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16. Abstract

The study sought to define the important parametric sensitivities which affect the directional performance limits of commercial vehicles. Rollover, unstable yaw response of the lead unit (spinout), and lightly damped yaw response of trailing units (rearward amplification) are identified as the three major response modes limiting directional performance. It is noted that both yaw response modes may precipitate rollover. The significant parametric sensitivities of commercial vehicles to each performance mode are identified by analytical means. Computer simulations of example vehicles, chosen for their peculiar susceptibility to one or more of the limiting performance modes, are used to demonstrate the parametric sensitivities.

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TABLE OF CONTENTS

APPENDIX A - MODIFICATION OF THE CLASSICAL STEADY-TURNING EQUATION TO REFLECT TIRE CORNERING STIFFNESS DEPENDENCE UPON VERTICAL LOAD

Consider, from Reference (13), the conventional front wheel angle-path curvature relationship for a straight truck with tandem-axle rear suspension:

$$\delta = \frac{\ell_e}{R} + K\frac{U^2}{R} \quad \text{(A-1)}$$

$$= \frac{\ell}{R}\left[1 + \left(\frac{\Delta^2 + D^2 C_{s_R}/C_{\alpha_R}}{\ell^2}\right)\left(1 + \frac{C_{\alpha_R}}{C_{\alpha_1}}\right)\right] + M\frac{U^2}{R}\left[\frac{b}{\ell C_{\alpha_1}} - \frac{a}{\ell C_{\alpha_R}}\right] \quad \text{(A-2)}$$

where

- δ is the front-wheel angle
- R is the path radius
- U is forward velocity

and

$$\ell_e = \ell\left[1 + \left(\frac{\Delta^2 + D^2 C_{s_R}/C_{\alpha_R}}{\ell^2}\right)\left(1 + \frac{C_{\alpha_R}}{C_{\alpha_1}}\right)\right] \quad \text{(A-3)}$$

is the "effective" wheelbase, related to

- ℓ the actual vehicle wheelbase (front suspension to center of rear tandem suspension),
- Δ one-half the rear tandem axle spread,
- D one-half the dual tire spacing,
- C_{s_R} the total rear tire longitudinal stiffness,
- C_{α_R} the total rear tire cornering stiffness, and
- C_{α_1} the total front tire cornering stiffness,

and,

$$K = M\left[\frac{b}{\ell C_{\alpha_1}} - \frac{a}{\ell C_{\alpha_R}}\right] \quad \text{(A-4)}$$

or

$$K = \left[\frac{W_1}{C_{\alpha_1}} - \frac{W_R}{C_{\alpha_R}}\right]/g \quad \text{(A-5)}$$

is the classical understeer gradient, related to the additional parameters

- M the vehicle mass
- a the distance from the c.g. to the front axle
- b the distance from the c.g. to the center of the rear suspension
- W_1 the static front suspension load
- W_R the static rear suspension load
- g the gravitational acceleration

For an actual vehicle, K is, of course, comprised of additional elements contributing understeer, such as steering system compliance and roll-steer effects, which are being ignored in the analysis that follows.

If the cornering stiffness, C'_α, of a single tire is now treated as the following function of vertical load, F_z,

$$C'_\alpha(F_z) = C0_\alpha + C1_\alpha(F_z - F_{z_o}) + C2_\alpha(F_z - F_{z_o})^2 \quad \text{(A-6)}$$

where

- $C0_\alpha$ is the cornering stiffness prevailing at the static or nominal load, F_{z_o}
- $C1_\alpha$ is the linear variation of cornering stiffness with vertical load, about the nominal load, F_{z_o}
- and $C2_\alpha$ is the quadratic variation of cornering stiffness with vertical load, about the nominal load, F_{z_o}

most C_α versus F_z measurements for heavy truck tires can be accurately represented by this relationship. Hence, the cornering stiffness of a single tire on the front suspension can be expressed as:

$$C'_{\alpha_1}(F_{z_1}) = C0_{\alpha_1} + C1_{\alpha_1}(F_{z_1} - F_{z_{o_1}}) + C2_{\alpha_1}(F_{z_1} - F_{z_{o_1}})^2 \quad \text{(A-7)}$$

and, for a single tire on the rear suspension:

$$C'_{\alpha_R}(F_{z_R}) = C0_{\alpha_R} + C1_{\alpha_R}(F_{z_R} - F_{z_{o_R}}) + C2_{\alpha_R}(F_{z_R} - F_{z_{o_R}})^2 \quad \text{(A-8)}$$

Introduction of lateral acceleration-induced front and rear side-to-side load transfers, ΔW_1 and ΔW_R, proportioned by fore/aft roll stiffness distributions, produces the <u>conservative</u> static approximations:

$$\Delta W_1 = \frac{h}{T_1} W\left(\frac{K_1}{K_1 + K_R}\right) a_y \quad \text{(A-9)}$$

$$\Delta W_R = \frac{h}{T_R} W\left(\frac{K_R}{K_1 + K_R}\right) a_y \quad \text{(A-10)}$$

where

- h is the total vehicle c.g. height
- $T_{1,R}$ are the front, rear track distances
- W is the total vehicle weight
- K_1 is the front suspension roll stiffness
- K_R is the rear suspension roll stiffness
- and a_y is the vehicle lateral acceleration in g's.

Left and right prevailing loads for both the front and rear axles can therefore be approximated for any given vehicle lateral acceleration, a_y, as:

$$F_{z_1}(a_y)\Big|_{left} = F_{z_{o_1}} + \Delta W_1(a_y)$$

$$F_{z_1}(a_y)\Big|_{right} = F_{z_{o_1}} - \Delta W_1(a_y)$$

and

$$F_{z_R}(a_y)\Big|_{left} = F_{z_{o_R}} + \Delta W_R(a_y)$$

$$F_{z_R}(a_y)\Big|_{right} = F_{z_{o_R}} - \Delta W_R(a_y)$$

Substitution of the above left and right vertical tire loads, deriving from load transfer during cornering, into the $C'_\alpha(F_z)$ expressions, Equations (A-7, A-8), and noting that the linear vertical load variations cancel one another when summed side to side, we are left with the following total cornering stiffness equations for front and rear suspensions expressed as a quadratic function of vehicle lateral acceleration, a_y:

$$C_{\alpha_1} = 2\left[CO_{\alpha_1} + C2_{\alpha_1}\Delta W_1^2\right]$$

$$= 2\left[CO_{\alpha_1} + C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2 W^2\left(\frac{K_1}{K_1 + K_R}\right)^2 a_y^2\right] \quad \text{(A-11)}$$

and

$$C_{\alpha_R} = N\left[CO_{\alpha_R} + C2_{\alpha_R}\left(\frac{\Delta W_R}{N/2}\right)^2\right]$$

$$= N \cdot CO_{\alpha_R} + 4C2_{\alpha_R}\Delta W_R^2/N$$

$$= N \cdot CO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2 W^2\left(\frac{K_R}{K_1 + K_R}\right)^2 a_y^2 \quad \text{(A-12)}$$

where N is the total number of tires on the rear suspension.

Substitution of these total front and rear cornering stiffnesses into Equations (A-3) and (A-5) produces the following lateral acceleration-dependent counterparts, ℓ'_e and K':

$$\ell'_e = \ell \cdot [1 + A \cdot B] \quad \text{(A-13)}$$

where

$$A = \frac{\Delta^2 + D^2 C_{s_R} / \left[N \cdot CO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2 W^2\left(\frac{K_R}{K_1+K_R}\right)^2 a_y^2\right]}{\ell^2}$$

$$B = 1 + \frac{N \cdot CO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2 W^2\left(\frac{K_R}{K_1+K_R}\right)^2 a_y^2}{2\left[CO_{\alpha_1} + C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2 W^2\left(\frac{K_1}{K_1+K_R}\right)^2 a_y^2\right]}$$

and

$$K' = \frac{W_1}{2\left[CO_{\alpha_1} + C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2 W^2\left(\frac{K_1}{K_1+K_R}\right)^2 a_y^2\right]g} - \frac{W_R}{\left[N \cdot CO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2 W^2\left(\frac{K_R}{K_1+K_R}\right)^2 a_v^2\right]g}$$

(A-1)

APPENDIX B - DIRECTIONAL STABILITY AND CALCULATION OF CRITICAL FORWARD VELOCITY

Rewriting Equation (A-1) as

$$\delta = \frac{\ell'_e}{R} + K'\frac{U^2}{R} \qquad \text{(B-1)}$$

where ℓ'_e and K' represent the lateral acceleration-dependent expressions (A-13) and (A-14), produces, by rearrangement, the path curvature-steer angle relationship:

$$\frac{1/R}{\delta} = \frac{1}{\ell'_e + K'U^2} \qquad \text{(B-2)}$$

Consideration of infinite path curvature for finite steer levels during steady turning leads to the stability condition

$$\frac{\partial(\delta)}{\partial(\frac{1}{R})} > 0 \qquad \text{(B-3)}$$

or

$$U^2 \cdot \frac{\partial(\delta)}{\partial(a_y)} = \ell'_e + K'U^2 + \left(\frac{\partial \ell'_e}{\partial a_y} + \frac{\partial K'}{\partial a_y}U^2\right)a_y > 0 \qquad \text{(B-4)}$$

The critical forward velocity, U_c, above which the vehicle becomes directionally unstable is obtained by solving Equation (B-4) for its zero condition:

$$U_c = \left[\frac{-\ell'_e - \frac{\partial \ell'_e}{\partial a_y}\cdot a_y}{K' + \frac{\partial K'}{\partial a_y}\cdot a_y}\right]^{1/2} \qquad \text{(B-5)}$$

The partial derivatives appearing in Equation (B-5) are obtained by differentiating Equations (A-13) and (A-14) with respect to a_y, viz.:

$$\frac{\partial \ell'_e}{\partial a_y} = [\frac{\partial A}{\partial a_y}\cdot B + \frac{\partial B}{\partial a_y}\cdot A]\cdot \ell$$

$$= \left\{\frac{-8D^2C_{s_R}C2_{\alpha_R}\left(\frac{h}{T_R}\right)^2W^2\left(\frac{K_R}{K_1+K_R}\right)^2a_y}{N\ell\left[NCO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2W^2\left(\frac{K_R}{K_1+K_R}\right)^2a_y^2\right]^2}\right\}$$

$$\cdot\left\{1 + \frac{NCO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2W^2\left(\frac{K_R}{K_1+K_R}\right)^2a_y^2}{2\left[CO_{\alpha_1} + C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2W^2\left(\frac{K_1}{K_1+K_R}\right)^2a_y^2\right]}\right.$$

$$+ \frac{8\frac{C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2W^2\left(\frac{K_R}{K_1+K_R}\right)^2a_y}{2\left[CO_{\alpha_1} + C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2W^2\left(\frac{K_1}{K_1+K_R}\right)^2a_y^2\right]}$$

$$\left. - \frac{\left[4C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2W^2\left(\frac{K_1}{K_1+K_R}\right)^2\cdot a_y\right]\left[NCO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2W^2\left(\frac{K_R}{K_1+K_R}\right)^2a_y^2\right]}{2\left[CO_{\alpha_1} + C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2W^2\left(\frac{K_1}{K_1+K_R}\right)^2a_y^2\right]^2}\right\}$$

$$\cdot\left\{\frac{\Delta^2 + D^2C_{s_R}/\left[NCO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2W^2\left(\frac{K_R}{K_1+K_R}\right)^2a_y^2\right]}{\ell}\right\} \qquad \text{(B-6)}$$

$$\frac{\partial K'}{\partial a_y} = -\frac{W_1C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2W^2\left(\frac{K_1}{K_1+K_R}\right)^2a_y}{\left[CO_{\alpha_1} + C2_{\alpha_1}\left(\frac{h}{T_1}\right)^2W^2\left(\frac{K_1}{K_1+K_R}\right)^2a_y^2\right]^2g}$$

$$+ \frac{8W_R\cdot\frac{C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2W^2\left(\frac{K_R}{K_1+K_R}\right)^2a_y}{\left[NCO_{\alpha_R} + \frac{4C2_{\alpha_R}}{N}\left(\frac{h}{T_R}\right)^2W^2\left(\frac{K_R}{K_1+K_R}\right)^2a_y^2\right]^2g} \qquad \text{(B-7)}$$

Note that Equation (B-5) is the classical critical velocity expression except for the additional partial derivative terms denoting dependence of ℓ'_e and K' (and hence U_c) upon lateral acceleration. Recall that the dependence of ℓ'_e and K' on lateral acceleration is directly related to the side-to-side load transfer assumption and variation of tire cornering stiffness with vertical load discussed in Appendix A.

APPENDIX C

SIMPLIFIED LINEAR ANALYSIS OF REARWARD AMPLIFICATION

This appendix presents a technical discussion of simplified equations developed for predicting rearward amplification. The equations of motion pertaining to a conventional dolly are examined in order to explain the lateral force "decoupling" achieved through the use of a dolly. Then frequency domain techniques and ideas from feedback control theory are employed to study the lateral acceleration gain between the pintle hitch and the center of gravity location of a full trailer. Finally, expressions defining the portion of the overall rearward amplification due to properties of towing units are derived.

C.1 Force Decoupling Achieved Through the Use of Steerable Dollies

In this section, an examination of the equations of motion of a truck-full trailer combination (as illustrated in Figure C.1) is used to indicate why F_A, the lateral force of constraint at the pintle hitch, is small.

For a truck-full trailer combination, the linearized equations of motion are as listed below. (These equations and symbols are the same as those used in [19].)

Truck-Full-Trailer Equations: (The full trailer consists of a dolly and a semitrailer)

$$m_1(\dot{v}_1+ur_1) = \sum_i F_{1i} - F_A \text{ (Truck lateral motion equation)} \tag{C.1}$$

$$m_2(\dot{v}_2+ur_2) = \sum_i F_{2i} + F_A - F_B \text{ (Dolly lateral motion equation)} \tag{C.2}$$

$$m_3(\dot{v}_3+ur_3) = \sum_i F_{3i} + F_B \text{ (Semitrailer lateral motion equation)} \tag{C.3}$$

where F_A is the force of constraint at the pintle hitch, F_B is the force of constraint at the fifth wheel or turntable, and the F_{ji} are tire forces that are linearly related to their slip angles.

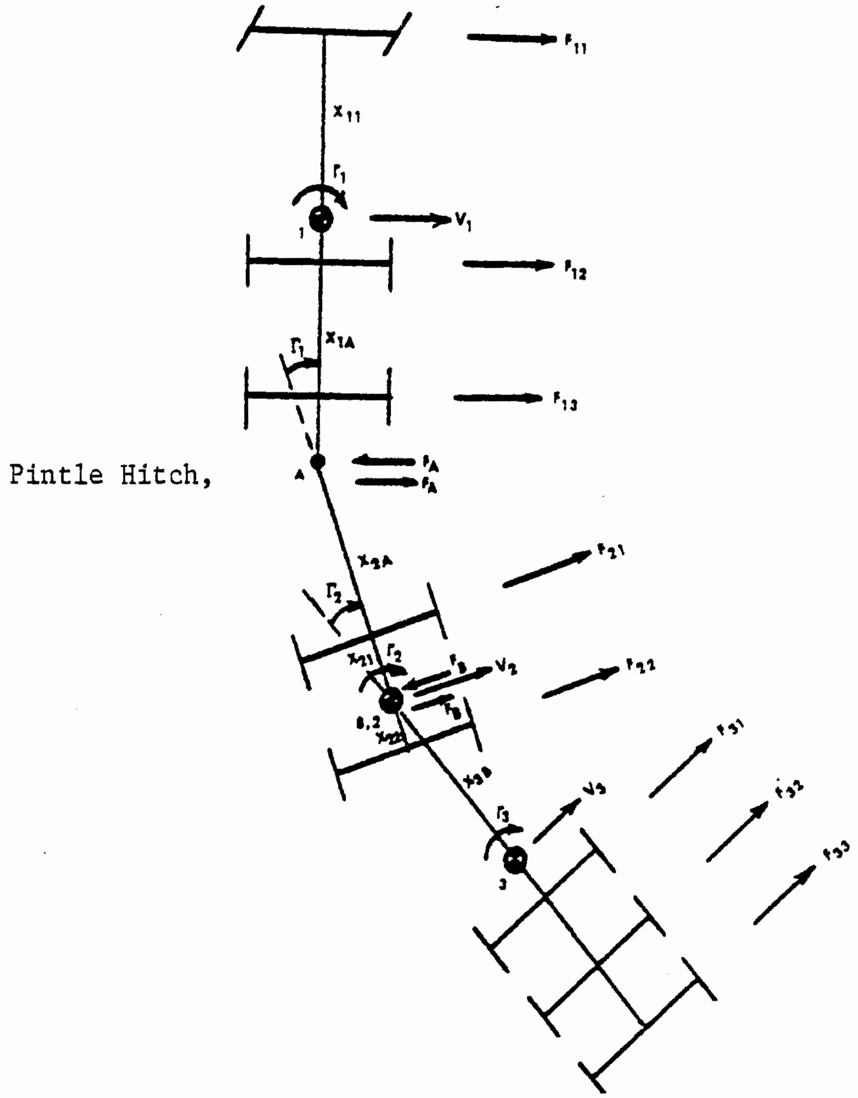


Figure C.1. Truck-full trailer combination.

$$I_1\dot{r}_1 = x_{11}F_{11} - x_{12}F_{12} - x_{13}F_{13} + x_{1A}F_A \quad \text{(Truck rotational equation)} \tag{C.4}$$

$$I_2\dot{r}_2 = +F_A x_{2A} + F_{21}x_{21} - F_{22}x_{22} \quad \text{(Dolly rotational equation)} \tag{C.5}$$

(F_B does not appear in (C.5) because $x_{2B} \approx 0$; that is, for the dolly, the center of gravity and the turntable location are approximately in the same place.)

$$I_3\dot{r}_3 = +F_B x_{3B} - F_{31}x_{31} - F_{32}x_{32} - F_{33}x_{33} \quad \text{(Semitrailer rotational equation)} \tag{C.6}$$

Expanding Equation (C.5) with $x_{21} \approx x_{22}$ and $C_{\alpha 21} \approx C_{\alpha 22}$ yields

$$F_A = \left(\frac{+I_2\dot{r}_2 + \dfrac{2C_{\alpha 21}x_{21}^2 r_2}{u}}{x_{2A}} \right) \tag{C.7}$$

The term, $2C_{\alpha 21}x_{21}^2 r_2/u$ in (C.7) represents the yaw moment contribution from the dolly's tire forces acting about the turntable center for a dolly with tandem axles.

In Equation (C.7) observe that:

a) $\dot{r}_2$ is not large because the dolly is tied to two large masses that do not move quickly

b) I_2 is not large (the dolly does not have a large moment of inertia)

c) $x_{21}r_2 << u$ (the forward speed is very much greater than this lateral velocity factor)

d) $x_{21}/x_{2A} < 0.3$ (even for dollies with short tongues)

As a consequence of items (a) through (d) above, $|F_A| < 200$ lbs for typical dollies.

The preceding discussion applies not only to truck-full-trailer combinations, but also to any combination vehicle employing full trailers with conventional pintle hitch connections between the dolly tongue and its towing unit. The configuration of the combination vehicle (i.e., double, triple, or truck-full trailer) will not alter the basic factors and equations pertaining to the yaw moment balance for the dolly.

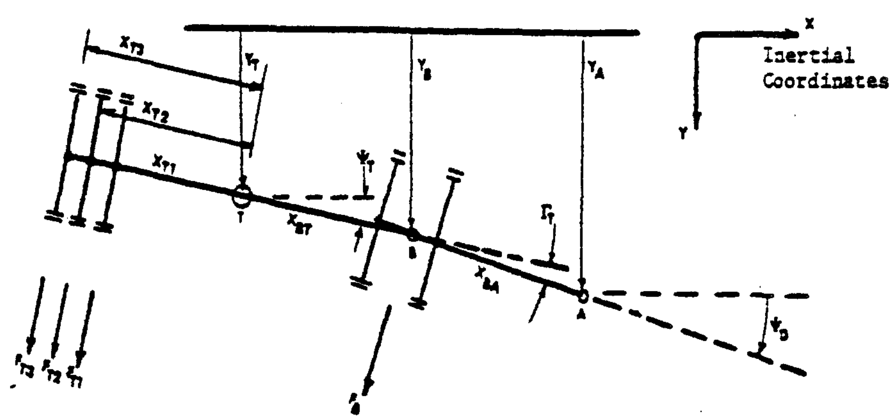
C.2 Analysis of the Full Trailer

For the idealized dolly employed in the analysis presented next, the force of constraint, F_A, at the pintle hitch is set equal to zero. This simplification and approximation separates the towing unit from the full trailer in that (a) the motion of the trailer does not influence the towing unit's motion and (2) the input to the trailer is the movement of the pintle hitch connection to the unit immediately ahead of the full trailer. (Also, if the full trailer being analyzed is the towing unit for another full trailer, such as in a triples combination, the properties of the trailer being towed will have no influence on the trailer being analyzed.)

Instead of comparing the last unit's lateral acceleration to that of the leading unit to obtain a rearward amplification factor for an entire vehicle, the acceleration or displacement of the center of mass of the full trailer may be compared to the acceleration or displacement of its hitch point to obtain an amplification factor for the full trailer alone. The overall amplification factor will then consist of the product of the full trailer's amplification factor with other amplification factors existing in the combination vehicle.

Figure C.2 illustrates and defines the points, geometric quantities, dimensions, and forces used in this analysis of the full trailer. (A nomenclature defining symbols and subscripts is given in Table 5 of the main body of this report.)

Using the approximation that $F_A = 0$, the steering of the dolly tongue can be determined from the motions of (1) the hitch point, A, and (2) the full trailer as influenced by (a) the side force from the dolly's tires acting at point B and (b) the side forces from the rear tires of the trailer acting at their corresponding axle locations.



x_{BA} = dolly tongue length

ψ_D = heading angle of the dolly tongue

ψ_T = heading angle of the trailer body

Γ_T = articulation angle at the trailer turntable

y_A = lateral displacement of the pintle hitch (point A)

y_B = lateral displacement of the turntable (point B)

y_T = lateral displacement of the trailer's center of mass

x_{BT} = the distance from point B to the c.g. of the full trailer (point T)

x_{Ti} = locations of the trailer's rear axles with respect to its c.g.

