  can best be studied by evaluating  $\Delta A$  numerically. Nevertheless, certain observations follow from recognizing that all of the expressions given for  $\Delta A$  in Table 4 may be stated in the following form:

$$\Delta A(\omega) = \frac{\frac{-x_{pc}}{u} j\omega (j\omega\tau + 1)}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + 2\zeta \frac{\omega}{\omega_n}} \quad (6.5)$$

where the specific definitions of  $\tau$ ,  $\omega_n$ , and  $\zeta$  depend upon whether the towing unit is a straight truck, a tractor-semitrailer, or a full trailer. The quantity  $\tau$  in the numerator of (6.5) represents the quotient of the forward velocity divided by the generalized cornering coefficient for the towing unit and, generally, the magnitude of  $\Delta A$  at a given forward velocity can be reduced if the unit's cornering coefficient is increased. The denominator of (6.5) corresponds to that of a classical second-order system which can resonate at a frequency near  $\omega_n$  if  $\zeta$  is small. For heavy commercial vehicles,  $\omega_n$  tends to be above 6 to 7 rad/sec—frequencies well above the maneuvering range that the driver can effectively utilize. However, if  $\zeta$  is small, the influences of the combinations of parameters appearing in the denominator of (6.5) can be important in the range of frequencies corresponding to emergency maneuvers, that is, around 3 rad/sec. These general observations are illustrated in the following results for a "California" truck-full trailer.

The truck in the "California" truck-full trailer is represented by the parameters given in Table 7. As indicated by the numerical results for  $\Delta A$  and  $A$  (also given in Table 7), these quantities ( $\Delta A$  and  $A$ ) are complex variables that are functions of frequency. The manner in which  $\Delta A$  varies with frequency is illustrated in Figure 50 which also shows how the vector,  $\Delta A(3)$ , combines with the unit vector to form the vector  $A = 1 + \Delta A$ , when  $\omega = 3$  rad/sec.

The form of the  $\Delta A$  function and the magnitude of the amplification factor,  $A$ , can be related to the unit constants ( $\omega_n$ ,  $\zeta$ , and  $1/\tau$ ) given in Table 7. The value of  $1/\tau$  indicates the frequency at which the magnitude of  $\Delta A$  will increase rapidly due to the terms in the numerator of Equation (6.5). In this case, the cornering coefficient,  $\Sigma C_{\alpha}/m$  and the forward velocity,  $u$ , have values such that  $1/\tau = 3.2$  rad/sec, which is in the range of frequencies that may be excited in emergency (lane-change) maneuvers. By making  $\Sigma C_{\alpha}/m$  larger (increasing the cornering coefficient) or by reducing  $u$  (velocity),  $1/\tau$  could be made to occur at a higher frequency, thereby reducing the amplification in the emergency maneuvering range.

Since  $\omega_n$  equals 7.5 rad/sec, the denominator terms in Equation (6.5) are smallest at frequencies well above the emergency (lane-change)

Table 7.  
STRAIGHT TRUCK-TOWING UNIT

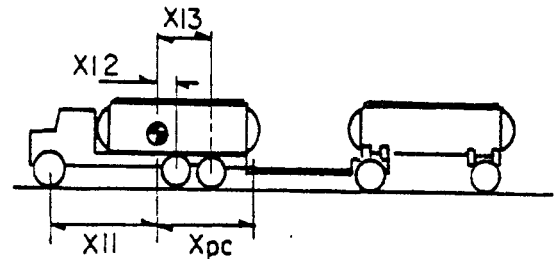
UNIT I.D.: T-1-"CALIFORNIA" TRUCK-FULL TRAILER (See Fig. 14)

INPUT PARAMETERS:

TRUCK WEIGHT, LBS.	W = 41840	1 m = 39.37 in
TRUCK YAW Inertia, in-lb-sec <sup>2</sup>	I = 1060000	1 N = 0.2248 lb
CG. TO FRONT AXLE, IN.	X11 = 177	
CG. TO 1ST. REAR AXLE, IN.	X12 = 32	
CG. TO 2ND. REAR AXLE, IN.	X13 = 84	
CG. TO PINTLE HITCH, IN.	XPC = 163	
FRONT AXLE C <sub>α</sub> , LB/DEG	C11 = 1300	
1ST. REAR AXLE C <sub>α</sub> , LB/DEG	C12 = 2300	
2ND. REAR AXLE C <sub>α</sub> , LB/DEG	C13 = 2300	
FORWARD VELOCITY, MPH	V = 55 (88.5 k/hr)	

UNIT CONSTANTS:

	WN = 7.513 RAD/SEC
ζ =	ZETA = .22
ΣC <sub>α</sub> /m u = 1/τ =	1/TAU = 3.225 RAD/SEC



FREQ. RAD/SEC	DELTA-A (ΔA)		GAIN (A = 1+ΔA)	
	REAL	IMAGINARY	MAGNITUDE	PHASE (DEG)
0	0	0	1	0
.5	.01	-.085	1.014	-4.802
1	.042	-.174	1.057	-9.473
1.5	.097	-.272	1.13	-13.921
2	.176	-.385	1.237	-18.113
2.5	.281	-.52	1.382	-22.082
3	.414	-.688	1.573	-25.927
3.5	.579	-.905	1.82	-29.803
4	.774	-1.194	2.138	-33.926
4.5	.993	-1.591	2.55	-38.587
5	1.211	-2.15	3.083	-44.184
5.5	1.358	-2.941	3.769	-51.264
6	1.274	-4.028	4.625	-60.547
6.5	.858	-5.351	5.601	-72.779
7	-.788	-6.483	6.485	-88.125
7.5	-2.817	-6.697	6.938	-105.18
*7.6	-3.214	-6.596	6.957	-108.556
8	-4.532	-5.628	6.813	-121.214
8.5	-5.411	-4.627	6.32	-134.259
9	-5.641	-3.364	5.73	-144.067
9.5	-5.549	-2.493	5.186	-151.273
10	-5.34	-1.878	4.727	-156.601

\* INDICATES MAXIMUM GAIN VALUE

$\Delta A$  versus  $\omega$  for the truck part of a California truck full trailer operating at 55 mph. (88.5 k/hr)

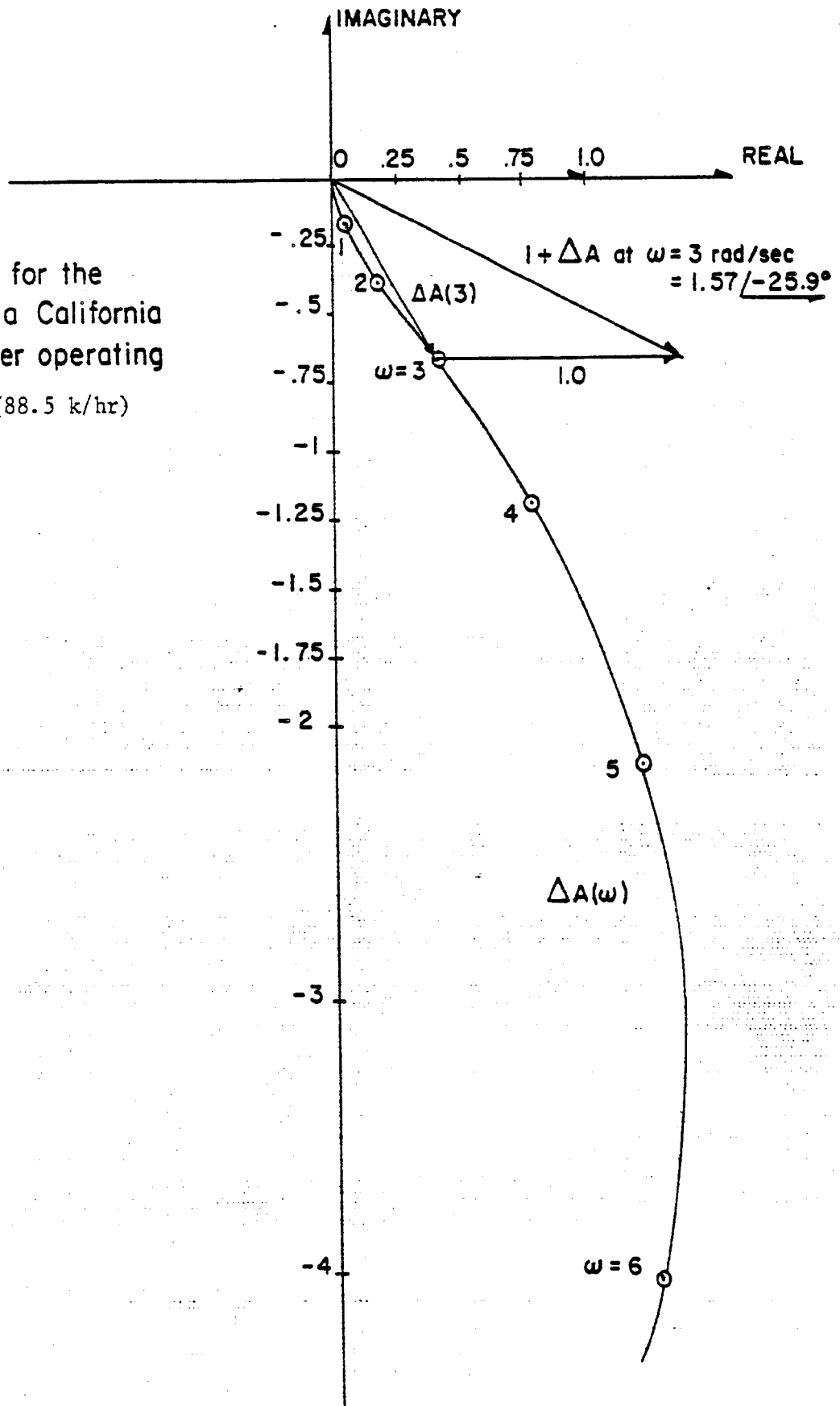


Figure 50.  $\Delta A$  versus frequency,  $\omega$ , in the complex plane.

maneuvering range. However,  $\zeta$ , the damping ratio, is small so that the denominator terms can contribute significantly to the magnitude of the  $\Delta A$  vector even at frequencies in the neighborhood of 3 rad/sec. For example, a second-order system with a damping ratio of 0.22 has a gain of approximately 1.25 at a frequency corresponding to one-half of its natural frequency. The point of this discussion is that the parameters contributing to  $\zeta$  can have a noticeable, possibly important, influence on the amplification factor for the towing unit.

Continuing with the straight truck example (but still noting that this example is similar to the situation for other types of towing units), the influences of design parameters on amplification can be discussed further by examining the expression for  $\zeta$  as given below:

$$\zeta = \frac{1}{2} \left( \frac{\Sigma x^2 C_{\alpha}}{u} \right) \frac{1}{(I x_{11} \Sigma C_{\alpha})^{1/2}} \quad (6.6)$$

where  $\frac{\Sigma x^2 C_{\alpha}}{u}$  is the damping in yaw (i.e., the influence of tire stiffnesses and axle location in opposing yaw motion)

$I$  is the yaw moment of inertia of the truck

$x_{11}$  is the distance from the c.g. to the front (steering axle)

$\Sigma C_{\alpha}$  is the total cornering stiffness (the sum of the cornering stiffnesses of all the tires)

As indicated by (6.6), desirable parametric combinations (with respect to making  $\zeta$  large) are present in vehicles that may be described as follows:

a) The damping-in-yaw ( $\Sigma x^2 C_{\alpha}/u$ ) is large because the axles are well spread away from the c.g. location and the tires are stiff. (As is typical for all pneumatic-tired vehicles, the damping-in-yaw is inversely proportional to forward velocity, indicating that the worst case occurs at high speed.)

b) The moment of inertia is small for the wheelbase—meaning that the unit does not have large, heavy sections that extend outside its wheelbase.

c) The distance from the c.g. to the steering axle or axles is not a large fraction of the wheelbase, i.e.,  $x_{11}$  is relatively small.

d) The total cornering stiffness is sufficiently large (although  $(\Sigma C_{\alpha})^{1/2}$  occurs in the denominator of (6.6), the damping-in-yaw also contains the cornering stiffnesses such that increasing the overall sum of the cornering stiffnesses tends to increase the damping ratio,  $\zeta$ ).

Based on the above considerations concerning  $x_{pc}$ ,  $\Sigma C_{\alpha}/m$ , and  $\zeta$ , the "California" tank truck sketched in Figure 51 is an example of a towing unit that will have a substantial rearward amplification at frequencies above 2 rad/sec (see Table 7). The center of gravity of the tank truck (shown towing a full trailer in Figure 51) lies just in front of the tires installed on the middle axle of the towing unit. Without knowing anything about the installed tires, but accepting them as satisfactory, one would expect the damping-in-yaw to be small because the rear axles with dual tires are close to the c.g. while the front wheels, although they are relatively far from the c.g., are equipped with single tires. The fact that the (front) steered wheels are far from the c.g. is unfavorable with regard to rearward amplification, and the large overhang of the tank beyond the rear axle is also unfavorable because of its implications concerning the yaw moment of inertia. Finally, since the hitch point is located well behind the end of the tank, the distance,  $x_{pc}$ , is large. This example illustrates that if (1) the c.g. location is calculated from axle loads and (2) a picture (or the vehicle itself) is available for examination, an estimate of whether the unit will have a large amplification factor can be readily made.

Intuitively, the basic idea used in identifying towing units with substantial lateral acceleration gains reduces to considering the situations in which the c.g. of the unit responds laterally with much less gain than the pintle hitch does. When a towing unit yaws significantly with respect to the amount of lateral translation occurring at the c.g., the motion of the pintle hitch, which is the input to the towed unit, will be a considerably



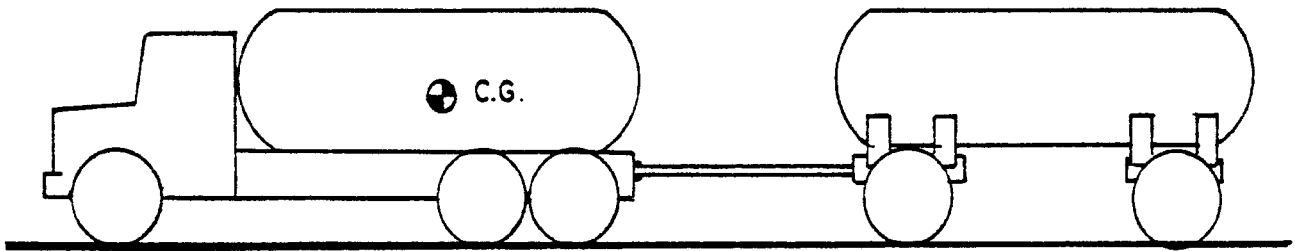


Figure 51. Sketch, California truck-full trailer.

amplified version of the motion of the c.g. (if the pintle hitch is located at a significant distance away from the c.g.). With the exception of  $x_{pc}$ , the parametric sensitivities discussed in this section are simply those that contribute to large ratios of yaw acceleration to lateral acceleration gain. In the simplest terms, if (a) tire and inertial characteristics are approximately equivalent and (b) the values of  $x_{pc}$  are nearly equal, long wheelbase vehicles will have less rearward amplification than that experienced by short wheelbase vehicles.

6.3.3 Matching of Towing and Towed Units. The overall rearward amplification for a combination vehicle is a complex variable that is a function of frequency. The magnitude of the total rearward amplification depends upon the product of the magnitudes of the amplification factors for each of the units comprising the combination vehicle. This product is carried out at each frequency of interest to account for how the frequency response characteristics of all units influence the overall amplification. Clearly, if several units in a combination have high gains at the same frequency, then a very high overall gain will result. On the other hand, if the frequencies at which high gains occur are separated from each other, the overall gain may not be very much larger than the maximum component gain.

