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PARAMETRIC ANALYSIS OF HEAVY DUTY TRUCK DYNAMIC STABILITY

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The study sought to define the important parametric sensitivities which affect the directional performance limits of commercial vehicles. Roll- over, unstable yaw response of the lead unit (spinout), and lightly damped yaw response of trailing units (rearward amplification) are identified as the three major response modes limiting directional performance. It is noted that both yaw response modes may precipitate rollover. The signi- ficant parametric sensitivities of commercial vehicles to each performance mode are identified by analytical means. Computer simulations of example vehicles, chosen for their peculiar susceptibility to one or more of the limiting performance modes, are used to demonstrate the parametric sensitivities.					
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APPENDIX A - MODIFICATION OF THE CLASSICAL STEADY-TURNING EQUATION TO REFLECT TIRE CORNER-ING STIFFNESS DEPENDENCE UPON VERTICAL LOAD

Consider, from Reference (19), the conventional front wheel angle-path curvature relationship for a straight truck with tandem-axle rear suspension:

$$\delta = \frac{\lambda_{e}}{R} + K \frac{U^{2}}{R}$$

$$= \frac{\lambda}{R} \left[1 + \left(\frac{\Delta^{2} + D^{2}C_{s_{R}}/C_{\alpha_{R}}}{\lambda^{2}} \right) \left(1 + \frac{C_{\alpha_{R}}}{C_{\alpha_{1}}} \right) \right]$$

$$+ M \frac{U^{2}}{R} \left[\frac{b}{\lambda C_{\alpha_{1}}} - \frac{a}{\lambda C_{\alpha_{R}}} \right]$$
(A-1)
(A-1)
(A-1)
(A-1)

where

- ô is the front-wheel angle
- R is the path radius
- U is forward velocity

and

$$\ell_{e} = \ell \left[1 + \left(\frac{\Delta^{2} + D^{2}C_{s_{R}}/C_{\alpha_{R}}}{\ell^{2}} \right) \left(1 + \frac{C_{\alpha_{R}}}{C_{\alpha_{1}}} \right) \right]$$
(A-3)

is the "effective" wheelbase, related to

1 the actual vehicle wheelbase (front suspension to center of rear tandem suspension),

one-half the rear tandem axle spread,

D one-half the dual tire spacing,

C the total rear tire longitudinal stiffness, S_R^R the total rear tire cornering stiffness, and $C_{a_1}^R$ the total front tire cornering stiffness,

and,

$$K = M \left[\frac{b}{\lambda C_{\alpha_1}} - \frac{a}{\lambda C_{\alpha_R}} \right]$$
 (A-4)

or

$$K = \left[\frac{W_1}{C_{\alpha_1}} - \frac{W_R}{C_{\alpha_R}}\right]/g \qquad (A-5)$$

is the classical understeer gradient, related to the additional parameters

M the vehicle mass

- a the distance from the c.g. to the front axle
- b the distance from the c.g. to the center of the rear suspension
- W_1 the static front suspension load
- ${\tt W}_{\rm R}$ the static rear suspension load
- g the gravitational acceleration

For an actual vehicle, K is, of course, comprised of additional elements contributing understeer, such as steering system compliance and roll-steer effects, which are being ignored in the analysis that follows.

If the cornering stiffness, $C^{\,\prime}_{\alpha},$ of a single tire is now treated as the following function of vertical load, $F_{\rm Z},$

$$C'_{\alpha}(F_{z}) = CO_{\alpha} + Cl_{\alpha}(F_{z}-F_{z}) + C2_{\alpha}(F_{z}-F_{z})^{2} (A-6)$$

where

- $^{\rm CO}_{\alpha}$ is the cornering stiffness prevailing at the static or nominal load, $\rm F_{Z_{\rm C}}$
- Cl is the linear variation of cornering stiffness with vertical load, about the nominal load, F_z
- and C2 is the quadratic variation of cornering stiffness with vertical load, about the nominal load, F_Z

most C_{α} versus F_{z} measurements for heavy truck tires can be accurately represented by this relationship. Hence, the cornering stiffness of a single tire on the front suspension can be expressed as:

$$C'_{x_{1}}(F_{z_{1}}) = CO_{x_{1}} + CI_{\alpha_{1}}(F_{z_{1}} - F_{z_{0}}) + C2_{\alpha_{1}}(F_{z_{1}} - F_{z_{0}})^{2}$$
(A-7)

and, for a single tire on the rear suspension:

$$C'_{u_{R}}(F_{z_{R}}) = CO_{u_{R}} + Cl_{u_{R}}(F_{z_{R}} - F_{z_{o_{R}}})$$

+ $C2_{u_{R}}(F_{z_{R}} - F_{z_{o_{R}}})^{2}$ (A-8)

Introduction of lateral acceleration-induced front and rear side-to-side load transfers, ΔW_1 and ΔW_R , proportioned by fore/aft roll stiffness distributions, produces the <u>conservative</u> static approximations:

$$\Delta W_{1} = \frac{h}{T_{1}} W \left(\frac{K_{1}}{K_{1} + K_{R}} \right) a_{y} \qquad (A-9)$$

$$\Delta W_{\rm R} = \frac{h}{T_{\rm R}} W \left(\frac{K_{\rm R}}{K_{\rm L} + K_{\rm R}} \right) a_{\rm y} \qquad (A-10)$$

where

1

h is the total vehicle c.g. height $T_{1,R}$ are the front, rear track distances W is the total vehicle weight K_1 is the front suspension roll stiffness K_R is the rear suspension roll stiffness and a is the vehicle lateral acceleration in g's. Left and right prevailing loads for both the front and rear axles can therefore be approximated for any given vehicle lateral acceleration, ay, as:

$$F_{z_{1}}(a_{y}) | = F_{z_{0_{1}}} + \Delta W_{1}(a_{y})$$

left
$$F_{z_{1}}(a_{y}) | = F_{z_{0_{1}}} - \Delta W_{1}(a_{y})$$

right
$$f_{z_{0_{1}}}(a_{y}) | = F_{z_{0_{1}}} - \Delta W_{1}(a_{y})$$

and

$$F_{z_{R}}(a_{y}) | = F_{z_{R}} + \Delta W_{R}(a_{y})$$

$$F_{z_{R}}(a_{y}) | = F_{z_{R}} - \Delta W_{R}(a_{y})$$

$$F_{z_{R}}(a_{y}) | = F_{z_{R}} - \Delta W_{R}(a_{y})$$

$$F_{z_{R}}(a_{y}) | = F_{z_{R}} - \Delta W_{R}(a_{y})$$

where



$$B = 1 + \frac{N \cdot CO_{\alpha_R} + \frac{\pi C2_{\alpha_R}}{N} \left(\frac{h}{T_R}\right)^2 W^2 \left(\frac{K_R}{K_1 + K_R}\right)^2 a_y^2}{2 \left[CO_{\alpha_1} + C2_{\alpha_1} \left(\frac{h}{T_1}\right)^2 W^2 \left(\frac{K_1}{K_1 + K_R}\right)^2 a_y^2\right]}$$

and



$$C_{\alpha_{1}} = 2 \left[CO_{\alpha_{1}} + C2_{\alpha_{1}} \Delta W_{1}^{2} \right]$$
$$= 2 \left[CO_{\alpha_{1}} + C2_{\alpha_{1}} \left(\frac{h}{T_{1}} \right)^{2} W^{2} \left(\frac{K_{1}}{K_{1} + K_{R}} \right)^{2} a_{y}^{2} \right] (A-11)$$

and

a_v:

$$C_{\alpha_{R}} = N \left[CO_{\alpha_{R}} + C2_{\alpha_{R}} \left(\frac{\Delta W_{R}}{N/2} \right)^{2} \right]$$

= N · CO_{\alpha_{R}} + 4C2_{\alpha_{R}} \Delta W_{R}^{2}/N
= N · CO_{\alpha_{R}} + \frac{4C2_{\alpha_{R}}}{N} \left(\frac{h}{T_{R}} \right)^{2} W^{2} \left(\frac{K_{R}}{K_{1} + K_{R}} \right)^{2} a_{y}^{2}
(A-12)

where N is the total number of tires on the rear suspension.