F_{Ti} = lateral tire forces at each rear axle location

F_B = the resultant of all of the lateral forces from the dolly tires. This resultant force acts at the center of the turntable.

Figure C.2. Full trailer representation.

Let

I_T ≃ the yaw moment of inertia of the full trailer

m_T = the mass of the full trailer

x_{BT} = the distance from the trailer's turntable to the c.g. of the full trailer

The side force, F_B, acting at point B on the full trailer (see Fig. C.2) is equal to F_{21} plus F_{22} for a dolly with tandem axles. Hence, in the linear range,

$$F_B = -C_{\alpha 21}\alpha_{21} - C_{\alpha 22}\alpha_{22}$$

For $C_{\alpha 21} = C_{\alpha 22}$ and $x_{21} = x_{22}$,

$$F_B = -2C_{\alpha 21}(\alpha_{21} + \alpha_{22})$$

where

$$\alpha_{21} = \frac{v_2 + x_{21}r_2}{u} \quad (C.8)$$

$$\alpha_{22} = \frac{v_2 - x_{21}r_2}{u} \quad (C.9)$$

or

$$F_B = -2C_{\alpha 21}(\frac{v_2}{u}) \quad (C.10)$$

Points B and 2 are coincident according to the condition that the c.g. of the dolly and the location of the turntable center are approximately at the same point.

For the system shown in Figure C.2, the equations of motion are

$$m_T(\dot{v}_T + x_A r_T) = F_B + \sum_i F_{Ti} \quad (C.11)$$

$$I_T\dot{r}_T = x_{BT}F_B - \sum_i x_{Ti}F_{Ti} \quad (C.12)$$

The slip angles needed for expressing the tire forces can be determined for small angles (such as those occurring in a lane-change maneuver) from the geometry illustrated in Figure C.1 and the appropriate kinematic relationships per the following equations:

$\dot{x}_A$ = u (the entire vehicle is moving longitudinally with approximately the same forward velocity, u)

$$\psi_D = \frac{y_A - y_B}{x_{BA}}$$

$$y_B = y_T + x_{BT}\psi_T$$

$$\psi_T = \int_0^t r_T dt$$

$$\Gamma_T = \psi_D - \psi_T = \frac{y_A - y_T - (x_{BT}+x_{BA})\psi_T}{x_{BA}}$$

$$\ddot{y}_T = \dot{v}_T + ur_T$$

$$\dot{y}_T = v_T + \psi_T\dot{x}_A$$

$$y_T = \int_0^t \dot{y}_T dt$$

$$v_2 = v_T + x_{BT}r_T - \dot{x}_A\Gamma_T$$

$$\frac{v_2}{u} = \frac{v_T + x_{BT}r_T}{\dot{x}_A} - \Gamma_T \tag{C.13}$$

$$\alpha_{Ti} = \frac{v_T - r_T x_{Ti}}{\dot{x}_A} \tag{C.14}$$

Using Equation (C.14) with i = 1,2,3 and assigning cornering stiffness values to the tires, the force and moment summations appearing in Equations (C.11) and (C.12) may be expressed as follows:

$$\sum_{i=1}^{3} F_{Ti} = -\frac{v_T}{\dot{x}_A}\sum_i C_{\alpha Ti} + \frac{r_T}{\dot{x}_A}\sum_i x_{Ti}C_{\alpha_{Ti}}$$

$$\sum_{i=1}^{3} x_{Ti}F_{Ti} = -\frac{v_T}{\dot{x}_A}\sum_i x_{Ti}C_{\alpha Ti} + \frac{r_T}{\dot{x}_A}\sum_i x_{Ti}^2 C_{\alpha Ti}$$

The force, F_B, is given by the following equation

$$F_B = -2C_{\alpha 21}\left(\frac{v_T + x_{BT}r_T}{\dot{x}_A} - r_T\right)$$

which is equivalent to

$$F_B = -2C_{\alpha 21}\frac{v_T}{\dot{x}_A} - \frac{2C_{\alpha 21}x_{BT}r_T}{\dot{x}_A} - \frac{2C_{\alpha 21}(x_{BT}+x_{BA})\psi_T}{x_{BA}} + \frac{2C_{\alpha 21}}{x_{BA}}(y_A - y_T)$$

Substituting these forces and moment expressions into the equations of motion ((C.11) and (C.12)) yields Equations (C.15) and (C.16) given in Figure C.3.

To obtain y_T and ψ_T as needed to complete the solution of (C.15) and (C.16), the following kinematic equations are employed:

$$\dot{\psi}_T = r_T \tag{C.17}$$

$$\dot{y}_T = v_T + \psi_T\dot{x}_A \tag{C.18}$$

By (a) combining Equations (C.15), (C.16), (C.17), and (C.18), (b) using p to replace the differential operator, d/dt, and (c) rearranging the equations, the following two equations are obtained:

$$(m_T p + F_v)v_T + (m_T u + F_r + \frac{F_\psi}{p})r_T = F_y(y_A - y_T) \tag{C.19}$$

$$T_v v_T + (I_T p + T_r + \frac{T_\psi}{p})r_T = T_y(y_A - y_T) \tag{C.20}$$

Since the quantity $(y_A - y_T)$ appears on the right sides of (C.19) and (C.20), the control engineer's notion of an open-loop transfer function comes to mind and provides the basis for additional development. Figure C.4 illustrates the idea that will be exploited here.

$$m_T(\dot{v}_T + \dot{x}_A r_T) = - F_v v_T - F_r r_T - F_\psi \psi_T + F_y(y_A - y_T) \qquad \text{(C.15)}$$

$$I_T \dot{r}_T = - T_v v_T - T_r r_T - T_\psi \psi_T + T_y(y_A - y_T) \qquad \text{(C.16)}$$

where

$$F_v = \left(\frac{2C_{\alpha 21} + \sum_i C_{\alpha Ti}}{\dot{x}_A}\right) \overset{=}{\text{def.}} \frac{\Sigma C_\alpha}{\dot{x}_A}$$

$$F_r = \left(\frac{2C_{\alpha 21} x_{BT} - \sum_i x_{Ti} C_{\alpha Ti}}{\dot{x}_A}\right) \overset{=}{\text{def.}} \frac{x_{BT} \sum_f C_\alpha - \sum_r x_T C_{\alpha T}}{\dot{x}_A}$$

$$F_\psi = \left(\frac{2C_{\alpha 21}(x_{BT} + x_{BA})}{x_{BA}}\right) = F_y(x_{BT} + x_{BA})$$

$$F_y = \left(\frac{2C_{\alpha 21}}{x_{BA}}\right) = \frac{F_\psi}{(x_{BT} + x_{BA})}$$

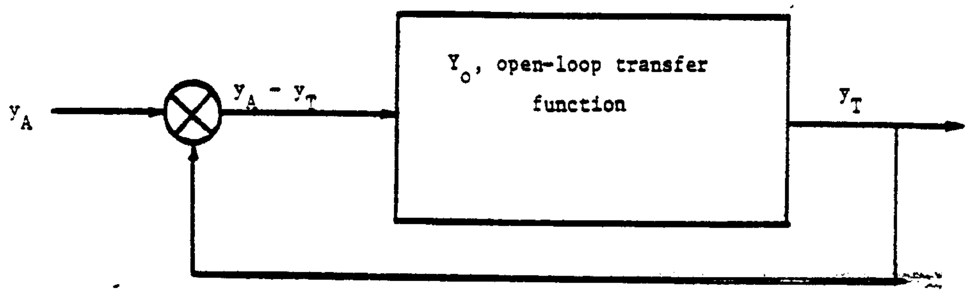
$$T_v = \left(\frac{2C_{\alpha 21} x_{BT} - \sum_i x_{Ti} C_{\alpha Ti}}{\dot{x}_A}\right) = F_r$$

$$T_r = \left(\frac{2C_{\alpha 21} x_{BT}^2 + \sum_i x_{Ti}^2 C_{\alpha Ti}}{\dot{x}_A}\right) \overset{=}{\text{def.}} \frac{\Sigma x^2 C_\alpha}{\dot{x}_A}$$

$$T_\psi = \frac{2C_{\alpha 21}(x_{BT} + x_{BA}) x_{BT}}{x_{BA}} = x_{BT} F_\psi = x_{BT}(x_{BT} + x_{BA}) F_y$$

$$T_y = \frac{2C_{\alpha 21} x_{BT}}{x_{BA}} = F_y x_{BT}$$

Figure C.3. Linearized equations of motion for a full trailer.



$$y_T = Y_o(y_A - y_T)$$

The closed-loop transfer function $Y_c = \frac{Y_o}{1 + Y_o}$

$$Y_o = \left(\frac{y_T}{y_A - y_T}\right) = \frac{[F_v I_T p^2 + (F_v T_r - T_v F_r)p + T_v u F_v - T_v u F_v]}{p[(m_T p + F_v)(I_T p^2 + T_r p + T_\psi) - (T_v)((m_T u + F_r)p + F_\psi)]}$$

Figure C.4. Feedback control diagram as applied to a full trailer.

Proceeding in this manner, consider the full trailer as a mechanical servomechanism in which the output, y_T, is expected to follow the input, y_A. (However, in this case, $y_T(t + ((x_{BT}+x_{BA})/\dot{x}_A))$ should equal $y_A(t)$ if the trailer c.g. is going to follow the same path as the hitch point—more about this later after completing the analysis of the open-loop transfer function.)

Formally solving (C.19) and (C.20) for v_T and r_T yields

$$v_T = D_1/D \tag{C.21}$$

and

$$r_T = D_2/D \tag{C.22}$$

where

$$D = (m_T p + F_v)(I_T p + T_r + \frac{T_\psi}{p}) - T_v(m_T u + F_r + \frac{F_\psi}{p}) ,$$

$$D_1 = \left[F_y(I_T p + T_r + \frac{T_\psi}{p}) - T_y(m_T u + F_r + \frac{F_\psi}{p})\right] (y_A - y_T) ,$$

and $D_2 = [(m_T p + F_v)T_y - T_v F_y](y_A - y_T)$

Noting that

$$y_T = \iint(\dot{v}_T + ur_T)dtdt = \frac{pD_1 + uD_2}{p^2D}$$

allows the open-loop transfer function to be expressed as

$$\frac{y_T}{(y_A - y_T)} = \frac{[(F_y)(I_Tp^2 + T_rp + T_\psi) - (T_y)(m_Tu+F_r)p + F_\psi) + T_yu(m_Tp+F_v)-T_vuF_y]}{p[(m_Tp+F_v)(I_Tp^2 + T_rp + T_\psi) - (T_v)((m_Tu+F_r)p + F_\psi)]} \tag{C.23}$$

or by rearranging the numerator and noting that $F_yT_\psi - T_yF_\psi = 0$

$$\frac{y_T}{(y_A - y_T)} = \frac{[F_yI_Tp^2 + (F_yT_r - T_yF_r)p + T_yu\ F_v - T_vu\ F_y]}{p^2D} \tag{C.24}$$

Although (C.24) can be expanded further by substituting the definitions that accompany Equations (C.15) and (C.16), the result does not appear to be particularly illuminating. Rather, observe that most trailers are loaded with approximately equal loads on all axles and equipped with similar tires on all wheels. Under these circumstances, $T_v = F_r = 0$, that is, the trailer may be described as approximately "neutral steer." For the "typical" trailer

$$Y_o = \frac{y_T}{y_A - y_T} = \frac{F_yI_Tp^2 + F_yT_rp + T_yu\ F_v}{p(m_Tp + F_v)(I_Tp^2 + T_rp + T_\psi)} \tag{C.25}$$

Upon substituting for the force and moment coefficients and rearranging, (C.25) becomes Equations (C.26), (C.27), and (C.28), expressed as follows:

$$Y_o = \frac{\left(\frac{\dot{x}_A}{x_{BT} + x_{BA}}\right)(Y_z)}{p\left(\frac{\dot{x}_Am_T}{\Sigma C_\alpha}p + 1\right)(Y_{po})} \tag{C.26}$$

where

$$Y_z = \frac{I_T p^2}{x_{BT}\Sigma C_\alpha} + \frac{(\Sigma x^2 C_\alpha)p}{\dot{x}_A x_{BT}\Sigma C_\alpha} + 1 \tag{C.27}$$

$$Y_{po} = \frac{I_T x_{BA}\, p^2}{(\Sigma_f C_\alpha)(x_{BT} + x_{BA})x_{BT}} + \frac{(\Sigma x^2 C_\alpha)x_{BA}\, p}{\dot{x}_A(\Sigma_f C_\alpha)(x_{BT} + x_{BA})x_{BT}} + 1 \tag{C.28}$$

The natural frequencies and damping ratios of Y_z and Y_{po} are nearly equal if $uF_v \approx F_\psi$, that is, if

$$\Sigma C_\alpha \approx \Sigma_f C_\alpha \left(\frac{x_{BT} + x_{BA}}{x_{BA}}\right) \tag{C.29}$$

For many trailers, Y_z and Y_{po} represent complex zero and pole pairs that are lightly damped and approximately equal (i.e., (C.29) is a reasonable approximation).

In addition to the pole-zero pair represented by Y_z/Y_{po}, the open-loop transfer function has a pole at the origin and another real pole at $p = - F_v/m_T = - \Sigma C_\alpha/\dot{x}_A m_T$. These poles and zeros are sketched in the complex plane in a manner typical of control system analysis (see Figure C.5) with arrows indicating the locus of the closed-loop poles as the "gain" is increased. Of course, the "gain" is not variable in the sense ordinarily used by the control system analyst. Rather, it is given by $\dot{x}_A/(x_{BA}+x_{BT})$ with the asterisks in Figure C.5 illustrating representative locations for the closed-loop poles. As indicated in the figure, the open-loop poles due to Y_{po} approach the zeros due to Y_z, giving a closed-loop pole-zero pair in the neighborhood of the open-loop zeros. The two open-loop poles on the real axis (one at the origin and the other at $-F_v/m_T$) become a complex conjugate set of closed-loop poles. This set of poles is the primary factor determining the frequency response of the closed-loop system (since the pole-zero pair effectively cancel each other). Hence, to a first approximation, the closed-loop transfer function, Y_c (where $Y_c = Y_o/1+Y_o = y_T/y_A$ or A_{yT}/A_{yA}) is given by

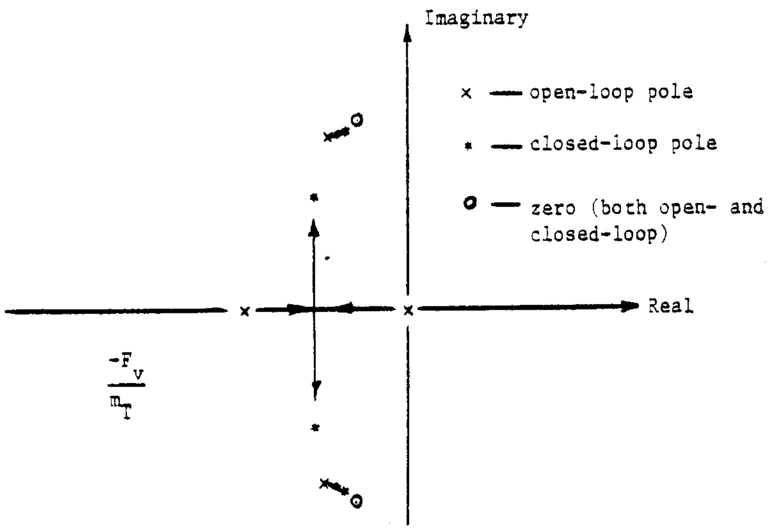


Figure C.5. Root locus diagram for a full trailer.

$$Y_c = \left[\frac{1}{\frac{p^2}{\omega_{nc}^2} + \frac{2\zeta_c}{\omega_{nc}} p + 1} \right]$$

where

$$\omega_{nc} = \left(\frac{(\Sigma C_\alpha)}{(x_{TB} + x_{BA}) m_T} \right)^{1/2}$$

$$\zeta_c = \frac{1}{2\dot{x}_A} \left(\frac{(\Sigma C_\alpha)(x_{TB} + x_{BA})}{m_T} \right)^{1/2} \qquad \text{(C.30)}$$

For $\omega < 0.3\ \omega_{nc}$, the phase shift, ϕ, is approximately given by

$$\phi = \frac{-2\zeta_c \omega}{\omega_{nc}}$$

where

$$\frac{2\zeta_c}{\omega_{nc}} = \frac{x_{BA} + x_{TB}}{\dot{x}_A}$$

The ideal phase shift, ϕ_I, required for the trailer's c.g. to track the hitch point is given by

$$\phi_I = \frac{-(x_{BA} + x_{TB})}{\dot{x}_A} \omega$$

Also, for small ω, $Y_c \simeq 1.0$. Hence, at low frequencies of sinusoidal excitation at the hitch point, the path of the trailer's c.g. will pass very closely to the path of the hitch point—an expected and desired result and one that adds some intuitive confirmation to the analysis.

With regard to amplification, a second-order system such as that represented by Y_c, has its maximum response at a frequency given by

$$\omega_{max} = \omega_n \sqrt{1 - 2\zeta^2}$$

and the gain at ω_{max} is given by

$$G_{max} = \frac{1}{2\zeta \sqrt{1 - \zeta^2}}$$

Sometimes the transfer function, Y_{c2}, corresponding to A_{yT}/y_A may be of interest in relating the path of the hitch to the acceleration of the trailer. In this case,

$$Y_{c2} = p^2 Y_c$$

However, the frequency of maximum response, ω_{max2}, is now given by

$$\omega_{max2} = \frac{\omega_n}{\sqrt{1 - 2\zeta^2}}$$

and the maximum gain is given by

$$G_{max2} = \frac{\omega_n^2}{2\zeta \sqrt{1 - \zeta^2}}$$

The basic results of the full trailer analysis are summarized by the expression for the damping ratio, ζ_c, i.e., Equation (C.30). As indicated by (C.30), the damping ratio decreases (thereby causing the rearward amplification to increase) if (a) the forward velocity, $\dot{x}_A$, is increased, (b) the total cornering coefficient, $\Sigma C_\alpha / m_T$, is decreased, or (c) the distance from the c.g. to the hitch, $(x_{BT} + x_{BA})$, is decreased.

Items (b) and (c) above follow a square root relationship, while the damping ratio is inversely proportional to velocity. Hence, the magnitude of forward velocity is a critical consideration when examining rearward amplification.

Clearly, the analysis of the rearward amplification of the full trailer has been reduced to three fundamental parameters, namely, velocity, cornering coefficient, and the distance from the c.g. to the hitch point. The first two of these parameters are expected to be important when considering the damping of the directional response of any highway vehicle. A length parameter is, also, expected to be important and in this case this length parameter involves the dolly's tongue length, x_{BA}, plus x_{BT}, which is a loading/wheelbase type of parameter for the full trailer.

The validity of the basic predictions of amplification can be checked by computing the quantities $F_r(Tv)$, Y_z, and Y_{po} to verify the reasonableness of the simplifications employed. Furthermore, more detailed analyses and/or complete simulations can be used to examine the characteristics of vehicles for which a basic analysis indicates large amounts of rearward amplification.

C.3 Towing Unit Amplification

If the hitch point of a towing unit is not located at the c.g. of the towing unit, the lateral displacement or acceleration of the hitch point may differ from that of the c.g. Specifically, for a unit with a yaw acceleration $\dot{r}$ and a distance x_{1A} from its c.g. to the hitch point

$$A_{yA} = A_{y1} - x_{1A}\dot{r}$$

(where x_{1A} is positive for the hitch point, A, being behind point 1 (i.e., behind the c.g. of the towing unit)). Or, similarly, in terms of displacements

$$y_A = y_1 - x_{1A}\psi$$

where ψ is the heading angle of the towing unit.

In these cases, the amplification, A, is simply A_{yA}/A_{y1} or y_A/y_1; that is, $A = 1 + \Delta A$ where $\Delta A = -x_{1A}\psi/y_1$ or $-x_{1A}\dot{r}/A_{y1}$. For example, consider a straight truck whose response to steering is described by the following equations:

$$m(\dot{v} + ur) = -F_v v - F_r r + F_\delta \delta \qquad (C.31)$$

$$I\dot{r} = -T_v v - T_r r + T_\delta \delta \qquad (C.32)$$

These equations can be used to write transfer functions for v and r, having the following forms:

$$\frac{r}{\delta} = \frac{N_r}{D} \qquad (C.33)$$

and

$$\frac{v}{\delta} = \frac{N_v}{D} \qquad (C.34)$$

where N_r is the numerator of the yaw rate transfer function; N_v is the numerator of the sideslip transfer function; and the denominator, D, is the same in both cases. The quantity ΔA is expressed in terms of the numerators of (C.33) and (C.34) by the following equation:

$$\Delta A = \frac{-x_{1A}\dot{r}}{A_{y1}} = \frac{-x_{1A}p\, r}{pv + ur} = \frac{-x_{1A}p\, N_r}{pN_v + uN_r} \qquad (C.35)$$

Equation (C.35) shows that the amplification factor for the towing vehicle depends upon the numerators of the yaw rate and side velocity transfer functions, that is, it depends upon the zeros of these transfer functions and not the denominator, the eigenvalues, or the characteristic equation as is studied to determine the stability of the vehicle.