For example, the simplified calculations for the full trailer of the "California" truck-full trailer indicate that  $G_{max} = 1.22$  at a frequency of 2.67 rad/sec, while the tank truck has a maximum amplification of almost 7.0 at 7.6 rad/sec. However, the overall amplification determined by the products of the appropriate transfer functions pertaining to the towing and towed units does not exceed 2.0 (see Figure 52). In this case, the lateral acceleration gain of the full trailer falls off rapidly enough above

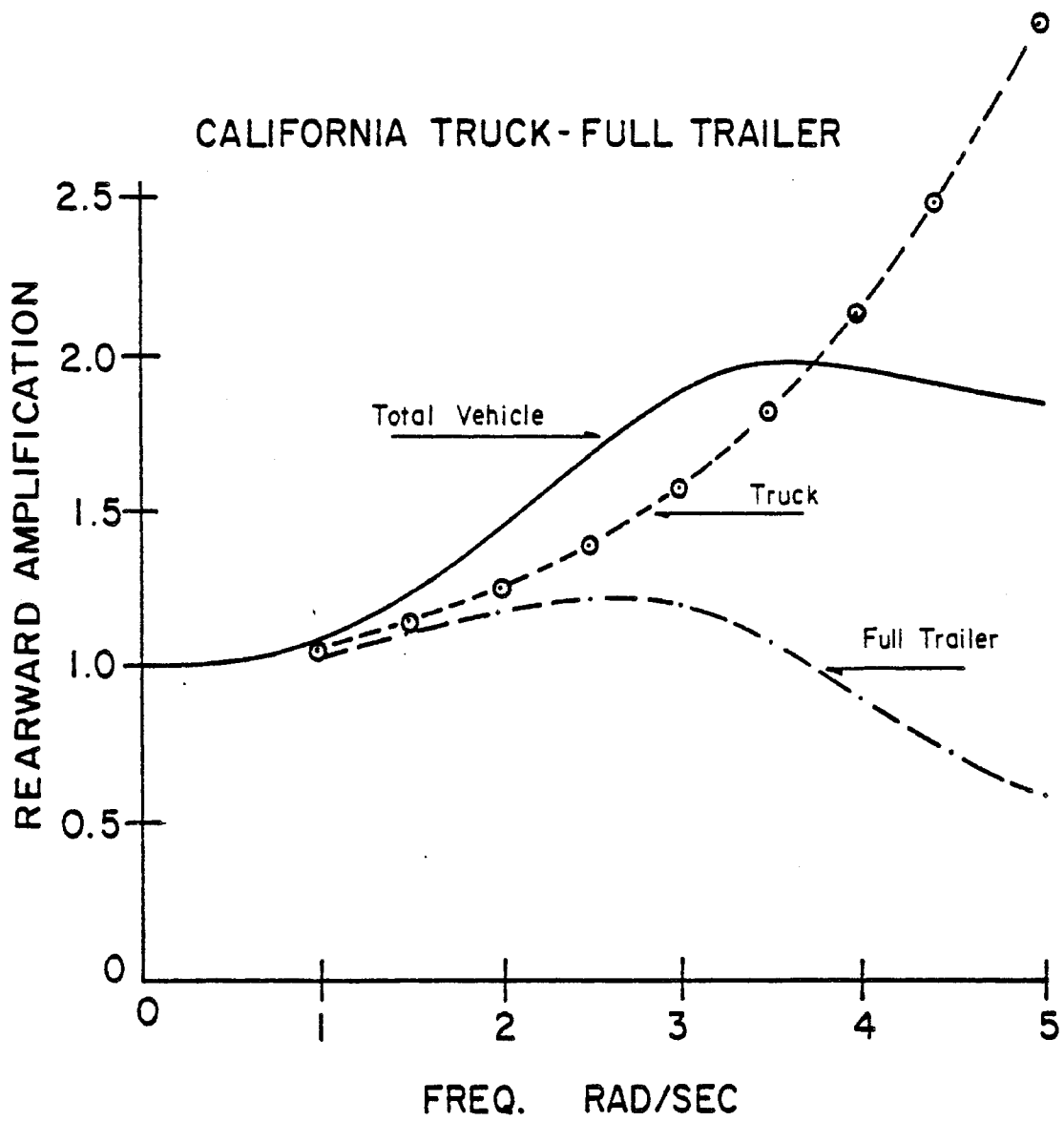


Figure 52. The influence of truck and full trailer amplification factors on rearward amplification.

3.5 rad/sec to compensate for the rapid increase in the amplification factor of the towing unit in the frequency range from 3 to 5 rad/sec (again see Figure 52).

(Results for frequencies above 5 rad/sec are generally not considered to be meaningful with regard to driver capabilities, and 3.14 rad/sec (0.5 Hz) is sometimes used as the maximum frequency of interest [21].)

The results shown in Figure 52 illustrate that the parameters describing the full trailer have less influence on the overall amplification factor than the parameters describing the tank truck. In this case, changes in full trailer parameters (excluding those changes pertinent to double drawbar arrangements) are not likely to be highly beneficial, but the selection of a different towing unit with a lower amplification factor might be a viable improvement. With regard to the tank truck, the desires to make (a) the quantity  $x_{pc}$  small and (b) the towing vehicle long with a centrally located c.g. (i.e., the front axle not greatly removed from the c.g.) are in conflict. That is, if the pintle hitch is placed at the rear of the towing unit, then attempts to reduce yaw response by lengthening the wheelbase or moving the c.g. closer to the front axle will increase  $x_{pc}$ , thereby tending to offset the improvement gained by reducing the yaw response. However, changes in design that decrease the yaw moment of inertia and reduce the overhang of the tank behind the rear axle appear to be good possibilities for obtaining moderate reductions in rearward amplification. Nevertheless, unless some novel approach (such as the double drawbar arrangement) is used to reduce  $x_{pc}$ , it will be difficult to achieve major reductions in rearward amplification by changing the geometric layout of the tow vehicle.

With regard to vehicles with multiple units (such as in a triples combination), the overall rearward amplification can be very large. For example, a conventional triple consisting of a tractor and three 27-foot trailers [20] has been found to have the following amplification factors at a frequency of 3.36 rad/sec corresponding to  $\omega_{max}$  for each of the nearly identical full trailers [22]:

Table 8. Rearward Amplification Factors Pertaining to a Triples Combination.

Tractor c.g. to semi c.g.	1.15
Semi c.g. to pintle hitch	1.41
1st full trailer pintle hitch to c.g.	1.28
1st full trailer c.g. to pintle hitch	1.40
2nd full trailer pintle hitch to c.g.	1.28

---

The product of these factors equals 3.72 which is a large amount of amplification to predict at 3.36 rad/sec.

Note that when all the units are nearly identical, the individual maximums all occur at nearly the same frequency, thereby producing the maximum overall amplification. Also, note that amplification increases by a multiplicative factor of 1.8 (i.e.,  $1.40 \times 1.28$ ) when going from a doubles to a triples combination (see Table 8 describing the conventional triple). Clearly, when several identical units with high individual amplification factors are coupled together, a very high overall rearward amplification will occur.

#### 6.4 Nonlinear Simulation Results for Selected Vehicles Performing Obstacle-Avoidance Maneuvers

As a result of the survey effort conducted in the initial part of this project, two multiply-articulated vehicles were selected for detailed study. These vehicles were a "California" truck-full trailer and a five-axle "dirt" truck pulling a six-axle full trailer (see Table 2 and Figures 14 and 15).

In addition, three truck-semitrailer combinations with unconventional hitching arrangements were selected. One of these vehicles consisted of an empty three-axle dump truck employing a pintle hitch to tow a lowboy semi-trailer loaded with a backhoe. This combination was seen as being (1) prevalent in the construction industry and (2) susceptible to enough rearward amplification to cause premature rollover of the semitrailer due to the use of a pintle hitch located well aft of the rearmost axle.

In this case, large lateral forces are present at the pintle hitch because there is not a steerable dolly as in the case of a full trailer. Nevertheless, the study of automobile-recreational semitrailer combinations indicates that this type of vehicle arrangement may have a very oscillatory directional behavior [27,28]. Furthermore, in contrast to a fifth wheel, the pintle hitch does not provide roll support to the semitrailer. Hence, in an obstacle-avoidance maneuver, the likelihood of rollover of the semitrailer can be a problem for this type of vehicle.

The other two vehicles selected because of their atypical hitching arrangements are a so-called "California dromedary" and a car hauler (see Figures 13 and 12). Both of these vehicles employ fifth wheels placed near or at the rear of the towing unit and in both cases the towing unit carries part of the load (see Figure 53). The fifth wheel provides a roll constraint (as contrasted to a pintle hitch), nevertheless, due to the rearward location of the articulation joint, the possibility for unusually high rearward amplification exists.

Due to the fundamental differences between vehicles employing steerable dollies and truck-semitrailer vehicles, the following material has been divided into separate subsections with the first subsection treating the truck-full trailer type of vehicle (to which the earlier simplified analysis pertains) and the second subsection presenting example results for an empty truck-lowboy combination, a "California" dromedary, and a car hauler with a stinger fifth wheel.

6.4.1 Truck-Full Trailer Examples. The "California" truck-full trailer has served as a primary example in the discussion of parametric sensitivities based on a simplified linear analysis (see Section 6.3). The linear analysis predicts a maximum rearward amplification of approximately 2.0 at frequencies in the neighborhood of 3.4 rad/sec for this truck-full trailer. Hence, if the full trailer has a load with a high center of gravity so that it is susceptible to rolling over, the simplified analysis indicates that the full trailer may be expected to roll over in a severe avoidance maneuver.

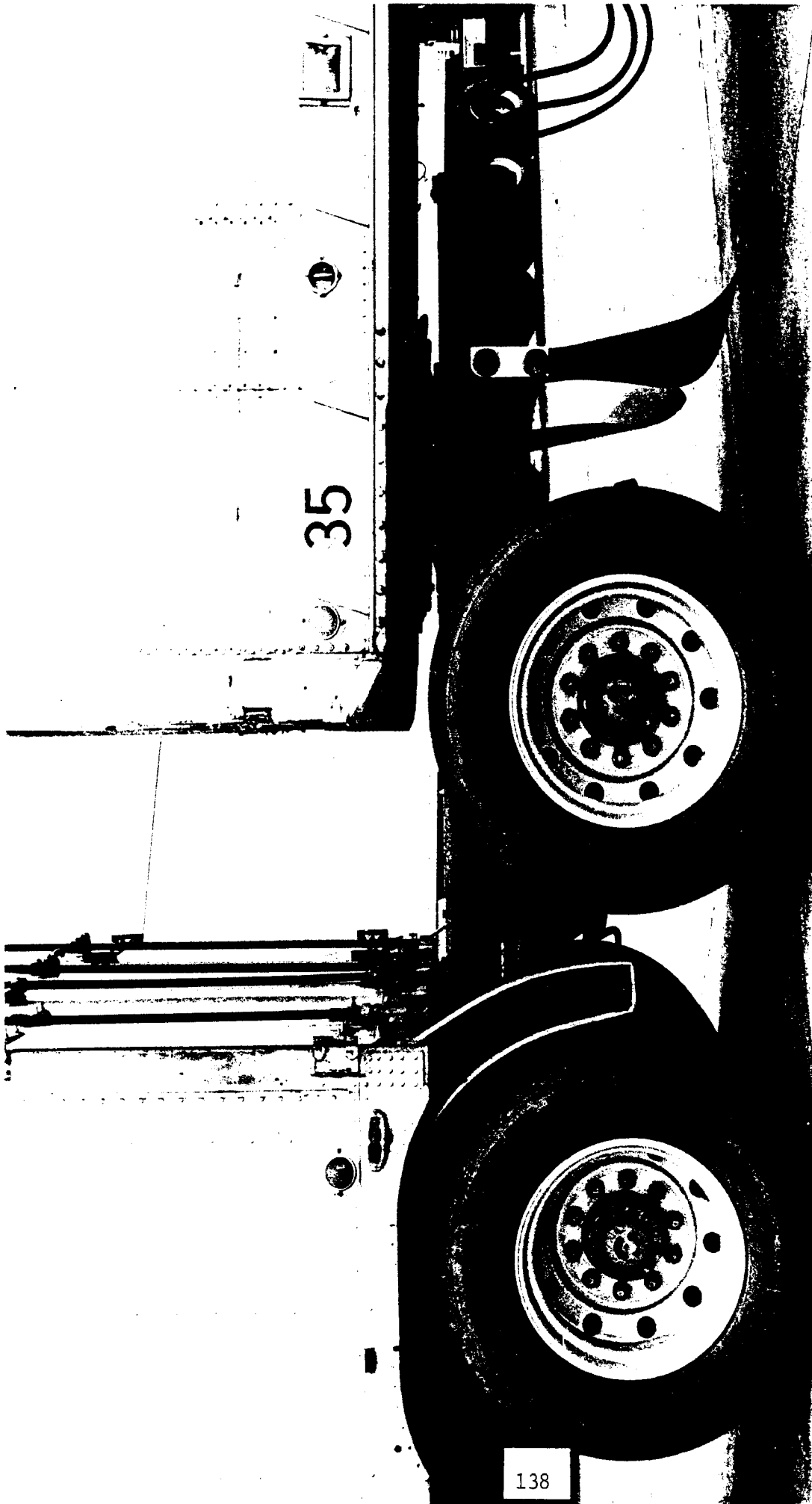


Figure 53a. California dromedary, 5th wheel on extended frame rails behind the trailing suspension on the towing unit.

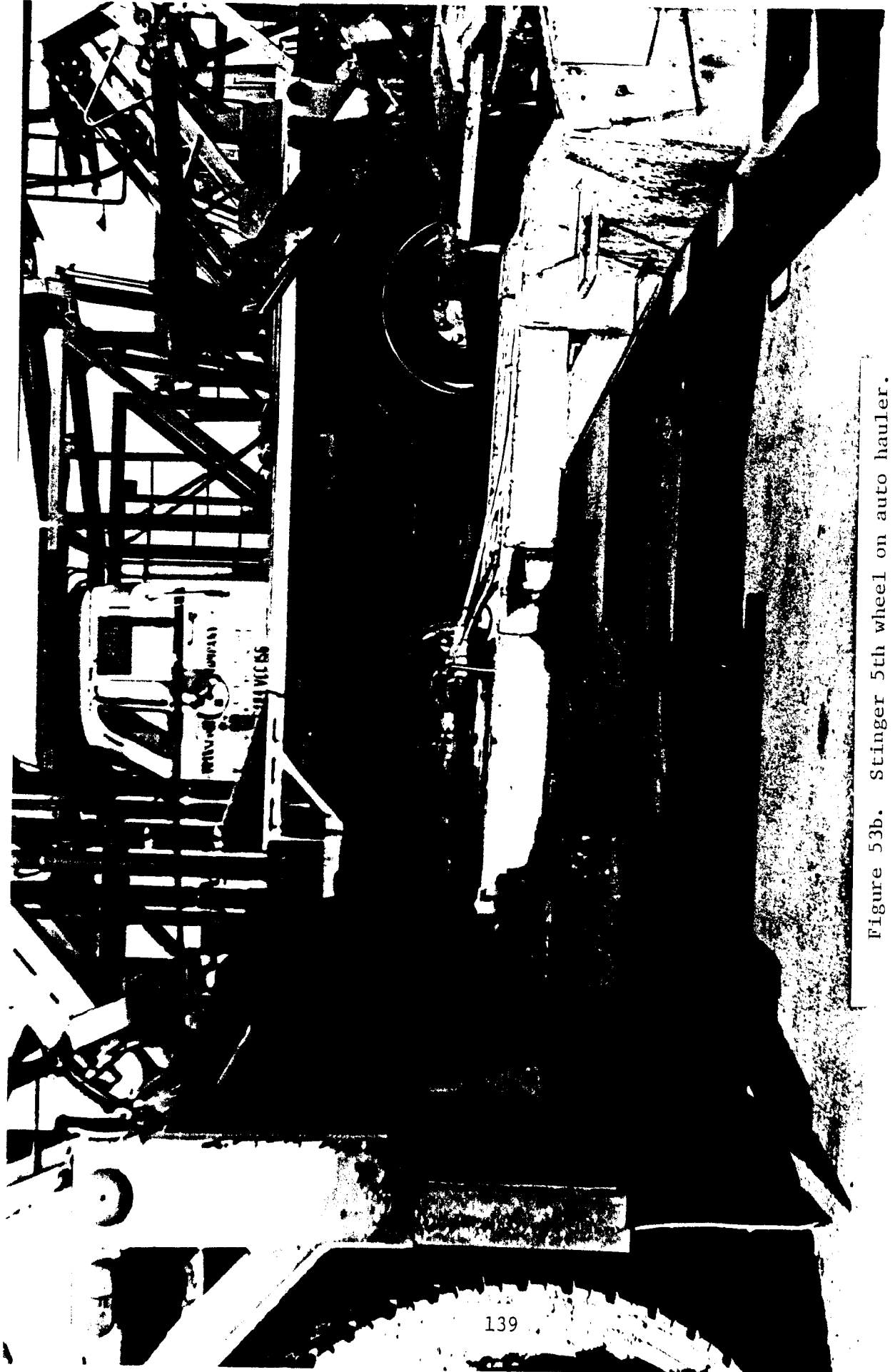


Figure 53b. Stinger 5th wheel on auto hauler.