Substitution of these total front and rear cornering stiffnesses into Equations (A-3) and (A-5) produces the following lateral acceleration-dependent counterparts, l_e^{\dagger} and K':

$$\lambda'_{a} = \lambda \cdot [1 + A \cdot B]$$
 (A-13)



(A-1)

APPENDIX B - DIRECTIONAL STABILITY AND CALCULA-TION OF CRITICAL FORWARD VELOCITY

Rewriting Equation (A-1) as

$$\delta = \frac{2'_e}{R} + K' \frac{U^2}{R}$$
 (B-1)

where λ'_e and K' represent the lateral acceleration-dependent expressions (A-13) and (A-14), produces, by rearrangement, the path curvature-steer angle relationship:

$$\frac{1/R}{\delta} = \frac{1}{\lambda'_{a} + K'U^{2}}$$
(B-2)

Consideration of infinite path curvature for finite steer levels during steady turning leads to the stability condition

$$\frac{\partial(\delta)}{\partial(\frac{1}{R})} > 0 \qquad (B-3)$$

or

$$U^{2} \cdot \frac{\partial(\delta)}{\partial(a_{y})} = \ell_{e}^{\prime} + K^{\prime}U^{2} + \left(\frac{\partial\ell_{e}^{\prime}}{\partial a_{y}} + \frac{\partial K^{\prime}}{\partial a_{y}}U^{2}\right)a_{y} > 0$$
(B-4)

The critical forward velocity, U_c , above which the vehicle becomes directionally unstable is obtained by solving Equation (B-4) for its zero condition:

$$U_{c} = \left[\frac{-\iota'_{e} - \frac{\partial \iota'_{e}}{\partial a_{y}} \cdot a_{y}}{K' + \frac{\partial K'}{\partial a_{y}} \cdot a_{y}} \right]^{1/2}$$
(B-5)

The partial derivatives appearing in Equation (B-5) are obtained by differentiating Equations (A-13) and (A-14) with respect to a_y , viz.:

$$\frac{\partial \mathcal{L}_{a}^{\dagger}}{\partial a_{y}} = \left[\frac{\partial A}{\partial a_{y}} \cdot B + \frac{\partial B}{\partial a_{y}} \cdot A\right] \cdot 2$$

$$= \left\{ \frac{-\partial D^{2}C_{s_{R}}C^{2}\alpha_{R}\left(\frac{h}{T_{R}}\right)^{2}W^{2}\left(\frac{K_{R}}{K_{1}+K_{R}}\right)^{2}a_{y}}{N^{2}\left[NCO_{\alpha_{R}} + \frac{4C^{2}\alpha_{R}}{N}\left(\frac{h}{T_{R}}\right)^{2}W^{2}\left(\frac{K_{R}}{K_{1}+K_{R}}\right)^{2}a_{y}^{2}\right]^{2}} \right\}$$

$$\cdot \left\{ 1 + \frac{NCO_{\alpha_{R}} + \frac{4C^{2}\alpha_{R}}{N}\left(\frac{h}{T_{R}}\right)^{2}W^{2}\left(\frac{K_{R}}{K_{1}+K_{R}}\right)^{2}a_{y}^{2}}{2\left[CO_{\alpha_{1}} + C^{2}\alpha_{1}\left(\frac{h}{T_{1}}\right)^{2}W^{2}\left(\frac{K_{1}+K_{R}}{K_{1}+K_{R}}\right)^{2}a_{y}^{2}\right]} \right\}$$

$$+ \left\{ \frac{8 \frac{C^{2}_{\alpha_{R}}}{N} \left(\frac{h}{T_{R}}\right)^{2} W^{2} \left(\frac{K_{R}}{K_{1}+K_{R}}\right)^{2} a_{y}}{2 \left[CO_{\alpha_{1}} + C2_{\alpha_{1}} \left(\frac{h}{T_{1}}\right)^{2} W^{2} \left(\frac{K_{1}}{K_{1}+K_{R}}\right)^{2} a_{y}^{2}\right]} - \left[4C2_{\alpha_{1}} \left(\frac{h}{T_{1}}\right)^{2} W^{2} \left(\frac{K_{1}}{K_{1}+K_{R}}\right)^{2} \cdot a_{y}\right] \left[NCO_{\alpha_{R}} + \frac{4C2}{N} a_{R} \left(\frac{h}{T_{R}}\right)^{2} W^{2} \left(\frac{K_{R}}{K_{1}+K_{R}}\right)^{2} a_{y}^{2}\right]}{2 \left[CO_{\alpha_{1}} + C2_{\alpha_{1}} \left(\frac{h}{T_{1}}\right)^{2} W^{2} \left(\frac{K_{1}}{K_{1}+K_{R}}\right)^{2} a_{y}^{2}\right]^{2}} \right\}$$
$$\cdot \left\{ \frac{\Delta^{2} + D^{2}C_{s_{R}} / \left[NCO_{\alpha_{R}} + \frac{4C2_{\alpha_{R}}}{N} \left(\frac{h}{T_{R}}\right)^{2} W^{2} \left(\frac{K_{R}}{K_{1}+K_{R}}\right)^{2} a_{y}^{2}\right]}{2} \right\}$$



Note that Equation (B-5) is the classical critical velocity expression except for the additional partial derivative terms denoting dependence of λ'_{a} and K' (and hence U_{c}) upon lateral acceleration. Recall that the dependence of λ'_{e} and K' on lateral acceleration is directly related to the side-to-side load transfer assumption and variation of tire cornering stiffness with vertical load discussed in Appendix A.

APPENDIX C

SIMPLIFIED LINEAR ANALYSIS OF REARWARD AMPLIFICATION

This appendix presents a technical discussion of simplified equations developed for predicting rearward amplification. The equations of motion pertaining to a conventional dolly are examined in order to explain the lateral force "decoupling" achieved through the use of a dolly. Then frequency domain techniques and ideas from feedback control theory are employed to study the lateral acceleration gain between the pintle hitch and the center of gravity location of a full trailer. Finally, expressions defining the portion of the overall rearward amplification due to properties of towing units are derived.

C.1 Force Decoupling Achieved Through the Use of Steerable Dollies

In this section, an examination of the equations of motion of a truck-full trailer combination (as illustrated in Figure C.l) is used to indicate why F_A , the lateral force of constraint at the pintle hitch, is small.

For a truck-full trailer combination, the linearized equations of motion are as listed below. (These equations and symbols are the same as those used in [19].)

<u>Truck-Full-Trailer Equations</u>: (The full trailer consists of a dolly and a semitrailer)

$$m_1(v_1+ur_1) = \sum_i F_{1i} - F_A$$
(Truck lateral motion equation) (C.1)

$$m_2(\dot{v}_2 + ur_2) = \sum_i F_{2i} + F_A - F_B$$
 (Dolly lateral motion equation) (C.2)

$$m_3(v_3+ur_3) = \sum_{i=3}^{\infty} F_{3i} + F_{B}$$
 (Semitrailer lateral motion equation) (C.3)

where F_A is the force of constraint at the pintle hitch, F_B is the force of constraint at the fifth wheel or turntable, and the F_{ji} are tire forces that are linearly related to their slip angles.



•

Figure C.1. Truck-full trailer combination.

$$I_1 r_1 = x_{11} F_{11} x_{12} F_{12} x_{13} F_{13} x_{1A} F_A$$
 (Truck rotational (C.4)
equation)

$$I_2 r_2 = +F_A x_{2A} + F_{21} x_{21} - F_{22} x_{22}$$
 (Dolly rotational equation) (C.5)

(F_B does not appear in (C.5) because $x_{2B} \approx 0$; that is, for the dolly, the center of gravity and the turntable location are approximately in the same place.)

$$I_{3}r_{3} = +F_{B}x_{3B}-F_{31}x_{31}-F_{32}x_{32}-F_{33}x_{33}$$
 (Semitrailer rotational (C.6)
equation)

Expanding Equation (C.5) with $x_{21} \approx x_{22}$ and $C_{\alpha 21} \approx C_{\alpha 22}$ yields

$$F_{A} = \left(\frac{+I_{2}\dot{r}_{2} + \frac{2C_{\alpha 21}x_{21}^{2}r_{2}}{u}}{x_{2A}}\right)$$
(C.7)

The term, $2C_{\alpha 21}x_{21}^2r_2/u$ in (C.7) represents the yaw moment contribution from the dolly's tire forces acting about the turntable center for a dolly with tandem axles.

In Equation (C.7) observe that:

- a) r₂ is not large because the dolly is tied to two large masses that do not move quickly
- b) I₂ is not large (the dolly does not have a large moment of inertia)
- c) x₂₁r₂ << u (the forward speed is very much greater than this lateral velocity factor)
- d) $x_{21}/x_{2A} < 0.3$ (even for dollies with short tongues)

As a consequence of items (a) through (d) above, $|F_A| < 200$ lbs for typical dollies.

The preceding discussion applies not only to truck-full-trailer combinations, but also to any combination vehicle employing full trailers with conventional pintle hitch connections between the dolly tongue and its towing unit. The configuration of the combination vehicle (i.e., double, triple, or truck-full trailer) will not alter the basic factors and equations pertaining to the yaw moment balance for the dolly.

C.2 Analysis of the Full Trailer

For the idealized dolly employed in the analysis presented next, the force of constraint, F_A , at the pintle hitch is set equal to zero. This simplification and approximation separates the towing unit from the full trailer in that (a) the motion of the trailer does not influence the towing unit's motion and (2) the input to the trailer is the movement of the pintle hitch connection to the unit immediately ahead of the full trailer. (Also, if the full trailer being analyzed is the towing unit for another full trailer, such as in a triples combination, the properties of the trailer being towed will have no influence on the trailer being analyzed.)

Instead of comparing the last unit's lateral acceleration to that of the leading unit to obtain a rearward amplification factor for an entire vehicle, the acceleration or displacement of the center of mass of the full trailer may be compared to the acceleration or displacement of its hitch point to obtain an amplification factor for the full trailer alone. The overall amplification factor will then consist of the product of the full trailer's amplification factor with other amplification factors existing in the combination vehicle.

Figure C.2 illustrates and defines the points, geometric quantities, dimensions, and forces used in this analysis of the full trailer. (A nomenclature defining symbols and subscripts is given in Table 5 of the main body of this report.)