Using Equations (C.31) and (C.32) to evaluate (C.35) yields

$$\Delta A = \frac{-\frac{x_{1A}}{u}\left(\frac{m\,T_\delta}{F_vT_\delta - T_cF_\delta}\,p + 1\right)p}{\left[\frac{F_\delta I\,p^2}{u(F_vT_\delta - T_vF_\delta)} + \left(\frac{T_rF_\delta - T_\delta F_r}{u(F_vT_\delta - T_vF_\delta)}\right)\,p + 1\right]} \tag{C.36}$$

For typical vehicles

$$F_vT_\delta \gg T_vF_\delta$$

and

$$T_rF_\delta \gg T_\delta F_r$$

implying that

$$\Delta A = \left[\frac{\frac{-x_{1A}}{u}\,(p)\,\left(\frac{m}{F_v}\,p + 1\right)}{\frac{F_\delta I}{uF_vT_\delta}\,p^2 + \frac{T_rF_\delta}{uF_vT_\delta}\,p + 1}\right]$$

Substituting for the force and moment coefficients in terms of design parameters yields

$$\Delta A = \left[\frac{\frac{-x_{1A}}{u}\,(p)\,\left(\frac{mu}{\Sigma C_\alpha}\,p + 1\right)}{Y_{zA}}\right] \tag{C.37}$$

where x_{11} is the distance from the c.g. to the front axle and

$$Y_{zA} = \frac{p^2}{\omega^2_{nzA}} + \frac{2\zeta_{zA}}{\omega_{nzA}}\,p + 1,$$

with

$$\omega_{nzA} = \left(\frac{x_{11}\Sigma C_\alpha}{I}\right)^{1/2} ,$$

and

$$\zeta_{zA} = \frac{1}{2}\left(\frac{\Sigma x^2 C_\alpha}{u}\right)\frac{1}{(I\ x_{11}\Sigma C_\alpha)^{1/2}}$$

In the frequency domain, ΔA is a vector of the form $Ke^{j\phi}$, which must be added to the unit vector (1.0) to obtain A at any particular frequency of interest, that is, K and ϕ are functions of the frequency, ω.

In general,

$$A = 1 + \Delta A = \frac{p^2\left(\frac{1}{\omega_{nzA}^2} - \alpha\tau\right) + p\left(\frac{2\zeta_{zA}}{\omega_{nzA}} - \alpha\right) + 1}{\frac{p^2}{\omega_{nzA}^2} + \frac{2\zeta_{zA}p}{\omega_{nzA}} + 1} \qquad (C.38)$$

where ω_{nzA} and ζ_{zA} are defined as before

and $\alpha = x_{1A}/u$

and $\tau = mu/\Sigma C_\alpha$.

At high frequencies

$$A = 1 - \alpha\tau\omega_{nzA}^2 = 1 - \frac{m\ x_{1A}x_{11}}{I}$$

Note that the analysis performed for the straight truck also applies to a full trailer that is the towing unit for another full trailer (as in a triples combination). In this case, the articulation angle between the dolly's longitudinal axis and the longitudinal axis of the semitrailer portion of the full trailer plays a role that is analogous to the role of the steering angle, δ, employed in the discussion of the straight truck. The results for a full trailer, that is acting as a towing unit, are presented in Table 4 in Chapter 6.

The semitrailer of a tractor-semitrailer towing unit is coupled by a fifth wheel arrangement that provides a large lateral force of constraint. Hence, it is not possible to separate the analysis of the tractor from that of the semitrailer. Nevertheless, the dynamics of tractor-semitrailer vehicles have been studied extensively and the lateral acceleration gain between the c.g. of the tractor and the c.g. of the semitrailer may be known or readily determined for many tractor-semitrailer combinations [20]. Assuming that the "c.g. to c.g." amplification factor is known, the material presented next contains a derivation of the rearward amplification between the c.g. of a semitrailer and the location of its pintle hitch.

A single-axle semitrailer, illustrated in Figure C.6, will be considered first.

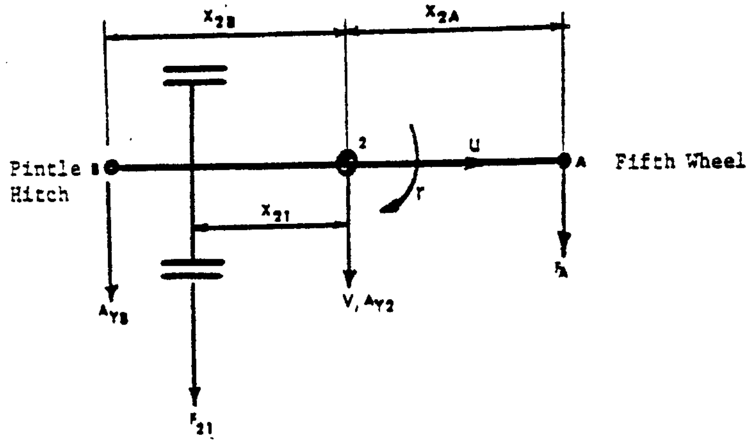


Figure C.6. Single-axle semitrailer.

The lateral acceleration, A_{yB}, of the pintle hitch differs from the lateral acceleration, A_{y2}, of the c.g. because of the yaw acceleration, $\dot{r}$, i.e.,

$$A_{yB} = A_{y2} - x_{2B}\dot{r} \tag{C.39}$$

Consequently, the amplification, A, defined by A_{yB}/A_{y2} may be expressed in the following manner.

$$A = \frac{A_{yB}}{A_{y2}} = 1 - \frac{x_{2B}\dot{r}}{A_{y2}} = 1 + \Delta A \tag{C.40}$$

where

$$\Delta A = \frac{-x_{2B}\dot{r}}{A_{y2}}$$

In order to evaluate ΔA it is necessary to consider the equations of motion for the semitrailer, that is,

$$m_2(A_{y2}) = m_2(\dot{v} + ur) = F_{21} + F_A \tag{C.41}$$

and

$$I_2\dot{r} = F_A x_{2A} - F_{21}x_{21} \tag{C.42}$$

where

$$F_{21} = -C_{\alpha 21}\left(\frac{v}{u} - \frac{x_{21}r}{u}\right) \tag{C.43}$$

Upon using the equations of motion to evaluate ΔA, the force of constraint at the fifth wheel may be eliminated from the resulting expression, thereby producing the following result:

$$\Delta A = \left(\frac{\frac{-x_{2B}p}{u} [\tau p + 1]}{\frac{p^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} p + 1} \right) \tag{C.44}$$

where

$$\tau = \frac{x_{2A} m_2 u}{(x_{2A}+x_{21}) C_{\alpha 21}}$$

$$\omega_n = \left(\frac{(x_{2A}+x_{21}) C_{\alpha 21}}{I_2} \right)^{1/2}$$

$$\zeta = \frac{x_{21}}{2u} \omega_n$$

For semitrailers with multiple rear axles, the expression for ΔA is derived in a manner similar to that used in deriving Equation (C.44). The result is as follows:

$$\Delta A = \left(\frac{\frac{-x_{2B}p}{u} (\tau p + 1)}{\frac{p^2}{\omega^2} + \frac{2\zeta}{\omega_n} p + 1} \right) \tag{C.45}$$

where for N rear axles

$$\tau = \frac{u\ m_2 x_{2A}}{\sum_{i=1}^{N} (x_{2A}+x_{2i}) C_{\alpha 2i}}$$

$$\omega_n = \left(\frac{\sum_{i=1}^{N} (x_{2A}+x_{2i}) C_{\alpha 2i}}{I_2} \right)^{1/2}$$

$$\zeta = \frac{\omega_n}{2u} \left(\frac{\sum_{i=1}^{N} x_{2i} (x_{2A}+x_{2i}) C_{\alpha 2i}}{\sum_{i=1}^{N} (x_{2A}+x_{2i}) C_{\alpha 2i}} \right)$$

An alternative approach for analyzing the influence of semitrailer properties on rearward amplification consists of evaluating the amplification factor between the fifth wheel and pintle hitch locations on the semitrailer. In this case, the motion of the fifth wheel, which is a point on both the tractor and the semitrailer, is taken as the input motion to which the amplification is referenced; that is, if the motion of the fifth wheel is presumed to be reasonable and satisfactory, will the motion of the pintle hitch be a greatly amplified version of that motion?

Using the notation illustrated in Figure C.6, the ratio of the lateral acceleration of the pintle hitch to that of the fifth wheel may be expressed as follows:

$$\frac{A_{yB}}{A_{yA}} = \frac{A_{y2} - x_{2B}\dot{r}}{A_{y2} + x_{2A}\dot{r}}$$

or

$$\frac{A_{yB}}{A_{yA}} = \frac{1 + \Delta A}{1 - \frac{x_{2A}}{x_{2B}} \Delta A}$$

where ΔA is given by Equation (C.44).

Or, upon substituting for ΔA and using the previously defined quantities τ, ω_n, and ζ,

$$A_{AB} = \frac{A_{yA}}{A_{yB}} = \left[\frac{p^2\left(\frac{1}{\omega_n^2} - \frac{x_{2B}\tau}{u}\right) + p\left(\frac{2\zeta}{\omega_n} - \frac{x_{2B}}{u}\right) + 1}{p^2\left(\frac{1}{\omega_n^2} + \frac{x_{2A}\tau}{u}\right) + p\left(\frac{2\zeta}{\omega_n} + \frac{x_{2A}}{u}\right) + 1}\right] \qquad (C.46)$$

APPENDIX D

DERIVATION OF THE EQUATIONS OF MOTION FOR THE DYNAMIC MODEL OF MULTIPLE-ARTICULATED VEHICLES (YAW/ROLL MODEL)

The equations of motion are derived by the application of the Newton's laws of motion. The derivation is organized under the following sub-headings:

1) Axis Systems

2) Equations of Motion for the Sprung and Unsprung Masses

3) Suspension Forces

4) Constraint Forces and Moments

5) Tire Forces

A brief outline of the computer code is presented at the end of the appendix.

D.1. Axis Systems

Three types of axis systems are used in the process of developing the equations of motion. They are: (1) an inertial axis system fixed in space, (2) an axis system fixed to each of the sprung masses, and (3) an axis system fixed to each of the unsprung masses. For example, Figure D.1 shows the axis systems for a four-axle, multiple-articulated vehicle with two articulation points, C_1 and C_2, respectively.

Euler angles are used to define the orientation of the sprung and unsprung masses with respect to the inertial axis system. Since all sprung mass axis systems are defined alike, the axis transformation equations are given below for only one sprung mass. For the same reason, the transformation equations for the unsprung mass axis systems are derived for a single unsprung mass.

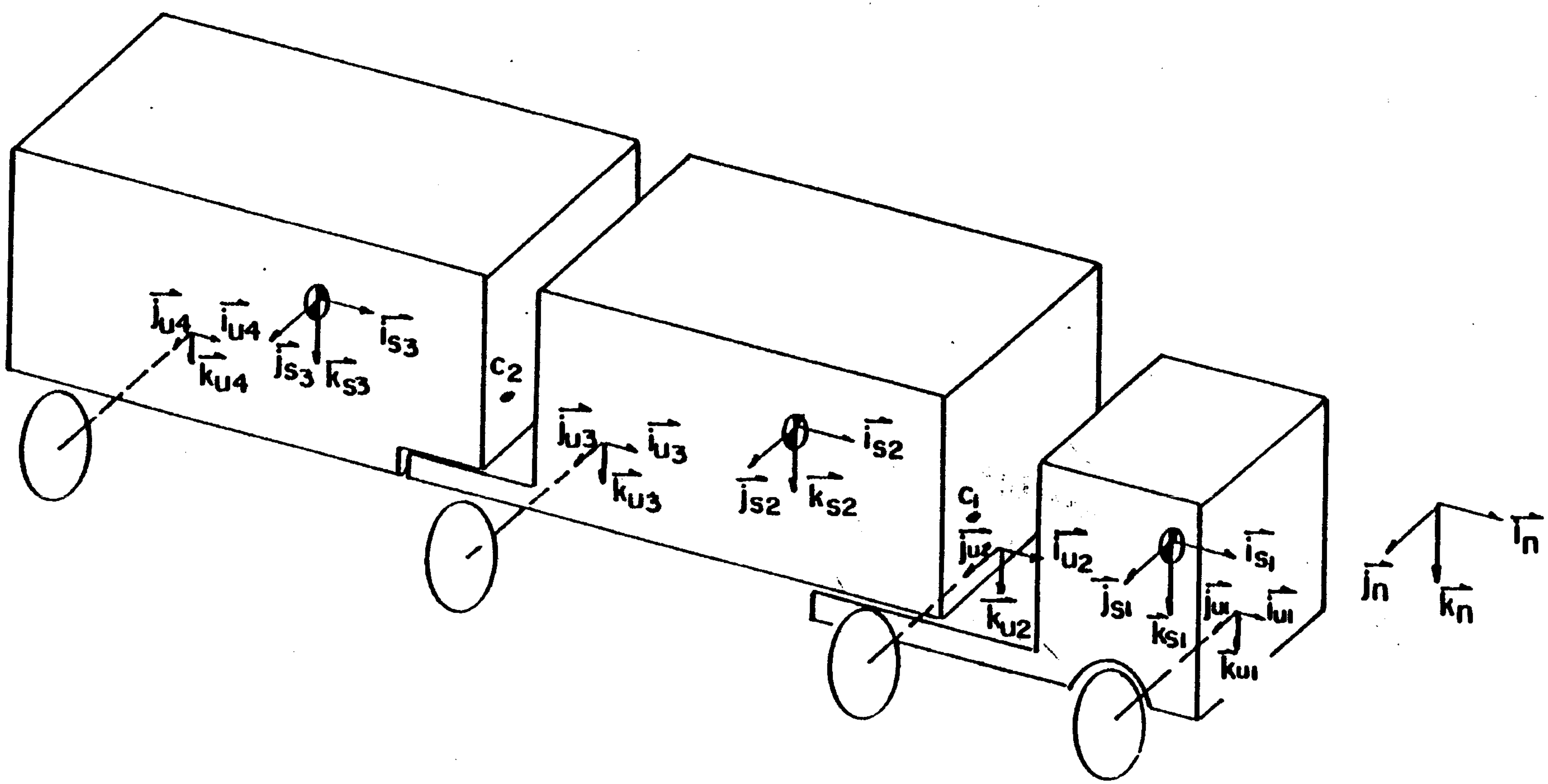


Figure D.1. Axis systems for an articulated vehicle with three sprung masses and four unsprung masses.

D.1.1 Sprung Mass Axis System. The three Euler angles of yaw (ψ_s), pitch (θ_s), and roll (ϕ_s) which are needed to describe the orientation of each of the sprung mass axis systems are shown in Figures D.2, D.3, and D.4, respectively.

The transformation equation between the inertial and sprung mass axis systems can be derived using the three sequential steps of rotation which are illustrated. For the yaw rotation, ψ_s

$$\begin{Bmatrix} \vec{i}_n \\ \vec{j}_n \\ \vec{k}_n \end{Bmatrix} = \begin{bmatrix} \cos\psi_s & -\sin\psi_s & 0 \\ \sin\psi_s & \cos\psi_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \vec{i}_1 \\ \vec{j}_1 \\ \vec{k}_1 \end{Bmatrix} \qquad (1)$$

or

$$\{\vec{i}_n, \vec{j}_n, \vec{k}_n\}^T = [a_{ij}] \{\vec{i}_1, \vec{j}_1, \vec{k}_1\}^T \qquad (2)$$

For the rotation, θ_s, illustrated in Figure C.3,

$$\begin{Bmatrix} \vec{i}_1 \\ \vec{j}_1 \\ \vec{k}_1 \end{Bmatrix} = \begin{bmatrix} \cos\theta_s & 0 & \sin\theta_s \\ 0 & 1 & 0 \\ -\sin\theta_s & 0 & \cos\theta_s \end{bmatrix} \begin{Bmatrix} \vec{i}_2 \\ \vec{j}_2 \\ \vec{k}_2 \end{Bmatrix} \qquad (3)$$

or

$$\{\vec{i}_1, \vec{j}_1, \vec{k}_1\}^T = [b_{ij}] \{\vec{i}_2, \vec{j}_2, \vec{k}_2\}^T \qquad (4)$$

Proceeding along similar lines, the roll rotation illustrated in Figure C.4 yields

$$\begin{Bmatrix} \vec{i}_2 \\ \vec{j}_2 \\ \vec{k}_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_s & -\sin\phi_s \\ 0 & \sin\phi_s & \cos\phi_s \end{bmatrix} \begin{Bmatrix} \vec{i}_s \\ \vec{j}_s \\ \vec{k}_s \end{Bmatrix} \qquad (5)$$

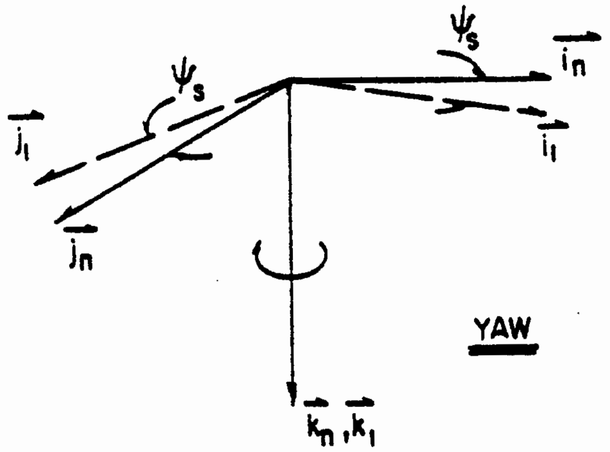


Figure D.2

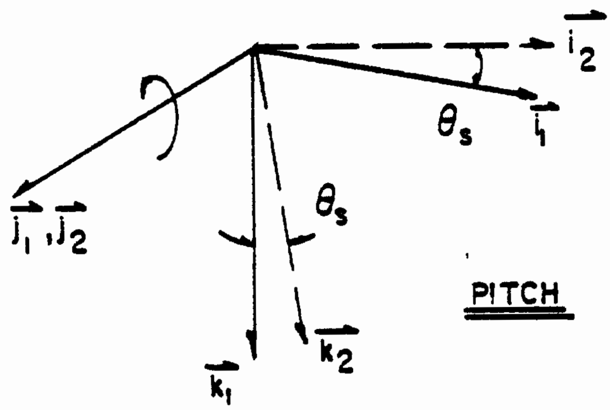


Figure D.3

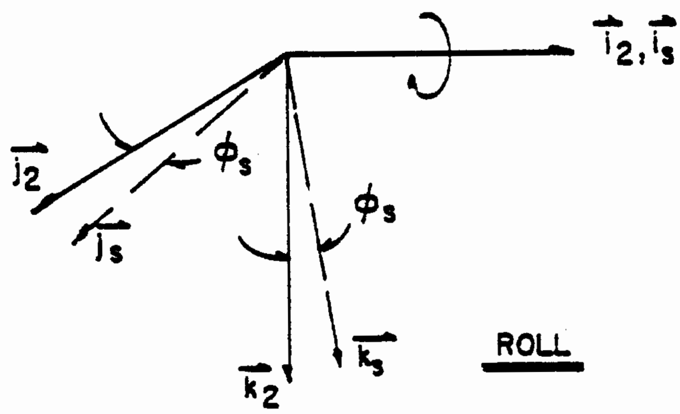


Figure D.4

Euler angles needed to define the orientation of each of the sprung mass axis systems.

or

$$\{\vec{i}_2,\vec{j}_2,\vec{k}_2\}^T = [c_{ij}]\ \{\vec{i}_s,\vec{j}_s,\vec{k}_s\}^T \tag{6}$$

The transformation matrix which is needed to relate the sprung mass axis system and the inertial axis system can now be obtained by combining (2), (4), and (6). Doing so, we get

$$\{\vec{i}_n,\vec{j}_n,\vec{k}_n\}^T = [A_{ij}]\ \{\vec{i}_s,\vec{j}_s,\vec{k}_s\}^T \tag{7}$$

where $[A_{ij}] = [a_{ij}]\ [b_{ij}]\ [c_{ij}]$

During directional maneuvers, the pitch angles of sprung masses are usually restricted to very small values, hence the transformation equations can be simplified by replacing sin θ_s by θ_s and cos θ_s by 1.0. Expanding Equation (7) and applying the small pitch angle assumption, we get:

$$\begin{Bmatrix}\vec{i}_n\\ \vec{j}_n\\ \vec{k}_n\end{Bmatrix} = \begin{bmatrix}\cos\psi_s & -\sin\psi_s\cos\phi_s+\cos\psi_s\theta_s\sin\phi_s & \sin\psi_s\sin\phi_s+\cos\psi_s\theta_s\cos\phi_s\\ \sin\psi_s & \cos\psi_s\cos\phi_s+\sin\psi_s\theta_s\sin\phi_s & -\cos\psi_s\sin\phi_s+\sin\psi_s\theta_s\cos\phi_s\\ -\theta_s & \sin\phi_s & \cos\phi_s\end{bmatrix}\begin{Bmatrix}\vec{i}_s\\ \vec{j}_s\\ \vec{k}_s\end{Bmatrix} \tag{8}$$

Also

$$\begin{Bmatrix} \vec{i}_s \\ \vec{j}_s \\ \vec{k}_s \end{Bmatrix} =$$

$$\begin{bmatrix} \cos\psi_s & \sin\psi_s & -\theta_s \\ -\sin\psi_s\cos\phi_s+\cos\psi_s\theta_s\sin\phi_s & \cos\psi_s\cos\phi_s+\sin\psi_s\theta_s\sin\phi_s & \sin\phi_s \\ \sin\psi_s\sin\phi_s+\cos\psi_s\theta_s\cos\phi_s & -\cos\psi_s\sin\phi_s+\sin\psi_s\theta_s\cos\phi_s & \cos\phi_s \end{bmatrix} \begin{Bmatrix} \vec{i}_n \\ \vec{j}_n \\ \vec{k}_n \end{Bmatrix} \tag{9}$$

Sprung Mass Angular Velocities:

The equations of motion of each sprung mass are written in terms of the body-fixed angular velocities (p_s,q_s,r_s) and their derivatives. In order to determine the Euler angles, the Euler angular velocities $(\dot{\phi}_s,\dot{\theta}_s,\dot{\psi}_s)$ have to be calculated from the body-fixed angular velocities (p_s,q_s,r_s) and then integrated numerically. The Euler angular velocities $(\dot{\phi}_s,\dot{\theta}_s,\dot{\psi}_s)$ are defined along the $(\vec{i}_s,\vec{j}_2,\vec{k}_n)$ directions. Therefore, equating the body-fixed and Euler angular velocities, we get

$$p_s\vec{i}_s + q_s\vec{j}_s + r_s\vec{k}_s = \dot{\phi}_s\vec{i}_s + \dot{\theta}_s\vec{j}_2 + \dot{\psi}_s\vec{k}_n \tag{10}$$

From Equation (5) we note that

$$\vec{j}_2 = \cos\phi_s\vec{j}_s - \sin\phi_s\vec{k}_s \tag{11}$$

Also, Equation (8) indicates that

$$\vec{k}_n = -\theta_s\vec{i}_s + \sin\phi_s\vec{j}_s + \cos\phi_s\vec{k}_s \tag{12}$$

Substituting Equations (11) and (12) back into (10) we get

$$p_s \vec{i}_s = (\dot{\phi}_s - \theta_s \dot{\psi}_s)\vec{i}_s \qquad (13)$$

$$q_s \vec{j}_s = (\dot{\theta}_s \cos \phi_s + \sin \phi_s \dot{\psi}_s)\vec{j}_s \qquad (14)$$

$$r_s \vec{k}_s = (-\dot{\theta}_s \sin \phi_s + \dot{\psi}_s \cos \phi_s)\vec{k}_s \qquad (15)$$

The above three equations can also be written for solving the Euler angular velocities in terms of the body-fixed angular velocities (p_s, q_s, r_s). In doing so, we get:

$$\dot{\phi}_s = p_s + (q_s \sin \phi_s + r_s \cos \phi_s)\theta_s \qquad (16)$$

$$\dot{\theta}_s = q_s \cos \phi_s - r_s \sin \phi_s \qquad (17)$$

$$\dot{\psi}_s = q_s \sin \phi_s + r_s \cos \phi_s \qquad (18)$$

Therefore, Equations (16)-(18) can be numerically integrated to obtain the Euler angles at any time t of the simulation.