To pursue this prediction in detail, a nonlinear simulation model (see Appendix D) has been used in studying the yaw and roll dynamics of the vehicle. Simulation runs have been made to provide a characterization of vehicle response in an emergency obstacle-avoidance maneuver. The desired trajectory for the maneuver is given by an equation of the form

$$y(X) = \begin{cases} \left(\frac{D}{T}\right)\left(\frac{X}{V}\right) & \text{for } X < VT \\ D & \text{for } X \geq VT \end{cases} \quad (6.7)$$

where

$y$  is the instantaneous lateral displacement

$X$  is the longitudinal position

$D$  is the total lateral displacement

$T$  is the period of the maneuver

and  $V$  is the forward velocity

Using the ground path given by (6.7), a steering control model, that approximates driver control characteristics, has been used to cause the vehicle to follow the desired path [17]. To illustrate the limit response of the vehicle in avoiding an obstacle, a series of simulation runs were made at 55 mph (88.5 k/hr) with  $D$  equal to a lane width (i.e., 12 feet (3.66 m)) and  $VT$  varied from 150 to 100 feet (45.7 m to 30.5 m).

In the simulation run with  $VT = 100$  feet (30.5 m), the full trailer rolled over (see Figure 54 and Table 9). Note that, although the simulation is a nonlinear time-domain calculation of vehicle response, the simulation results correspond qualitatively to the predictions of the simplified linear analysis. First, the force at the pintle hitch is small throughout the maneuver. Second, even though this type of vehicle has a very limited range of linear operation, the linear analysis does indicate the influence of the parameters that it includes—even in the nonlinear case. None of the nonlinearities contribute to improving the directional response characteristics of the vehicle. As predicted, the lateral acceleration at the c.g. of the full trailer is much larger than at the c.g. of the truck.



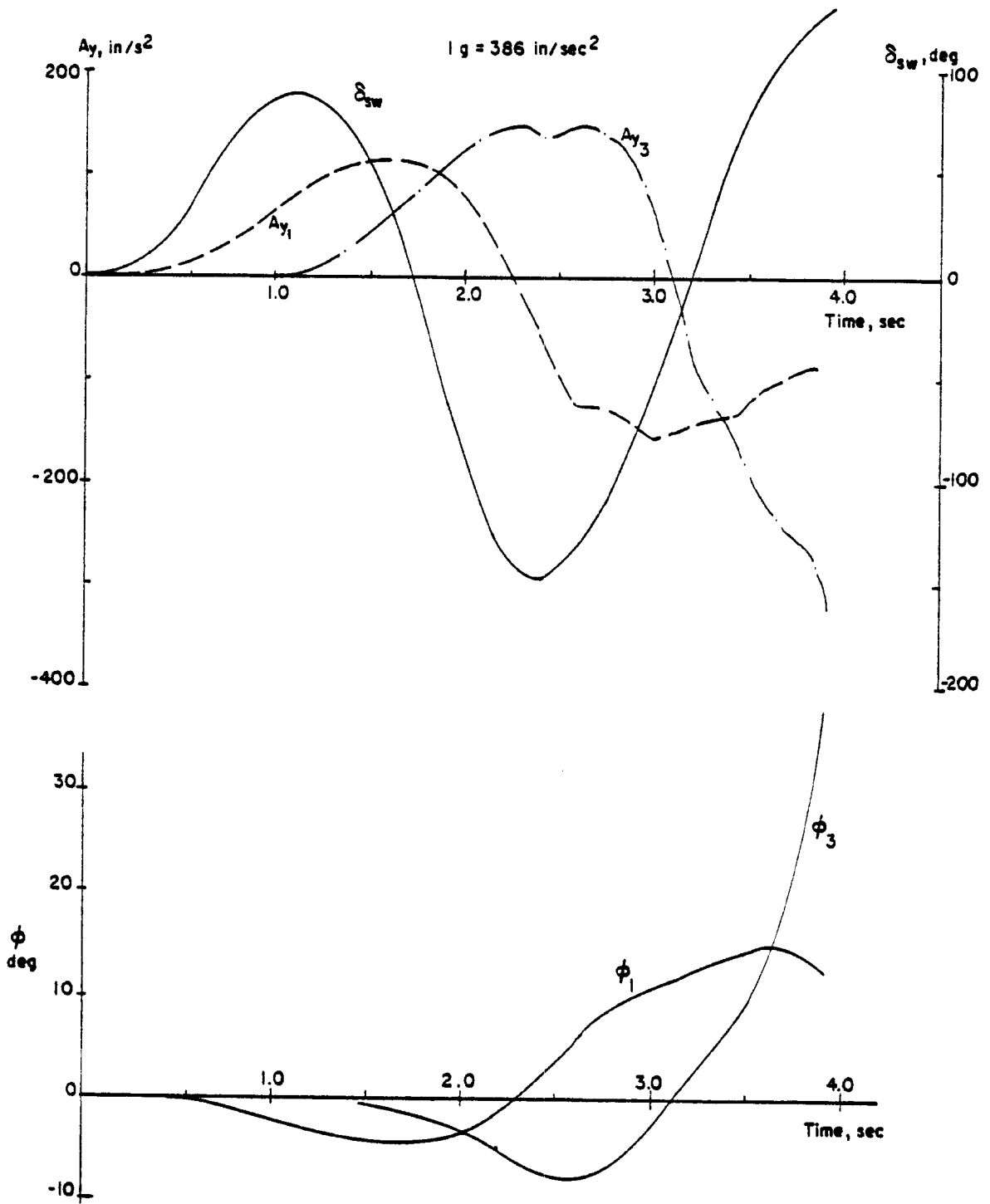


Figure 54. Obstacle-avoidance, California truck-full trailer.

Table 9

SAXLE - CALIFORNIA - TRUCK - FULL TRAILER

SPRINGS MASS \* 1 (TRUCK SPRUNG MASS)

PAWIE  
HITCH  
/ WGT  
(LBS)

TIME (SEC)	FORWARD POSITION (IN)	LATERAL POSITION (IN)	VEDICAL POSITION (IN)	ROLL ANGLE (DEG)	YAW ANGLE (DEG)	PITCH ANGLE (DEG)	FORWARD VEL (IN/SEC)	LATERAL VEL (IN/SEC)	ROLL RATE (DEG/SEC)	YAW RATE (DEG/SEC)	PITCH RATE (DEG/SEC)	LATERAL ACCN (IN/SEC**2)	STEEER ANGLE DEG
0.0	0.0	0.0	0.0	0.0	0.0	0.0	968.04	0.0	0.0	0.0	0.0	0.0	0.0
0.10	96.80	0.00	0.000	0.01	0.00	0.000	968.04	0.01	0.01	0.09	0.05	0.00	0.51
0.20	193.60	0.01	0.000	-0.01	0.01	-0.000	968.04	-0.01	-0.01	-0.09	0.10	0.00	0.75
0.30	290.40	0.02	0.000	0.02	0.02	0.000	968.04	-0.02	0.02	-0.14	0.15	0.00	1.31
0.40	387.20	0.05	0.000	-0.06	0.05	-0.000	968.04	0.32	-0.32	0.54	0.38	0.00	4.42
0.50	484.00	0.13	0.000	-0.14	0.13	-0.000	968.04	-0.75	-1.18	1.18	0.86	0.00	9.50
0.60	580.77	0.30	0.001	-0.30	0.29	-0.000	968.04	-1.81	-3.20	3.07	1.89	-0.01	17.20
0.70	677.54	0.64	0.001	-0.56	0.50	-0.000	968.04	-3.84	-6.27	6.15	3.07	0.03	29.95
0.80	774.31	1.24	0.003	-0.87	0.87	0.000	968.04	-7.01	-11.15	10.97	4.35	-0.07	36.54
0.90	871.08	2.21	0.009	-1.52	1.37	0.001	968.04	-11.15	-15.91	15.72	6.75	-0.14	49.77
1.00	967.85	3.68	0.022	-2.19	1.99	0.002	968.04	-15.91	-20.96	20.83	9.51	-0.25	64.52
1.10	1064.62	5.79	0.045	-2.84	2.71	0.002	968.04	-20.96	-26.17	26.04	12.73	-0.40	78.89
1.20	1161.38	8.69	0.073	-3.31	3.53	0.001	968.04	-26.17	-31.36	31.23	16.51	-0.54	91.61
1.30	1258.11	12.50	0.094	-3.73	4.41	0.005	968.04	-31.36	-36.36	36.23	20.32	-0.63	101.69
1.40	1354.82	17.33	0.103	-3.94	5.33	0.007	968.04	-36.36	-41.07	40.94	24.19	-0.65	108.35
1.50	1451.49	23.25	0.107	-4.10	6.27	0.005	968.04	-41.07	-45.02	44.89	28.06	-0.64	111.95
1.60	1548.10	30.29	0.114	-4.24	7.17	0.004	968.04	-45.02	-48.19	48.06	31.92	-0.68	113.14
1.70	1644.64	38.45	0.107	-4.35	7.99	0.009	968.04	-48.19	-50.61	50.48	35.79	-0.71	110.56
1.80	1741.09	47.72	0.074	-4.41	8.66	0.020	968.04	-50.61	-52.27	52.14	39.66	-0.73	103.27
1.90	1837.43	58.02	0.042	-4.24	9.13	0.018	968.04	-52.27	-53.10	52.97	43.53	-0.73	93.60
2.00	1933.64	69.26	0.125	-3.68	9.30	0.007	968.04	-53.10	-53.10	52.97	47.40	-0.73	81.77
2.10	2029.72	81.30	0.108	-2.83	9.13	0.014	968.04	-53.10	-53.10	52.97	51.27	-0.73	68.80
2.20	2125.62	93.90	0.007	-1.65	8.57	0.014	968.04	-53.10	-53.10	52.97	55.14	-0.73	55.14
2.30	2221.62	106.76	-0.075	-0.09	7.66	0.014	968.04	-53.10	-53.10	52.97	59.01	-0.73	41.54
2.40	2317.55	119.48	-0.014	1.84	6.48	0.023	968.04	-53.10	-53.10	52.97	62.88	-0.73	27.97
2.50	2413.50	131.64	0.105	3.86	5.11	0.042	968.04	-53.10	-53.10	52.97	66.75	-0.73	14.40
2.60	2509.78	142.83	0.105	5.56	3.65	0.014	968.04	-53.10	-53.10	52.97	70.62	-0.73	0.83
2.70	2606.17	152.85	-0.194	7.39	2.17	-0.196	968.04	-53.10	-53.10	52.97	74.49	-0.73	-12.74
2.80	2702.76	161.62	-0.511	9.06	0.65	-0.353	968.04	-53.10	-53.10	52.97	78.36	-0.73	-25.69
2.90	2799.55	169.04	-0.588	10.02	-0.87	-0.410	968.04	-53.10	-53.10	52.97	82.23	-0.73	-38.64
3.00	2896.52	174.99	-0.665	10.73	-2.34	-0.466	968.04	-53.10	-53.10	52.97	86.10	-0.73	-51.59
3.10	2992.44	179.37	-0.853	11.38	-3.70	-0.559	968.04	-53.10	-53.10	52.97	90.00	-0.73	-64.54
3.20	3090.94	182.32	-1.146	12.12	-4.96	-0.684	968.04	-53.10	-53.10	52.97	93.87	-0.73	-77.49
3.30	3188.32	183.78	-1.476	13.83	-6.03	-0.821	968.04	-53.10	-53.10	52.97	97.75	-0.73	-90.44
3.40	3285.74	183.85	-1.796	14.55	-6.90	-0.967	968.04	-53.10	-53.10	52.97	101.62	-0.73	-103.39
3.50	3383.17	182.61	-2.056	14.94	-7.54	-1.107	968.04	-53.10	-53.10	52.97	105.50	-0.73	-116.34
3.60	3480.54	180.14	-2.208	14.85	-7.92	-1.207	968.04	-53.10	-53.10	52.97	109.37	-0.73	-129.29
3.70	3577.80	176.53	-2.229	14.85	-8.04	-1.250	968.04	-53.10	-53.10	52.97	113.25	-0.73	-142.24
3.80	3674.90	171.84	-2.092	14.13	-7.87	-1.125	968.04	-53.10	-53.10	52.97	117.12	-0.73	-155.19
3.90	3771.80	166.09	-1.747	12.57	-7.38	-0.925	968.04	-53.10	-53.10	52.97	121.00	-0.73	-168.14

Table 9 (Cont.)

SAXLE CALIFORNIA TRUCK FULL TRAILER

SPRING MASS # 3  
 .....  
 (SPRING MASS OF THE LOADED PORTION OF THE FULL TRAILER.)