Using the approximation that $F_A = 0$, the steering of the dolly tongue can be determined from the motions of (1) the hitch point, A, and (2) the full trailer as influenced by (a) the side force from the dolly's tires acting at point B and (b) the side forces from the rear tires of the trailer acting at their corresponding axle locations.

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Figure C.2. Full trailer representation.

The side force, F_B , acting at point B on the full trailer (see Fig. C.2) is equal to F_{21} plus F_{22} for a dolly with tandem axles. Hence, in the linear range,

$$F_{B} = -C_{\alpha 21} \alpha_{21} - C_{\alpha 22} \alpha_{22}$$

For $C_{\alpha 21} = C_{\alpha 22}$ and $x_{21} = x_{22}$,
 $F_{B} = -2C_{\alpha 21} (\alpha_{21} + \alpha_{22})$

where

$$\alpha_{21} = \frac{v_2 + x_{21}r_2}{u}$$
(C.8)

$$\alpha_{22} = \frac{v_2 - x_{21}r_2}{u}$$
(C.9)

or

$$F_{\rm B} = -2C_{\alpha 21}(\frac{v_2}{u})$$
(C.10)

Points B and 2 are coincident according to the condition that the c.g. of the dolly and the location of the turntable center are approximately at the same point.

For the system shown in Figure C.2, the equations of motion are

$$m_{T}(\dot{v}_{T} + x_{A}r_{T}) = F_{B} + \sum_{i} F_{Ti}$$
(C.11)

$$I_{T}\dot{r}_{T} = x_{BT}F_{B} - \sum_{i} x_{Ti}F_{Ti}$$
(C.12)

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The slip angles needed for expressing the tire forces can be determined for small angles (such as those occurring in a lane-change maneuver) from the geometry illustrated in Figure C.l and the appropriate kinematic relationships per the following equations:

- \dot{x}_{A} = u (the entire vehicle is moving longitudinally with approximately the same forward velocity, u)
- $\psi_{D} = \frac{y_{A} y_{B}}{x_{BA}}$ $y_{B} = y_{T} + x_{BT}\psi_{T}$ $\psi_{T} = \int_{0}^{t} r_{T}dt$ $r_{T} = \psi_{D} \psi_{T} = \frac{y_{A} y_{T} (x_{BT} + x_{BA})\psi_{T}}{x_{BA}}$ $\ddot{y}_{T} = \dot{v}_{T} + ur_{T}$ $\dot{y}_{T} = v_{T} + \psi_{T}\dot{x}_{A}$ $y_{T} = \int_{0}^{t} \dot{y}_{T}dt$ $v_{2} = v_{T} + x_{BT}r_{T} \dot{x}_{A}r_{T}$ $\frac{v_{2}}{u} = \frac{v_{T} + x_{BT}r_{T}}{\dot{x}_{A}} r_{T}$ $\alpha_{Ti} = \frac{v_{T} r_{T}x_{Ti}}{\dot{x}_{A}}$

Using Equation (C.14) with i = 1, 2, 3 and assigning cornering stiffness values to the tires, the force and moment summations appearing in Equations (C.11) and (C.12) may be expressed as follows:

(C.13)

(C.14)

$$\sum_{i=1}^{3} F_{Ti} = -\frac{v_{T}}{\dot{x}_{A}} \sum_{i}^{2} C_{\alpha Ti} + \frac{r_{T}}{\dot{x}_{A}} \sum_{i}^{2} x_{Ti} C_{\alpha Ti}$$

$$\sum_{i=1}^{3} x_{Ti}^{F} F_{Ti} = -\frac{v_{T}}{\dot{x}_{A}} \sum_{i}^{2} x_{Ti} C_{\alpha Ti} + \frac{r_{T}}{\dot{x}_{A}} \sum_{i}^{2} x_{Ti}^{2} C_{\alpha Ti}$$

The force, $F_{\rm B}$, is given by the following equation

$$F_{B} = -2C_{\alpha 21} \left(\frac{v_{T} + x_{BT}r_{T}}{\dot{x}_{A}} - \Gamma_{T} \right)$$

which is equivalent to

$$F_{B} = -2C_{\alpha 21} \frac{v_{T}}{\dot{x}_{A}} - \frac{2C_{\alpha 21}x_{BT}r_{T}}{\dot{x}_{A}} - \frac{2C_{\alpha 21}(x_{BT}+x_{BA})\psi_{T}}{x_{BA}} + \frac{2C_{\alpha 21}}{x_{BA}}(y_{A} - y_{T})$$

Substituting these forces and moment expressions into the equations of motion ((C.11) and (C.12)) yields Equations (C.15) and (C.16) given in Figure C.3.

To obtain y_T and ψ_T as needed to complete the solution of (C.15) and (C.16), the following kinematic equations are employed:

$$\dot{\psi}_{T} = r_{T}$$
(C.17)
$$\dot{y}_{T} = v_{T} + \psi_{T} \dot{x}_{A}$$
(C.18)

By (a) combining Equations (C.15), (C.16), (C.17), and (C.18), (b) using p to replace the differential operator, d/dt, and (c) rearranging the equations, the following two equations are obtained:

$$(m_T^p + F_v)v_T + (m_T^u + F_r + \frac{F_\psi}{p})r_T = F_y(y_A - y_T)$$
 (C.19)

$$T_v v_T + (I_T p + T_r + \frac{T_{\psi}}{p})r_T = T_y (y_A - y_T)$$
 (C.20)

Since the quantity $(y_A - y_T)$ appears on the right sides of (C.19) and (C.20), the control engineer's notion of an open-loop transfer function comes to mind and provides the basis for additional development. Figure C.4 illustrates the idea that will be exploited here.

$$\mathbf{m}_{T}(\mathbf{\dot{v}}_{T} + \mathbf{\dot{x}}_{A}\mathbf{r}_{T}) = -\mathbf{F}_{v}\mathbf{v}_{T} + \mathbf{F}_{r}\mathbf{r}_{T} - \mathbf{F}_{\psi}\psi_{T} + \mathbf{F}_{y}(\mathbf{y}_{A} - \mathbf{y}_{T}) \quad (C.15)$$

$${}^{1}\mathbf{T}^{\mathbf{r}}\mathbf{T} = {}^{-}\mathbf{T}_{\mathbf{v}}\mathbf{v}_{\mathbf{T}} - {}^{\mathbf{T}}\mathbf{r}_{\mathbf{T}}\mathbf{T} - {}^{\mathbf{T}}\psi\psi_{\mathbf{T}} + {}^{\mathbf{T}}\mathbf{y}(\mathbf{y}_{\mathbf{A}}-\mathbf{y}_{\mathbf{T}})$$
(C.16)

where $F_{v} = \left(\frac{2C_{a21} + \frac{v}{1} - C_{aT1}}{\dot{x}_{A}}\right) \overline{def}. \quad \frac{\Sigma C_{a}}{\ddot{x}_{A}}$ $F_{T} = \left(\frac{2C_{a21}x_{BT} - \frac{v}{1}x_{T1}C_{aT1}}{\dot{x}_{A}}\right) \overline{def}. \quad \frac{x_{BT} \frac{v}{f} - \frac{v}{s}x_{T}C_{aT}}{\dot{x}_{A}}$ $F_{\psi} = \left(\frac{2C_{a21}(x_{BT} + x_{BA})}{\overline{x}_{BA}}\right) = F_{v}(x_{BT} + x_{BA})$ $F_{v} = \left(\frac{2C_{a21}}{x_{BA}}\right) = \frac{F_{\psi}}{(x_{BT} + x_{BA})}$ $T_{v} = \left(\frac{2C_{a21}x_{BT} - \frac{v}{1}x_{T1}C_{aT1}}{\dot{x}_{A}}\right) = F_{r}$ $T_{r} = \left(\frac{2C_{a21}x_{BT}^{2} + \frac{v}{1}x_{T1}^{2}C_{aT1}}{\dot{x}_{A}}\right) = \overline{def}. \quad \frac{\Sigma - x^{2}C_{a}}{\dot{x}_{A}}$ $T_{\psi} = \frac{2C_{a21}(x_{BT} + x_{BA})x_{BT}}{x_{BA}} = x_{BT}F_{\psi} = x_{BT}(x_{BT} + x_{BA})F_{v}$ $T_{v} = \left(\frac{2C_{a21}x_{BT}^{2} + x_{BA}x_{BT}}{x_{BA}}\right) = F_{r}$



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$$Y_{0} = (\frac{y_{T}}{y_{A} - y_{T}}) = \frac{[F_{v}I_{T}p^{2} + (F_{v}T_{v} - T_{v}F_{v})p + T_{v}uF_{v} - T_{v}uF_{v}]}{p[(m_{T}p+F_{v})(I_{T}p^{2} + T_{v}p + T_{\psi}) - (T_{v})((m_{T}u+F_{v})p + F_{\psi})]}$$

Figure C.4. Feedback control diagram as applied to a full trailer.

Proceeding in this manner, consider the full trailer as a mechanical servomechanism in which the output, y_T , is expected to follow the input, y_A . (However, in this case, $y_T(t + ((x_{BT}+x_{BA})/\dot{x}_A))$ should equal $y_A(t)$ if the trailer c.g. is going to follow the same path as the hitch point—more about this later after completing the analysis of the open-loop transfer function.)