D.1.2 Unsprung Mass Axis System. Each unsprung mass is permitted only to roll and bounce with respect to the sprung mass to which it is attached. The orientation of the unsprung mass with respect to the inertial axis system is therefore defined by the yaw angle, ψ_s, and the roll angle, ϕ_u, which are shown in Figures D.5 and D.6, respectively.

Figure D.6 indicates that

$$\begin{Bmatrix} \vec{i}_u \\ \vec{j}_u \\ \vec{k}_u \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_u & \sin \phi_u \\ 0 & -\sin \phi_u & \cos \phi_u \end{bmatrix} \begin{Bmatrix} \vec{i}_1 \\ \vec{j}_1 \\ \vec{k}_1 \end{Bmatrix} \qquad (19)$$

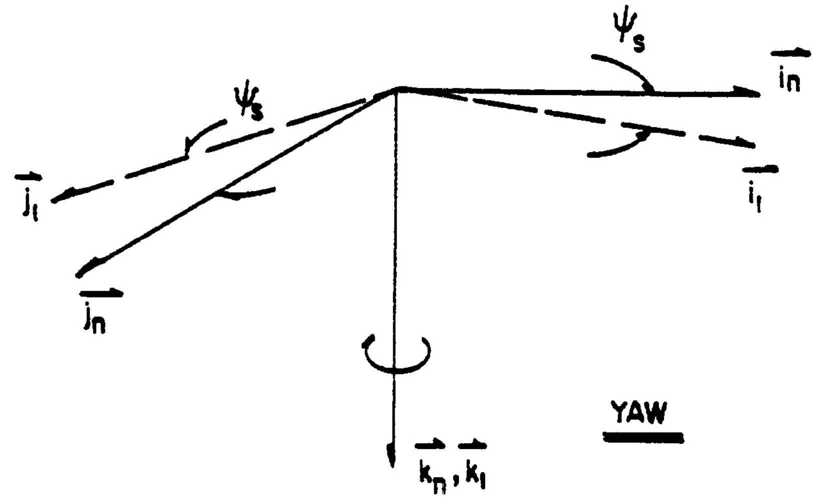


Figure D.5

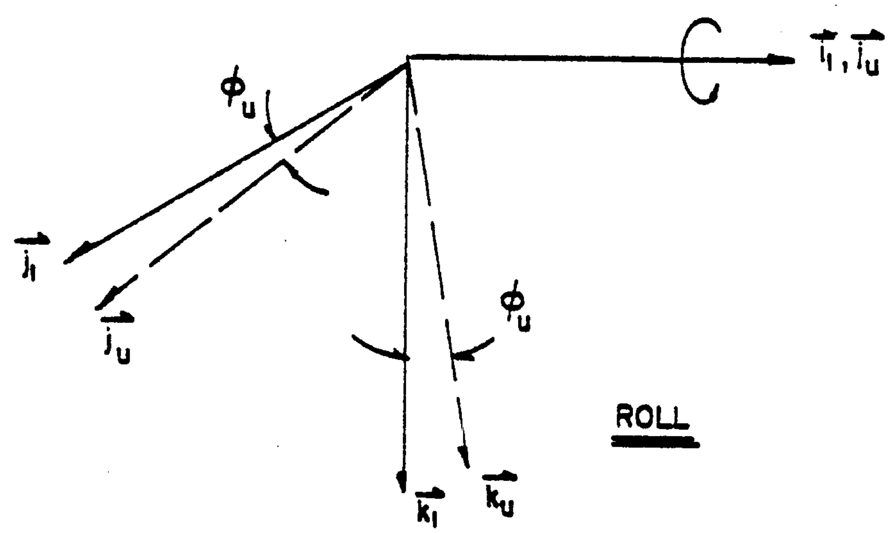


Figure D.6

Euler angles needed to define the orientation of each of the unsprung masses.

When Equations (3) and (5) are combined, we have

$$\begin{Bmatrix} \vec{i}_1 \\ \vec{j}_1 \\ \vec{k}_1 \end{Bmatrix} = [b_{ij}]\,[c_{ij}] \begin{Bmatrix} \vec{i}_s \\ \vec{j}_s \\ \vec{k}_s \end{Bmatrix} \qquad (20)$$

Therefore, combining Equations (19) and (20) and substituting for $[b_{ij}]$ and $[c_{ij}]$, we get the transformation equation which relates the sprung and unsprung mass axis systems.

$$\begin{Bmatrix} \vec{i}_u \\ \vec{j}_u \\ \vec{k}_u \end{Bmatrix} = \begin{bmatrix} 1 & \theta_s \sin\phi_s & \theta_s \cos\phi_s \\ -\theta_s \sin\phi_u & \cos(\phi_s-\phi_u) & -\sin(\phi_s-\phi_u) \\ -\theta_s \cos\phi_u & \sin(\phi_s-\phi_u) & \cos(\phi_s-\phi_u) \end{bmatrix} \begin{Bmatrix} \vec{i}_s \\ \vec{j}_s \\ \vec{k}_s \end{Bmatrix} \qquad (21)$$

D.2. Equations of Motion

With each sprung mass assumed to possess five degrees of freedom and each unsprung mass assumed to possess two degrees of freedom, the number of differential equations required to describe the directional and roll behavior of a multiple-articulated vehicle is given by

$$k = 5n + 2m$$

where n = number of sprung masses

m = number of unsprung masses

D.2.1 Equations of Motion for the Sprung Masses.

Application of Newton's laws of motion leads to five equations for each of the sprung masses possessed by the assumed vehicle system, viz.:

Lateral Force Equation:

$$m_s\dot{v}_s - m_s(p_s w_s - r_s u_s) = \Sigma\ \vec{j}_s \text{ component of constraint forces} + \Sigma\ \vec{j}_s \text{ component of the suspension forces} + m_s g \sin\phi_s \tag{22}$$

Vertical Force Equation:

$$m_s\dot{w}_s - m_s(q_s u_s - p_s v_s) = \Sigma\ \vec{k}_s \text{ component of constraint forces} + \Sigma\ \vec{k}_s \text{ component of the suspension forces} + m_s g \cos\phi_s \tag{23}$$

Rolling Moment Equation:

$$I_{xx_s}\dot{p}_s - (I_{yy_s} - I_{zz_s})q_s r_s = \Sigma \text{ roll moments from the constraints} + \Sigma \text{ roll moments from the suspensions} \tag{24}$$

Pitching Moment Equation:

$$I_{yy_s}\dot{q}_s - (I_{zz_s} - I_{xx_s})p_s r_s = \Sigma \text{ pitching moments from the constraints} + \Sigma \text{ pitching moments from the suspensions} \tag{25}$$

Yawing Moment Equation:

$$I_{zz}\dot{r}_s - (I_{xx_s} - I_{yy_s})p_s q_s = \Sigma \text{ yawing moments from the constraints} + \Sigma \text{ yawing moments from the suspensions} \tag{26}$$

Note:

In the above equation, the "constraint forces" are the forces which arise at the points of connection between adjacent sprung masses. The "suspension forces" are defined as the forces acting between an axle and the sprung mass.

D.2.2 Equations of Motion for the Unsprung Masses. Two equations can be written for the roll and bounce degrees of freedom possessed by each of the unsprung masses:

$$I_{xx_{u_i}} \dot{p}_{u_i} = \text{roll moment produced by the suspension forces + roll moment produced by the tire forces} \quad (27)$$

$$m_{u_i} \vec{a}_{m_{u_i}} \cdot \vec{k}_{u_i} = \vec{k}_{u_i} \text{ component of suspension forces} + \vec{k}_{u_i} \text{ component of the tire forces} + m_{u_i} g \cos \phi_{u_i} \quad (28)$$

In order to evaluate the right-hand side of Equations (22) through (28), the forces produced by the suspension, hitching mechanisms and tires need to be determined. The manner in which these forces are treated is outlined in the following sections.

D.3. Suspension Forces

Each suspension is assumed to consist of a pair of linear springs and linkages which establish a roll center, R_i. Figure D.7 is a schematic diagram showing that the suspension springs are assumed to remain parallel to the $\vec{k}_{u_i}$ axis of the unsprung mass, and are capable of transmitting either compressive or tensile forces only. All roll plane forces which are perpendicular to the suspension springs are assumed to act through the roll center, R_i. The roll center, R_i, is located at a fixed distance, z_{R_i}, beneath the sprung mass, and is permitted to slide along the $\vec{k}_{u_i}$ axis of the unsprung mass.

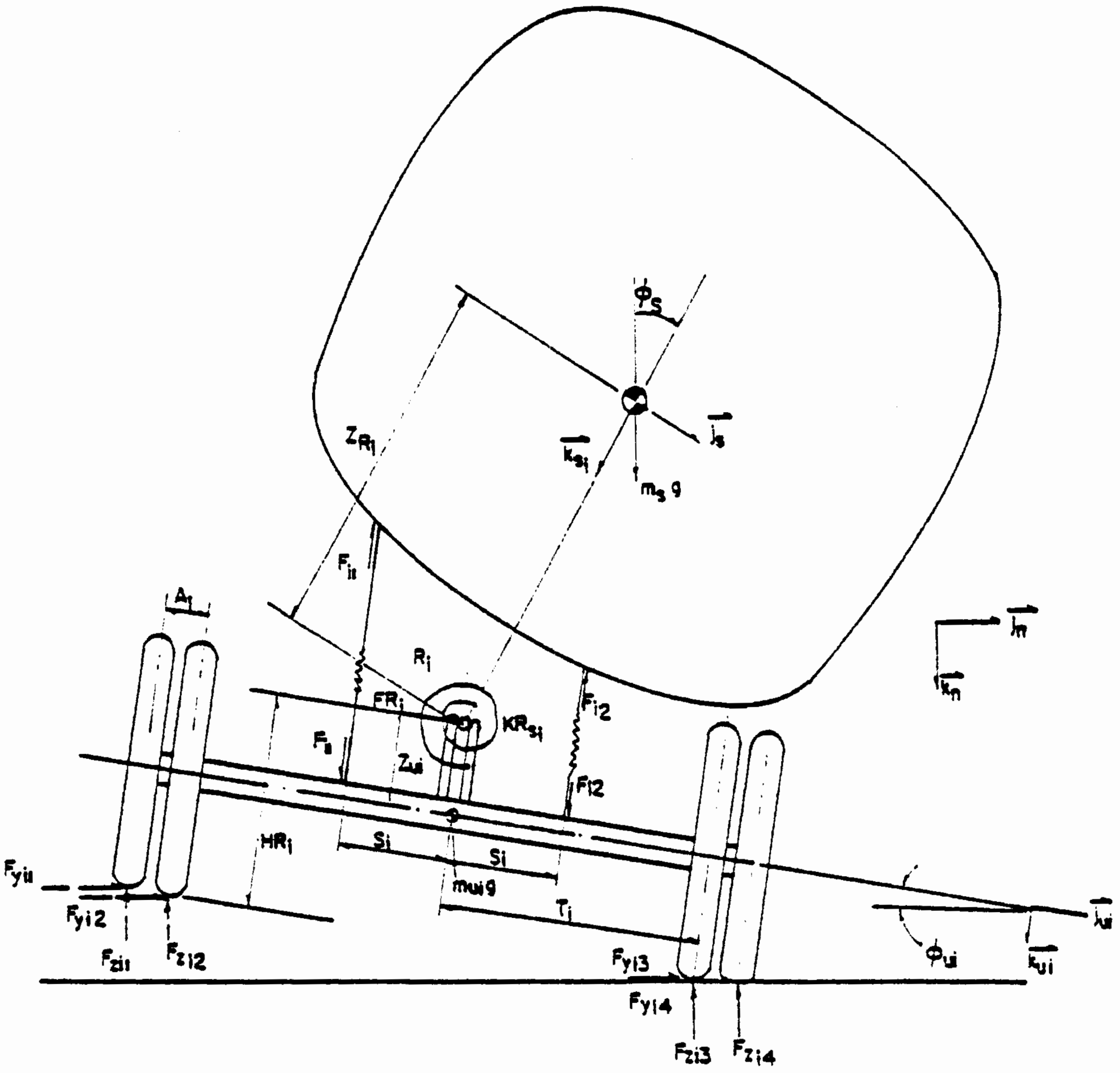


Figure D.7. Suspension and tire forces at each axle.

Figure D.7 shows that the suspension forces transmitted to the sprung mass from any given axle, i, are

$$F_{susp_i} = F_{R_i}\vec{j}_{u_i} - (F_{i1}+F_{i2})\vec{k}_{u_i} \tag{29}$$

The suspension forces can be defined in the sprung mass coordinate system by applying the coordinate transformation expressed by Equation (21). Upon applying the transformation, we get

$$\begin{aligned} F_{susp_i} = {} & [- F_{R_i}\theta_s \sin\phi_{u_i} + (F_{i1}+F_{i2})\theta_s \cos\phi_{u_i}]\vec{i}_s + [F_{R_i}\cos(\phi_s-\phi_{u_i}) \\ & - (F_{i1}+F_{i2})\sin(\phi_s-\phi_{u_i})]\vec{j}_s - [F_{R_i}\sin(\phi_s-\phi_{u_i}) \\ & + (F_{i1}+F_{i2})\cos(\phi_s-\phi_{u_i})]\vec{k}_s \end{aligned} \tag{30}$$

The compressive or tensile forces, F_{ij}, produced by the suspension springs are calculated using a suitable suspension spring model. On the other hand, the force, F_{R_i}, acting through the roll center, R_i, is an internal force which can be eliminated by inspecting the dynamic equilibrium of the axle in the $\vec{j}_{u_i}$ direction. Upon writing the equation for the lateral equilibrium of the axle, and rearranging, we get:

$$\begin{aligned} F_{R_i} = {} & - m_{u_i}[\vec{a}_{m_{u_i}} \cdot \vec{j}_{u_i}] + \sum_{j=1}^{4} F_{y_{ij}} \cos\phi_{u_i} \\ & - \sum_{j=1}^{4} F_{z_{ij}} \sin\phi_{u_i} + m_{u_i} g \sin\phi_{u_i} \end{aligned} \tag{31}$$

Of the terms in the right-hand side of (31), the only unknown is the acceleration, $\vec{a}_{m_{u_i}}$, of the unsprung mass. Since the acceleration of the unsprung mass can be defined relative to the sprung mass to which it is attached, it can be written as:

$$\vec{a}_{m_{u_i}} = \vec{a}_{m_s} + \vec{a}_{R_i/m_s} + \vec{a}_{m_{u_i}/R_i} \tag{32}$$

where $\vec{a}_{m_s}$ is the acceleration at the c.g. of the sprung mass

$\vec{a}_{R_i/m_s}$ is the relative acceleration at the roll center, R_i, with respect to the sprung mass c.g.

and $\vec{a}_{m_{u_i}/R_i}$ is the relative acceleration at the c.g. of the axle with respect to the roll center, R_i.

We shall now derive expressions for each of the three terms in the right-hand side of (32).

The acceleration of the sprung mass along the body-fixed coordinates $(\vec{i}_s, \vec{j}_s, \vec{k}_s)$ is given by:

$$\vec{a}_{m_s} = (\dot{u}_s + q_s w_s - r_s v_s)\vec{i}_s + (\dot{v}_s + u_s r_s - p_s w_s)\vec{j}_s + (\dot{w}_s + p_s v_s - q_s u_s)\vec{k}_s \tag{33}$$

Since the roll center, R_i, is at a fixed distance from the sprung mass c.g., the acceleration of R_i with respect to the sprung mass c.g. ($\vec{a}_{R_i/m_s}$) can be derived as follows:

$$\vec{r}_{R_i/m_s} = x_{R_i}\vec{i}_s + z_{R_i}\vec{k}_s \tag{34}$$

$$\vec{v}_{R_i/m_s} = \dot{\vec{r}}_{R_i/m_s} = (z_{R_i} q_s)\vec{i}_s + (-p_s z_{R_i} + x_{R_i} r_s)\vec{j}_s - x_{R_i} q_s \vec{k}_s \tag{35}$$

$$\vec{a}_{R_i/m_s} = \dot{\vec{v}}_{R_i/m_s} = [\dot{q}_s z_{R_i} - x_{R_i} q_s^2 + p_s r_s z_{R_i} - x_{R_i} r_s^2]\vec{i}_s + [-\dot{p}_s z_{R_i} + x_{R_i}\dot{r}_s + z_{R_i} q_s r_s + x_{R_i} q_s p_s]\vec{j}_s + [-p_s^2 z_{R_i} + x_{R_i} r_s p_s - z_{R_i} q_s^2 - x_{R_i}\dot{q}_s]\vec{k}_s \tag{36}$$

The third term in Equation (32), $\vec{a}_{m_{u_i}/R_i}$, can be derived along the same lines as $\vec{a}_{R_i/m_s}$, viz.:

$$\vec{r}_{m_{u_i}/R_i} = z_{u_i}\vec{k}_{u_i} \tag{37}$$

$$\vec{v}_{m_{u_i}/R_i} = \dot{\vec{r}}_{m_{u_i}/R_i} = \dot{z}_{u_i}\vec{k}_{u_i} - p_{u_i}z_{u_i}\vec{j}_{u_i} \tag{38}$$

$$\begin{aligned}\vec{a}_{m_{u_i}/R_i} = \dot{\vec{v}}_{m_{u_i}/R_i} &= \ddot{z}_{u_i}\vec{k}_{u_i} - (\dot{p}_{u_i}z_{u_i} + 2p_{u_i}\dot{z}_{u_i})\vec{j}_{u_i} \\ &- p_{u_i}^2 z_{u_i}\vec{k}_{u_i} + p_{u_i}r_{u_i}z_{u_i}\vec{i}_{u_i}\end{aligned} \tag{39}$$

Hence, combining Equations (33), (36), and (39) and transforming the acceleration defined in the sprung mass coordinate system to the unsprung mass coordinate system, we get:

$$\begin{aligned}\vec{a}_{m_{u_i}} \cdot \vec{j}_{u_i} = &-(\dot{u}_s + q_s w_s - r_s v_s + \dot{q}_s z_{R_i} - x_{R_i} q_s^2 + p_s r_s z_{R_i} \\ &- x_{R_i} r_s^2)\theta_s \sin\phi_{u_i} + (\dot{v}_s + u_s r_s - p_s w_s - \dot{p}_s z_{R_i} \\ &+ x_{R_i}\dot{r}_s + z_{R_i} q_s r_s + x_{R_i} q_s p_s)\cos(\phi_s - \phi_u) \\ &- [\dot{w}_s + p_s v_s - q_s u_s - p_s^2 z_{R_i} + x_{R_i} r_s p_s \\ &- z_{R_i} q_s^2 - x_{R_i}\dot{q}_s]\sin(\phi_s - \phi_u) - \dot{p}_{u_i} z_{u_i} - 2p_{u_i}\dot{z}_{u_i}\end{aligned} \tag{40}$$

On substituting the right-hand side of (40) for the term $(\vec{a}_{m_{u_i}} \cdot \vec{j}_{u_i})$ in Equation (31), we get the following result for F_{R_i}:

$$F_{R_i} = -m_{u_i}\{-[\dot{u}_s + q_s w_s - r_s v_s + \dot{q}_s z_{R_i} - x_{R_i} q_s^2 + p_s r_s z_{R_i}$$

$$+ x_{R_i} r_s^2]\theta_s \sin\phi_{u_i} + (\dot{v}_s + u_s r_s - p_s w_s - \dot{p}_s z_{R_i}$$

$$+ x_{R_i}\dot{r}_s + z_{R_i} q_s r_s + x_{R_i} q_s p_s]\cos(\phi_s - \phi_{u_i}) - (\dot{w}_s + p_s v_s$$

$$- q_s u_s - p_s^2 z_{R_i} + x_{R_i} r_s p_s - z_{R_i} q_s^2 - x_{R_i}\dot{q}_s]\sin(\phi_s - \phi_{u_i})$$

$$- \dot{p}_{u_i} z_{u_i} - 2p_{u_i}\dot{z}_{u_i}\} + \sum_{j=1}^{4} F_{y_{ij}} \cos\phi_{u_i} - \sum_{j=1}^{4} F_{z_{ij}} \sin\phi_{u_i}$$

$$+ m_{u_i} g \sin\phi_{u_i} \qquad (41)$$

D.4. Constraint Equations

The differential equations which describe the motion of the sprung mass (Equations (22)-(26)) contain terms which are related to the forces and moments which arise at the points of connection between the various sprung masses. These forces and moments can be determined from the kinematic equations which define the constraints. Alternatively, the constraint forces and moments could be evaluated by considering the coupling mechanisms to be compliant such that the forces/moments transmitted through the coupling becomes a function of the relative displacement (linear and angular) between the lead and trailing elements of the coupling mechanism.