LINE (SEC)	FORWARD POSITION (IN)	LATERAL POSITION (IN)	VERTICAL POSITION (IN)	ROLL ANGLE (DEG)	YAW ANGLE (DEG)	PITCH ANGLE (DEG)	FORWARD VEL (IN/SEC)	LATERAL VEL (IN/SEC)	ROLL RATE (DEG/SEC)	YAW RATE (DEG/SEC)	PITCH RATE (DEG/SEC)	LATERAL ACCH (IN/SEC**2)	ROLL ACCH (IN/SEC**2)	YAW ACCH (IN/SEC**2)	PITCH ACCH (IN/SEC**2)
0.0	425.40	0.0	0.0	0.0	0.0	0.0	968.04	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.10	328.58	-0.00	0.000	0.00	0.00	0.00	968.04	0.00	0.00	-0.00	0.00	0.00	0.00	-0.00	0.00
0.20	231.77	-0.00	0.000	0.00	0.00	0.00	968.04	0.00	0.03	-0.02	0.00	0.15	0.01	0.01	0.01
0.30	134.96	-0.00	0.000	0.01	0.00	0.00	968.04	0.01	0.03	-0.03	0.00	-0.26	0.01	0.01	0.01
0.40	-38.16	-0.00	0.000	0.01	0.01	0.00	968.04	0.05	0.09	-0.05	0.00	-1.19	0.01	0.01	0.01
0.50	58.65	-0.01	0.000	0.03	0.01	0.00	968.06	0.10	0.21	-0.11	0.00	-2.33	0.11	0.07	0.07
0.60	155.45	-0.02	0.000	0.05	0.03	0.00	968.10	0.22	0.35	-0.23	0.00	-3.61	0.14	0.14	0.14
0.70	252.26	-0.04	0.001	0.09	0.06	0.00	968.19	0.44	0.36	-0.37	0.00	-4.40	0.10	0.10	0.10
0.80	349.06	-0.09	0.000	0.12	0.10	0.00	968.34	0.73	0.17	-0.44	0.00	-3.71	0.02	0.02	0.02
0.90	445.88	-0.17	0.000	0.12	0.14	0.00	968.56	0.98	0.28	0.32	0.00	-0.75	0.24	0.24	0.24
1.00	542.70	-0.29	0.001	0.06	0.16	0.00	968.86	1.00	0.93	0.06	0.00	5.09	0.52	0.52	0.52
1.10	639.54	-0.45	0.001	0.08	0.12	0.00	969.25	0.54	1.75	0.75	0.00	14.33	0.87	0.87	0.87
1.20	736.41	-0.62	0.000	0.09	0.08	0.00	969.68	0.58	2.60	1.73	0.00	27.03	1.26	1.26	1.26
1.30	833.32	-0.73	0.000	0.09	0.23	0.00	970.17	-2.50	-3.49	2.98	0.03	42.37	1.69	1.69	1.69
1.40	930.28	-0.69	0.004	0.60	0.23	0.00	970.17	-5.29	-4.17	4.41	-0.03	59.05	2.16	2.16	2.16
1.50	1027.28	-0.39	0.013	0.99	0.60	0.00	970.57	-8.90	-4.39	5.82	0.13	76.18	2.67	2.67	2.67
1.60	1124.33	0.35	0.020	-1.42	1.12	0.00	971.29	-13.22	-4.38	7.29	-0.23	92.69	3.19	3.19	3.19
1.70	1221.41	1.67	0.017	-1.86	1.77	0.00	971.46	-18.49	-4.48	8.91	-0.40	108.08	3.65	3.65	3.65
1.80	1318.50	3.76	0.012	-2.30	2.58	0.00	971.46	-24.97	-4.56	10.57	-0.57	121.56	4.03	4.03	4.03
1.90	1415.60	6.77	0.022	-2.76	3.56	0.00	971.24	-32.68	-4.65	12.04	-0.68	131.53	4.34	4.34	4.34
2.00	1512.66	10.86	0.043	-3.21	4.69	0.00	970.87	-41.43	-6.55	13.22	-1.05	136.87	4.42	4.42	4.42
2.10	1609.65	16.16	0.074	-3.72	5.96	0.00	970.38	-51.16	-15.61	14.16	-1.31	147.31	4.04	4.04	4.04
2.20	1706.58	22.77	0.205	-4.76	7.33	0.00	970.00	-61.72	-15.53	15.14	-1.23	139.63	3.43	3.43	3.43
2.30	1803.39	30.71	0.614	-6.63	8.80	0.00	969.90	-73.15	-2.54	15.14	-1.95	144.53	2.41	2.41	2.41
2.40	1900.15	40.09	1.017	-7.33	10.33	0.00	968.83	-84.43	-3.16	14.41	-2.46	144.53	1.30	1.30	1.30
2.50	1998.79	50.91	1.126	-7.95	11.83	0.00	968.25	-93.27	-2.50	12.97	-2.65	126.36	0.69	0.69	0.69
2.60	2093.21	63.14	1.126	-7.95	13.23	0.00	965.47	-98.45	4.60	10.49	-2.24	104.86	0.07	0.07	0.07
2.70	2189.33	76.83	1.044	-7.90	14.43	0.00	966.02	-99.71	13.12	7.05	-0.36	84.11	0.01	0.01	0.01
2.80	2285.18	91.96	0.918	-6.98	15.33	0.00	967.09	-96.15	21.14	1.94	2.88	62.42	0.01	0.01	0.01
2.90	2380.90	109.42	0.654	-5.27	15.79	0.00	968.11	-82.28	32.05	0.86	0.30	42.37	0.01	0.01	0.01
3.00	2476.43	125.99	0.136	2.63	15.55	0.00	971.32	-56.43	20.87	-18.45	2.25	19.46	0.01	0.01	0.01
3.10	2570.67	141.14	0.365	-0.04	14.31	0.00	974.44	-20.57	20.72	-27.62	2.33	15.18	0.01	0.01	0.01
3.20	2667.43	162.78	0.101	2.02	11.93	0.00	979.05	18.61	18.94	-29.69	2.33	10.42	0.01	0.01	0.01
3.30	2763.46	180.49	0.475	3.86	9.03	0.00	981.44	56.72	31.05	-30.03	5.05	6.53	0.01	0.01	0.01
3.40	2860.10	197.13	0.664	6.57	6.02	0.00	980.65	92.12	27.06	-31.50	5.05	2.29	0.01	0.01	0.01
3.50	2957.42	212.36	0.877	9.32	2.83	0.00	982.85	124.70	42.80	-31.31	5.43	0.01	0.01	0.01	0.01
3.60	3055.32	225.87	-1.834	12.68	0.30	0.00	980.65	152.13	72.96	-28.65	9.33	252.11	0.01	0.01	0.01
3.70	3153.56	237.70	-4.010	18.41	3.41	0.00	970.01	171.49	98.24	-24.01	14.30	260.15	0.01	0.01	0.01
3.80	3252.08	249.27	-7.103	27.05	6.31	0.00	964.47	178.38	132.61	-18.62	17.36	311.95	0.01	0.01	0.01
3.90	3350.99	258.22	-10.127	36.45	8.97	0.00									

The roll characteristics are obviously important in the simulation. The simulation run is stopped after rollover of the full trailer is certain (but before it hits the ground). The roll dynamics and the parameters significantly influencing rollover (e.g., c.g. height to track width, roll center heights, suspension roll stiffnesses, suspension freeplay, etc. see Chapter 4)) are important primarily in determining rollover but, also, they contribute to the lateral accelerations of each of the sprung masses, thereby making the determination of rearward amplification more complicated than in the simplified linear analysis. Clearly, the simplified linear analysis is much easier to use to understand and predict whether a vehicle may have a tendency towards large amounts of rearward amplification. The nonlinear simulation provides the means for producing detailed time histories that can be used to (1) confirm the predictions of the linear analysis and (2) study roll and yaw interactions directly and simultaneously.

To further verify the predictions of the simplified analysis, the simulation model has been used to study the "California" truck-full trailer with an idealized double drawbar arrangement such that  $x_{pc} = 0$ , that is, the effective drawbar is connected from the turntable of the full trailer to a point below the c.g. of the sprung mass of the tank truck (see Figure 55). Clearly, in this case the simulation is representing a linkage arrangement that can only be approximated in practice. Nevertheless, the results (see Table 10 and Figure 56) confirm in a qualitative manner the predictions of the simplified theory. Specifically, the rearward amplification (see Table 10) is approximately 1.0 for this vehicle in this maneuver. Furthermore, the simulated vehicle does not roll over (although it comes close). In this case, the trailer tracks the truck's path with much less overshoot than in the original case with the conventional drawbar (see Figure 56). The improvement in tracking is approximately 50 inches (1.27 m). Hence, the simulation results indicate that the double drawbar should be a successful improvement for extending the obstacle-avoidance capability of this vehicle.

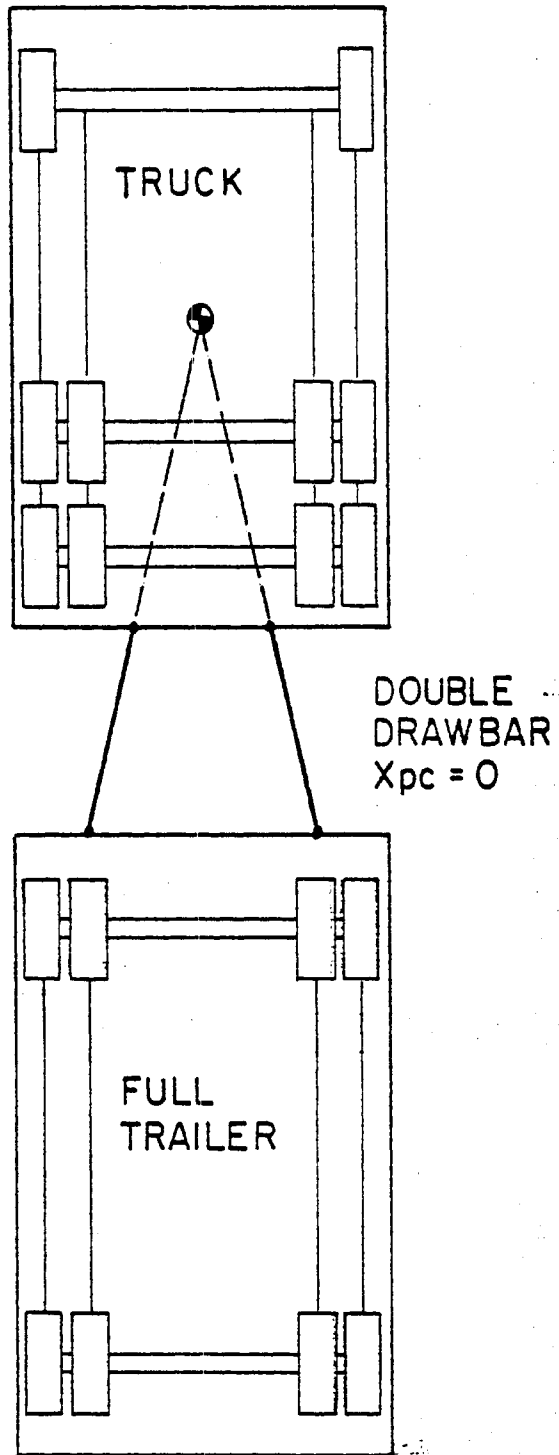


Figure 55

Table 10

SAXLE - CALIFORNIA - TRUCK - FULL TRAILER (XPC = 0)

SPRUNG MASS # 1  
\*\*\*\*\*

PITCH  
HITCH  
FORCE  
(LBS)

TIME (SEC)	FORWARD POSITION (IN)	LATERAL POSITION (IN)	VERTICAL POSITION (IN)	ROLL ANGLE (DEG)	YAW ANGLE (DEG)	PITCH ANGLE (DEG)	FORWARD VEL IN/SEC	LATERAL VEL IN/SEC	ROLL RATE DEG/SEC	YAW RATE DEG/SEC	PITCH RATE DEG/SEC	LATERAL ACCN. IN/SEC**2	STEER ANGLE DEG
0.0	0.0	0.0	0.0	0.0	0.0	0.0	968.04	0.0	0.0	0.0	0.0	0.0	-0.0
-0.0	0.10	96.80	0.00	0.000	-0.01	0.00	968.04	-0.01	-0.09	0.05	-0.00	0.51	1.67
-0.1	0.20	193.60	0.01	0.000	-0.01	0.01	968.04	-0.08	-0.09	0.10	0.00	0.75	1.65
-0.2	0.30	290.40	0.02	0.000	-0.02	0.02	968.04	-0.18	-0.14	0.15	-0.00	1.63	4.31
-0.6	0.40	387.20	0.05	0.000	-0.06	0.05	968.04	-0.32	-0.54	0.38	-0.00	4.41	13.19
-1.1	0.50	484.00	0.13	0.000	-0.14	0.11	968.04	-0.76	-1.18	0.96	-0.00	9.48	25.91
-1.9	0.60	580.77	0.30	0.001	-0.30	0.25	968.04	-1.83	-2.04	1.90	-0.01	17.17	41.35
-3.1	0.70	677.54	0.64	0.001	-0.56	0.50	968.04	-3.86	-3.21	3.07	-0.03	25.95	56.29
-4.5	0.80	774.31	1.24	0.003	-0.96	0.87	968.04	-7.03	-4.87	4.35	-0.07	36.55	69.17
-6.2	0.90	871.08	2.21	0.009	-1.52	1.37	968.04	-11.18	-6.27	5.61	-0.14	49.79	78.86
-8.2	1.00	967.85	3.67	0.023	-2.19	1.99	968.04	-15.92	-6.83	6.74	-0.25	64.56	85.09
-10.4	1.10	1064.62	5.79	0.045	-2.88	2.71	968.04	-20.96	-6.12	7.71	-0.40	78.92	87.72
-13.1	1.20	1161.38	8.69	0.073	-3.37	3.52	968.04	-26.13	-4.36	8.49	0.53	91.62	86.41
-16.2	1.30	1258.11	12.50	0.094	-3.72	4.40	968.04	-31.26	-2.70	9.02	0.63	101.68	81.42
-19.1	1.40	1354.82	17.34	0.103	-3.94	5.32	968.04	-36.21	-1.74	9.27	-0.64	108.28	72.93
-20.7	1.50	1451.49	23.25	0.107	-4.10	6.25	968.04	-40.79	-1.53	9.17	-0.63	111.85	59.95
-22.3	1.60	1548.10	30.29	0.114	-4.23	7.14	968.04	-44.60	-1.03	8.61	-0.64	112.94	38.85
-23.5	1.70	1644.64	38.45	0.108	-4.33	7.95	968.04	-47.04	-0.95	7.43	-0.65	110.29	11.58
-24.2	1.80	1741.08	47.71	0.080	-4.37	8.62	968.04	-47.43	0.42	5.65	-0.50	103.21	-19.21
-24.6	1.90	1837.42	58.01	0.083	-4.18	9.06	968.04	-45.12	3.97	3.21	-0.05	93.52	-51.87
-25.9	2.00	1933.63	69.24	0.125	-3.63	9.23	968.04	-39.11	8.73	-0.04	0.15	79.87	83.36
-25.1	2.10	2029.71	81.26	0.093	-2.80	9.04	968.04	-28.98	10.08	-3.79	0.00	55.98	-110.76
-23.6	2.20	2125.69	93.82	-0.013	-1.60	8.47	968.04	-15.23	14.01	-7.48	0.06	25.40	-130.80
-20.9	2.30	2221.61	106.64	-0.064	-0.01	7.56	968.04	0.80	17.81	-10.63	0.27	-14.61	-141.80
-17.4	2.40	2317.55	119.30	-0.007	1.92	6.38	968.04	17.04	20.45	-12.81	-0.09	-59.26	-143.83
-14.4	2.50	2413.60	131.38	0.085	3.92	5.03	968.04	31.88	18.47	-14.07	-1.09	-98.58	-138.60
-8.3	2.60	2509.81	142.49	0.091	5.60	3.58	968.04	44.75	16.55	-14.50	-2.64	-125.37	-128.09
-5.1	2.70	2606.21	152.43	-0.209	7.41	2.10	968.04	56.37	19.55	-14.41	-3.93	-128.08	-112.39
2.2	2.80	2702.80	161.13	-0.510	9.04	0.61	968.04	68.23	11.60	-14.83	-3.14	-132.50	-91.75
12.5	2.90	2799.60	168.47	-0.574	9.97	-0.90	968.04	79.04	7.43	-14.60	-2.99	-148.16	-75.53
20.4	3.00	2896.58	174.35	-0.636	10.63	-2.39	968.04	87.82	6.04	-13.77	-3.30	-155.30	52.47
26.8	3.10	2993.73	178.69	-0.812	11.25	-3.71	968.04	94.55	6.27	-12.49	-3.59	-152.75	-25.24
33.2	3.20	3091.01	181.52	-1.089	11.94	-4.93	968.04	99.36	7.37	-10.91	-3.56	-147.05	1.96
36.5	3.30	3188.37	182.89	-1.406	12.73	-5.98	968.04	101.79	8.03	-8.95	-3.37	-140.21	28.87
39.9	3.40	3285.78	182.86	-1.713	13.53	-6.82	968.04	101.43	7.31	-6.69	-3.01	-131.05	51.28
40.0	3.50	3383.17	181.52	-1.955	14.15	-7.42	968.04	98.15	4.71	-4.28	2.28	-119.91	77.16
41.8	3.60	3479.27	178.99	-2.084	14.43	-7.75	968.04	92.00	0.54	-1.90	-0.98	-108.96	96.82
49.5	3.70	3577.69	175.23	-2.075	14.21	-7.82	968.04	82.63	-5.19	0.63	0.83	-98.79	112.76
41.1	3.80	3674.72	170.41	-1.889	13.31	-7.58	968.04	69.73	-13.00	3.40	2.53	-91.70	124.25
43.7	3.90	3771.55	164.55	-1.471	11.51	-7.04	968.04	52.61	-23.24	6.29	4.28	-87.81	131.04
47.7	4.00	3868.18	157.66	-0.715	8.57	-6.18	968.04	30.71	-35.92	9.22	6.15	-89.00	131.96
50.9	4.10	3964.66	149.84	0.399	4.22	-5.06	968.04	3.88	-50.01	12.03	5.54	-99.46	126.99
48.3	4.20	4061.07	141.35	0.710	-0.84	-3.72	968.04	-23.47	-47.87	14.30	-1.66	16.58	120.40
35.6	4.30	4157.48	132.96	-0.222	-5.18	-2.26	968.04	-41.05	-36.96	14.14	-5.96	95.17	107.41
32.7	4.40	4253.89	125.35	-1.004	-8.34	-0.80	968.04	-55.92	-23.06	14.41	-4.83	90.74	90.94
29.1	4.50	4350.64	118.74	-1.005	-10.11	0.70	968.04	-69.56	-13.40	14.61	-3.39	124.41	73.66
26.3	4.60	4447.49	113.41	-0.722	-11.23	2.17	968.04	-79.95	-8.81	14.05	-2.72	151.84	54.38
24.4	4.70	4544.50	109.53	-0.632	-11.95	3.54	968.04	-86.82	-5.20	12.72	-2.84	163.10	27.13
22.2	4.80	4641.61	107.19	-0.922	-12.35	4.76	968.04	-91.16	-2.74	10.86	-3.33	147.37	-0.97
20.6	4.90	4738.80	106.31	-1.386	-12.59	5.80	968.04	-93.70	-1.80	8.76	-3.50	131.94	-28.83
19.9	5.00	4835.99	106.81	-1.769	-12.77	6.62	968.04	-94.32	-1.40	6.72	-2.59	117.75	-54.28

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Table 10 (Cont.)