Formally solving (C.19) and (C.20) for $\boldsymbol{v}_{T}^{}$ and $\boldsymbol{r}_{T}^{}$ yields

$$v_{\rm T} = D_1/D \tag{C.21}$$

and

$$r_{\rm T} = D_{\rm 2}/D \tag{C.22}$$

where

$$D = (m_T p + F_v) (I_T p + T_r + \frac{T_{\psi}}{p}) - T_v (m_T u + F_r + \frac{F_{\psi}}{p}) ,$$

$$D_1 = \left[F_y (I_T p + T_r + \frac{T_{\psi}}{p}) - T_y (m_T u + F_r + \frac{F_{\psi}}{p})\right] (y_A - y_T) ,$$

and $D_2 = [(m_T p + F_v)T_y - T_v F_y](y_A - y_T)$

Noting that

$$y_{T} = \int \int (\dot{v}_{T} + ur_{T}) dt dt = \frac{pD_{1} + uD_{2}}{p^{2}D}$$

allows the open-loop transfer function to be expressed as

$$\frac{y_{T}}{(y_{A} - y_{T})} = \frac{\left[(F_{y})(I_{T}p^{2} + T_{r}p + T_{\psi}) - (T_{y})(m_{T}u + F_{r})p + F_{\psi}) + T_{y}u(m_{T}p + F_{v}) - T_{v}uF_{y}\right]}{p\left[(m_{T}p + F_{v})(I_{T}p^{2} + T_{r}p + T_{\psi}) - (T_{v})((m_{T}u + F_{r})p + F_{\psi})\right]}$$
(C.23)

or by rearranging the numerator and noting that $F_{y\psi} - T_{y\psi} = 0$

$$\frac{y_{T}}{(y_{A} - y_{T})} = \frac{[F_{y}I_{T}p^{2} + (F_{y}T_{r} - T_{y}F_{r})p + T_{y}u F_{v} - T_{v}u F_{y}]}{p^{2}D}$$
(C.24)

Although (C.24) can be expanded further by substituting the definitions that accompany Equations (C.15) and (C.16), the result does not appear to be particularly illuminating. Rather, observe that most trailers are loaded with approximately equal loads on all axles and equipped with similar tires on all wheels. Under these circumstances, $T_v = F_r = 0$, that is, the trailer may be described as approximately "neutral steer." For the "typical" trailer

$$Y_{o} = \frac{y_{T}}{y_{A} - y_{T}} = \frac{F_{y}I_{T}p^{2} + F_{y}T_{r}p + T_{y}uF_{y}}{p(m_{T}p + F_{y})(I_{T}p^{2} + T_{r}p + T_{\psi})}$$
(C.25)

Upon substituting for the force and moment coefficients and rearranging, (C.25) becomes Equations (C.26), (C.27), and (C.28), expressed as follows:

$$Y_{o} = \frac{\left(\frac{\dot{x}_{A}}{x_{BT} + x_{BA}}\right)(Y_{z})}{p\left(\frac{\dot{x}_{A}^{m}T}{\Sigma C_{\alpha}} p + 1\right)(Y_{po})}$$
(C.26)

where

$$Y_{z} = \frac{I_{T}p^{2}}{x_{BT}^{\Sigma}C_{\alpha}} + \frac{(\Sigma x^{2}C_{\alpha})p}{x_{A}x_{BT}^{\Sigma}C_{\alpha}} + 1$$
(C.27)

$$Y_{po} = \frac{I_{T} x_{BA} p^{2}}{(\Sigma_{f} C_{\alpha}) (x_{BT} + x_{BA}) x_{BT}} + \frac{(\Sigma x^{2} C_{\alpha}) x_{BA} p}{x_{A} (\Sigma_{f} C_{\alpha}) (x_{BT} + x_{BA}) x_{BT}} + 1$$
(C.28)

The natural frequencies and damping ratios of Y and Y are nearly equal if $uF_v \approx F_\psi$, that is, if

$$\Sigma C_{\alpha} \approx \Sigma_{f} C_{\alpha} \left(\frac{x_{BT} + x_{BA}}{x_{BA}} \right)$$
(C.29)

For many trailers, Y_z and Y_po represent complex zero and pole pairs that are lightly damped and approximately equal (i.e., (C.29) is a reasonable approximation).

In addition to the pole-zero pair represented by Y_z/Y_{po} , the openloop transfer function has a pole at the origin and another real pole at $p = -F_v/m_T = -\Sigma C_\alpha/\dot{x}_A m_T$. These poles and zeros are sketched in the complex plane in a manner typical of control system analysis (see Figure C.5) with arrows indicating the locus of the closed-loop poles as the "gain" is increased. Of course, the "gain" is not variable in the sense ordinarily used by the control system analyst. Rather, it is given by $\dot{x}_{A}^{/}(x_{BA}^{+}+x_{BT}^{-})$ with the asterisks in Figure C.5 illustrating representative locations for the closed-loop poles. As indicated in the figure, the openloop poles due to Y_{po} approach the zeros due to Y_{z} , giving a closed-loop pole-zero pair in the neighborhood of the open-loop zeros. The two openloop poles on the real axis (one at the origin and the other at $-F_v/m_T$) become a complex conjugate set of closed-loop poles. This set of poles is the primary factor determining the frequency response of the closed-loop system (since the pole-zero pair effectively cancel each other). Hence, to a first approximation, the closed-loop transfer function, Y_c (where $Y_{c} = Y_{o}/1+Y_{o} = y_{T}/y_{A} \text{ or } A_{yT}/A_{yA})$ is given by



Figure C.5. Root locus diagram for a full trailer.

$$Y_{c} = \left[\frac{1}{\frac{p^{2}}{\omega_{nc}^{2}} + \frac{2\zeta_{c}}{\omega_{nc}}p + 1}\right]$$

where

$$\omega_{nc} = \left(\frac{(\Sigma C_{\alpha})}{(x_{TB} + x_{BA})m_{T}}\right)^{1/2}$$

$$\zeta_{c} = \frac{1}{2\dot{x}_{A}} \left(\frac{(\Sigma C_{\alpha})(x_{TB} + x_{BA})}{m_{T}}\right)^{1/2}$$
(C.30)

For ω < 0.3 $\omega_{nc},$ the phase shift, $\phi,$ is approximately given by

$$\phi = \frac{-2\zeta_c\omega}{\omega_{nc}}$$

where

$$\frac{2\zeta_{c}}{\omega_{nc}} = \frac{x_{BA} + x_{TB}}{\dot{x}_{A}}$$

The ideal phase shift, ϕ_{I} , required for the trailer's c.g. to track the hitch point is given by

$$\phi_{I} = \frac{-(x_{BA} + x_{TB})}{\dot{x}_{A}} \omega$$

Also, for small ω , $Y_c \approx 1.0$. Hence, at low frequencies of sinusoidal excitation at the hitch point, the path of the trailer's c.g. will pass very closely to the path of the hitch point—an expected and desired result and one that adds some intuitive confirmation to the analysis.

With regard to amplification, a second-order system such as that represented by Y_{c} , has its maximum response at a frequency given by

$$\omega_{\max} = \omega_n \sqrt{1 - 2\zeta^2}$$

and the gain at $\omega_{\mbox{max}}$ is given by

$$G_{\max} = \frac{1}{2\zeta \sqrt{1-\zeta^2}}$$

Sometimes the transfer function, Y_{c2} , corresponding to A_{yT}/y_A may be of interest in relating the path of the hitch to the acceleration of the trailer. In this case,

$$Y_{c2} = p^2 Y_{c}$$

However, the frequency of maximum response, ω_{max2} , is now given by

$$\omega_{\max 2} = \frac{\omega_n}{\sqrt{1 - 2\zeta^2}}$$

and the maximum gain is given by

$$G_{\max 2} = \frac{\omega_n^2}{2\zeta \sqrt{1-\zeta^2}}$$

The basic results of the full trailer analysis are summarized by the expression for the damping ratio, ζ_c , i.e., Equation (C.30). As indicated by (C.30), the damping ratio decreases (thereby causing the rearward amplification to increase) if (a) the forward velocity, $\dot{\mathbf{x}}_A$, is increased, (b) the total cornering coefficient, $\Sigma C_{\alpha}/m_T$, is decreased, or (c) the distance from the c.g. to the hitch, $(\mathbf{x}_{\rm BT} + \mathbf{x}_{\rm BA})$, is decreased.

Items (b) and (c) above follow a square root relationship, while the damping ratio is inversely proportional to velocity. Hence, the magnitude of forward velocity is a critical consideration when examining rearward amplification.

Clearly, the analysis of the rearward amplification of the full trailer has been reduced to three fundamental parameters, namely, velocity, cornering coefficient, and the distance from the c.g. to the hitch point. The first two of these parameters are expected to be important when considering the damping of the directional response of any highway vehicle. A length parameter is, also, expected to be important and in this case this length parameter involves the dolly's tongue length, $x_{BA}^{}$, plus $x_{BT}^{}$, which is a loading/wheelbase type of parameter for the full trailer.

The validity of the basic predictions of amplification can be checked by computing the quantities $F_r(Tv)$, Y_z , and Y_{po} to verify the reasonableness of the simplifications employed. Furthermore, more detailed analyses and/or complete simulations can be used to examine the characteristics of vehicles for which a basic analysis indicates large amounts of rearward amplification.