Examination of the geometric configuration of the coupling units used on heavy-duty trucks and the structures to which they are attached indicates that these coupling elements are relatively rigid with respect to translation but relatively compliant with respect to rotation. Accordingly, two different schemes were adopted to represent the constraint forces/moments that appear on the right-hand side of Equations (22) through (26). Specifically, the forces were determined from kinematic expressions which state, in effect, that the acceleration at a coupling point is the same for both the lead and the trailing units of the coupling. The moments, on the other hand, were calculated as a function of the angular displacement of the lead and trailing elements of a given coupling mechanism.

Four particular coupling mechanisms were of interest. These mechanisms are diagrammed in Figure D.8. Note that the fifth wheel and the inverted fifth wheel arrangement permit the lead and the trailing units to yaw and pitch with respect to one another, but are stiff in roll. The kingpin-type connection permits only yaw motions. In the case of the pintle hook connection, the trailing unit is permitted to roll, bounce, yaw, and pitch with respect to the lead unit, the only constraint being that which requires the lateral position of the lead coupler to be the same as the lateral position of the forward end of the drawbar.

Let us consider, first, the procedure by which the unknown constraint forces can be determined on the basis of the kinematic conditions which must be satisfied. If the set of k second-order differential equations of motion corresponding to n sprung masses and m unsprung masses are written in matrix notation (recall that $k = 5n + 2m$), we obtain:

$$M \ddot{\vec{x}} = \vec{y} + N \vec{f}_c \qquad (42)$$

where

M is the inertia matrix of size $k \times k$

$\dot{\vec{x}}$ is a vector of the first derivative of the motion variables of size k:

$[(v_i, w_i, r_i, q_i, p_i)\ i=1,\ldots,n\ ,\ (p_{u_i}, z_{u_i})\ i=1,\ldots,m$

$\vec{y}$ is a vector of size k, which is a function of $\vec{x}$, $\dot{\vec{x}}$ and the dimension of the vehicle

$\vec{f}_c$ is a vector of j unknown constraint forces and moments

N is a matrix of size $k \times k$ which is a function of vehicle dimensions and $\vec{x}$.

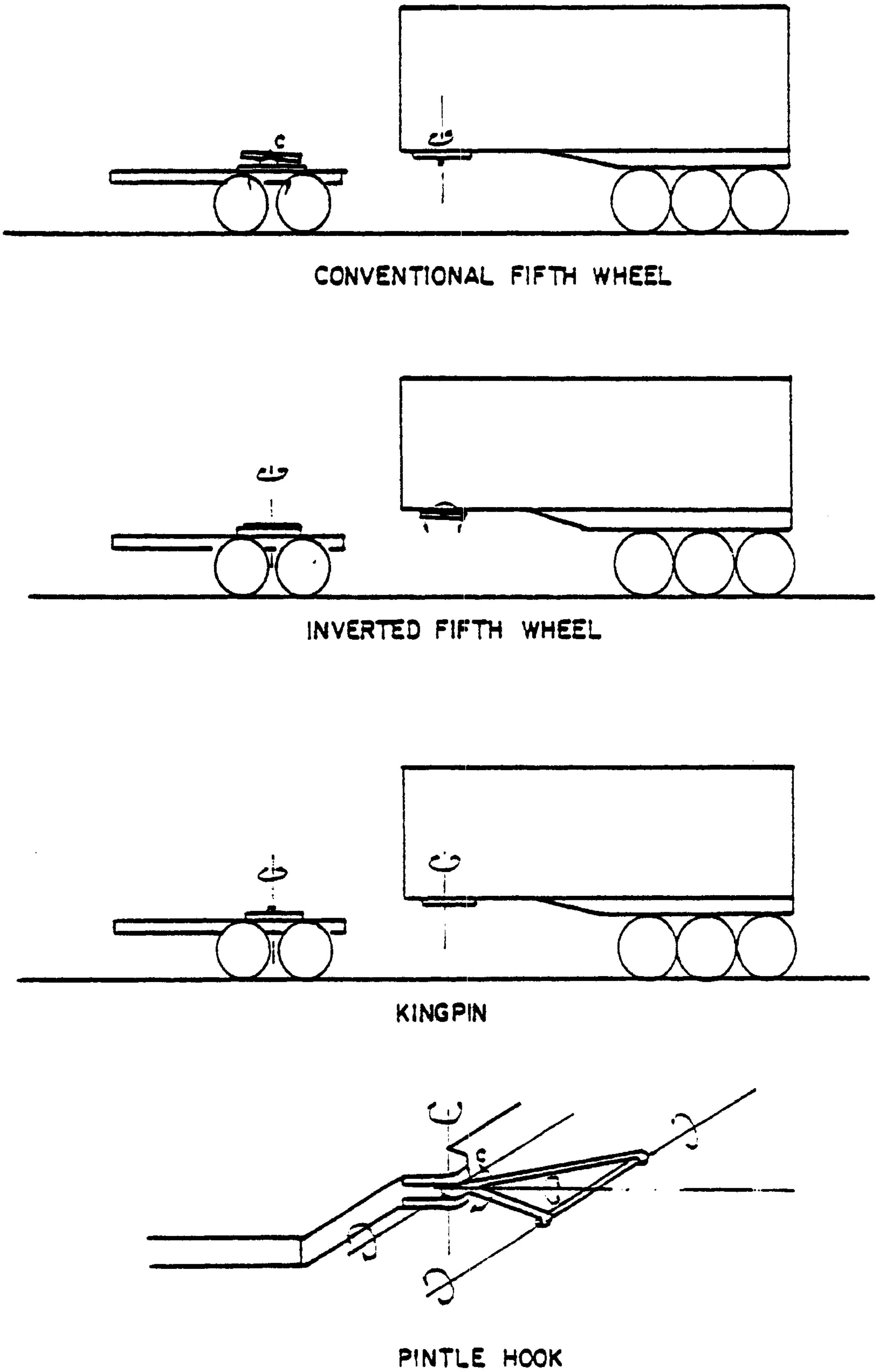


Figure D.8. Schematic diagrams of four coupling mechanisms commonly used on commercial vehicles.

The kinematic constraints that exist at the various connecting points, when written as a set of acceleration constraint equations, are of the form:

$$C \ddot{\vec{x}} = \vec{d} \qquad (43)$$

where

C is a matrix of size j×k, which is a function of the vehicle dimensions and $\vec{x}$

$\vec{d}$ is a vector of size j, which is a function of $\vec{x}$, $\dot{\vec{x}}$, and the dimensions of the vehicle.

If we solve Equation (42) for $\ddot{\vec{x}}$, we obtain

$$\ddot{\vec{x}} = M^{-1}\vec{y} + M^{-1}N \vec{f}_c \qquad (44)$$

On substituting Equation (44) in (43), we get

$$C M^{-1}\vec{y} + C M^{-1}N \vec{f}_c = \vec{d} \qquad (45)$$

which, on being solved for the constraint forces, yields:

$$\vec{f}_c = [C M^{-1}N]^{-1} \{\vec{d} - C M^{-1}\vec{y}\} \qquad (46)$$

The set of differential equations given by Equation (42) can now be solved by substituting Equation (46) back into (42). Upon doing so, we obtain:

$$\ddot{\vec{x}} = M^{-1}\vec{y} + M^{-1}N[C M^{-1}N]^{-1} \{\vec{d} - C M^{-1}\vec{y}\} \qquad (47)$$

Since all the terms in the right-hand side of (47) are known, Equation (47) can be integrated by any standard integration subroutine.

Each of the four connections considered here are single-point constraints. Specifically, there is a point C common to both the lead and the trailing units, about which articulation takes place. (See, for example, Figure D.9.) The required equations of constraint are obtained by equating the acceleration of point C on the lead unit to the acceleration of the same point on the trailing unit.

With reference to Figure D.9, the acceleration of point C on the lead unit is seen to be:

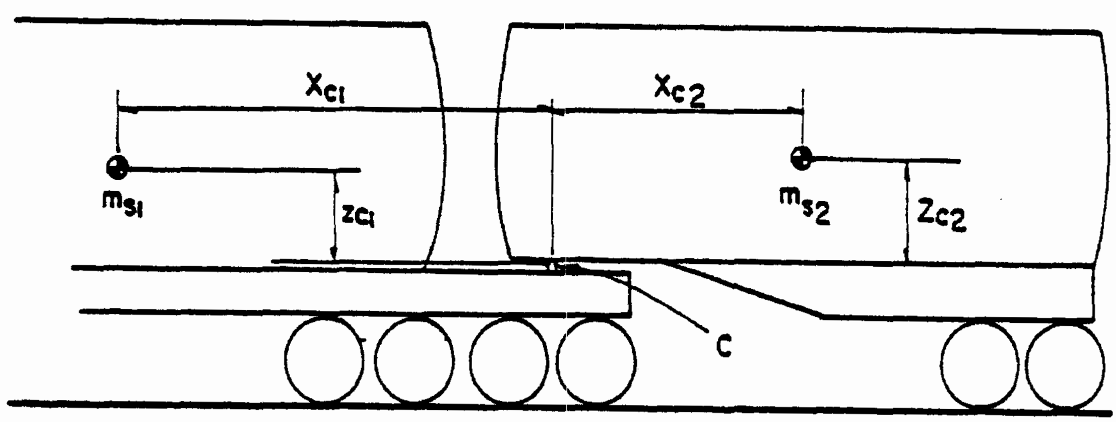


Figure D.9. A single point constraint in which the articulation takes place about point C.

$$
\begin{aligned}
\vec{a}_c = [&\dot{u}_{s_1} + q_{s_1}w_{s_1} - r_{s_1}v_{s_1} + \dot{q}_{s_1}z_{c_1} - x_{c_1}q_{s_1}^2 + p_{s_1}r_{s_1}z_{c_1} \\
&- x_{c_1}r_{s_1}^2]\vec{i}_{s_1} + [\dot{v}_{s_1} + u_{s_1}r_{s_1} - p_{s_1}w_{s_1} - \dot{p}_{s_1}z_{c_1} + x_{c_1}\dot{r}_{s_1} \\
&+ z_{c_1}q_{s_1}r_{s_1} + z_{c_1}q_{s_1}r_{s_1} + x_{c_1}q_{s_1}r_{s_1}]\vec{j}_{s_1} + [\dot{w}_{s_1} + p_{s_1}v_{s_1} \\
&- q_{s_1}u_{s_1} - x_{c_1}\dot{q}_{s_1} - p_{s_1}^2 z_{c_1} + x_{c_1}r_{s_1}p_{s_1} - z_{c_1}q_{s_1}^2]\vec{k}_{s_1} \\
&= a_1\vec{i}_{s_1} + b_1\vec{j}_{s_1} + c_1\vec{k}_{s_1}
\end{aligned}
\tag{48}
$$

The acceleration of the same point in terms of the motion variables of the trailing unit is:

$$
\begin{aligned}
\vec{a}_c = [&\dot{u}_{s_2} + q_{s_2}w_{s_2} - r_{s_2}v_{s_2} + \dot{q}_{s_2}z_{c_2} - x_{c_2}q_{s_2}^2 + p_{s_2}r_{s_2}z_{c_2} \\
&- x_{c_2}r_{s_2}^2]\vec{i}_{s_2} + [\dot{v}_{s_2} + u_{s_2}r_{s_2} - p_{s_2}w_{s_2} - \dot{p}_{s_2}z_{c_2} + x_{c_2}\dot{r}_{s_2} \\
&+ z_{c_2}q_{s_2}r_{s_2} + x_{c_2}q_{s_2}p_{s_2}]\vec{j}_{s_2} + [\dot{w}_{s_2} + p_{s_2}v_{s_2} - q_{s_2}u_{s_2} \\
&- x_{c_2}\dot{q}_{s_2} - p_{s_2}^2 z_{c_2} + x_{c_2}r_{s_2}p_{s_2} - z_{c_2}q_{s_2}^2]\vec{k}_{s_2} \\
&= a_2\vec{i}_{s_2} + b_2\vec{j}_{s_2} + c_2\vec{k}_{s_2}
\end{aligned}
\tag{49}
$$

Equations (48) and (49) can be equated to each other after transforming the coordinate system of the lead unit to that of the trailing unit, or vice versa.

Referring to Equation (7), we note that:

$$
\begin{Bmatrix} \vec{i}_n \\ \vec{j}_n \\ \vec{k}_n \end{Bmatrix} = [A_{ij}]_1 \{\vec{i}_{s_1}, \vec{j}_{s_1}, \vec{k}_{s_1}\}^T \tag{50}
$$

But

$$\begin{Bmatrix} \vec{i}_{s_2} \\ \vec{j}_{s_2} \\ \vec{k}_{s_2} \end{Bmatrix} = [A_{ij}]_2^T \{\vec{i}_n, \vec{j}_n, \vec{k}_n\}^T \tag{51}$$

Upon combining Equations (50) and (51), we get:

$$\begin{Bmatrix} \vec{i}_{s_2} \\ \vec{j}_{s_2} \\ \vec{k}_{s_2} \end{Bmatrix} = [A_{ij}]_2^T [A_{ij}]_1 \{\vec{i}_{s_1}, \vec{j}_{s_1}, \vec{k}_{s_1}\}^T = [T_{ij}]\{\vec{i}_{s_1}, \vec{j}_{s_1}, \vec{k}_{s_1}\}^T \tag{52}$$

where:

$$T_{11} = \cos(\psi_{s_2} - \psi_{s_1})$$

$$T_{12} = \sin(\psi_{s_2} - \psi_{s_1})\cos\phi_{s_1} - \theta_{s_2}\sin\phi_{s_1} + \sin\phi_{s_1}\theta_{s_1}\cos(\psi_{s_2} - \psi_{s_1})$$

$$T_{13} = -\sin(\psi_{s_2} - \psi_{s_1})\sin\phi_{s_1} - \theta_{s_2}\cos\phi_{s_1} + \cos\phi_{s_1}\theta_{s_1}\cos(\psi_{s_2} - \psi_{s_1})$$

$$T_{21} = -\cos\phi_{s_2}\sin(\psi_{s_2} - \psi_{s_1}) - \theta_{s_1}\sin\phi_{s_2} + \sin\phi_{s_2}\theta_{s_2}\cos(\psi_{s_2} - \psi_{s_1})$$

$$\begin{aligned} T_{22} = {} & \cos\phi_{s_1}\cos\phi_{s_2}\cos(\psi_{s_2} - \psi_{s_1}) + \sin\phi_{s_1}\sin\phi_{s_2} \\ & - \sin\phi_{s_1}\theta_{s_1}\cos\phi_{s_2}\sin(\psi_{s_2} - \psi_{s_1}) \\ & + \sin\phi_{s_2}\theta_{s_2}\cos\phi_{s_1}\sin(\psi_{s_2} - \psi_{s_1}) \end{aligned}$$

$$\begin{aligned} T_{23} = {} & -\sin\phi_{s_1}\cos\phi_{s_2}\cos(\psi_{s_2} - \psi_{s_1}) + \cos\phi_{s_1}\sin\phi_{s_2} \\ & - \cos\phi_{s_1}\cos\phi_{s_2}\theta_{s_1}\sin(\psi_{s_2} - \psi_{s_1}) \\ & - \sin\phi_{s_1}\sin\phi_{s_2}\theta_{s_2}\sin(\psi_{s_2} - \psi_{s_1}) \end{aligned}$$

$$T_{31} = \sin\phi_{s_2}\sin(\psi_{s_2} - \psi_{s_1}) - \cos\phi_{s_2}\theta_{s_1} + \cos\phi_{s_2}\theta_{s_2}\cos(\psi_{s_2} - \psi_{s_1})$$

$$T_{32} = -\cos\phi_{s_1}\sin\phi_{s_2}\cos(\psi_{s_2}-\psi_{s_1}) + \cos\phi_{s_2}\sin\phi_{s_1}$$

$$+ \sin\phi_{s_1}\sin\phi_{s_2}\theta_{s_1}\sin(\psi_{s_2}-\psi_{s_1})$$

$$+ \cos\phi_{s_1}\cos\phi_{s_2}\theta_{s_2}\sin(\psi_{s_2}-\psi_{s_1})$$

$$T_{33} = \sin\phi_{s_1}\sin\phi_{s_2}\cos(\psi_{s_2}-\psi_{s_1}) + \cos\phi_{s_1}\cos\phi_{s_2}$$

$$+ \cos\phi_{s_1}\sin\phi_{s_2}\theta_{s_1}\sin(\psi_{s_2}-\psi_{s_1})$$

$$- \sin\phi_{s_1}\cos\phi_{s_2}\theta_{s_2}\sin(\psi_{s_2}-\psi_{s_1}) \qquad (53)$$

After the transformation is applied to Equation (51), one obtains the following constraint equations:

$$b_2\vec{j}_{s_2} = (a_1T_{21} + b_1T_{22} + c_1T_{23})\vec{j}_{s_2} \qquad (54)$$

$$c_2\vec{k}_{s_2} = (a_1T_{31} + b_1T_{32} + c_1T_{33})\vec{k}_{s_2} \qquad (55)$$

Note that Equations (54) and (55) are equivalent to Equation (43) above, since the quantities, a_1, b_1, and c_1, etc., are accelerations. Equations (54) and (55) serve to determine the lateral and vertical forces acting at the coupling point C. In the case of the pintle hook connection, only Equation (54) is required since a constraint force cannot exist in the vertical direction.

D.4.1 Roll and Pitch Moments for a Conventional Fifth Wheel Connection. Figure D.10 presents both the side and rear views of a conventional fifth wheel arrangement. It is observed that the conventional fifth wheel arrangement permits free rotational motions of the trailing unit along the pitch axis, $\vec{j}_{s_1}$, of the lead unit, and along the yaw axis, $\vec{k}_{s_2}$, of the trailing unit. When the two units are in line, the pitch axis, $\vec{j}_{s_2}$, of the trailing unit coincides with the $\vec{j}_{s_1}$ axis. Therefore, when the relative yaw angle is zero,

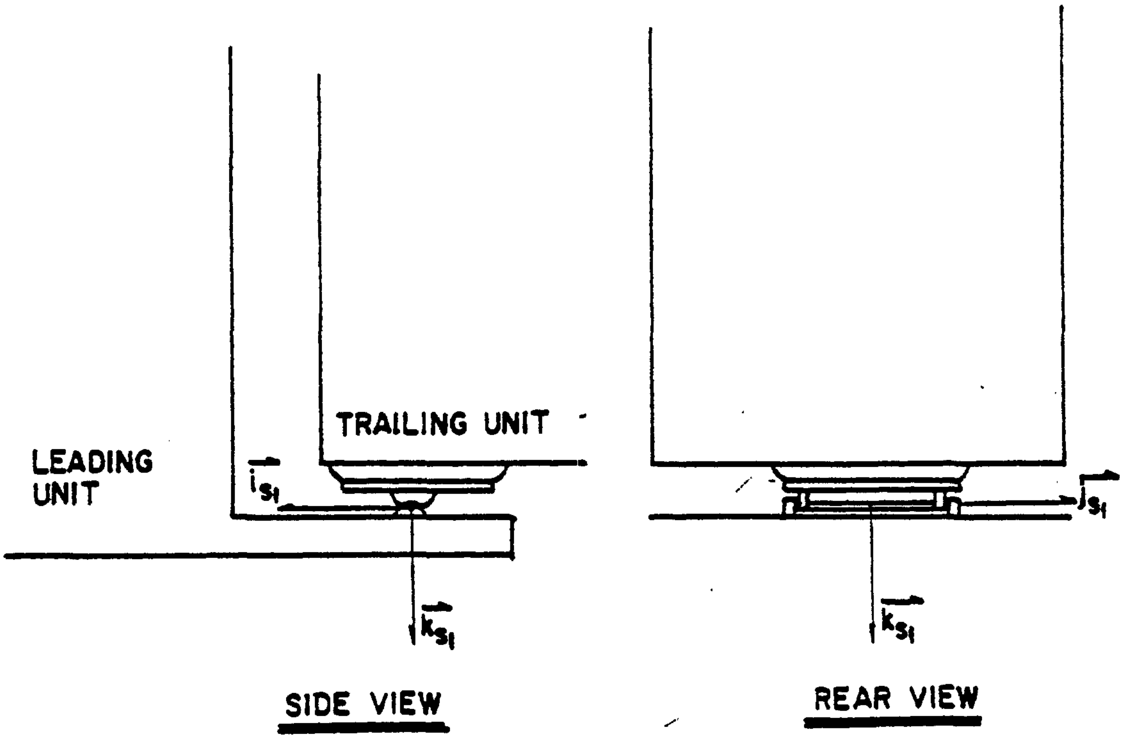


Figure D ..10. Conventional fifth wheel arrangement.

the trailing unit is free to pitch with respect to the lead unit. When the relative yaw angle between the two units reaches 90 degrees, the roll axis, $\vec{i}_{s_2}$, of the trailing unit coincides with the pitch axis, $\vec{j}_{s_1}$, making the trailing unit free to roll with respect to the lead unit.