SAXLE-CALIFORNIA-TRUCK-FULLTRAILER (X<sub>PC</sub> = ϕ)

SPRING MASS # 3  
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TIME (SEC)	FORWARD POSITION (IN)	LATERAL POSITION (IN)	VERTICAL POSITION (IN)	ROLL ANGLE (DEG)	YAW ANGLE (DEG)	PITCH ANGLE (DEG)	FORWARD VEL (IN/SEC)	LATERAL VEL (IN/SEC)	ROLL RATE (DEG/SEC)	YAW RATE (DEG/SEC)	PITCH RATE (DEG/SEC)	LATERAL ACCN. (IN/SEC**2)	ARTIC ANGLE (DEG)
0.0	-425.40	0.0	0.0	0.0	0.0	0.0	968.04	0.0	0.0	0.0	0.0	0.0	0.0
0.10	-328.58	0.00	0.000	-0.00	0.00	-0.000	968.04	-0.00	-0.00	0.00	-0.00	0.01	0.00
0.20	-231.77	0.00	0.000	-0.00	0.00	-0.000	968.04	-0.00	-0.01	0.00	-0.00	0.03	0.00
0.30	-134.96	0.00	0.000	-0.00	0.00	-0.000	968.04	-0.01	-0.01	0.01	-0.00	0.08	0.00
0.40	-38.16	0.00	0.000	-0.00	0.00	-0.000	968.04	-0.01	-0.03	0.02	-0.00	0.19	0.01
0.50	58.65	0.01	0.000	-0.01	0.00	-0.000	968.04	-0.03	-0.08	0.04	-0.00	0.45	0.03
0.60	155.45	0.01	-0.000	-0.02	0.01	-0.000	968.04	-0.08	-0.18	0.04	0.00	1.04	0.06
0.70	252.25	0.03	-0.001	-0.05	0.03	-0.000	968.04	-0.19	-0.35	0.22	-0.00	2.20	0.12
0.80	349.05	0.07	-0.000	-0.10	0.06	-0.000	968.04	-0.41	-0.60	0.42	-0.00	4.18	0.21
0.90	445.84	0.16	-0.000	-0.17	0.12	-0.000	968.06	-0.80	-0.92	0.73	0.00	7.45	0.33
1.00	542.63	0.31	-0.002	-0.28	0.21	-0.000	968.10	-1.44	-1.26	1.17	0.00	12.05	0.49
1.10	639.41	0.60	-0.004	-0.42	0.35	-0.000	968.20	-2.39	-1.65	1.73	0.02	18.15	0.67
1.20	736.20	1.06	-0.007	-0.61	0.56	-0.000	968.35	-3.69	-2.01	2.40	-0.03	25.66	0.86
1.30	833.01	1.78	-0.010	-0.82	0.84	-0.000	968.55	-5.35	-2.32	3.13	-0.04	34.44	1.06
1.40	929.85	2.84	-0.012	-1.07	1.19	-0.000	968.79	-7.37	-2.91	3.89	-0.08	44.08	1.26
1.50	1026.70	4.35	-0.014	-1.22	1.61	-0.000	969.07	-9.68	-2.48	4.61	-0.11	53.71	1.48
1.60	1123.57	6.39	-0.015	-1.36	2.11	-0.000	969.39	-12.21	-2.38	5.31	-0.14	63.19	1.70
1.70	1220.46	9.07	-0.019	-1.80	2.68	-0.001	969.70	-15.06	-2.28	6.07	-0.19	72.06	1.92
1.80	1317.37	12.47	-0.014	-2.02	3.32	-0.001	969.99	-18.34	-2.27	6.84	-0.24	80.87	2.10
1.90	1414.27	16.68	-0.014	-2.25	4.04	-0.000	970.10	-22.03	-2.20	7.53	-0.31	89.03	2.20
2.00	1511.14	21.77	-0.017	-2.46	4.82	-0.001	969.98	-26.01	-2.03	8.07	-0.35	96.03	2.19
2.10	1607.94	27.82	-0.026	-2.65	5.65	-0.000	969.64	-30.05	-1.64	8.36	-0.37	101.66	2.08
2.20	1704.64	34.89	-0.032	-2.78	6.49	-0.000	969.12	-33.85	-1.08	8.37	-0.41	105.44	1.84
2.30	1801.22	42.99	-0.033	-2.86	7.30	-0.001	968.52	-37.02	-0.35	7.92	-0.41	106.85	1.45
2.40	1897.65	52.16	-0.038	-2.84	8.06	-0.002	967.92	-39.13	0.71	7.07	-0.36	104.90	0.91
2.50	1993.93	62.36	-0.039	-2.71	8.70	-0.002	967.51	-39.75	1.88	5.68	-0.27	98.92	0.23
2.60	2090.07	73.53	-0.039	-2.46	9.17	-0.003	967.27	-38.31	3.20	3.70	-0.16	88.87	-0.55
2.70	2186.07	85.58	-0.028	-2.04	9.42	-0.000	967.19	-34.34	5.31	1.89	0.01	73.39	-1.78
2.80	2281.99	98.36	-0.010	-1.39	9.39	-0.003	968.07	-27.60	7.45	-1.99	0.03	48.29	-2.72
2.90	2377.94	111.62	-0.012	-0.57	9.01	-0.001	969.21	-17.96	8.85	-5.53	0.02	16.26	-3.08
3.00	2473.96	125.05	-0.012	0.41	8.30	-0.000	970.41	-6.09	10.21	-8.61	-0.03	-21.41	-3.08
3.10	2570.13	138.29	-0.003	1.50	7.32	-0.000	971.74	6.17	10.21	-10.65	-0.29	-62.66	-3.32
3.20	2666.52	150.92	-0.019	2.35	6.19	-0.000	973.17	17.43	6.60	-11.93	-0.51	-93.86	-3.54
3.30	2763.19	162.66	-0.038	2.91	4.84	-0.005	974.43	28.22	4.88	-12.94	-0.68	-112.61	-3.74
3.40	2860.13	173.29	-0.049	3.40	3.61	-0.006	975.39	38.90	5.19	-13.67	-0.86	-126.83	-3.88
3.50	2957.31	182.68	-0.100	4.05	2.22	-0.018	975.84	49.11	5.14	-14.04	-1.20	-174.32	-3.93
3.60	3053.44	190.65	-0.347	5.50	0.81	-0.030	975.43	59.77	20.06	-14.32	-1.27	-140.02	-3.75
3.70	3152.15	197.45	-0.821	7.08	-0.67	0.004	975.31	70.16	7.01	-14.68	-1.50	-145.40	-3.49
3.80	3249.77	202.70	-1.119	7.46	-2.13	0.005	974.46	80.85	3.51	-14.29	-2.10	-138.73	-3.21
3.90	3347.41	206.52	-1.191	7.98	-3.55	-0.032	972.71	90.38	6.98	-13.74	-2.40	-149.28	-2.83
4.00	3444.98	208.84	-1.249	8.79	-4.92	-0.078	970.44	97.50	8.73	-13.01	-2.42	-161.43	-2.31
4.10	3542.41	209.62	-1.558	9.70	-6.19	-0.106	968.08	102.45	9.70	-11.87	-2.16	-160.59	-1.65
4.20	3639.69	208.88	-2.172	10.79	-7.34	-0.105	966.49	106.03	12.26	-10.57	-1.81	-148.22	-0.95
4.30	3736.89	206.73	-2.950	12.19	-8.32	-0.096	965.19	108.27	15.83	-8.76	-1.88	-133.57	-0.25
4.40	3833.90	203.31	-3.729	13.90	-9.12	-0.107	964.45	108.65	17.79	-6.71	-1.90	-120.01	0.51
4.50	3930.85	198.74	-4.408	15.66	-9.71	-0.137	963.98	105.76	16.89	-4.60	-1.61	-106.88	1.30
4.60	4027.67	193.15	-4.934	17.16	-10.09	-0.160	963.36	102.83	12.42	-2.54	-0.90	-92.77	2.14
4.70	4124.31	186.63	-5.258	18.04	-10.25	-0.163	962.79	98.79	4.85	-0.37	-0.11	-77.88	3.00
4.80	4220.71	179.25	-5.317	18.03	-10.16	-0.162	962.50	88.22	-5.08	0.61	-0.27	-68.27	3.81
4.90	4316.82	171.04	-5.044	16.95	-9.83	-0.172	962.69	76.18	-17.28	4.48	1.14	-66.18	4.52
5.00	4412.96	161.95	-4.350	14.49	-9.23	-0.193	963.47	59.56	-32.53	7.16	1.64	-72.03	5.09

California Truck - Full-Trailer  
Conventional Hitch

1 m = 39.37 in

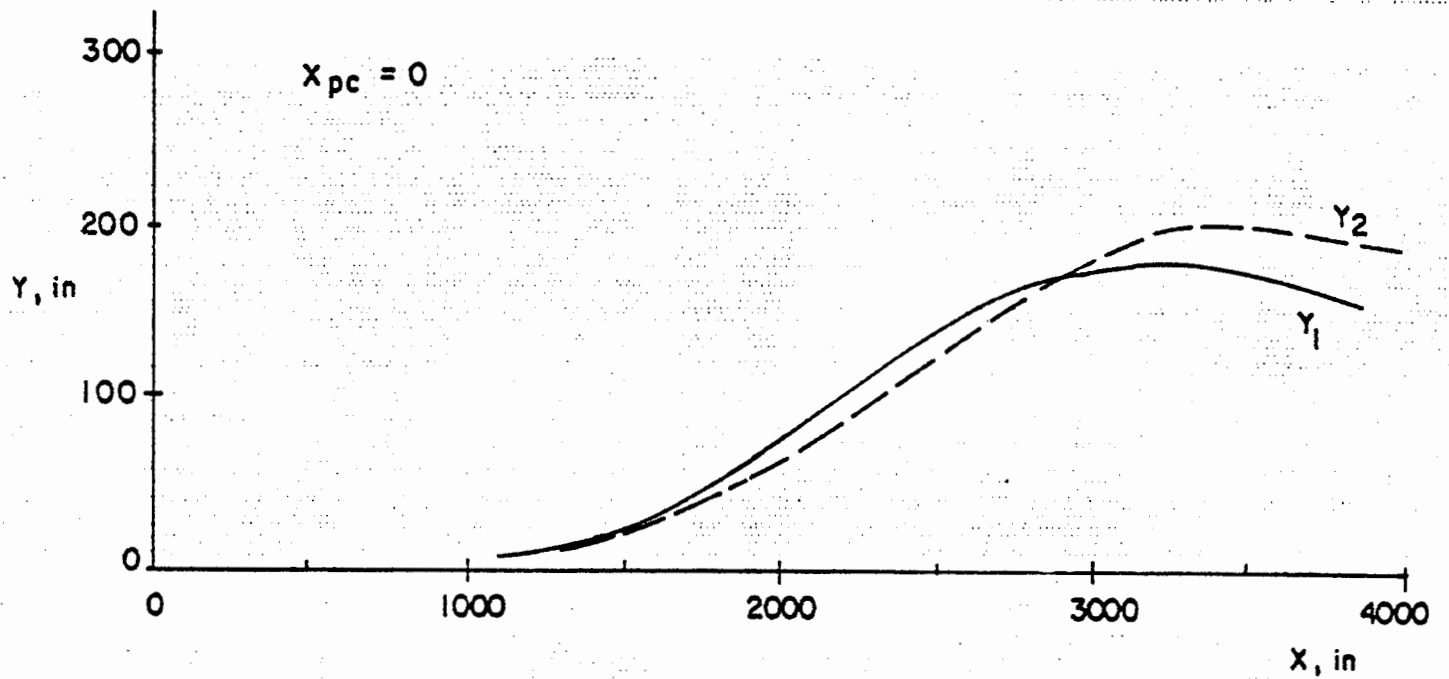
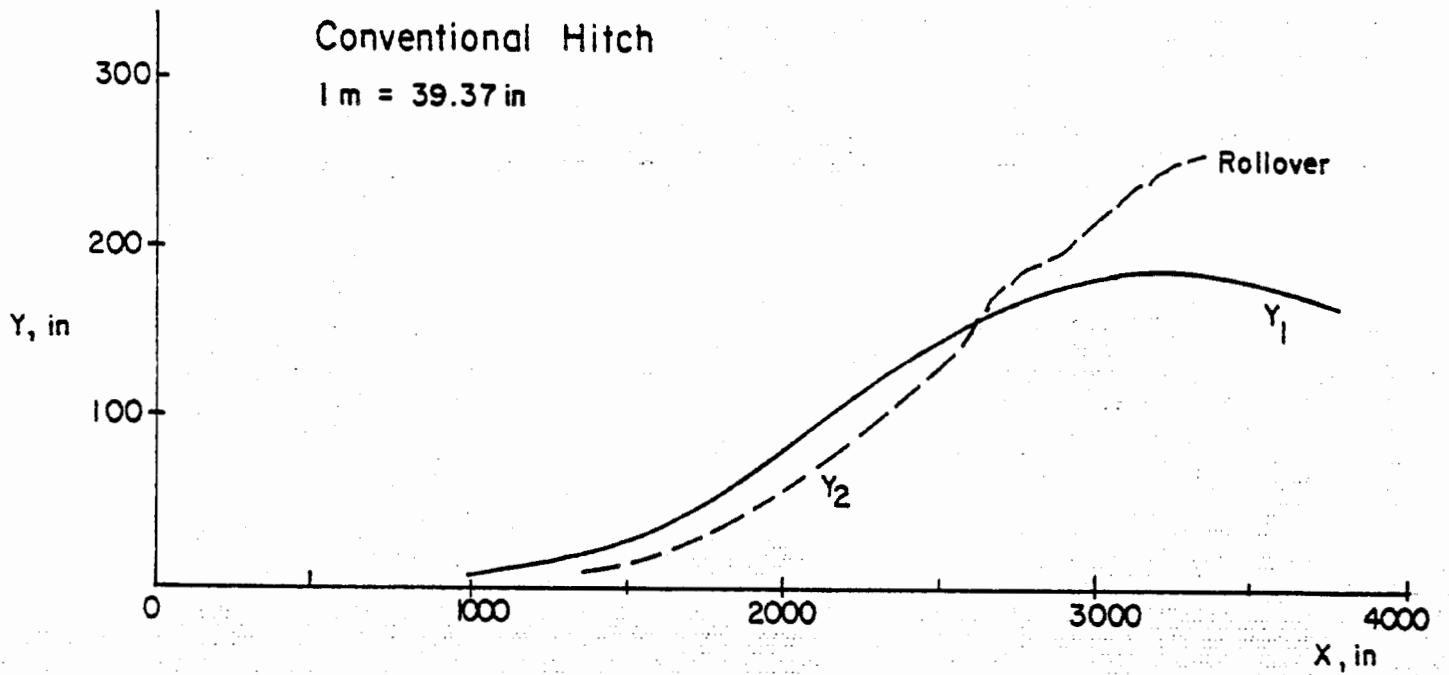


Figure 56. Influence of double drawbar on trajectories.



The other truck-full trailer selected for simulation study is a five-axle truck pulling a six-axle full trailer with a three-axle dolly (see Figure 15).

The simplified analysis of this vehicle indicates that it will have a maximum rearward amplification of 2.46 at 3.95 rad/sec (see Figure 57). At 3.95 rad/sec, the magnitudes of the amplification factors for the towing and towed units are 2.05 and 1.20, respectively.