C.3 Towing Unit Amplification

If the hitch point of a towing unit is not located at the c.g. of the towing unit, the lateral displacement or acceleration of the hitch point may differ from that of the c.g. Specifically, for a unit with a yaw acceleration $\dot{\mathbf{r}}$ and a distance \mathbf{x}_{1A} from its c.g. to the hitch point

$$A_{yA} = A_{y1} - x_{1A}r$$

(where x_{1A} is positive for the hitch point, A, being behind point 1 (i.e., behind the c.g. of the towing unit)). Or, similarly, in terms of displacements

$$y_A = y_1 - x_{1A}\psi$$

where ψ is the heading angle of the towing unit.

In these cases, the amplification, A, is simply A_{yA}/A_{y1} or y_A/y_1 ; that is, $A = 1 + \Delta A$ where $\Delta A = -x_{1A}\psi/y_1$ or $-x_{1A}\dot{r}/A_{y1}$. For example, consider a straight truck whose response to steering is described by the following equations:

$$m(\mathbf{v} + \mathbf{ur}) = -\mathbf{F}_{\mathbf{v}}\mathbf{v} - \mathbf{F}_{\mathbf{r}}\mathbf{r} + \mathbf{F}_{\delta}\delta \qquad (C.31)$$

$$\mathbf{I} \, \mathbf{\dot{r}} = - \mathbf{T}_{\mathbf{v}} \mathbf{v} - \mathbf{T}_{\mathbf{r}} \mathbf{r} + \mathbf{T}_{\delta} \delta \tag{C.32}$$

These equations can be used to write transfer functions for v and r, having the following forms:

$$\frac{\mathbf{r}}{\delta} = \frac{N_{\mathbf{r}}}{D}$$
(C.33)

and

$$\frac{\mathbf{v}}{\delta} = \frac{\mathbf{N}}{\mathbf{D}}$$
(C.34)

where N_r is the numerator of the yaw rate transfer function; N_v is the numerator of the sideslip transfer function; and the denominator, D, is the same in both cases. The quantity ΔA is expressed in terms of the numerators of (C.33) and (C.34) by the following equation:

$$\Delta A = \frac{-x_{1A}r}{A_{y1}} = \frac{-x_{1A}p r}{pv + ur} = \frac{-x_{1A}p N_{r}}{pN_{v} + uN_{r}}$$
(C.35)

Equation (C.35) shows that the amplification factor for the towing vehicle depends upon the numerators of the yaw rate and side velocity transfer functions, that is, it depends upon the zeros of these transfer functions

and not the denominator, the eigenvalues, or the characteristic equation as is studied to determine the stability of the vehicle.

Using Equations (C.31) and (C.32) to evaluate (C.35) yields

$$\Delta A = \frac{-\frac{x_{1A}}{u} \left(\frac{m T_{\delta}}{F_{v}T_{\delta} - T_{c}F_{\delta}} p + 1\right) p}{\left[\frac{F_{\delta}I p^{2}}{u(F_{v}T_{\delta} - T_{v}F_{\delta})} + \left(\frac{T_{r}F_{\delta} - T_{\delta}F_{r}}{u(F_{v}T_{\delta} - T_{v}F_{\delta})}\right) p + 1\right]}$$
(C.36)

For typical vehicles

$$F_v T_\delta >> T_v F_\delta$$

and

implying that

$$\Delta A = \left[\frac{\frac{-\mathbf{x}_{1A}}{\mathbf{u}} (\mathbf{p}) \quad (\frac{\mathbf{m}}{\mathbf{F}} \mathbf{p} + 1)}{\frac{\mathbf{v}}{\frac{\mathbf{F}_{\delta}\mathbf{I}}{\mathbf{u}\mathbf{F}_{v}\mathbf{T}_{\delta}} \mathbf{p}^{2} + \frac{\mathbf{T}_{r}\mathbf{F}_{\delta}}{\mathbf{u}\mathbf{F}_{v}\mathbf{T}_{\delta}} \mathbf{p} + 1} \right]$$

Substituting for the force and moment coefficients in terms of design parameters yields

$$\Delta A = \begin{bmatrix} \frac{-x_{1A}}{u} (p) (\frac{mu}{\Sigma C_{\alpha}} p + 1) \\ \frac{Y_{zA}}{zA} \end{bmatrix}$$
(C.37)

where x_{11} is the distance from the c.g. to the front axle and

$$Y_{zA} = \frac{p^2}{\omega_{nzA}^2} + \frac{2\zeta_{zA}}{\omega_{nzA}} p + 1,$$

with

$$\omega_{nzA} = \left(\frac{x_{11} \Sigma C_{\alpha}}{I}\right)^{1/2}$$

and

$$\zeta_{zA} = \frac{1}{2} \left(\frac{\Sigma x^2 C_{\alpha}}{u} \right) \frac{1}{\left(I x_{11} \Sigma C_{\alpha} \right)^{1/2}}$$

In the frequency domain, ΔA is a vector of the form Ke^{j ϕ}, which must be added to the unit vector (1.0) to obtain A at any particular frequency of interest, that is, K and ϕ are functions of the frequency, ω .

In general,

$$A = 1 + \Delta A = \frac{p^2 \left(\frac{1}{\omega_{nzA}^2} - \alpha \tau\right) + p \left(\frac{2\zeta_{zA}}{\omega_{nzA}} - \alpha\right) + 1}{\frac{p^2}{\omega_{nzA}^2} + \frac{2\zeta_{zA}p}{\omega_{nzA}} + 1}$$
(C.38)

where ω_{nzA} and ζ_{zA} are defined as before and $\alpha = x_{1A}/u$ and $\tau = mu/\Sigma C_{\alpha}$.

At high frequencies

$$A = 1 - \alpha \tau \omega_{nzA}^{2} = 1 - \frac{m x_{1A}^{2} 11}{I}$$

Note that the analysis performed for the straight truck also applies to a full trailer that is the towing unit for another full trailer (as in a triples combination). In this case, the articulation angle between the dolly's longitudinal axis and the longitudinal axis of the semitrailer portion of the full trailer plays a role that is analogous to the role of the steering angle, δ , employed in the discussion of the straight truck. The results for a full trailer, that is acting as a towing unit, are presented in Table 4 in Chapter 6. The semitrailer of a tractor-semitrailer towing unit is coupled by a fifth wheel arrangement that provides a large lateral force of constraint. Hence, it is not possible to separate the analysis of the tractor from that of the semitrailer. Nevertheless, the dynamics of tractor-semitrailer vehicles have been studied extensively and the lateral acceleration gain between the c.g. of the tractor and the c.g. of the semitrailer may be known or readily determined for many tractor-semitrailer combinations [20]. Assuming that the "c.g. to c.g." amplification factor is known, the material presented next contains a derivation of the rearward amplification between the c.g. of a semitrailer and the location of its pintle hitch.

A single-axle semitrailer, illustrated in Figure C.6, will be considered first.



Figure C.6. Single-axle semitrailer.

The lateral acceleration, $A_{yB}^{},$ of the pintle hitch differs from the lateral acceleration, $A_{y2}^{},$ of the c.g. because of the yaw acceleration, r, i.e.,

$$A_{yB} = A_{y2} - x_{2B}r$$
 (C.39)

Consequently, the amplification, A, defined by $A_{yB}^{}/A_{y2}^{}$ may be expressed in the following manner.

$$A = \frac{A_{yB}}{A_{y2}} = 1 - \frac{x_{2B}\dot{r}}{A_{y2}} = 1 + \Delta A$$
 (C.40)

where

$$\Delta A = \frac{-x_{2B}r}{A_{y2}}$$

In order to evaluate ΔA it is necessary to consider the equations of motion for the semitrailer, that is,

$$m_2(A_{y2}) = m_2(v + ur) = F_{21} + F_A$$
 (C.41)

and

$$I_2 \dot{r} = F_A x_{2A} - F_{21} x_{21}$$
 (C.42)

where

$$F_{21} = -C_{\alpha 21} \left(\frac{v}{u} - \frac{x_{21}r}{u} \right)$$
(C.43)

Upon using the equations of motion to evaluate ΔA , the force of constraint at the fifth wheel may be eliminated from the resulting expression, thereby producing the following result:

$$\Delta A = \left(\frac{-x_{2B}^{p}}{\frac{u}{\omega_{1}^{2}} + \frac{2\zeta}{\omega_{n}} p + 1}\right)$$

where

$$\tau = \frac{\frac{x_{2A}m_{2}u}{(x_{2A}+x_{21})C_{\alpha 21}}}{\left(\frac{(x_{2A}+x_{21})C_{\alpha 21}}{I_{2}}\right)^{1/2}}$$
$$\omega_{n} = \left(\frac{\frac{(x_{2A}+x_{21})C_{\alpha 21}}{I_{2}}}{I_{2}}\right)^{1/2}$$
$$\zeta = \frac{x_{21}}{2u}\omega_{n}$$