It is assumed that frictional couples which exist along the $\vec{j}_{s_1}$ and $\vec{k}_{s_2}$ directions are sufficiently small that they can be neglected. Thus, the only constraining moment that can act on the lead unit is a roll moment along the $\vec{i}_{s_1}$ direction. The roll compliance which exists both in the tractor and trailer structures and in the coupling device is lumped to constitute the torsional stiffness, K_1, shown in Figure D.11. A second set of axes ($\vec{i}'_{s1}$, $\vec{j}'_{s1}$, $\vec{k}'_{s1}$) affixed to the fifth wheel are also defined in Figure C.11. This axis system has the same yaw and pitch angles as those of the lead unit, but has a different roll angle, ϕ'_{s1}. The

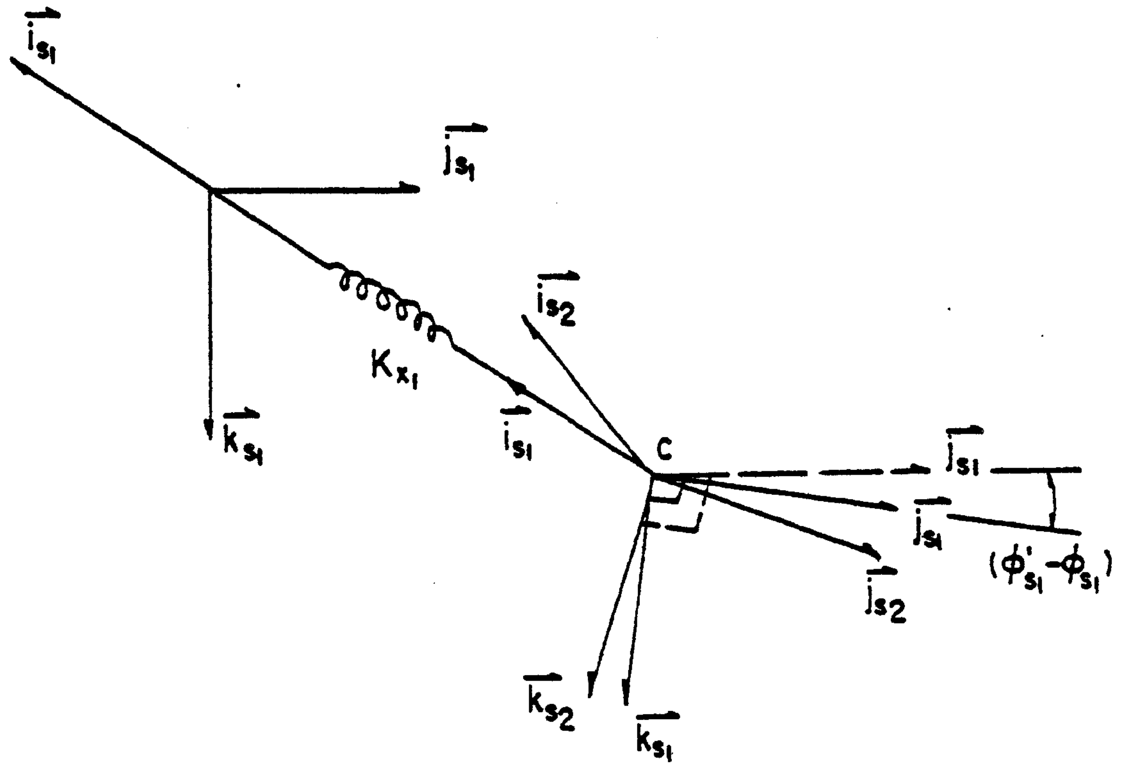


Figure D.11. Representation of the conventional fifth wheel arrangement in the yaw/roll model.

difference in the roll angle ($\phi'_{s1}-\phi_{s1}$) represents the structural compliance. The roll moment acting through the fifth wheel can therefore be expressed as:

$$M_{x_1} = K_1(\phi'_{s_1}-\phi_{s_1}) \tag{56}$$

The construction of the fifth wheel is such that the pitch axis, $\vec{j}'_{s1}$, is always perpendicular to the yaw axis, $\vec{k}_{s2}$. In terms of unit vectors, this condition can be written as:

$$\vec{j}'_{s_1} \cdot \vec{k}_{s_2} = 0 \tag{57}$$

Both $\vec{j}'_{s1}$ and $\vec{k}_{s2}$ can be expressed in terms of the inertial unit vector ($\vec{i}_n$, $\vec{j}_n$, $\vec{k}_n$) using the transform equation (9). Upon doing so, carrying out the dot product and solving for ϕ'_{s1}, we find that Equation (56) can be expressed as

$$M_{x_1} = K_{x_1}\left\{\tan^{-1}\left[\frac{\sin\phi_{s_2}\cos(\psi_{s_2}-\psi_{s_1})-\theta_{s_2}\cos\phi_{s_2}\sin(\psi_{s_2}-\psi_{s_1})}{\theta_{s_1}\sin\phi_{s_2}\sin(\psi_{s_2}-\psi_{s_1})+\cos\phi_{s_2}}\right]-\phi_{s_1}\right\} \tag{58}$$

The constraining moments acting on the trailing unit are

$$M_{x_2} = -M_{x_1}T_{11} \tag{59}$$

and

$$M_{y_2} = -M_{x_1}T_{21} \tag{60}$$

where T_{11} and T_{21} are defined in Equation (53).

D.4.2 Roll and Pitch Moments for an Inverted Fifth Wheel Arrangement. The inverted fifth wheel is an arrangement in which the lower and upper halves of a conventional fifth wheel coupling are reversed. The inverted fifth wheel arrangement is shown in Figure D.12.

The coupler permits free rotational motion of the trailing unit along the pitch axis, $\vec{j}_{s_2}$, of the trailing unit and the yaw axis, $\vec{k}_{s_1}$, of the lead unit. Unlike the conventional fifth wheel arrangement, the pitch axis of the inverted coupler is always lined up with the pitch axis of the trailer for all values of articulation angles. The inverted fifth wheel coupling can therefore transmit a roll-resisting moment from the lead unit to the trailing unit for all values of the relative yaw angle between the lead and the trailing

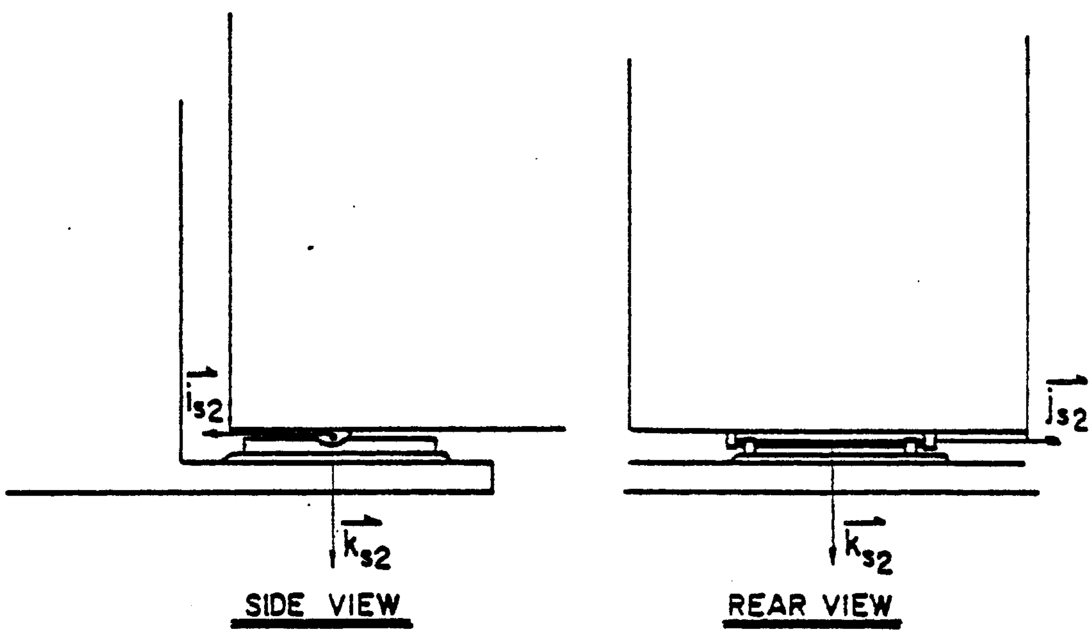


Figure D.12. The inverted fifth wheel arrangement.

units. In the case of the inverted fifth wheel, the structural compliance in roll is modeled by a torsional spring of stiffness K_{x_1}, oriented along the $\vec{i}_{s_2}$ axis of the trailing unit. Upon carrying out the derivation, we get:

$$M_{x_2} = K_{x_1}\left\{\tan^{-1}\left[\frac{\sin\phi_{s_1}\cos(\psi_{s_1}-\psi_{s_2})-\theta_{s_1}\cos\phi_{s_1}\sin(\psi_{s_1}-\psi_{s_2})}{\theta_{s_2}\sin\phi_{s_1}\sin(\psi_{s_1}-\psi_{s_2})+\cos\phi_{s_1}}\right]-\phi_{s_2}\right\} \tag{61}$$

The roll and pitch moment acting on the lead unit are given by

$$M_{x_1} = -M_{x_2}T_{11} \tag{62}$$

$$M_{y_1} = -M_{x_2}T_{12} \tag{63}$$

where T_{11} and T_{12} are once again defined in Equation (53).

D.4.3 Roll and Pitch Moments for a Kingpin-Type Connection. In a kingpin-type arrangement, only yaw motion is permitted between the lead and the trailing units. Hence, constraining moments act in both the pitch and yaw directions. The structural compliance is therefore represented by torsional springs, K_{x_1} and K_{y_1}, along the pitch and roll axes. Shown in Figure D.13 is an axis system ($\vec{i}'_{s_1}$, $\vec{j}'_{s_1}$, $\vec{k}'_{s_1}$) which has the same yaw angle, ψ_{s_1}, as the lead unit, but different roll and pitch angles, ϕ'_{s_1} and θ'_{s_1}, respectively. The axis system is so oriented that the k'_{s_1} axis is parallel to the $\vec{k}_{s_2}$ axis of the trailing unit. Therefore, the vector equations

$$\vec{i}'_{s_1} \cdot \vec{k}_{s_2} = 0 \tag{64}$$

and

$$\vec{j}'_{s_1} \cdot \vec{k}_{s_2} = 0 \tag{65}$$

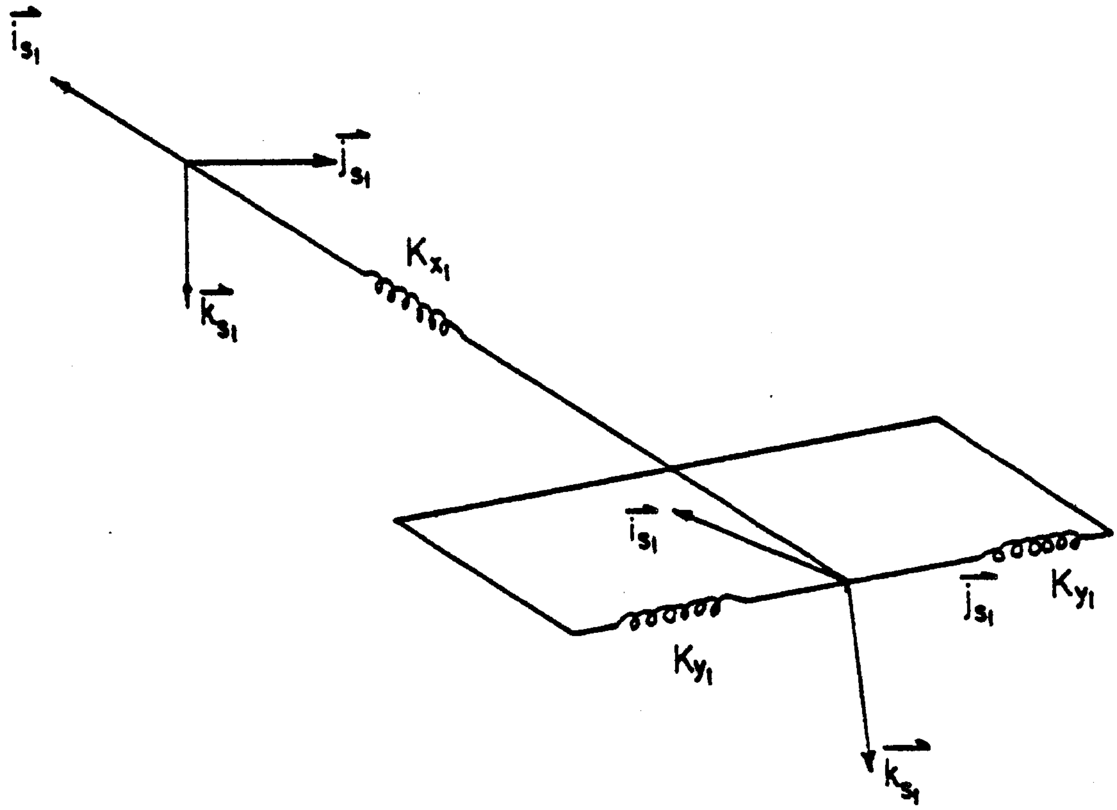


Figure D.13. Representation of the kingpin-type connection in the yaw/roll model.

have to be satisfied. Equation (64) yields the pitch angle

$$\theta'_{s_1} = \theta_{s_2}\cos(\psi_{s_2}-\psi_{s_1}) + \tan\phi_{s_2}\sin(\psi_{s_2}-\psi_{s_1}) \qquad (66)$$

Therefore

$$M_{y_1} = K_{y_1}(\theta'_{s_1}-\theta_{s_1})$$
$$= K_{y_1}[\theta_{s_2}\cos(\psi_{s_2}-\psi_{s_1}) + \tan\phi_{s_2}\sin(\psi_{s_2}-\psi_{s_1}) - \theta_{s_1}] \qquad (67)$$

Equation (65) yields a result which is identical to (58), therefore

$$M_{x_1} = K_{x_1}\left[\tan^{-1}\left(\frac{\sin\phi_{s_2}\cos(\psi_{s_2}-\psi_{s_1})-\theta_{s_2}\cos\phi_{s_2}\sin(\psi_{s_2}-\psi_{s_1})}{\theta'_{s_1}\sin\phi_{s_2}\sin(\psi_{s_2}-\psi_{s_1}) + \cos\phi_{s_2}}\right) - \phi_{s_1}\right] \quad (68)$$

The constraining moments, M_{x_2} and M_{y_2}, which act on the trailing unit are now given by

$$M_{x_2} = - M_{x_1}T_{11} - M_{y_1}T_{12} \quad (69)$$

and

$$M_{y_2} = - M_{x_1}T_{21} - M_{y_1}T_{22} \quad (70)$$

where T_{11}, T_{12}, T_{21}, and T_{22} are once again defined in Equation (53).

D.5 Forces and Moments at the Tire-Road Interface

In order to obtain a high degree of accuracy in predicting roll/yaw behavior, the computer code utilizes measured tire data for computing the lateral forces and aligning moments generated at the tire-road interface. If the sideslip angle and the vertical load acting on a tire are known, the lateral force and aligning moment can be computed by a linear interpolation of the tabulated tire data. Expressions for the sideslip angle and the vertical load at the tire-road interface will now be derived in terms of the velocities and displacements of the sprung and unsprung masses.

D.5.1 Sideslip Angles. Let us first express the sideslip angle at the tire-road interface in terms of the body-fixed velocities of the sprung mass and the axle. The sideslip angle at the j-th tire on axle i is given by the expression:

$$\alpha_{ij} = \tan^{-1}(v_{axle_i}/u_{tire_{ij}}) - \delta_i \tag{71}$$

where

$$v_{axle_i} = [v_s - z_{R_i}p_s]\cos\phi_s + x_{u_i}r_s/\cos\phi_s - p_{u_i}HR_i\cos\phi_{u_i} \tag{72}$$

$$u_{tire_{i1}} = u_s + (T_i + A_i)r_s \tag{73}$$

$$u_{tire_{i2}} = u_s + T_i r_s \tag{74}$$

$$u_{tire_{i3}} = u_s - T_i r_s \tag{75}$$

$$u_{tire_{i4}} = u_s - (T_i + A_i)r_s \tag{76}$$

(Note that the term δ_i in Equation (71) represents the angle made by the wheel plane with respect to the longitudinal axis of the sprung mass coordinate system.)

D.5.2 Vertical Loads. The vertical compliance in the tires is modeled by linear springs, KT_{ij}. Therefore, if the vertical deflection, Δ_{ij}, at the tire is known, the vertical tire load, $F_{z_{ij}}$, can be calculated from the expression:

$$F_{z_{ij}} = KT_{ij}\Delta_{ij} \tag{77}$$

The vertical deflection of the tires can be expressed in terms of the deflection of the sprung and unsprung masses. The deflection of the outer left tire on axle i is given by the equation:

$$\Delta_{i1} = \Delta_{i0} + \Delta z_s - z_{R_i}(1 - \cos\phi_s) + z_{u_i}\cos\phi_{u_i} - z_{u_{i0}} - (T_i + A_i)\sin\phi_{u_i} - x_{u_i}\theta_s \tag{78}$$

where

Δz_s is the vertical deflection of the sprung mass c.g. along the inertial axis $\vec{k}_n$

$$\Delta z_s = 0.0 \text{ at time } t = 0.0$$

$z_{u_{i0}}$ is the vertical distance between the roll center, R_i, and the axle c.g. at time $t = 0.0$

Δ_{i0} is the static deflection of the tires at time $t = 0.0$.

The deflection of the other three tires on axle i is given by:

$$\Delta_{i2} = \Delta_{i1} + A_i \sin \phi_{u_i} \tag{79}$$

$$\Delta_{i3} = \Delta_{i2} + 2T_i \sin \phi_{u_i} \tag{80}$$

$$\Delta_{i4} = \Delta_{i3} + A_i \sin \phi_{u_i} \tag{81}$$

Equations (78) through (81) yield the vertical load on a given tire which, in combination with Equation (71), permits one to perform a linear interpolation of tabulated tire data to determine the value of lateral force and aligning moment corresponding to the prevailing load and slip angle.

APPENDIX E

VEHICLE PARAMETER SETS

```
*********************************
*DIRECTIONAL RESPONSE SIMULATION*
*********************************

CEMENT MIXER 4 AXLES

# OF SPRUNG MASSES        =  1

TOTAL # OF AXLES          =  4

GROSS VEHICLE WEIGHT      =   68000.00  LB.

FORWARD VELOCITY          =      55.00  M.P.H

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                       =      25.00

STEERING STIFFNESS (IN.LB/DEG)            =   25000.00

TIE ROD STIFFNESS (IN.LB/DEG)            =   25000.00

MECHANICAL TRAIL (IN)                    =       1.00

# OF POINTS IN STEER TABLE                = 3

      TIME         STEERING WHEEL
      SEC              DEGREES
       0.0           0.0
       1.00         20.00
       6.00         20.00
```

CEMENT MIXER 4 AXLES

UNIT # 1
****

OF AXLES ON THIS UNIT = 4

WEIGHT OF SPRUNG MASS = 60400.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 1250000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 1250000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 70.00 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE # 4	AXLE #			
	*********	*********	*********	*********	*********	*********	*********	*********
LOAD ON EACH AXLE (LB.)	18000.00	19000.00	19000.00	12000.00				
AXLE WEIGHT (LB.)	1500.00	2300.00	2300.00	1500.00				
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	3700.00	4500.00	4500.00				
X DIST FROM SP MASS CG (IN)	205.31	-15.24	-68.20	-123.00				
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	21.00	20.00	20.00	20.00				
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.50	29.00	29.00	29.00				
HALF SPRING SPACING (IN)	16.00	19.00	19.00	19.00				
HALF TRACK - INNER TIRES (IN)	37.00	37.00	37.00	36.00				
DUAL TIRE SPACING (IN)	0.0	0.0	0.0	0.0				
STIFFNESS OF EACH TIRE (LB/IN)	8000.00	8000.00	8000.00	5000.00				
ROLL STEER COEFFICIENT	0.0	0.0	0.0	0.0				
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0	0.0				
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	750.00	750.00	750.00				
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	0.0				
SPRING TABLE #	1	2	1	2				
CORNERING FORCE TABLE #	1	1	1	1				
ALIGNING TORQUE TABLE #	1	1	1	1				

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-5.00
10000.00	5.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-10.00
0.0	0.0
10000.00	10.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-7500.00	-1.75
0.0	-0.75
0.0	0.0
[illegible]	[illegible]

CORNERING FORCE TABLE # 1
********* ***** *********

LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	780.00	1440.00	2040.00	2580.00	3600.00
8000.00	960.00	1840.00	2560.00	3280.00	4640.00
10000.00	1200.00	2200.00	3100.00	3900.00	5500.00

ALIGNING TORQUE TABLE # 1
******** ****** *********

ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	1944.00	3500.00	5100.00	6444.00	9000.00
8000.00	2400.00	4596.00	6396.00	8196.00	11604.00
10000.00	3000.00	5496.00	7752.00	9756.00	13752.00

```
                    ********************************
                    *DIRECTIONAL RESPONSE SIMULATION*
                    ********************************

5-AXLE-CEMENT-MIXER

# OF SPRUNG MASSES          =  1

TOTAL # OF AXLES            =  5

GROSS VEHICLE WEIGHT        =   69600.00  LB.

FORWARD VELOCITY            =      55.00  M.P.H

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                       =      25.00

STEERING STIFFNESS (IN.LB/DEG)            =   25000.00

TIE ROD STIFFNESS (IN.LB/DEG)             =   25000.00

MECHANICAL TRAIL (IN)                     =       1.00

# OF POINTS IN STEER TABLE                = 3

     TIME          STEERING WHEEL
     SEC               DEGREES
      0.0            0.0
      1.00          25.00
     20.00         225.00
```

5-AXLE-CEMENT-MIXER

UNIT # 1

OF AXLES ON THIS UNIT = 5

WEIGHT OF SPRUNG MASS = 61000.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 71.70 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE # 4	AXLE # 5	AXLE #		
LOAD ON EACH AXLE (LB.)	18000.00	9600.00	18000.00	18000.00	6000.00			
AXLE WEIGHT (LB.)	1500.00	1500.00	2300.00	2300.00	1000.00			
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	3700.00	4500.00	4500.00	3000.00			
X DIST FROM SP MASS CG (IN)	140.90	14.90	-30.10	-80.10	-143.10			
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	21.00	20.00	20.00	20.00	15.00			
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	22.00	29.00	29.00	17.00			
HALF SPRING SPACING (IN)	16.00	19.00	19.00	19.00	19.00			
HALF TRACK - INNER TIRES (IN)	49.50	42.00	29.00	29.00	36.00			
DUAL TIRE SPACING (IN)	0.0	0.0	13.00	13.00	0.0			
STIFFNESS OF EACH TIRE (LB/IN)	8000.00	5000.00	5000.00	5000.00	5000.00			
ROLL STEER COEFFICIENT	0.0	0.0	0.0	0.0	0.0			
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	100000.00	0.0	0.0	10000.00			
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	250.00	750.00	750.00	250.00			
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	0.0	0.0			
SPRING TABLE #	1	2	3	3	2			
CORNERING FORCE TABLE #	1	2	2	2	2			
ALIGNING TORQUE TABLE #	1	2	2	2	2			

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-5.00
10000.00	5.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-10.00
0.0	0.0
10000.00	10.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-7500.00	-1.75
0.0	-0.75
0.0	0.0
7500.00	1.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	780.00	1440.00	2040.00	2580.00	3600.00
8000.00	960.00	1840.00	2560.00	3280.00	4640.00
10000.00	1200.00	2200.00	3100.00	3900.00	5500.00

CORNERING FORCE TABLE # 2
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
8000.00	720.00	1360.00	1920.00	2400.00	3360.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	1944.00	3600.00	5100.00	6444.00	9000.00
8000.00	2400.00	4596.00	6396.00	8196.00	11604.00
10000.00	3000.00	5496.00	7752.00	9756.00	13752.00

ALIGNING TORQUE TABLE # 2
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	984.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
8000.00	864.00	1632.00	2304.00	2880.00	4032.00

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********************************
*DIRECTIONAL RESPONSE SIMULATION*
********************************

CEMENT MIXER 4 AXLES- TAG STEERING

# OF SPRUNG MASSES        =  1

TOTAL # OF AXLES          =  4

GROSS VEHICLE WEIGHT      =   68000.00  LB.