The results from the simplified analysis of the "dirt" truck-full trailer are qualitatively similar to those for the "California" truck-full trailer and, hence, the simulation results for these two vehicles are expected to be similar. Although the "dirt" truck-full trailer vehicle has many axles, its performance is similar to the "California" truck-full trailer because its axles are heavily loaded, that is, its cornering coefficients are not large. As indicated in Table 11, the simulated directional response of the "dirt" truck-full trailer is indeed comparable to that of the "California" truck-full trailer. The lateral acceleration of the full trailer is much larger than that of the "dirt" truck and the full trailer rolls over in an emergency maneuver requiring a lateral displacement of 12 feet (3.66 m). In addition, further calculations with an idealized double drawbar arrangement (such that  $x_{pc} = 0$ ) show that in the emergency maneuver the full trailer will not roll over (the rearward amplification is close to 1.0) and the path of the full trailer overshoots the path of the tractor by 30 inches (.76 m) when  $x_{pc} = 0$  compared to a 75-inch (1.9-m) overshoot for the conventional hitching arrangement. Although the maximum force at the pintle hitch can be on the order of five percent of the maximum force at the turntable for this three-axle dolly, the simplified analysis provides approximate results that can be used to predict the qualitative nature of the vehicle's directional response in an obstacle-avoidance maneuver. In summary, the simulation results for the "dirt" truck-full trailer provide further evidence supporting the previous conclusions that (1) the simplified analysis provides the basis for useful predictions and (2) the double drawbar arrangement has potential for improving directional performance in obstacle-avoidance maneuvers.

Vehicle Identification: Dirt Truck-Full Trailer

Max. Amplification Gain for  $W \leq 5.00$  rad/sec:

GM = 2.467 at  $W = 3.95$  rad/sec

Amplification Gain Components at  $W = 3.95$  rad/sec:

Straight truck, c.g. to pintle hook,  $G_2 = 2.05$

1st full trailer, pintle eye to c.g.,  $G_3 = 1.203$

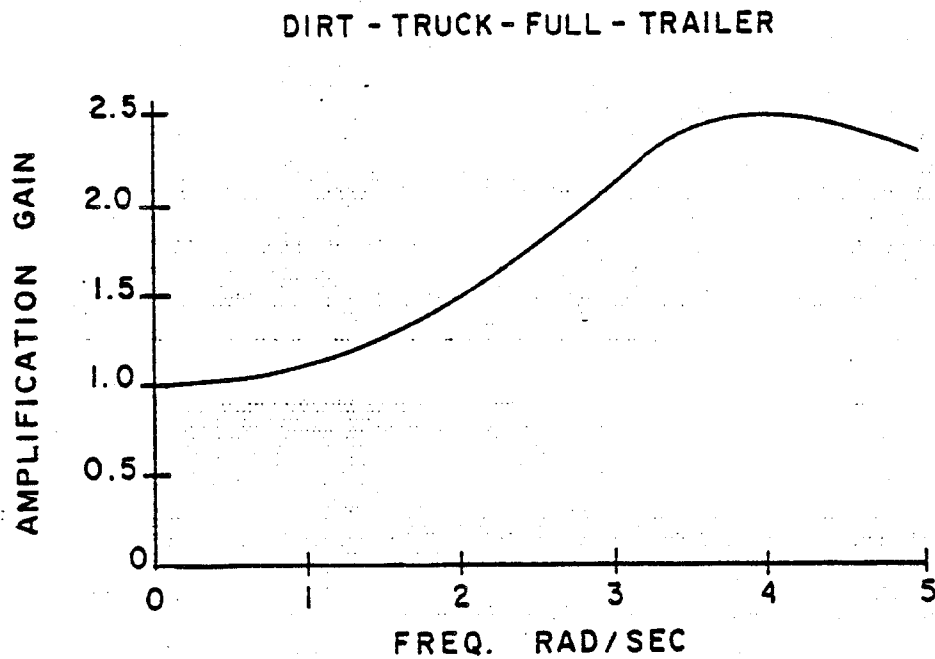


Figure 57. Five-axle "dirt" truck pulling a six-axle trailer.

Table 11

5-axle dirt truck + 6-axle full trailer

SPRING MASS # 1  
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TIME (SEC)	FORWARD POSITION (IN)	LATERAL POSITION (IN)	VERTICAL POSITION (IN)	ROLL ANGLE (DEG)	YAW ANGLE (DEG)	PITCH ANGLE (DEG)	FORWARD VEL IN/SEC	LATERAL VEL IN/SEC	ROLL RATE DEG/SEC	YAW RATE DEG/SEC	PITCH RATE DEG/SEC	LATERAL ACCN IN/SEC**2	STEEP ANGLE DEG
0.0	0.0	0.0	0.0	0.0	0.0	0.0	968.04	0.0	0.0	0.0	0.0	0.0	-0.0
0.10	96.80	0.00	0.000	-0.01	0.00	0.000	968.04	0.00	-0.09	0.08	0.00	0.46	1.63
0.20	193.60	0.01	0.000	-0.01	0.01	0.000	968.04	-0.14	-0.10	0.12	-0.00	0.78	1.62
0.30	290.40	0.02	0.000	-0.03	0.03	0.000	968.04	-0.27	-0.15	0.17	-0.00	1.65	4.22
0.40	387.20	0.05	0.000	-0.06	0.06	0.000	968.04	-0.51	-0.51	0.48	-0.01	4.27	12.87
0.50	484.00	0.13	0.000	-0.14	0.14	0.000	968.04	-1.24	-1.18	1.20	-0.01	17.31	25.28
0.60	580.77	0.29	0.000	-0.30	0.31	0.000	968.04	-2.86	-2.10	2.31	-0.01	17.31	40.40
0.70	677.54	0.64	0.000	-0.56	0.61	0.000	968.04	-5.64	-3.11	3.63	-0.03	27.85	54.96
0.80	774.32	1.25	0.003	-0.93	1.04	0.001	968.04	-9.53	-4.23	4.96	-0.07	40.08	67.16
0.90	871.09	2.27	0.009	-1.40	1.59	0.002	968.04	-14.28	-5.05	6.18	-0.15	53.96	75.98
1.00	967.87	3.63	0.020	-1.91	2.27	0.002	968.04	-19.50	-4.96	7.22	-0.24	68.63	81.44
1.10	1064.64	6.08	0.037	-2.37	3.03	0.001	968.04	-24.86	-4.10	8.04	-0.34	82.28	83.48
1.20	1161.40	9.15	0.056	-2.72	3.87	0.000	968.04	-30.19	-2.93	8.67	-0.44	93.52	82.44
1.30	1258.15	13.15	0.083	-2.99	4.76	0.006	968.04	-35.48	-3.03	9.06	-0.62	99.20	77.48
1.40	1354.86	18.15	0.120	-3.33	5.68	0.028	968.04	-40.70	-3.78	9.20	-0.73	104.14	69.33
1.50	1451.54	24.19	0.164	-3.72	6.59	0.042	968.04	-45.47	-3.70	8.99	-0.70	108.23	57.28
1.60	1548.17	31.31	0.212	-4.04	7.46	0.056	968.04	-49.17	-2.45	8.22	-0.74	109.82	38.09
1.70	1644.72	39.53	0.263	-4.16	8.22	0.066	968.04	-51.00	0.10	6.83	-0.53	109.04	13.42
1.80	1741.18	48.85	0.317	-4.05	8.80	0.063	968.04	-50.04	2.07	4.97	-0.31	104.45	14.77
1.90	1837.51	59.21	0.377	-3.59	9.11	0.058	968.04	-45.18	5.23	1.44	0.08	94.01	-43.96
2.00	1933.71	70.50	0.441	-2.95	9.07	0.020	968.04	-35.80	9.83	-2.29	0.69	78.27	-71.92
2.10	2029.77	82.57	0.516	-1.82	8.83	0.026	968.04	-21.51	11.98	-5.38	0.33	50.23	-96.07
2.20	2125.74	95.10	0.599	-0.57	8.01	0.017	968.04	-9.14	13.22	-8.91	-0.70	5.34	-113.99
2.30	2221.69	107.69	0.686	0.88	6.70	0.060	968.04	11.92	15.36	-12.12	0.01	-39.13	-122.26
2.40	2317.70	119.91	0.786	2.39	5.42	0.034	968.04	27.59	14.04	-13.25	0.61	-78.37	-121.48
2.50	2413.86	131.36	0.906	3.63	4.08	0.096	968.04	40.78	11.17	-13.39	-1.53	-109.14	-116.64
2.60	2510.18	141.75	1.053	4.62	2.73	0.086	968.04	51.71	8.50	-13.63	-3.19	-125.34	-110.33
2.70	2608.69	150.92	1.324	5.31	1.31	0.222	968.04	62.49	5.33	-14.46	-1.78	-134.56	-100.24
2.80	2703.39	158.76	1.725	5.81	-0.16	0.214	968.04	73.20	5.28	-14.50	-1.21	-142.91	-87.38
2.90	2800.27	165.17	2.258	6.44	-1.58	0.232	968.04	82.39	7.43	-13.52	-3.27	-147.49	-66.54
3.00	2896.03	170.09	2.931	7.31	-2.87	0.258	968.04	89.54	10.10	-12.29	-3.27	-142.87	-41.51
3.10	2993.26	173.68	3.754	8.48	-4.07	0.248	968.04	94.83	13.02	-10.66	-3.68	-141.57	-17.35
3.20	3090.53	175.91	4.727	9.92	-5.11	0.161	968.04	97.82	15.21	-8.93	-3.78	-137.25	5.26
3.30	3187.87	176.86	5.860	11.49	-5.94	0.985	968.04	97.83	15.77	-6.98	-3.72	-125.58	30.28
3.40	3285.24	176.64	7.153	13.00	-6.54	1.221	968.04	94.99	13.66	-4.19	-3.31	-112.26	49.77
3.50	3382.59	175.34	8.606	14.21	-6.90	1.440	968.04	89.16	10.34	-1.59	-2.66	-102.81	64.87
3.60	3479.88	173.03	10.239	15.06	-6.97	1.644	968.04	79.61	6.32	1.19	-1.29	-95.33	76.87
3.70	3577.04	169.78	13.004	15.40	-6.73	1.684	968.04	66.51	0.10	3.73	1.75	-81.39	86.55

Table 11 (Cont.)

5 axle dirt truck + 6 axle full trailer

SPRUNG MASS # 3  
\*\*\*\*\*

<i>PINCH HITCH FORCE (LBS)</i>	TIME (SEC)	FORWARD POSITION (IN)	LATERAL POSITION (IN)	VERTICAL POSITION (IN)	ROLL ANGLE (DEG)	YAW ANGLE (DEG)	PITCH ANGLE (DEG)	FORWARD VEL IN/SEC	LATERAL VEL IN/SEC	ROLL RATE DEG/SEC	YAW RATE DEG/SEC	PITCH RATE DEG/SEC	LATERAL ACCN. IN/SEC**2	ARTIC ANGLE DEG
	0.0	-358.60	0.0	0.0	0.0	0.0	0.0	968.04	0.0	0.0	0.0	0.0	0.0	0.0
3.4	0.10	-261.78	-0.00	0.000	0.00	-0.00	0.000	968.04	-0.00	0.00	-0.00	0.00	-0.02	-0.00
2.1	0.20	-164.98	-0.00	0.000	0.00	-0.00	0.000	968.04	0.00	0.02	-0.01	-0.00	-0.08	-0.01
1.7	0.30	-68.17	-0.00	0.000	0.00	-0.00	0.000	968.04	0.01	0.03	-0.02	-0.00	-0.16	-0.01
13.6	0.40	28.63	-0.00	0.000	0.01	-0.00	0.000	968.04	0.04	0.03	-0.03	-0.01	-0.30	-0.01
30.9	0.50	125.44	-0.01	-0.000	0.01	-0.01	-0.000	968.05	0.07	0.09	-0.06	-0.02	-0.65	-0.03
39.9	0.60	222.24	-0.02	0.000	0.03	-0.02	0.000	968.07	0.14	0.21	-0.14	0.03	-1.39	-0.07
22.3	0.70	319.04	-0.05	0.001	0.05	-0.04	0.001	968.12	0.29	0.35	-0.27	-0.01	-2.43	-0.11
-27.6	0.80	415.85	-0.10	0.001	0.09	-0.07	0.001	968.23	0.58	0.37	-0.40	-0.01	-3.39	-0.11
-88.5	0.90	512.67	-0.18	0.001	0.12	-0.11	0.001	968.40	0.92	0.11	-0.39	0.01	-3.39	-0.02
-157.0	1.00	609.49	-0.30	0.001	0.10	-0.14	0.001	968.55	1.12	-0.45	-0.11	-0.00	-1.55	0.15
-234.5	1.10	706.33	-0.43	0.001	0.02	-0.12	0.001	968.99	0.87	-1.28	0.51	-0.01	2.96	0.40
-321.6	1.20	803.22	-0.52	0.001	-0.16	-0.03	0.001	969.41	-0.14	-2.27	1.49	0.00	10.54	0.71
-463.3	1.30	900.15	-0.51	0.002	-0.44	0.18	0.001	969.82	-2.14	-3.41	2.80	-0.02	21.96	1.06
-505.2	1.40	997.12	-0.28	0.005	-0.84	0.54	0.001	970.25	-5.29	-4.68	4.40	-0.06	36.90	1.46
-530.5	1.50	1094.14	0.32	0.013	-1.38	1.07	0.001	970.73	-9.68	-6.01	6.14	-0.14	54.13	1.86
-549.9	1.60	1191.21	1.47	0.031	-2.02	1.77	0.003	971.19	-15.20	-6.68	7.88	-0.25	73.55	2.21
-556.5	1.70	1288.31	3.35	0.062	-2.68	2.64	0.005	971.52	-21.61	-6.35	9.50	-0.42	93.20	2.50
-560.8	1.80	1385.42	6.16	0.069	-3.35	3.66	-0.015	971.47	-28.85	-7.66	10.68	-1.31	105.91	2.75
-434.5	1.90	1482.50	10.01	-0.071	-4.32	4.78	-0.106	971.03	-36.84	-12.20	11.79	-1.41	111.92	2.95
-584.0	2.00	1579.51	14.98	-0.281	-5.48	6.06	-0.022	970.57	-46.63	-8.79	13.64	0.37	125.25	2.99
-562.7	2.10	1676.46	21.23	-0.209	-6.06	7.44	0.061	970.44	-56.54	-3.60	13.82	-1.72	145.40	3.22
-316.2	2.20	1773.35	28.96	-0.042	-6.45	8.86	-0.041	970.17	-64.72	-5.20	14.28	-3.04	163.37	3.49
51.4	2.30	1870.08	38.27	-0.132	-7.16	10.36	-0.168	968.98	-72.98	-8.19	14.95	-3.03	166.89	3.11
256.6	2.40	1966.58	49.18	-0.551	-8.05	11.86	-0.264	967.58	-81.68	-9.60	14.51	-2.70	158.08	1.88
391.0	2.50	2062.86	61.59	-1.189	-9.05	13.25	-0.299	966.33	-89.77	-9.74	12.53	-2.14	140.33	-0.06
446.3	2.60	2158.92	75.37	-1.819	-9.91	14.35	-0.317	965.86	-95.12	-6.61	8.89	-1.91	117.75	-2.37
525.3	2.70	2254.85	90.36	-2.177	-10.22	15.02	-0.386	966.59	-95.35	1.28	3.85	-1.77	96.52	-4.61
611.0	2.80	2350.64	106.44	-2.019	-9.50	15.15	-0.516	967.40	-88.14	13.91	-1.87	-1.13	84.32	-6.47
674.9	2.90	2446.23	123.52	-1.163	-7.29	14.67	-0.646	968.46	-71.43	30.79	-8.14	-0.04	84.20	-7.85
1506.8	3.00	2540.40	141.27	0.275	-3.20	13.44	-0.465	969.97	-43.72	49.40	-16.87	8.23	73.41	-8.41
1021.5	3.10	2635.79	159.76	0.835	1.75	11.11	0.413	973.82	-2.81	46.18	-29.07	4.68	-58.68	-8.15
652.7	3.20	2731.71	177.90	-0.368	5.79	7.99	0.194	978.84	39.54	37.78	-31.63	-11.21	-116.37	-7.73
614.0	3.30	2828.49	194.91	-1.376	9.05	4.61	-0.446	981.84	80.94	36.27	-34.08	-10.42	-167.63	-7.58
331.8	3.40	2925.97	210.49	-2.403	13.43	0.95	-0.668	980.02	122.79	52.70	-36.92	-7.96	-213.93	-6.56
64.3	3.50	3023.98	224.57	-3.961	19.80	-2.83	-0.393	974.89	162.85	73.50	-36.34	-9.12	-244.73	-4.33
-282.6	3.60	3122.54	237.42	-6.171	28.26	-6.34	-0.158	969.23	191.79	97.43	-28.95	-14.94	-299.79	-1.55
-169.1	3.70	3221.82	249.57	-8.622	39.36	-9.32	-0.321	964.89	199.48	123.42	-18.57	-20.19	-343.35	1.59