For semitrailers with multiple rear axles, the expression for ΔA is derived in a manner similar to that used in deriving Equation (C.44). The result is as follows:

$$\Delta A = \left(\frac{\frac{-x_{2B}p}{u}(\tau p + 1)}{\frac{p^2}{\omega^2} + \frac{2\zeta}{\omega_n}p + 1}\right)$$
(C.45)

(C.44)

where for N rear axles

-

$$\tau = \frac{\frac{u m_2 x_{2A}}{\sum_{i=1}^{N} (x_{2A} + x_{2i}) c_{\alpha 2i}}}{\left(\frac{\sum_{i=1}^{N} (x_{2A} + x_{2i}) c_{\alpha 2i}}{1_2}\right)^{1/2}}$$
$$\omega_n = \left(\frac{\sum_{i=1}^{N} (x_{2A} + x_{2i}) c_{\alpha 2i}}{\sum_{i=1}^{N} (x_{2A} + x_{2i}) c_{\alpha 2i}}\right)^{1/2}$$
$$\zeta = \frac{\omega_n}{2u} \left(\frac{\sum_{i=1}^{N} (x_{2A} + x_{2i}) c_{\alpha 2i}}{\sum_{i=1}^{N} (x_{2A} + x_{2i}) c_{\alpha 2i}}\right)$$

An alternative approach for analyzing the influence of semitrailer properties on rearward amplification consists of evaluating the amplification factor between the fifth wheel and pintle hitch locations on the semitrailer. In this case, the motion of the fifth wheel, which is a point on both the tractor and the semitrailer, is taken as the input motion to which the amplification is referenced; that is, if the motion of the fifth wheel is presumed to be reasonable and satisfactory, will the motion of the pintle hitch be a greatly amplified version of that motion?

Using the notation illustrated in Figure C.6, the ratio of the lateral acceleration of the pintle hitch to that of the fifth wheel may be expressed as follows:

$$\frac{A_{yB}}{A_{yA}} = \frac{A_{y2} - x_{2B}r}{A_{y2} + x_{2A}r}$$

or

$$\frac{A_{yB}}{A_{yA}} = \frac{1 + \Delta A}{1 - \frac{x_{2A}}{x_{2B}} \Delta A}$$

where ΔA is given by Equation (C.44).

Or, upon substituting for ΔA and using the previously defined quantities $\tau,~\omega_{\rm p},$ and $\zeta,$

$$A_{AB} = \frac{A_{yA}}{A_{yB}} = \left[\frac{p^2 \left(\frac{1}{\omega_n^2} - \frac{x_{2B}^{\tau}}{u} \right) + p \left(\frac{2\zeta}{\omega_n} - \frac{x_{2B}}{u} \right) + 1}{p^2 \left(\frac{1}{\omega_n^2} + \frac{x_{2A}^{\tau}}{u} \right) + p \left(\frac{2\zeta}{\omega_n} + \frac{x_{2A}}{u} \right) + 1} \right]$$
(C.46)

APPENDIX D

DERIVATION OF THE EQUATIONS OF MOTION FOR THE DYNAMIC MODEL OF MULTIPLE-ARTICULATED VEHICLES (YAW/ROLL MODEL)

The equations of motion are derived by the application of the Newton's laws of motion. The derivation is organized under the following sub-headings:

- 1) Axis Systems
- Equations of Motion for the Sprung and Unsprung Masses
- 3) Suspension Forces
- 4) Constraint Forces and Moments
- 5) Tire Forces

A brief outline of the computer code is presented at the end of the appendix.

D.1. Axis Systems

Three types of axis systems are used in the process of developing the equations of motion. They are: (1) an inertial axis system fixed in space, (2) an axis system fixed to each of the sprung masses, and (3) an axis system fixed to each of the unsprung masses. For example, Figure D.1 shows the axis systems for a four-axle, multiplearticulated vehicle with two articulation points, C_1 and C_2 , respectively.

Euler angles are used to define the orientation of the sprung and unsprung masses with respect to the inertial axis system. Since all sprung mass axis systems are defined alike, the axis transformation equations are given below for only one sprung mass. For the same reason, the transformation equations for the unsprung mass axis systems are derived for a single unsprung mass.



Axis systems for an articulated vehicle with three sprung masses and four unsprung masses.

Figure D.1.

D.1.1 <u>Sprung Mass Axis System</u>. The three Euler angles of yaw (ψ_s) , pitch (θ_s) , and roll (ϕ_s) which are needed to describe the orientation of each of the sprung mass axis systems are shown in Figures D.2, D.3, and D.4, respectively.

The transformation equation between the inertial and sprung mass axis systems can be derived using the three sequential steps of rotation which are illustrated. For the yaw rotation, ψ_c

$$\begin{vmatrix} \mathbf{i}_{n} \\ \mathbf{j}_{n} \\ \mathbf{k}_{n} \end{vmatrix} = \begin{bmatrix} \cos \psi_{s} & -\sin \psi_{s} & 0 \\ \sin \psi_{s} & \cos \psi_{s} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \mathbf{i}_{1} \\ \mathbf{j}_{1} \\ \mathbf{k}_{1} \end{pmatrix}$$
(1)

or

$$[\vec{i}_{n}, \vec{j}_{n}, \vec{k}_{n}]^{T} = [a_{ij}] \{\vec{i}_{1}, \vec{j}_{1}, \vec{k}_{1}\}^{T}$$
 (2)

For the rotation, θ_s , illustrated in Figure C.3,

$$\begin{pmatrix} \dot{i}_{1} \\ \dot{j}_{1} \\ \dot{k}_{1} \end{pmatrix} = \begin{bmatrix} \cos \theta_{s} & 0 & \sin \theta_{s} \\ 0 & 1 & 0 \\ -\sin \theta_{s} & 0 & \cos \theta_{s} \end{bmatrix} \begin{pmatrix} \dot{i}_{2} \\ \dot{j}_{2} \\ \dot{k}_{2} \end{pmatrix}$$
(3)

or

$$[\vec{i}_1, \vec{j}_1, \vec{k}_1]^T = [b_{ij}] \{\vec{i}_2, \vec{j}_2, \vec{k}_2\}^T$$
 (4)

Proceeding along similar lines, the roll rotation illustrated in Figure C.4 yields

$$\begin{cases} \dot{i}_{2} \\ \dot{j}_{2} \\ \dot{k}_{2} \end{cases} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_{s} & -\sin \phi_{s} \\ 0 & \sin \phi_{s} & \cos \phi_{s} \end{bmatrix} \begin{pmatrix} \dot{i}_{s} \\ \dot{j}_{s} \\ \dot{k}_{s} \end{pmatrix}$$
(5)


Figure D.2



Figure D.3



Figure D.4

Euler angles needed to define the orientation of each of the sprung mass axis systems.

$$\{\vec{i}_{2}, \vec{j}_{2}, \vec{k}_{2}\}^{T} = [c_{ij}] \{\vec{i}_{s}, \vec{j}_{s}, \vec{k}_{s}\}^{T}$$
 (6)

The transformation matrix which is needed to relate the sprung mass axis system and the inertial axis system can now be obtained by combining (2), (4), and (6). Doing so, we get

$$\{\vec{i}_{n}, \vec{j}_{n}, \vec{k}_{n}\}^{T} = [A_{ij}] \{\vec{i}_{s}, \vec{j}_{s}, \vec{k}_{s}\}^{T}$$
 (7)

where $[A_{ij}] = [a_{ij}] [b_{ij}] [c_{ij}]$

During directional maneuvers, the pitch angles of sprung masses are usually restricted to very small values, hence the transformation equations can be simplified by replacing $\sin \theta_s$ by θ_s and $\cos \theta_s$ by 1.0. Expanding Equation (7) and applying the small pitch angle assumption, we get:

$$\begin{pmatrix} \mathbf{i}_n \\ \mathbf{j}_n \\ \mathbf{k}_n \end{pmatrix}$$

 $\begin{bmatrix} \cos\psi_{s} & -\sin\psi_{s}\cos\phi_{s} + \cos\psi_{s}\theta_{s}\sin\phi_{s} & \sin\psi_{s}\sin\phi_{s} + \cos\psi_{s}\theta_{s}\cos\phi_{s} \\ \sin\psi_{s} & \cos\psi_{s}\cos\phi_{s} + \sin\psi_{s}\theta_{s}\sin\phi_{s} & -\cos\psi_{s}\sin\phi_{s} + \sin\psi_{s}\theta_{s}\cos\phi_{s} \\ -\theta_{s} & \sin\phi_{s} & \cos\phi_{s} \end{bmatrix} \begin{pmatrix} + \\ i_{s} \\ + \\ j_{s} \\ k_{s} \end{pmatrix}$

(8)

Also

$$\begin{cases} \vec{i}_{s} \\ \vec{j}_{s} \\ \vec{k}_{s} \end{cases} =$$

$$\begin{cases} cos\psi_{s} & sin\psi_{s} & -\theta_{s} \\ -sin\psi_{s}cos\phi_{s}+cos\psi_{s}\theta_{s}sin\phi_{s} & cos\psi_{s}cos\phi_{s}+sin\psi_{s}\theta_{s}sin\phi_{s} & sin\phi_{s} \\ sin\psi_{s}sin\phi_{s}+cos\psi_{s}\theta_{s}cos\phi_{s} & -cos\psi_{s}sin\phi_{s}+sin\psi_{s}\theta_{s}cos\phi_{s} & cos\phi_{s} \end{bmatrix} \begin{pmatrix} \vec{i}_{n} \\ \vec{j}_{n} \\ \vec{k}_{n} \end{pmatrix}$$

$$(9)$$

Sprung Mass Angular Velocities:

The equations of motion of each sprung mass are written in terms of the body-fixed angular velocities (p_s,q_s,r_s) and their derivatives. In order to determine the Euler angles, the Euler angular velocities $(\dot{\phi}_s, \dot{\theta}_s, \dot{\psi}_s)$ have to be calculated from the body-fixed angular velocities (p_s, q_s, r_s) and then integrated numerically. The Euler angular velocities $(\dot{\phi}_s, \dot{\theta}_s, \dot{\psi}_s)$ are defined along the $(\vec{i}_s, \vec{j}_2, \vec{k}_n)$ directions. Therefore, equating the body-fixed and Euler angular velocities, we get

$$p_{s}\vec{i}_{s} + q_{s}\vec{j}_{s} + r_{s}\vec{k}_{s} = \dot{\phi}_{s}\vec{i}_{s} + \dot{\theta}_{s}\vec{j}_{2} + \dot{\psi}_{s}\vec{k}_{n}$$
 (10)

From Equation (5) we note that

$$\vec{j}_2 = \cos \phi_s \vec{j}_s - \sin \phi_s \vec{k}_s$$
(11)

Also, Equation (8) indicates that

$$\vec{k}_{n} = -\theta_{s}\vec{i}_{s} + \sin\phi_{s}\vec{j}_{s} + \cos\phi_{s}\vec{k}_{s}$$
(12)

Substituting Equations (11) and (12) back into (10) we get

$$p_{s\dot{i}s} = (\dot{\phi}_{s} - \theta_{s}\dot{\psi}_{s})\dot{\bar{i}}_{s}$$
(13)

$$q_{s}\dot{j}_{s} = (\dot{e}_{s}\cos\phi_{s} + \sin\phi_{s}\dot{\psi}_{s})\dot{j}_{s}$$
(14)

$$r_{s}\vec{k}_{s} = (-\dot{\theta}_{s}\sin\phi_{s} + \dot{\psi}_{s}\cos\phi_{s})\vec{k}_{s}$$
(15)

The above three equations can also be written for solving the Euler angular velocities in terms of the body-fixed angular velocities (p_s,q_s,r_s) . In doing so, we get:

$$\dot{\phi}_{s} = p_{s} + (q_{s} \sin \phi_{s} + r_{s} \cos \phi_{s})\theta_{s}$$
(16)

$$\dot{\theta}_{s} = q_{s} \cos \phi_{s} - r_{s} \sin \phi_{s}$$
 (17)

$$\dot{\psi}_{s} = q_{s} \sin \phi_{s} + r_{s} \cos \phi_{s}$$
 (18)

Therefore, Equations (16)-(18) can be numerically integrated to obtain the Euler angles at any time t of the simulation.

D.1.2 Unsprung Mass Axis System. Each unsprung mass is permitted only to roll and bounce with respect to the sprung mass to which it is attached. The orientation of the unsprung mass with respect to the inertial axis system is therefore defined by the yaw angle, ψ_s , and the roll angle, ϕ_u , which are shown in Figures D.5 and D.6, respectively.

Figure D.6 indicates that

$$\begin{pmatrix} \vec{i}_{u} \\ \vec{j}_{u} \\ \vec{k}_{u} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_{u} & \sin \phi_{u} \\ 0 & -\sin \phi_{u} & \cos \phi_{u} \end{bmatrix} \begin{pmatrix} \vec{i}_{1} \\ \vec{j}_{1} \\ \vec{k}_{1} \end{pmatrix}$$
(19)







Figure D.6

Euler angles needed to define the orientation of each of the unsprung masses.

When Equations (3) and (5) are combined, we have

$$\begin{cases} \vec{i}_{1} \\ \vec{j}_{1} \\ \vec{k}_{1} \end{cases} = \begin{bmatrix} b_{ij} \end{bmatrix} \begin{bmatrix} c_{ij} \end{bmatrix} \begin{pmatrix} \vec{i}_{s} \\ \vec{j}_{s} \\ \vec{k}_{s} \end{cases}$$
(20)

Therefore, combining Equations (19) and (20) and substituting for $[b_{ij}]$ and $[c_{ij}]$, we get the transformation equation which relates the sprung and unsprung mass axis systems.

$$\begin{vmatrix} \dot{i}_{u} \\ \dot{j}_{u} \\ \dot{k}_{u} \end{vmatrix} = \begin{bmatrix} 1 & \theta_{s} \sin \phi_{s} & \theta_{s} \cos \phi_{s} \\ -\theta_{s} \sin \phi_{u} & \cos(\phi_{s} - \phi_{u}) & -\sin(\phi_{s} - \phi_{u}) \\ -\theta_{s} \cos \phi_{u} & \sin(\phi_{s} - \phi_{u}) & \cos(\phi_{s} - \phi_{u}) \end{bmatrix} \begin{pmatrix} \dot{i}_{s} \\ \dot{j}_{s} \\ \dot{k}_{s} \end{pmatrix}$$
(21)

D.2. Equations of Motion

With each sprung mass assumed to possess five degrees of freedom and each unsprung mass assumed to possess two degrees of freedom, the number of differential equations required to describe the directional and roll behavior of a multiple-articulated vehicle is given by

$$k = 5n + 2m$$

where n = number of sprung masses m = number of unsprung masses

D.2.1 <u>Equations of Motion for the Sprung Masses</u>. Application of Newton's laws of motion leads to five equations for each of the sprung masses possessed by the assumed vehicle system, viz.:

Lateral Force Equation:

$$m_{s}\dot{v}_{s} - m_{s}(p_{s}w_{s} - r_{s}u_{s}) = \Sigma \vec{j}_{s} \text{ component of constraint forces} + \Sigma \vec{j}_{s} \text{ component of the suspension forces} + m_{s}g \sin \phi_{s}$$
(22)

Vertical Force Equation:

$$m_{s}\dot{w}_{s} - m_{s}(q_{s}u_{s} - p_{s}v_{s}) = \Sigma \vec{k}_{s}$$
 component of constraint forces
+ $\Sigma \vec{k}_{s}$ component of the suspension forces
+ $m_{s}g\cos\phi_{s}$ (23)

Rolling Moment Equation:

 $I_{xx_s} \dot{p}_s - (I_{yy_s} - I_{zz_s})q_s r_s = \Sigma$ roll moments from the constraints $+ \Sigma$ roll moments from the suspensions

Pitching Moment Equation:

 $I_{yy}s^{a}s^{-}(I_{zz}s^{-}I_{xx}s^{})p_{s}s^{-}z_{s} = z$ pitching moments from the constraints + z pitching moments from the suspensions

(25)

Yawing Moment Equation:

 $I_{zz}r_{s} - (I_{xx} - I_{yy})_{s}r_{s} = \Sigma$ yawing moments from the constraints + Σ yawing moments from the suspensions

(26)

Note:

In the above equation, the "constraint forces" are the forces which arise at the points of connection between adjacent sprung masses. The "suspension forces" are defined as the forces acting between an axle and the sprung mass.

D.2.2 <u>Equations of Motion for the Unsprung Masses</u>. Two equations can be written for the roll and bounce degrees of freedom possessed by each of the unsprung masses:

$$m_{u_{i}}^{\vec{a}} \cdot \vec{k}_{u_{i}} = \vec{k}_{u_{i}} \text{ component of suspension forces} + \vec{k}_{u_{i}} \text{ component of the tire forces} + m_{u_{i}}^{g} \cos \phi_{u_{i}}$$
(28)

In order to evaluate the right-hand side of Equations (22) through (28), the forces produced by the suspension, hitching mechanisms and tires need to be determined. The manner in which these forces are treated is outlined in the following sections.

D.3. Suspension Forces

Each suspension is assumed to consist of a pair of linear springs and linkages which establish a roll center, R_i . Figure D.7 is a schematic diagram showing that the suspension springs are assumed to remain parallel to the \vec{k}_{u_i} axis of the unsprung mass, and are capable of transmitting either compressive or tensile forces only. All roll plane forces which are perpendicular to the suspension springs are assumed to act through the roll center, R_i . The roll center, R_i , is located at a fixed distance, 2R_i , beneath the sprung mass, and is permitted to slide along the \vec{k}_{u_i} axis of the unsprung mass.

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Figure D.7. Suspension and tire forces at each axle.