FORWARD VELOCITY          =      55.00  M.P.H


OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                    =      25.00

STEERING STIFFNESS (IN.LB/DEG)         =   25000.00

TIE ROD STIFFNESS (IN.LB/DEG)          =   25000.00

MECHANICAL TRAIL (IN)                  =       1.00

# OF POINTS IN STEER TABLE             = 3

     TIME         STEERING WHEEL
     SEC              DEGREES
      0.0           0.0
      1.00         20.00
      6.00         20.00
```

CEMENT MIXER 4 AXLES- TAG STEERING

UNIT # 1
****

OF AXLES ON THIS UNIT = 4

WEIGHT OF SPRUNG MASS = 60400.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 1250000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 1250000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 70.90 INCHES

	AXLE # 1 *********	AXLE # 2 *********	AXLE # 3 *********	AXLE # 4 *********	AXLE # *********	*********	*********	*********
LOAD ON EACH AXLE (LB.)	18000.00	19000.00	19000.00	12000.00				
AXLE WEIGHT (LB.)	1500.00	2300.00	2300.00	1500.00				
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	3700.00	4500.00	4500.00				
X DIST FROM SP MASS CG (IN)	205.31	-15.24	-68.28	-192.72				
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	21.00	20.00	20.00	20.00				
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	29.00	29.00	29.00				
HALF SPRING SPACING (IN)	16.00	19.00	19.00	19.00				
HALF TRACK - INNER TIRES (IN)	37.00	37.00	37.00	36.00				
DUAL TIRE SPACING (IN)	0.0	0.0	0.0	0.0				
STIFFNESS OF EACH TIRE (LB/IN)	8000.00	8000.00	8000.00	5000.00				
ROLL STEER COEFFICIENT	0.0	0.0	0.0	0.0				
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0	0.0				
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	750.00	750.00	750.00				
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	0.0				
SPRING TABLE #	1	3	3	2				
CORNERING FORCE TABLE #	1	1	1	2				
ALIGNING TORQUE TABLE #	1	1	1	2				

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-5.00
10000.00	5.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-10.00
0.0	0.0
10000.00	10.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-7500.00	-1.75
0.0	-0.75
0.0	0.0
7500.00	1.00

CORNERING FORCE TABLE # 1
********* ***** *********

LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	780.00	1440.00	2040.00	2580.00	3600.00
8000.00	960.00	1840.00	2560.00	3280.00	4640.00
10000.00	1200.00	2200.00	3100.00	3900.00	5500.00

CORNERING FORCE TABLE # 2
********* ***** *********

LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	0.0	0.0	0.0	0.0	0.0
4000.00	0.0	0.0	0.0	0.0	0.0
6000.00	0.0	0.0	0.0	0.0	0.0

ALIGNING TORQUE TABLE # 2
******** ****** *********

ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	0.0	0.0	0.0	0.0	0.0
4000.00	0.0	0.0	0.0	0.0	0.0
6000.00	0.0	0.0	0.0	0.0	0.0

ALIGNING TORQUE TABLE # 1
******** ****** *********

ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	1944.00	3600.00	5100.00	6444.00	9000.00
8000.00	2400.00	4596.00	6396.00	8196.00	11604.00
10000.00	3000.00	5496.00	7752.00	9756.00	13752.00

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********************************
*DIRECTIONAL RESPONSE SIMULATION*
********************************

3-AXLE EMPTY DUMP TRUCK

# OF SPRUNG MASSES       =  1

TOTAL # OF AXLES         =  3

GROSS VEHICLE WEIGHT     =   27000.00  LB.

FORWARD VELOCITY         =      55.00  M.P.H

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                      =      25.00

STEERING STIFFNESS  (IN.LB/DEG)          =   25000.00

TIE ROD STIFFNESS (IN.LB/DEG)          =  25000.00

MECHANICAL TRAIL  (IN)                 =       1.00

# OF POINTS IN STEER TABLE               = 3

     TIME         STEERING WHEEL
     SEC              DEGREES
      0.0          0.0
      1.00        25.00
     20.00       225.00
```

3-AXLE EMPTY DUMP TRUCK

UNIT # 1
****

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 21200.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 20000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 400000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 400000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 50.00 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE #				
	*********	*********	*********	*********	*********	*********	*********	*********
LOAD ON EACH AXLE (LB.)	10000.00	8500.00	8500.00					
AXLE WEIGHT (LB.)	1200.00	2300.00	2300.00					
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	4500.00	4500.00					
X DIST FROM SP MASS CG (IN)	126.30	-68.70	-110.70					
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	19.00	19.00	19.00					
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	23.00	29.00	29.00					
HALF SPRING SPACING (IN)	17.00	19.00	19.00					
HALF TRACK - INNER TIRES (IN)	40.50	29.50	29.50					
DUAL TIRE SPACING (IN)	0.0	13.00	13.00					
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	5000.00					
ROLL STEER COEFFICIENT	0.0	0.0	0.0					
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0					
SPRING COULOMB FRICTION - PER SPRING (LB)	200.00	500.00	500.00					
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0					
SPRING TABLE #	1	2	2					
CORNERING FORCE TABLE #	1	1	1					
ALIGNING TORQUE TABLE #	1	1	1					

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-6000.00	-5.00
6000.00	5.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-20000.00	-10.00
0.0	-1.00
0.0	0.0
60000.00	10.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
8000.00	720.00	1360.00	1920.00	2400.00	3360.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	984.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
8000.00	864.00	1632.00	2304.00	2880.00	4032.00

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                    *********************************
                    *DIRECTIONAL RESPONSE SIMULATION*
                    *********************************

5-AXLE-DIRT-TRUCK

# OF SPRUNG MASSES        =  1

TOTAL # OF AXLES          =  5

GROSS VEHICLE WEIGHT      =   70000.00  LB.

FORWARD VELOCITY          =      55.00  M.P.H

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                        =      25.00

STEERING STIFFNESS (IN.LB/DEG)             =   25000.00

TIE ROD STIFFNESS (IN.LB/DEG)              =   25000.00

MECHANICAL TRAIL (IN)                      =       1.00

# OF POINTS IN STEER TABLE                 = 3

     TIME         STEERING WHEEL
     SEC              DEGREES
      0.0           0.0
      1.00         25.00
     20.00        225.00
```

5-AXLE-DIRT-TRUCK

UNIT # 1

OF AXLES ON THIS UNIT = 5

WEIGHT OF SPRUNG MASS = 60900.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 78.30 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE # 4	AXLE # 5	AXLE #		
LOAD ON EACH AXLE (LB.)	18000.00	13000.00	13000.00	13000.00	13000.00			
AXLE WEIGHT (LB.)	1500.00	1500.00	2300.00	2300.00	1500.00			
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	3700.00	4500.00	4500.00	3700.00			
X DIST FROM SP MASS CG (IN)	137.80	21.80	-28.20	-78.20	-124.20			
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	21.00	20.00	20.00	20.00	20.00			
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	22.00	29.00	29.00	22.00			
HALF SPRING SPACING (IN)	16.00	19.00	19.00	19.00	19.00			
HALF TRACK - INNER TIRES (IN)	40.25	29.00	29.00	29.00	29.00			
DUAL TIRE SPACING (IN)	0.0	13.00	13.00	13.00	13.00			
STIFFNESS OF EACH TIRE (LB/IN)	8000.00	5000.00	5000.00	5000.00	5000.00			
ROLL STEER COEFFICIENT	0.0	0.0	0.0	0.0	0.0			
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	100000.00	0.0	0.0	100000.00			
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	250.00	750.00	750.00	250.00			
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	0.0	0.0			
SPRING TABLE #	1	3	2	2	3			
CORNERING FORCE TABLE #	1	2	2	2	2			
ALIGNING TORQUE TABLE #	1	2	2	2	2			

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-5.00
14250.00	7.13
40000.00	7.25

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-7500.00	-1.75
0.0	-0.75
0.0	0.0
7500.00	1.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-10.00
10000.00	10.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATFPAL FORCE VS. SLIP ANGLL

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	780.00	1440.00	2040.00	2580.00	3600.00
8000.00	960.00	1840.00	2560.00	3280.00	4640.00
10000.00	1200.00	2200.00	3100.00	3900.00	5500.00

CORNERING FORCE TABLE # 2
********* ***** *********
LATFPAL FORCE VS. SLIP ANGLL

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
8000.00	720.00	1360.00	1920.00	2400.00	3360.00

ALICHING TORQUE TABLE # 1
******** ****** *********
ALICHING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	1944.00	3600.00	5100.00	6444.00	9000.00
8000.00	2400.00	4596.00	6396.00	8196.00	11604.00
10000.00	3000.00	5496.00	7752.00	9756.00	13752.00

ALICHING TORQUE TABLE # 2
******** ****** *********
ALICHING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	984.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
8000.00	864.00	1632.00	2304.00	2800.00	4032.00

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**********************************
*DIRECTIONAL RESPONSE SIMULATION*
**********************************

3-AXLE-DUMPSTER

# OF SPRUNG MASSES       =  1

TOTAL # OF AXLES         =  3

GROSS VEHICLE WEIGHT     =   58000.00  LB.

FORWARD VELOCITY         =      55.00  M.P.H

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                    =      25.00

STEERING STIFFNESS (IN.LB/DEG)         =   25000.00

TIE ROD STIFFNESS (IN.LB/DEG)          =   25000.00

MECHANICAL TRAIL (IN)                  =       1.00

# OF POINTS IN STEER TABLE             = 3

     TIME        STEERING WHEEL
     SEC            DEGREES
      0.0          0.0
      1.00        25.00
     20.00       225.00
```

3-AXLE-DUMPSTER

UNIT # 1

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 51900.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100856.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 331790.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 331790.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 81.40 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE #
LOAD ON EACH AXLE (LB.)	18000.00	20000.00	20000.00	
AXLE WEIGHT (LB.)	1500.00	2300.00	2300.00	
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	4500.00	4500.00	
X DIST FROM SP MASS CG (IN)	118.00	-30.00	-80.00	
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	21.00	20.00	20.00	
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	29.00	29.00	
HALF SPRING SPACING (IN)	16.00	19.00	19.00	
HALF TRACK - INNER TIRES (IN)	40.25	29.00	29.00	
DUAL TIRE SPACING (IN)	0.0	13.00	13.00	
STIFFNESS OF EACH TIRE (LB/IN)	8000.00	5000.00	5000.00	
ROLL STEER COEFFICIENT	0.0	0.0	0.0	
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0	
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	750.00	750.00	
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	
SPRING TABLE #	1	2	2	
CORNERING FORCE TABLE #	1	2	2	
ALIGNING TORQUE TABLE #	1	2	2	

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-5.00
10000.00	5.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-15000.00	-2.75
0.0	-0.75
0.0	0.0
15000.00	2.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	780.00	1440.00	2040.00	2580.00	3600.00
8000.00	960.00	1840.00	2560.00	3280.00	4640.00
10000.00	1200.00	2200.00	3100.00	3900.00	5500.00

CORNERING FORCE TABLE # 2
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
8000.00	720.00	1360.00	1920.00	2400.00	3360.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	1944.00	3600.00	5100.00	6444.00	9000.00
8000.00	2400.00	4596.00	6396.00	8196.00	11604.00
10000.00	3000.00	5896.00	7752.00	9756.00	13752.00

ALIGNING TORQUE TABLE # 2
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	984.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
8000.00	864.00	1632.00	2304.00	2880.00	4032.00

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********************************
*DIRECTIONAL RESPONSE SIMULATION*
********************************

6-AXLE TRACTOR -SEMI (DUMP-SEMITRAILER)

# OF SPRUNG MASSES      =  2

TOTAL # OF AXLES        =  6

GROSS VEHICLE WEIGHT    =   80100.00  LB.

FORWARD VELOCITY        =      55.00  M.P.H

                                            DISTANCE AHEAD   HEIGHT BELOW   ROLL STIFFNESS    TYPE OF
                                            OF SPRUNG MASS   SPRUNG MASS    (IN.LB/DEG)       CONSTRAINT
                                            C.G. (INCHES)   C.G. (INCHES)

                 ON UNIT # 1                   -109.70           -6.00      999999.88            1
ARTICULATION PT # 1
                 ON UNIT # 2                    111.30           35.00

TYPE OF CONSTRAINT :  01  CONVENTIONAL 5TH WHEEL
                      02  INVERTED 5TH WHEEL
                      03  PINTLE HOOK
                      04  KING PIN(RIGID IN ROLL & PITCH)

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                 =     25.00

STEERING STIFFNESS (IN.LB/DEG)      =  25000.00

TIE ROD STIFFNESS (IN.LB/DEG)       =  25000.00

MECHANICAL TRAIL (IN)               =      1.00

# OF POINTS IN STEER TABLE          = 3

     TIME        STEERING WHEEL
     SEC            DEGREES
      0.0          0.0
      1.00        25.00
     20.00       225.00
```

6-AXLE TRACTOR -SEMI (DUMP-SEMITRAILER)
UNIT # 1
****

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 10000.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 20000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 65000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 65000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 44.00 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE #
LOAD ON EACH AXLE (LB.)	11779.00	14661.00	14661.00	
AXLE WEIGHT (LB.)	1500.00	2300.00	2300.00	
AXLE ROLL M.I (LB.IN.SEC**2)	4000.00	4500.00	4500.00	
X DIST FROM SP MASS CG (IN)	40.30	-102.70	-152.70	
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.00	20.00	20.00	
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	29.00	29.00	
HALF SPRING SPACING (IN)	17.00	19.00	19.00	
HALF TRACK - INNER TIRES (IN)	49.25	29.00	29.00	
DUAL TIRE SPACING (IN)	0.0	13.00	13.00	
STIFFNESS OF EACH TIRE (LB/IN)	8000.00	5000.00	5000.00	
ROLL STEER COEFFICIENT	0.0	0.0	0.0	
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0	
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	500.00	500.00	
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	
SPRING TABLE #	1	2	2	
CORNERING FORCE TABLE #	1	2	2	
ALIGNING TORQUE TABLE #	1	2	2	

6-AXLE TRACTOR -SEMI (DUMP-SEMITRAILER)
UNIT # 2
****

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 59500.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 80000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 400000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 400000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 85.00 INCHES

	AXLE # 4	AXLE # 5	AXLE # 6	AXLE #
LOAD ON EACH AXLE (LB.)	13000.00	13000.00	13000.00	
AXLE WEIGHT (LB.)	1500.00	1500.00	1500.00	
AXLE ROLL M.I (LB.IN.SEC**2)	4000.00	4000.00	4000.00	
X DIST FROM SP MASS CG (IN)	-38.70	-80.70	-122.70	
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.00	20.00	20.00	
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	29.00	29.00	29.00	
HALF SPRING SPACING (IN)	19.00	19.00	19.00	
HALF TRACK - INNER TIRES (IN)	29.00	29.00	29.00	
DUAL TIRE SPACING (IN)	13.00	13.00	13.00	
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	5000.00	
ROLL STEER COEFFICIENT	0.0	0.0	0.0	
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0	
SPRING COULOMB FRICTION - PER SPRING (LB)	500.00	500.00	500.00	
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	
SPRING TABLE #	3	3	3	
CORNERING FORCE TABLE #	2	2	2	
ALIGNING TORQUE TABLE #	2	2	2	

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-15000.00	-10.00
15000.00	10.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-20000.00	-10.00
0.0	-1.00
0.0	0.0
50000.00	10.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-30000.00	-10.00
0.0	-1.00
0.0	0.0
75000.00	10.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	780.00	1440.00	2040.00	2580.00	3600.00
8000.00	960.00	1840.00	2560.00	3280.00	4640.00
10000.00	1200.00	2200.00	3100.00	3900.00	5500.00

CORNERING FORCE TABLE # 2
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
8000.00	720.00	1360.00	1920.00	2400.00	3360.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	1944.00	3600.00	5100.00	6444.00	9000.00
8000.00	2400.00	4596.00	6396.00	8196.00	11604.00
10000.00	3000.00	5496.00	7752.00	9756.00	13752.00

ALIGNING TORQUE TABLE # 2
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	984.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
8000.00	864.00	1632.00	2304.00	2880.00	4032.00

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*******************************
*DIRECTIONAL RESPONSE SIMULATION*
*******************************

3-AXLE EMPTY DUMP TRUCK+LOWBOY+BACK-HOE SEMI-TRAILER

# OF SPRUNG MASSES          =  2

TOTAL # OF AXLES            =  6

GROSS VEHICLE WEIGHT        =   45900.00  LB.

FORWARD VELOCITY            =      55.00  M.P.H

                                            DISTANCE AHEAD   HEIGHT BELOW   ROLL STIFFNESS    TYPE OF
                                            OF SPRUNG MASS   SPRUNG MASS    (IN.LB/DEG)       CONSTRAINT
                                            C.G. (INCHES)    C.G. (INCHES)

                ON UNIT # 1                       -154.00          19.00          0.0             4
ARTICULATION PT # 1
                ON UNIT # 2                        192.00          30.00

TYPE OF CONSTRAINT :  01  CONVENTIONAL 5TH WHEEL
                      02  INVERTED 5TH WHEEL
                      03  PINTLE HOOK
                      04  KING PIN(RIGID IN ROLL & PITCH)

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                  =      25.00

STEERING STIFFNESS (IN.LB/DEG)       =   25000.00

TIE ROD STIFFNESS (IN.LB/DEG)        =   25000.00

MECHANICAL TRAIL (IN)                =       1.00

# OF POINTS IN STEER TABLE           = 3

     TIME        STEERING WHEEL
     SEC            DEGREES
      0.0          0.0
      1.00        25.00
     20.00       225.00
```

3-AXLE EMPTY DUMP TRUCK+LOWBOY+BACK-HOE SEMI-TRAILER

UNIT # 1

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 21200.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 20000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 400000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 400000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 50.00 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE #
LOAD ON EACH AXLE (LB.)	9400.00	9800.00	9800.00	
AXLE WEIGHT (LB.)	1200.00	2300.00	2300.00	
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	4500.00	4500.00	
X DIST FROM SP MASS CG (IN)	126.30	-68.70	-110.70	
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	19.00	19.00	19.00	
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	23.00	29.00	29.00	
HALF SPRING SPACING (IN)	17.00	19.00	19.00	
HALF TRACK - INNER TIRES (IN)	40.50	29.50	29.50	
DUAL TIRE SPACING (IN)	0.0	13.00	13.00	
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	5000.00	
ROLL STEER COEFFICIENT	0.0	0.0	0.0	
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0	
SPRING COULOMB FRICTION - PER SPRING (LB)	200.00	500.00	500.00	
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	
SPRING TABLE #	1	2	2	
CORNERING FORCE TABLE #	1	1	1	
ALIGNING TORQUE TABLE #	1	1	1	

3-AXLE EMPTY DUMP TRUCK+LOWBOY+BACK-HOE SEMI-TRAILER

UNIT # 2
****

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 18000.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 20000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 400000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 400000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 61.00 INCHES

	AXLE # 4	AXLE # 5	AXLE # 6	AXLE #				
	*********	*********	*********	*********	*********	*********	*********	*********
LOAD ON EACH AXLE (LB.)	5633.00	5633.00	5633.00					
AXLE WEIGHT (LB.)	300.00	300.00	300.00					
AXLE ROLL M.I (LB.IN.SEC**2)	1000.00	1000.00	1000.00					
X DIST FROM SP MASS CG (IN)	9.00	-24.00	-57.00					
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	13.50	13.50	13.50					
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	24.00	24.00	24.00					
HALF SPRING SPACING (IN)	24.00	24.00	24.00					
HALF TRACK - INNER TIRES (IN)	36.00	36.00	36.00					
DUAL TIRE SPACING (IN)	0.0	0.0	0.0					
STIFFNESS OF EACH TIRE (LB/IN)	2500.00	2500.00	2500.00					
ROLL STEER COEFFICIENT	0.0	0.0	0.0					
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0					
SPRING COULOMB FRICTION - PER SPRING (LB)	100.00	100.00	100.00					
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0					
SPRING TABLE #	3	3	3					
CORNERING FORCE TABLE #	2	2	2					
ALIGNING TORQUE TABLE #	2	2	2					

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-6000.00	-5.00
6000.00	5.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-20000.00	-10.00
0.0	-1.00
0.0	0.0
60000.00	10.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-10.00
0.0	-1.00
0.0	0.0
25000.00	10.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
8000.00	720.00	1360.00	1920.00	2400.00	3360.00

CORNERING FORCE TABLE # 2
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	4.00	8.00
1335.00	150.00	310.00	530.00	840.00
2050.00	225.00	420.00	750.00	1200.00
2800.00	260.00	500.00	910.00	1490.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	984.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
8000.00	864.00	1632.00	2304.00	2880.00	4032.00

ALIGNING TORQUE TABLE # 2
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	4.00	8.00
1335.00	180.00	298.00	372.00	348.00
2050.00	324.00	552.00	756.00	708.00
2800.00	456.00	920.00	1236.00	1236.00

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                    ********************************
                    *DIRECTIONAL RESPONSE SIMULATION*
                    ********************************

5-AXLE TRACTOR-SEMITRAILER (CAR HAULER, STINGER HITCH)

# OF SPRUNG MASSES       =  2

TOTAL # OF AXLES         =  5

GROSS VEHICLE WEIGHT     =   66000.00  LB.