6.4.2 Simulation Results for Truck-Semitrailer Combinations. The empty dump truck-loaded semitrailer combination was chosen because it has a pintle hitch connection located well behind the rear axle of the truck (see Figures 8 and 16). Parameters describing this vehicle in detail are given in Appendix E. The directional response of this vehicle is not at all like that of a typical tractor-semitrailer which employs a fifth wheel located in front of the rear axle of the tractor. In the typical tractor-semitrailer the rearward amplification is low and, in addition, the fifth wheel constrains the relative roll between the tractor and the semitrailer thereby improving the roll stability of the vehicle. In contrast, the dump-truck-pintle-hook-semitrailer has a large rearward amplification and the semitrailer may roll over while the tractor remains upright because the pintle hitch does not provide a roll constraint. Example results (time histories) from the simulation of a 12-foot (3.66-m) obstacle-avoidance maneuver are presented in Figure 58 to illustrate the rearward amplification in the combination and the eventual rollover of the semitrailer. Compared to the "California" truck-full trailer, this truck-semitrailer combination has approximately the same level of rearward amplification (around 2.0), however, the timing of the roll response is different. As shown in Figure 58, the lateral acceleration gain between the truck and semitrailer is large and the semitrailer's roll angle increases very quickly once the semitrailer starts to roll over. These simulated results demonstrate that this truck-semitrailer combination (1) has large amounts of rearward amplification and (2) is susceptible to rolling over the semitrailer in obstacle-avoidance maneuvers.

The simulation results for the "California" dromedary (Figure 13) and the car hauler (Figure 12) are not as dramatic as those for the empty-dump/loaded-lowboy vehicle (e.g., compare Figures 59 and 60 with Figure 58). Both the dromedary and the car hauler employ a fifth wheel which couples the towing and towed units in roll (see Figure 53), thereby allowing each unit to aid in providing roll support to the other unit during the various phases of an obstacle-avoidance maneuver. In contrast to the dump truck-semitrailer that uses a pintle connection, neither the dromedary nor the car hauler roll over in the simulated obstacle-avoidance maneuver. (See the roll time histories in Figures 59 and 60.) The rearward amplification

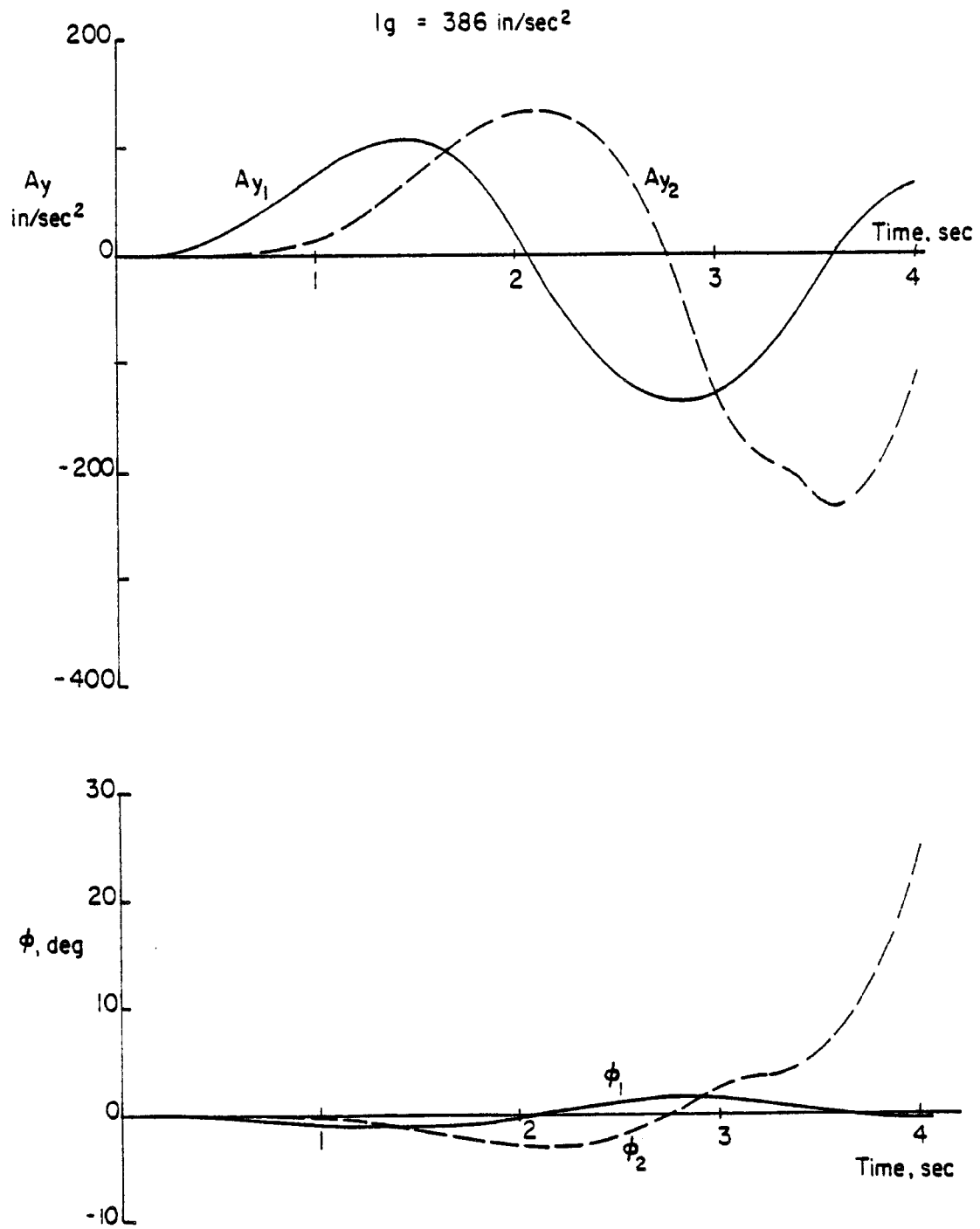


Figure 58. Empty dump truck, loaded lowboy semi.

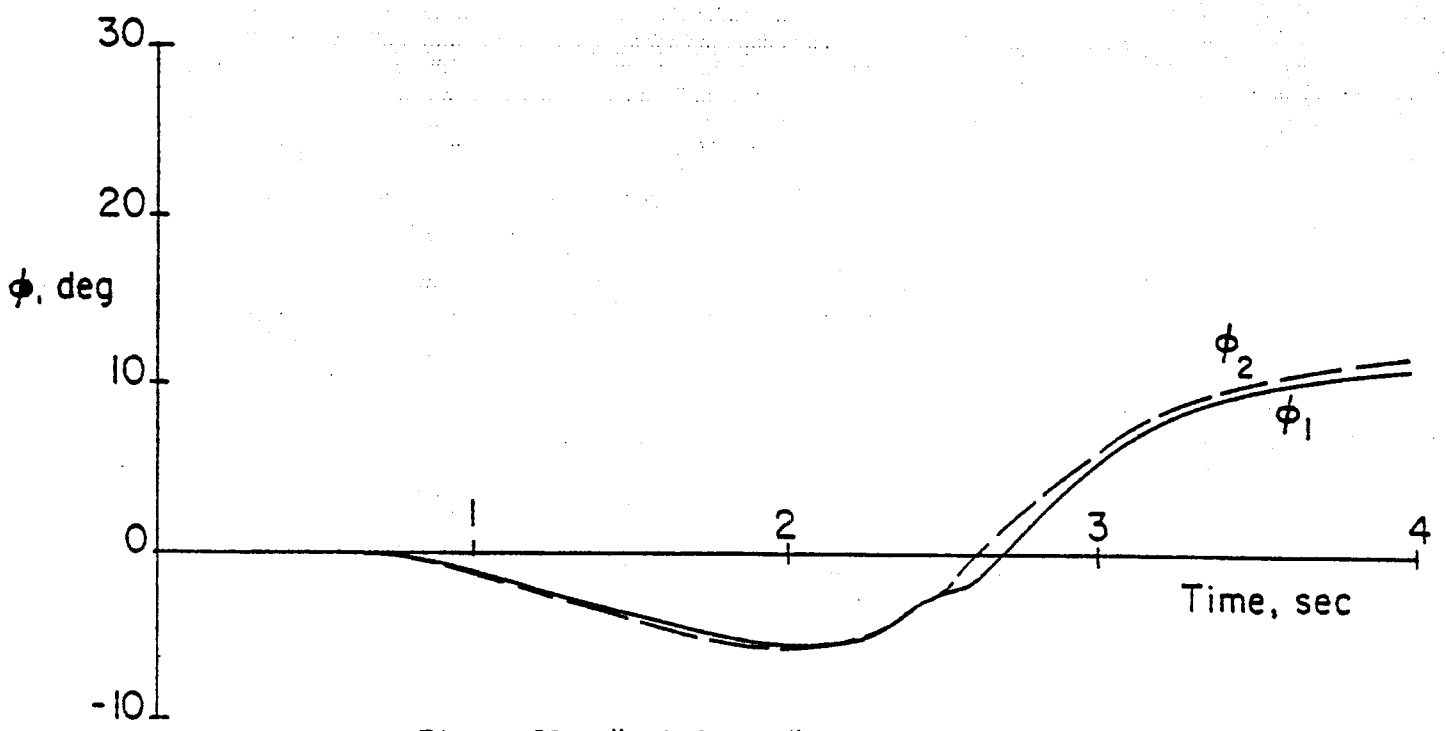
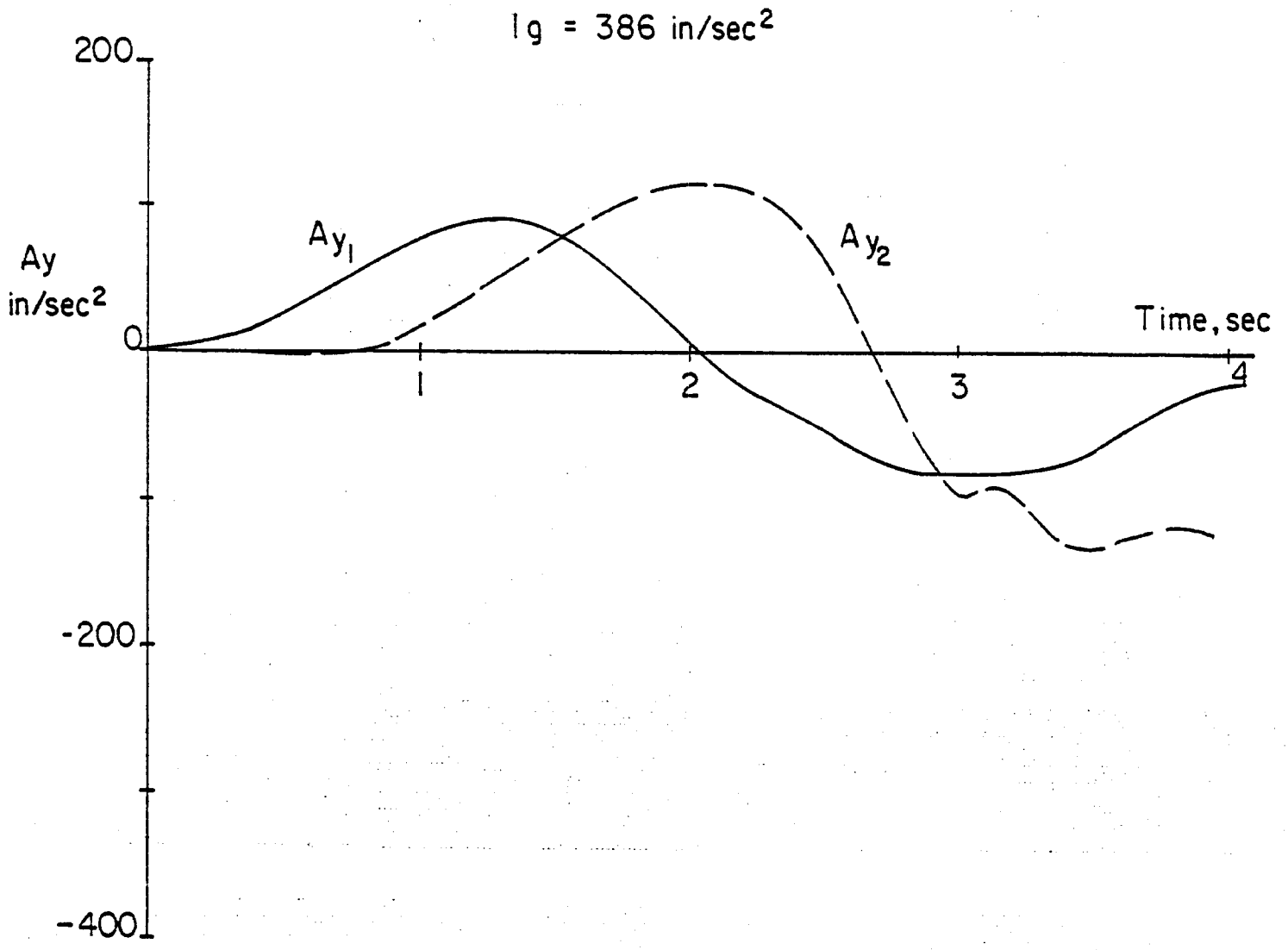


Figure 59. "California" dromedary.

$$1g = 386 \text{ in/sec}^2$$

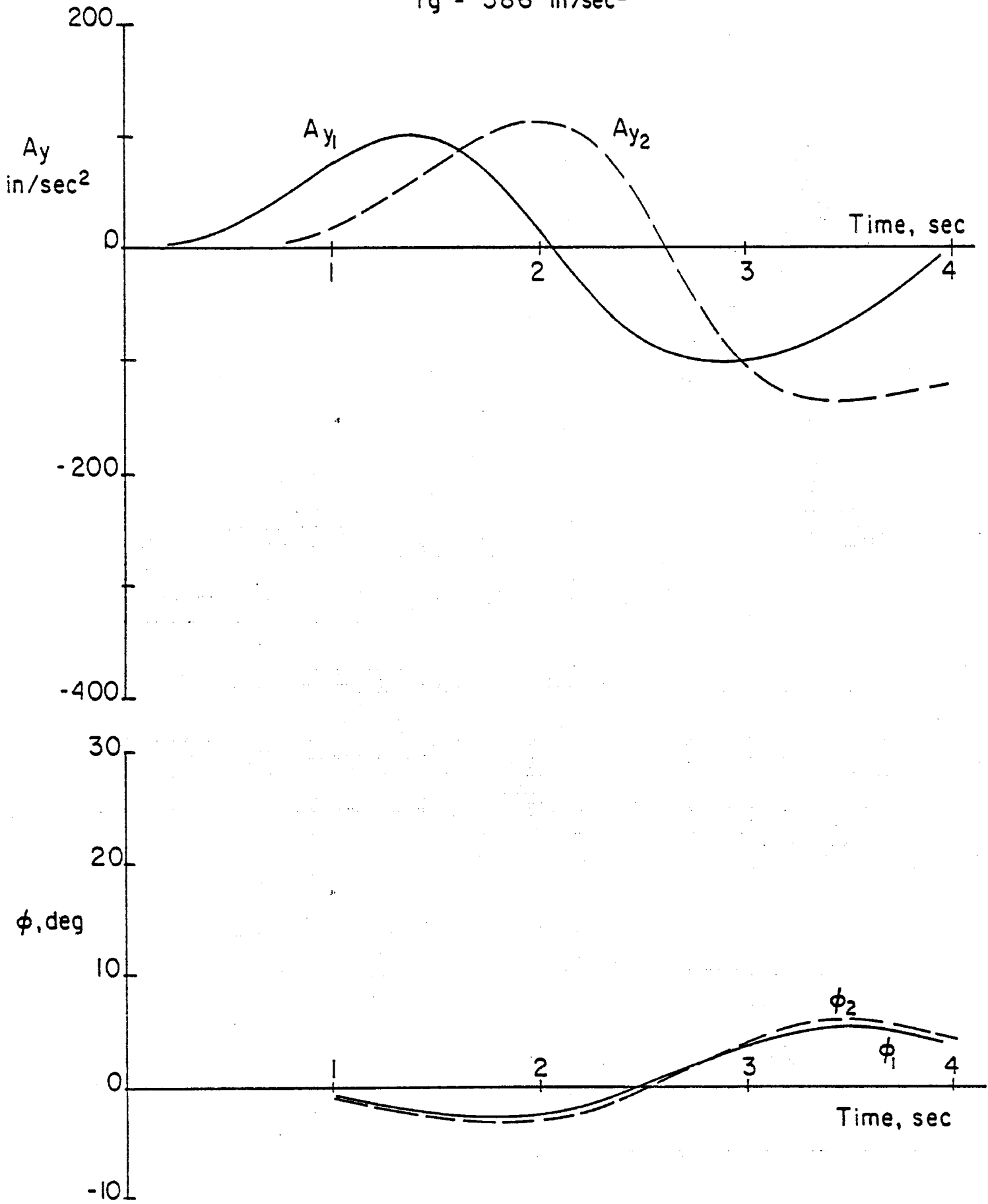


Figure 60. Car hauler.



factors for the dromedary and the car hauler (both approximately 1.4 in this maneuver) are slightly larger than those associated with typical tractor-semitrailers with towing units having comparable wheelbases. Nevertheless, the paths of the rear units do not differ greatly from the paths of the lead units for these two vehicles. Apparently, even though their fifth wheels are in unusual locations, these vehicles have tires, wheelbases, and loading arrangements such that they can execute emergency avoidance maneuvers reasonably well.

Note that reasonably good performance in avoiding obstacles depends upon using a fifth wheel rather than a pintle hitch. The following hypothetical results (see Figure 61) apply to a "California" dromedary in which the fifth wheel has been replaced by a pintle hitch in the simulation model. As illustrated in Figure 61, the semitrailer rolls over in this hypothetical situation which demonstrates the consequences of using a pintle hitch rather than a fifth wheel. Although the timing of the roll response is different from that achieved by the empty dump truck-loaded semitrailer combination, the rollover result occurs in both cases due to the lack of the roll constraint supplied by a fifth wheel.

#### 6.5 Concluding Statement for Chapter 6

The findings and results presented in this chapter have been summarized in Chapter 2. Recommendations from this study of rearward amplification are included in Chapter 7.

$1g = 386 \text{ in/sec}^2$

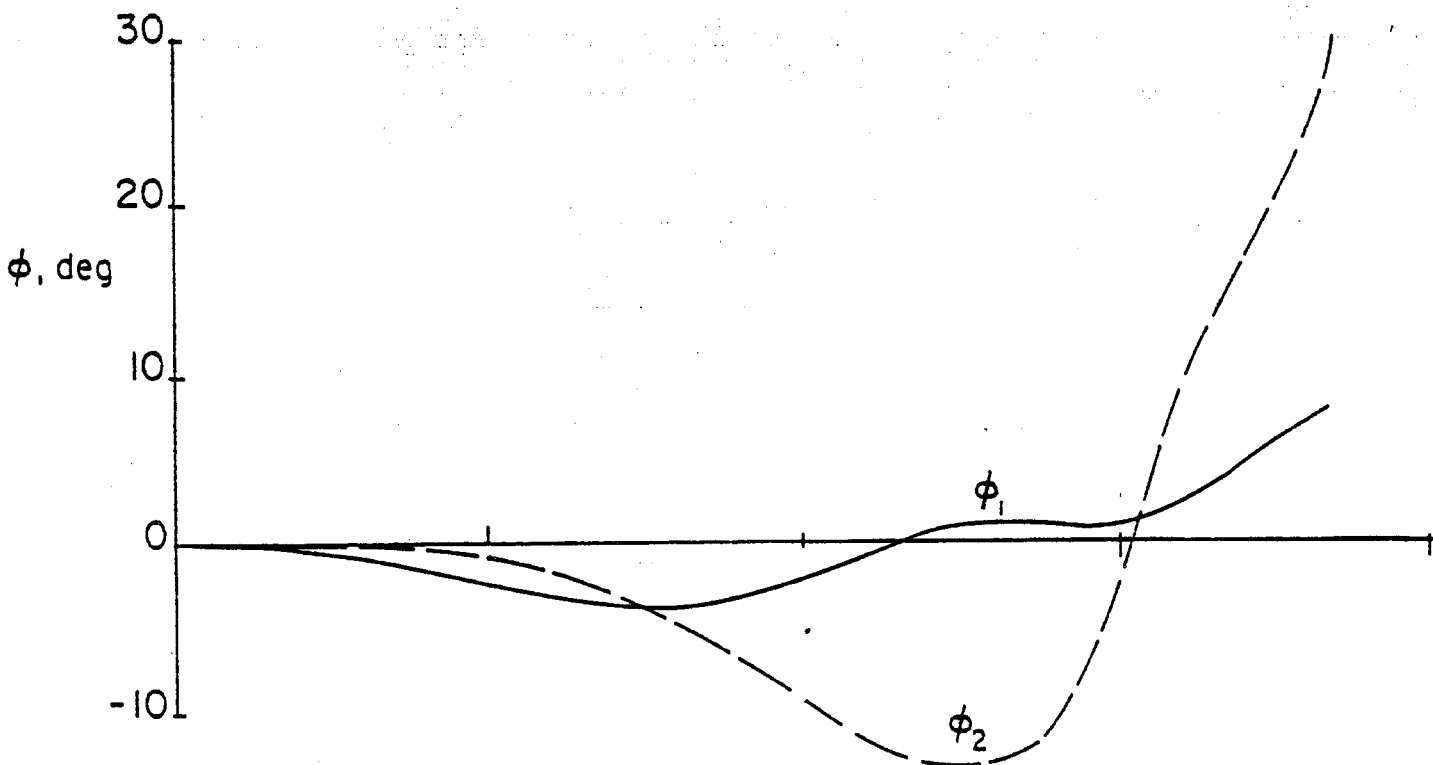
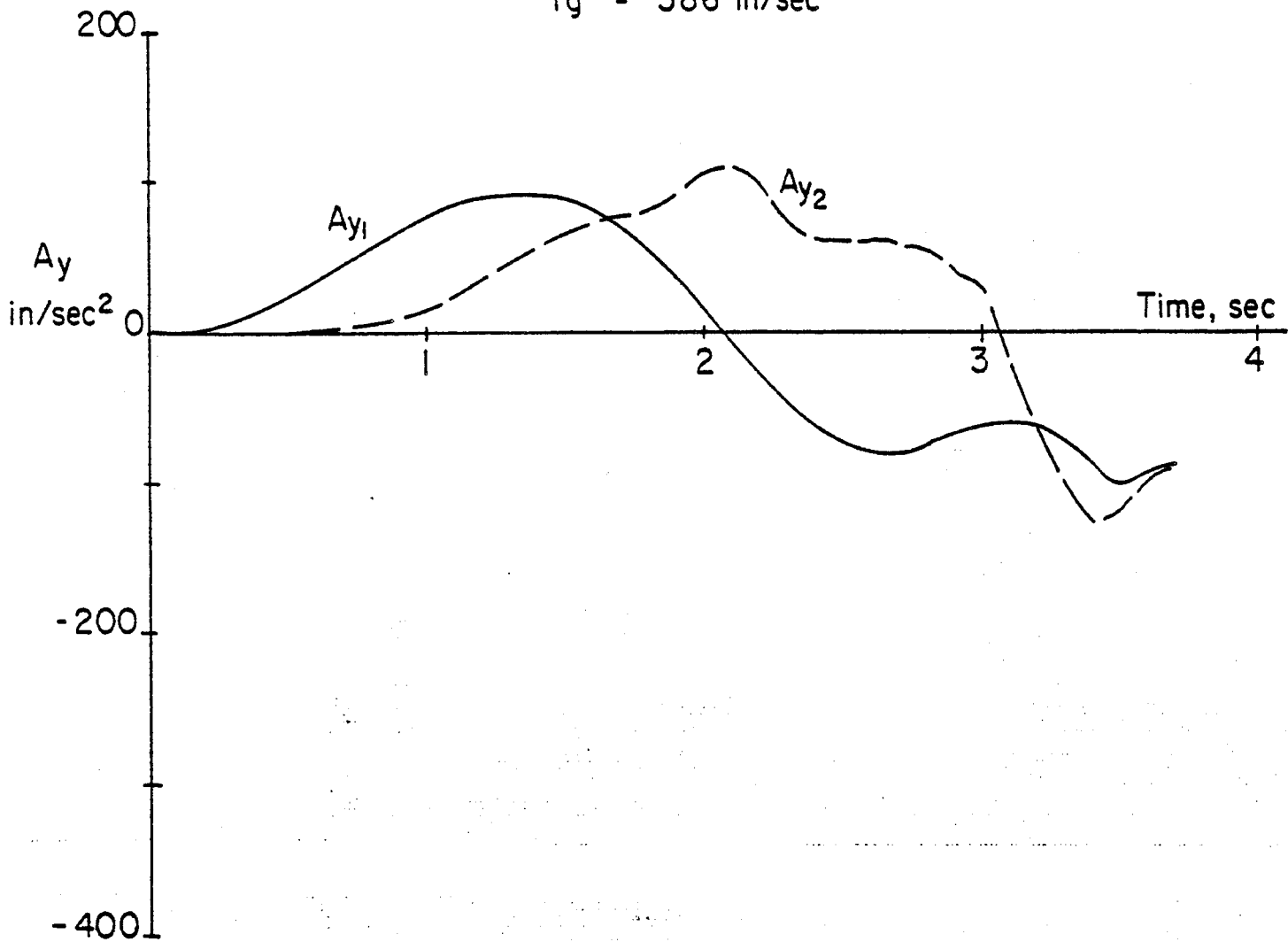


Figure 61. "California" dromedary.

## CHAPTER 7

### CONCLUSIONS AND RECOMMENDATIONS

The stated objective of this study was "to characterize, by analytical means, the bounds of safe vehicle design and the limits within which drivers must operate large trucks, as these considerations relate to the directional dynamic stability of large trucks." The study has endeavored to meet this objective by first identifying the primary safety-related problems associated with commercial vehicle directional dynamics and then to identify what vehicle and operating parameters tend to exacerbate these problems. In so doing, the study has helped to identify what types of vehicles may have what types of problems in what types of maneuvers. Further, the study has helped identify what actions can be taken to mitigate a given vehicle's safety problems.

The study has focused on three specific subjects related to "directional" performance, viz., (1) roll stability limits, (2) divergent yaw instability, and (3) lightly damped, oscillatory yaw response. In Chapters 4, 5, and 6, respectively, these subjects were discussed in terms of their underlying physical mechanisms and with respect to the specific vehicle and operating parameters by which they are influenced.

"Safety," of course, is a relative term. As such, it is always impossible to identify exactly at what level of performance a vehicle becomes "safe." Accordingly, this study has not attempted to define quantitatively, "safe" boundaries of performance relative to the three topics of interest. However, the major parametric sensitivities have been identified, such that, with respect to each of the three areas of performance, "less-safe" and "more-safe" vehicles can be recognized and changes in design or operating parameters leading to safer vehicle behavior can be identified.

The knowledge concerning the parametric sensitivities of commercial vehicle directional behavior reflected in this report holds potential for improving the safety performance of the U.S. commercial trucking fleet.

That potential cannot be fully realized unless this knowledge is delivered to the broad range of members of the trucking industry in a usable form. Herein lies a problem.

Presumably, most manufacturing entities of the trucking industry have the technical capability to interpret, qualitatively, the findings of this study and to provide themselves with the parametric data necessary to implement the findings in specific areas. This is probably not so, however, for the large majority of truck users. While many users may understand the findings of this study in a general sense, the parametric data necessary to implement these findings in specific cases is largely unavailable to the user. In many cases, component and vehicle manufacturers do not regularly generate such data and it is virtually never made readily available to the purchaser. It should be noted that a major reason for this is that such data has historically not been demanded in the marketplace. Accordingly, it is recommended that consideration be given to the distribution of the information contained herein, in appropriate forms, to the broad range of individuals involved in the U.S. trucking industry and that efforts be made to encourage its practical implementation.

Along more technical lines, the efforts of this particular project have been purely analytical. While we believe strongly that the findings herein are qualitatively correct, there is good reason to confirm the quantitative nature of the results by experimentation.

As regards the roll stability limit, full-scale vehicle testing has been found by experience to provide a relatively low level of fidelity in determining this performance limit. An alternative procedure for making this measurement on real vehicles is the laboratory method employing a tilt table. In this method, the vehicle in question is mounted on a table which can be tilted in roll. The precise roll angle at which the vehicle becomes unstable in roll can be directly related to the roll stability limit in terms of lateral acceleration. (For commercial vehicles, the required tilt angle is not so great as to seriously degrade the experiment due to radical changes in tire "vertical" loads, as is the problem with passenger cars.) Unfortunately, we know of no such facility suitable for heavy vehicles in North America (there being at least five in Europe and one in

Australia). UMTRI has recommended that such a facility be established in this country, and continues to do so.

Regarding divergent yaw instability, although the existence of divergent yaw instability has been established both analytically and experimentally for heavy trucks, the importance of this open-loop divergence in the concept of the closed-loop stability of the driver-vehicle system has not been completely evaluated. Computerized models for representing the control functions of drivers are available and they have been used to study the performance of the driver-vehicle system in path-following situations. Theoretical predictions obtained by employing these computerized models have produced realistic results and they certainly may be used to indicate reasonable bounds of closed-loop system stability. Nevertheless, little or no experimental work, dealing with the closed-loop control of the directional performance of heavy commercial vehicles, has been reported in the open literature. The progress that can be made analytically will necessarily be limited until confirming, experimental research can be accomplished.

Concerning lightly damped oscillatory yaw responses of commercial vehicles, the analytical findings of this study indicate that multiply-articulated vehicles as currently configured may have levels of rearward amplification that are large enough to cause emergency maneuvering to be exceptionally dangerous. Experimental results confirming this hazard for particular vehicles have been obtained in previous studies, but the underlying mechanisms and parameter sensitivities were not nearly as well understood in the past as they are now. Certainly, the knowledge now exists to configure vehicles, load them, and operate them so that the hazards of rearward amplification can be readily demonstrated for a variety of combination vehicles. More importantly, the theoretically predicted advantages of using double drawbar arrangements for connecting multiply-articulated vehicles should be tested in vehicle experiments involving doubles, triples, and truck-full trailer combinations.

Finally, the parametric sensitivities developed herein could be used in attempts to identify vehicles that might be "accident prone." Confirmation of this quality of accident proneness might be pursued in analyzing the accident record, but past experience indicates that great skill and care

would be needed to remove the influences of confounding or lurking variables from the results. Furthermore, the amount of detailed information needed to establish the values of the pertinent design parameters is not ordinarily obtained in collecting accident data. However, the work done in this study could aid in defining a basic set of parameters to be studied and the types of accident scenarios to look for. Due to the need for detailed parametric and operational information, the most successful users of accident proneness measures derived from the results of this study might be individuals or groups involved with particular trucking operations for which they had rather complete records specifying vehicle characteristics, how they are loaded, how they are used, and what types of accidents they have.

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