FORWARD VELOCITY         =      55.00  M.P.H

                                     DISTANCE AHEAD   HEIGHT BELOW   ROLL STIFFNESS   TYPE OF
                                     OF SPRUNG MASS   SPRUNG MASS    (IN.LB/DEG)      CONSTRAINT
                                     C.G. (INCHES)    C.G. (INCHES)

             ON UNIT # 1                  -132.80           40.00     999999.88           1
ARTICULATION PT # 1
             ON UNIT # 2                   250.80           65.00

TYPE OF CONSTRAINT :  01  CONVENTIONAL 5TH WHEEL
                      02  INVERTED 5TH WHEEL
                      03  PINTLE HOOK
                      04  KING PIN(RIGID IN ROLL & PITCH)

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                  =      25.00

STEERING STIFFNESS (IN.LB/DEG)       =   25000.00

TIE ROD STIFFNESS (IN.LB/DEG)        =   25000.00

MECHANICAL TRAIL (IN)                =       1.00

# OF POINTS IN STEER TABLE           = 3

     TIME        STEERING WHEEL
     SEC             DEGREES
      0.0          0.0
      1.00        25.00
     20.00       225.00
```

5-AXLE TRACTOR-SEMITRAILER (CAR HAULER, STINGER HITCH)
UNIT # 1
****

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 27000.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 50000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 150000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 150000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 60.00 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE #				
	*********	*********	*********	*********	*********	*********	*********	*********
LOAD ON EACH AXLE (LB.)	12000.00	14000.00	14000.00					
AXLE WEIGHT (LB.)	1200.00	2300.00	2300.00					
AXLE ROLL M.I (LB.IN.SEC**2)	3500.00	4500.00	4500.00					
X DIST FROM SP MASS CG (IN)	95.20	-61.80	-107.80					
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.00	20.00	20.00					
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	29.00	29.00					
HALF SPRING SPACING (IN)	17.00	19.00	19.00					
HALF TRACK - INNER TIRES (IN)	40.25	29.00	29.00					
DUAL TIRE SPACING (IN)	0.0	13.00	13.00					
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	5000.00					
ROLL STEER COEFFICIENT	0.0	0.0	0.0					
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0					
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	500.00	500.00					
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0					
SPRING TABLE #	1	2	2					
CORNERING FORCE TABLE #	1	1	1					
ALIGNING TORQUE TABLE #	1	1	1					

5-AXLE TRACTOR-SEMITRAILER (CAR HAULER, STINGER HITCH)
UNIT # 2
****

OF AXLES ON THIS UNIT = 2

WEIGHT OF SPRUNG MASS = 30200.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 1000000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 1000000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 85.00 INCHES

	AXLE # 4	AXLE # 5	AXLE #
LOAD ON EACH AXLE (LB.)	13000.00	13000.00	
AXLE WEIGHT (LB.)	1500.00	1500.00	
AXLE ROLL M.I (LB.IN.SEC**2)	3500.00	3500.00	
X DIST FROM SP MASS CG (IN)	-42.20	-116.20	
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.00	20.00	
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	29.00	29.00	
HALF SPRING SPACING (IN)	19.00	19.00	
HALF TRACK - INNER TIRES (IN)	29.00	29.00	
DUAL TIRE SPACING (IN)	13.00	13.00	
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	
ROLL STEER COEFFICIENT	0.0	0.0	
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	
SPRING COULOMB FRICTION - PER SPRING (LB)	500.00	500.00	
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	
SPRING TABLE #	3	3	
CORNERING FORCE TABLE #	1	1	
ALIGNING TORQUE TABLE #	1	1	

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-12000.00	-10.00
12000.00	10.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-20000.00	-10.00
0.0	-1.00
0.0	0.0
50000.00	10.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-30000.00	-10.00
0.0	-1.00
0.0	0.0
75000.00	10.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
8000.00	720.00	1360.00	1920.00	2400.00	3360.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	984.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
8000.00	864.00	1632.00	2304.00	2880.00	4032.00

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********************************
*DIRECTIONAL RESPONSE SIMULATION*
********************************

5-AXLE TRUCK-SEMITRAILER (DROMEDARY)

# OF SPRUNG MASSES        =  2

TOTAL # OF AXLES          =  5

GROSS VEHICLE WEIGHT      =   75228.00  LB.

FORWARD VELOCITY          =      55.00  M.P.H

                                       DISTANCE AHEAD   HEIGHT BELOW   ROLL STIFFNESS    TYPE OF
                                       OF SPRUNG MASS   SPRUNG MASS    (IN.LB/DEG)       CONSTRAINT
                                       C.G. (INCHES)    C.G. (INCHES)

                ON UNIT # 1               -153.60           23.00        999999.88          1
ARTICULATION PT # 1
                ON UNIT # 2                238.00           41.00

TYPE OF CONSTRAINT :  01  CONVENTIONAL 5TH WHEEL
                      02  INVERTED 5TH WHEEL
                      03  PINTLE HOOK
                      04  KING PIN(RIGID IN ROLL & PITCH)

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                =      25.00

STEERING STIFFNESS (IN.LB/DEG)     =  25000.00

TIE ROD STIFFNESS (IN.LB/DEG)      =  25000.00

MECHANICAL TRAIL (IN)              =      1.00

# OF POINTS IN STEER TABLE         = 3

     TIME       STEERING WHEEL
     SEC           DEGREES
      0.0         0.0
      1.00       25.00
     20.00      225.00
```

5-AXLE TRUCK-SEMITRAILER (DROMEDARY)

UNIT # 1
****

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 25000.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 50000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 150000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 150000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 75.00 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE #				
	*********	*********	*********	*********	*********	*********	*********	*********
LOAD ON EACH AXLE (LB.)	9076.00	17076.00	17076.00					
AXLE WEIGHT (LB.)	1200.00	2300.00	2300.00					
AXLE ROLL M.I (LB.IN.SEC**2)	3500.00	4500.00	4500.00					
X DIST FROM SP MASS CG (IN)	161.40	-81.60	-133.60					
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.00	20.00	20.00					
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	29.00	29.00					
HALF SPRING SPACING (IN)	17.00	19.00	19.00					
HALF TRACK - INNER TIRES (IN)	40.25	29.00	29.00					
DUAL TIRE SPACING (IN)	0.0	13.00	13.00					
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	5000.00					
ROLL STEER COEFFICIENT	0.0	0.0	0.0					
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0					
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	500.00	500.00					
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0					
SPRING TABLE #	1	2	2					
CORNERING FORCE TABLE #	1	1	1					
ALIGNING TORQUE TABLE #	1	1	1					

5-AXLE TRUCK-SEMITRAILER (DROMEDARY)

UNIT # 2

OF AXLES ON THIS UNIT = 2

WEIGHT OF SPRUNG MASS = 41428.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 1000000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 1000000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 93.00 INCHES

	AXLE # 4	AXLE # 5	AXLE #
LOAD ON EACH AXLE (LB.)	16000.00	16000.00	
AXLE WEIGHT (LB.)	1500.00	1500.00	
AXLE ROLL M.I (LB.IN.SEC**2)	3500.00	3500.00	
X DIST FROM SP MASS CG (IN)	-76.00	-128.00	
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.00	20.00	
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	29.00	29.00	
HALF SPRING SPACING (IN)	19.00	19.00	
HALF TRACK - INNER TIRES (IN)	29.00	29.00	
DUAL TIRE SPACING (IN)	13.00	13.00	
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	
ROLL STEER COEFFICIENT	0.0	0.0	
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	
SPRING COULOMB FRICTION - PER SPRING (LB)	500.00	500.00	
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	
SPRING TABLE #	3	3	
CORNERING FORCE TABLE #	1	1	
ALIGNING TORQUE TABLE #	1	1	

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-12000.00	-10.00
12000.00	10.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-20000.00	-10.00
0.0	-1.00
0.0	0.0
50000.00	10.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-30000.00	-10.00
0.0	-1.00
0.0	0.0
75000.00	10.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
6000.00	720.00	1360.00	1920.00	2400.00	3360.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	984.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
6000.00	864.00	1632.00	2304.00	2880.00	4032.00

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********************************
*DIRECTIONAL RESPONSE SIMULATION*
********************************

5AXLE-CALIFORNIA-TRUCK-FULLTRAILER

# OF SPRUNG MASSES        =  3

TOTAL # OF AXLES          =  5

GROSS VEHICLE WEIGHT      =   80000.00  LB.

FORWARD VELOCITY          =      55.00  M.P.H
```

	DISTANCE AHEAD OF SPRUNG MASS C.G. (INCHES)	HEIGHT BELOW SPRUNG MASS C.G. (INCHES)	ROLL STIFFNESS (IN.LB/DEG)	TYPE OF CONSTRAINT
ARTICULATION PT # 1 ON UNIT # 1	-163.00	26.10	0.0	3
ARTICULATION PT # 1 ON UNIT # 2	148.00	-1.00		
ARTICULATION PT # 2 ON UNIT # 2	0.0	-1.00	999999.88	4
ARTICULATION PT # 2 ON UNIT # 3	114.40	34.00		

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TYPE OF CONSTRAINT :  01  CONVENTIONAL 5TH WHEEL
                      02  INVERTED 5TH WHEEL
                      03  PINTLE HOOK
                      04  KING PIN(RIGID IN ROLL & PITCH)

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                 =     25.00

STEERING STIFFNESS (IN.LB/DEG)      =  25000.00

TIE ROD STIFFNESS (IN.LB/DEG)       =  25000.00

MECHANICAL TRAIL (IN)               =      1.00

# OF POINTS IN STEER TABLE          = 3
```

TIME SEC	STEERING WHEEL DEGREES
0.0	0.0
1.00	25.00
20.00	225.00

5AXLE-CALIFORNIA-TRUCK-FULLTRAILER

UNIT # 1

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 36040.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 53640.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 904000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 904000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 71.10 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE #				
LOAD ON EACH AXLE (LB.)	10500.00	15750.00	15750.00					
AXLE WEIGHT (LB.)	1300.00	2330.00	2330.00					
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	4500.00	4500.00					
X DIST FROM SP MASS CG (IN)	175.00	-34.00	-86.00					
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.60	20.60	20.60					
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	29.00	29.00					
HALF SPRING SPACING (IN)	16.00	19.00	19.00					
HALF TRACK - INNER TIRES (IN)	40.25	29.00	29.00					
DUAL TIRE SPACING (IN)	0.0	13.00	13.00					
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	5000.00					
ROLL STEER COEFFICIENT	0.0	0.0	0.0					
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0					
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	500.00	500.00					
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0					
SPRING TABLE #	1	2	2					
CORNERING FORCE TABLE #	1	1	1					
ALIGNING TORQUE TABLE #	1	1	1					

5AXLE-CALIFORNIA-TRUCK-FULLTRAILER

UNIT # 2
****

OF AXLES ON THIS UNIT = 1

WEIGHT OF SPRUNG MASS = 965.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 1900.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 2560.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 2560.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 44.00 INCHES

	AXLE # 4	AXLE #						
	*********	*********	*********	*********	*********	*********	*********	*********
LOAD ON EACH AXLE (LB.)	19000.00							
AXLE WEIGHT (LB.)	1500.00							
AXLE ROLL M.I (LB.IN.SEC**2)	4100.00							
X DIST FROM SP MASS CG (IN)	0.0							
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.60							
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	29.00							
HALF SPRING SPACING (IN)	19.00							
HALF TRACK - INNER TIRES (IN)	29.00							
DUAL TIRE SPACING (IN)	13.00							
STIFFNESS OF EACH TIRE (LB/IN)	5000.00							
ROLL STEER COEFFICIENT	0.0							
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0							
SPRING COULOMB FRICTION - PER SPRING (LB)	500.00							
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0							
SPRING TABLE #	3							
CORNERING FORCE TABLE #	1							
ALIGNING TORQUE TABLE #	1							

```
5AXLE-CALIFORNIA-TRUCK-FULLTRAILER
                                          UNIT #   3
                                          *********
# OF AXLES ON THIS UNIT =  1

WEIGHT OF SPRUNG MASS =    34035.00  LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS =    11725.00  LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS =  546000.00  LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS =  546000.00  LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND =      79.00  INCHES

                                 AXLE #  5  AXLE #
                                 *********  *********  *********  *********  *********  *********  *********  *********
LOAD ON EACH AXLE (LB.)           19000.00

AXLE WEIGHT (LB.)                  1500.00

AXLE ROLL M.I (LB.IN.SEC**2)       4100.00

X DIST FROM SP MASS CG (IN)       -108.10

HEIGHT OF AXLE C.G. ABOVE            20.60
           GROUND (INCHES)

HEIGHT OF ROLL CENTER ABOVE          29.00
           GROUND (INCHES)

HALF SPRING SPACING (IN)             19.00

HALF TRACK - INNER TIRES (IN)        29.00

DUAL TIRE SPACING (IN)               13.00

STIFFNESS OF EACH TIRE (LB/IN)     5000.00

ROLL STEER COEFFICIENT               0.0

AUX ROLL STIFFNESS (IN.LB/DEG)       0.0

SPRING COULOMB FRICTION -          500.00
           PER SPRING (LB)

VISCOUS DAMPING PER SPRING           0.0
              (LB.SEC/IN)

SPRING TABLE #                       3

  CORNERING FORCE TABLE #            1

  ALIGNING TORQUE TABLE #            1
```

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-7500.00	-5.00
7500.00	5.00

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-4000.00	-2.50
0.0	-1.50
0.0	0.0
6000.00	1.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-3.25
0.0	-1.00
0.0	0.0
1100.00	0.25
3200.00	0.50
9000.00	1.00
20000.00	1.75

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	3.00	4.00	5.00	7.00	10.00
2000.00	356.00	824.00	1018.00	1221.00	1502.00	1767.00
4000.00	580.00	1421.00	1770.00	2123.00	2612.00	3171.00
6000.00	701.00	1808.00	2259.00	2711.00	3378.00	4182.00
8000.00	767.00	2032.00	2583.00	3072.00	3849.00	4861.00
9000.00	784.00	2104.00	2674.00	3182.00	4020.00	5056.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	3.00	4.00	5.00	7.00	10.00
2000.00	372.00	528.00	552.00	672.00	732.00	468.00
4000.00	960.00	1716.00	1884.00	2268.00	2328.00	1896.00
6000.00	1560.00	3132.00	3588.00	4248.00	4476.00	3984.00
8000.00	2148.00	4644.00	5508.00	6384.00	6744.00	5676.00
9000.00	2400.00	5424.00	6396.00	7488.00	7800.00	6780.00

```
********************************
*DIRECTIONAL RESPONSE SIMULATION*
********************************

5-axle dirt truck + 6-axle full trailer

# OF SPRUNG MASSES        =  3

TOTAL # OF AXLES          = 11

GROSS VEHICLE WEIGHT      =  148000.00  LB.

FORWARD VELOCITY          =      55.00  M.P.H

                                          DISTANCE AHEAD   HEIGHT BELOW   ROLL STIFFNESS   TYPE OF
                                          OF SPRUNG MASS   SPRUNG MASS    (IN.LB/DEG)      CONSTRAINT
                                          C.G. (INCHES)    C.G. (INCHES)

                 ON UNIT # 1                  -154.60          51.30          0.0              3
ARTICULATION PT # 1
                 ON UNIT # 2                   132.00           9.00

                 ON UNIT # 2                     0.0           -9.00      999999.88            1
ARTICULATION PT # 2
                 ON UNIT # 3                    72.00          40.00

TYPE OF CONSTRAINT :  01  CONVENTIONAL 5TH WHEEL
                      02  INVERTED 5TH WHEEL
                      03  PINTLE HOOK
                      04  KING PIN(RIGID IN ROLL & PITCH)

OPEN LOOP STEER INPUT
*********************

STEERING GEAR RATIO                =     25.00

STEERING STIFFNESS (IN.LB/DEG)     =  25000.00

TIE ROD STIFFNESS (IN.LB/DEG)      =  25000.00

MECHANICAL TRAIL (IN)              =      1.00

# OF POINTS IN STEER TABLE         = 3

     TIME        STEERING WHEEL
     SEC            DEGREES
      0.0          0.0
      1.00        25.00
     20.00       225.00
```

5-axle dirt truck + 6-axle full trailer

UNIT # 1
****

OF AXLES ON THIS UNIT = 5

WEIGHT OF SPRUNG MASS = 60900.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 78.30 INCHES

	AXLE # 1	AXLE # 2	AXLE # 3	AXLE # 4	AXLE # 5	AXLE #		
	*********	*********	*********	*********	*********	*********	*********	*********
LOAD ON EACH AXLE (LB.)	18000.00	13000.00	13000.00	13000.00	13000.00			
AXLE WEIGHT (LB.)	1500.00	1500.00	2300.00	2300.00	1500.00			
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	3700.00	4500.00	4500.00	3700.00			
X DIST FROM SP MASS CG (IN)	137.80	21.80	-28.20	-78.20	-124.20			
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	21.00	20.00	20.00	20.00	20.00			
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	22.00	29.00	29.00	22.00			
HALF SPRING SPACING (IN)	16.00	19.00	19.00	19.00	19.00			
HALF TRACK - INNER TIRES (IN)	40.25	29.00	29.00	29.00	29.00			
DUAL TIRE SPACING (IN)	0.0	13.00	13.00	13.00	13.00			
STIFFNESS OF EACH TIRE (LB/IN)	8000.00	5000.00	5000.00	5000.00	5000.00			
ROLL STEER COEFFICIENT	0.0	0.0	0.0	0.0	0.0			
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	100000.00	0.0	0.0	100000.00			
SPRING COULOMB FRICTION - PER SPRING (LB)	250.00	250.00	750.00	750.00	250.00			
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	0.0	0.0			
SPRING TABLE #	1	3	2	2	3			
CORNERING FORCE TABLE #	1	2	2	2	2			
ALIGNING TORQUE TABLE #	1	2	2	2	2			

5-axle dirt truck + 6-axle full trailer

UNIT # 2
****

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 2500.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 4000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 8000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 8000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 36.00 INCHES

	AXLE # 6	AXLE # 7	AXLE # 8	AXLE #
LOAD ON EACH AXLE (LB.)	13417.00	13417.00	13417.00	
AXLE WEIGHT (LB.)	1500.00	1500.00	1500.00	
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	3700.00	3700.00	
X DIST FROM SP MASS CG (IN)	42.00	0.0	-42.00	
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.00	20.00	20.00	
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	22.00	22.00	
HALF SPRING SPACING (IN)	19.00	19.00	19.00	
HALF TRACK - INNER TIRES (IN)	29.00	29.00	29.00	
DUAL TIRE SPACING (IN)	13.00	13.00	13.00	
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	5000.00	
ROLL STEER COEFFICIENT	0.0	0.0	0.0	
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0	
SPRING COULOMB FRICTION - PER SPRING (LB)	750.00	750.00	750.00	
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	
SPRING TABLE #	2	2	2	
CORNERING FORCE TABLE #	2	2	2	
ALIGNING TORQUE TABLE #	2	2	2	

5-axle dirt truck + 6-axle full trailer

UNIT # 3

OF AXLES ON THIS UNIT = 3

WEIGHT OF SPRUNG MASS = 66500.00 LB.

ROLL MOMENT OF INERTIA OF SPRUNG MASS = 80000.00 LB.IN.SEC**2

PITCH MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2

YAW MOMENT OF INERTIA OF SPRUNG MASS = 800000.00 LB.IN.SEC**2

HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 85.00 INCHES

	AXLE # 9	AXLE # 10	AXLE # 11	AXLE #
LOAD ON EACH AXLE (LB.)	12583.00	12583.00	12583.00	
AXLE WEIGHT (LB.)	1500.00	1500.00	1500.00	
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	3700.00	3700.00	
X DIST FROM SP MASS CG (IN)	-30.00	-72.00	-114.00	
HEIGHT OF AXLE C.G. ABOVE GROUND (INCHES)	20.00	20.00	20.00	
HEIGHT OF ROLL CENTER ABOVE GROUND (INCHES)	22.00	22.00	22.00	
HALF SPRING SPACING (IN)	19.00	19.00	19.00	
HALF TRACK - INNER TIRES (IN)	29.00	29.00	29.00	
DUAL TIRE SPACING (IN)	13.00	13.00	13.00	
STIFFNESS OF EACH TIRE (LB/IN)	5000.00	5000.00	5000.00	
ROLL STEER COEFFICIENT	0.0	0.0	0.0	
AUX ROLL STIFFNESS (IN.LB/DEG)	0.0	0.0	0.0	
SPRING COULOMB FRICTION - PER SPRING (LB)	750.00	750.00	750.00	
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	
SPRING TABLE #	2	2	2	
CORNERING FORCE TABLE #	2	2	2	
ALIGNING TORQUE TABLE #	2	2	2	

SPRING TABLE # 1
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-5.00
14250.00	7.13
40000.00	7.25

SPRING TABLE # 2
****** *******

FORCE LB	DEFLECTION INCHES
-7500.00	-1.75
0.0	-0.75
0.0	0.0
7500.00	1.00

SPRING TABLE # 3
****** *******

FORCE LB	DEFLECTION INCHES
-10000.00	-10.00
10000.00	10.00

CORNERING FORCE TABLE # 1
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	780.00	1440.00	2040.00	2580.00	3600.00
8000.00	960.00	1840.00	2560.00	3280.00	4640.00
10000.00	1200.00	2200.00	3100.00	3900.00	5500.00

CORNERING FORCE TABLE # 2
********* ***** *********
LATERAL FORCE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
4000.00	520.00	960.00	1360.00	1720.00	2400.00
8000.00	720.00	1360.00	1920.00	2400.00	3360.00

ALIGNING TORQUE TABLE # 1
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
6000.00	1944.00	3600.00	5100.00	6444.00	9000.00
8000.00	2400.00	4596.00	6396.00	8196.00	11604.00
10000.00	3000.00	5496.00	7752.00	9756.00	13752.00

ALIGNING TORQUE TABLE # 2
******** ****** *********
ALIGNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	360.00	696.00	994.00	1224.00	1728.00
4000.00	624.00	1152.00	1632.00	2064.00	2880.00
8000.00	864.00	1532.00	2304.00	2880.00	4032.00