Figure D.7 shows that the suspension forces transmitted to the sprung mass from any given axle, i, are

$$F_{susp_{i}} = F_{R_{i}} \dot{J}_{u_{i}} - (F_{i1} + F_{i2}) \dot{K}_{u_{i}}$$
(29)

The suspension forces can be defined in the sprung mass coordinate system by applying the coordinate transformation expressed by Equation (21). Upon applying the transformation, we get

$$F_{susp_{i}} = [-F_{R_{i}} e_{s} \sin \phi_{u_{i}} + (F_{i1} + F_{i2}) e_{s} \cos \phi_{u_{i}}]_{i_{s}}^{\dagger} + [F_{R_{i}} \cos(\phi_{s} - \phi_{u_{i}})]$$

- $(F_{i1} + F_{i2}) \sin(\phi_{s} - \phi_{u_{i}})]_{j_{s}}^{\dagger} - [F_{R_{i}} \sin(\phi_{s} - \phi_{u_{i}})]$
+ $(F_{i1} + F_{i2}) \cos(\phi_{s} - \phi_{u_{i}})]_{k_{s}}^{\dagger}$ (30)

The compressive or tensile forces, F_{ij} , produced by the suspension springs are calculated using a suitable suspension spring model. On the other hand, the force, F_{R_i} , acting through the roll center, R_i , is an internal force which can be eliminated by inspecting the dynamic equilibrium of the axle in the \vec{j}_{U_i} direction. Upon writing the equation for the lateral equilibrium of the axle, and rearranging, we get:

$$F_{R_{i}} = -m_{u_{i}} [\vec{a}_{u_{i}} \cdot \vec{j}_{u_{i}}] + \sum_{j=1}^{4} F_{y_{ij}} \cos \phi_{u_{i}} - \sum_{j=1}^{4} F_{z_{ij}} \sin \phi_{u_{i}} + m_{u_{j}} g \sin \phi_{u_{i}}$$
(31)

Of the terms in the right-hand side of (31), the only unknown is the acceleration, $\vec{a}_{m_{u_i}}$, of the unsprung mass. Since the acceleration of the unsprung mass can be defined relative to the sprung mass to which it is attached, it can be written as:

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$$\vec{a}_{m_{u_{i}}} = \vec{a}_{m_{s}} + \vec{a}_{R_{i}}/m_{s} + \vec{a}_{m_{u_{i}}/R_{i}}$$
 (32)

where $\vec{a}_{m_{s}}$ is the acceleration at the c.g. of the sprung mass $\vec{a}_{R_{i}/m_{s}}$ is the relative acceleration at the roll center, R_{i} , swith respect to the sprung mass c.g.

and \vec{a}_{m_u/R_i} is the relative acceleration at the c.g. of the i_i axle with respect to the roll center, R_i .

We shall now derive expressions for each of the three terms in the right-hand side of (32).

The acceleration of the sprung mass along the body-fixed coordinates $(\vec{i}_s, \vec{j}_s, \vec{k}_s)$ is given by:

$$\vec{a}_{m_{s}} = (\dot{u}_{s} + q_{s}w_{s} - r_{s}v_{s})\vec{i}_{s} + (\dot{v}_{s} + u_{s}r_{s} - p_{s}w_{s})\vec{j}_{s} + (\dot{w}_{s} + p_{s}v_{s} - q_{s}u_{s})\vec{k}_{s}$$
(33)

Since the roll center, R_i , is at a fixed distance from the sprung mass c.g., the acceleration of R_i with respect to the sprung mass c.g. (\vec{a}_{R_i}/m_e) can be derived as follows:

$$\vec{r}_{R_{i}/m_{s}} = x_{R_{i}}\vec{r}_{s} + z_{R_{i}}\vec{k}_{s}$$
(34)

$$\vec{v}_{R_{i}/m_{s}} = \vec{r}_{R_{i}/m_{s}} = (z_{R_{i}}q_{s})\vec{r}_{s} + (-p_{s}z_{R_{i}} + x_{R_{i}}r_{s})\vec{j}_{s} - x_{R_{i}}q_{s}\vec{k}_{s}$$
(35)

$$\vec{a}_{R_{i}/m_{s}} = \vec{v}_{R_{i}/m_{s}} = [\dot{q}_{s}z_{R_{i}} - x_{R_{i}}q_{s}^{2} + p_{s}r_{s}z_{R_{i}} - x_{R_{i}}r_{s}^{2}]\vec{r}_{s}$$
(35)

$$\vec{a}_{R_{i}/m_{s}} = \vec{v}_{R_{i}/m_{s}} = [\dot{q}_{s}z_{R_{i}} - x_{R_{i}}q_{s}^{2} + p_{s}r_{s}z_{R_{i}} - x_{R_{i}}r_{s}^{2}]\vec{r}_{s}$$
(35)

$$\vec{a}_{R_{i}/m_{s}} = \vec{v}_{R_{i}/m_{s}} = [\dot{q}_{s}z_{R_{i}} - x_{R_{i}}q_{s}^{2} + p_{s}r_{s}z_{R_{i}} - x_{R_{i}}r_{s}^{2}]\vec{r}_{s}$$
(36)

ŝ

The third term in Equation (32), \vec{a}_{m_u/R_i} , can be derived along the same lines as \vec{a}_{R_i/m_s} , viz.: u_i

$$\vec{r}_{\mathfrak{m}_{i}}/R_{i} = z_{u_{i}}\vec{k}_{u_{i}}$$
(37)

$$\vec{v}_{m_{u_i}/R_i} = \vec{r}_{m_{u_i}/R_i} = \dot{z}_{u_i}\vec{k}_{u_i} - p_{u_i}z_{u_i}\vec{j}_{u_i}$$
 (38)

$$\vec{a}_{m_{u_{i}}/R_{i}} = \vec{v}_{m_{u_{i}}/R_{i}} = \vec{z}_{u_{i}}\vec{k}_{u_{i}} - (\dot{p}_{u_{i}}z_{u_{i}} + 2p_{u_{i}}\dot{z}_{u_{i}})\vec{j}_{u_{i}} - p_{u_{i}}^{2}z_{u_{i}}\vec{k}_{u_{i}} + p_{u_{i}}r_{u_{i}}z_{u_{i}}\vec{u}_{u_{i}}$$
(39)

Hence, combining Equations (33), (36), and (39) and transforming the acceleration defined in the sprung mass coordinate system to the unsprung mass coordinate system, we get:

$$\vec{a}_{m_{u_{i}}} \cdot \vec{j}_{u_{i}} = -(\dot{u}_{s} + q_{s}w_{s} - r_{s}v_{s} + \dot{q}_{s}z_{R_{i}} - x_{R_{i}}q_{s}^{2} + p_{s}r_{s}z_{R_{i}}$$

$$- x_{R_{i}}r_{s}^{2})\theta_{s}\sin\phi_{u_{i}} + (\dot{v}_{s} + u_{s}r_{s} - p_{s}w_{s} - \dot{p}_{s}z_{R_{i}}$$

$$+ x_{R_{i}}\dot{r}_{s} + z_{R_{i}}q_{s}r_{s} + x_{R_{i}}q_{s}p_{s})\cos(\phi_{s}-\phi_{u})$$

$$- [\dot{w}_{s} + p_{s}v_{s} - q_{s}u_{s} - p_{s}^{2}z_{R_{i}} + x_{R_{i}}r_{s}p_{s}$$

$$- z_{R_{i}}q_{s}^{2} - x_{R_{i}}\dot{q}_{s}]\sin(\phi_{s}-\phi_{u}) - \dot{p}_{u_{i}}z_{u_{i}} - 2p_{u_{i}}\dot{z}_{u_{i}}$$
(40)

On substituting the right-hand side of (40) for the term $(\vec{a}_m - \vec{j}_u)$ in Equation (31), we get the following result for F_{R_i} :

$$F_{R_{i}} = -m_{u_{i}} \{-[\dot{u}_{s} + q_{s}w_{s} - r_{s}v_{s} + \dot{q}_{s}z_{R_{i}} - x_{R_{i}}q_{s}^{2} + p_{s}r_{s}z_{R_{i}} + x_{R_{i}}r_{s}^{2}]\theta_{s}\sin\phi_{u_{i}} + (\dot{v}_{s} + u_{s}r_{s} - p_{s}w_{s} - \dot{p}_{s}z_{R_{i}} + x_{R_{i}}\dot{r}_{s} + z_{R_{i}}q_{s}r_{s} + x_{R_{i}}q_{s}p_{s}]\cos(\phi_{s}-\phi_{u_{i}}) - (\dot{w}_{s} + p_{s}v_{s} - q_{s}u_{s} - p_{s}^{2}z_{R_{i}} + x_{R_{i}}r_{s}p_{s} - z_{R_{i}}q_{s}^{2} - x_{R_{i}}\dot{q}_{s}]\sin(\phi_{s}-\phi_{u_{i}}) - \dot{v}_{u_{i}}z_{u_{i}} - 2p_{u_{i}}\dot{z}_{u_{i}}\} + \sum_{j=1}^{4} F_{y_{ij}}\cos\phi_{u_{i}} - \sum_{j=1}^{4} F_{z_{ij}}\sin\phi_{u_{i}} + m_{u_{i}}g\sin\phi_{u_{i}}$$

$$(41)$$

D.4. Constraint Equations

The differential equations which describe the motion of the sprung mass (Equations (22)-(26)) contain terms which are related to the forces and moments which arise at the points of connection between the various sprung masses. These forces and moments can be determined from the kinematic equations which define the constraints. Alternatively, the constraint forces and moments could be evaluated by considering the coupling mechanisms to be compliant such that the forces/moments transmitted through the coupling becomes a function of the relative displacement (linear and angular) between the lead and trailing elements of the coupling mechanism.

Examination of the geometric configuration of the coupling units used on heavy-duty trucks and the structures to which they are attached indicates that these coupling elements are relatively rigid with respect to translation but relatively compliant with respect to rotation. Accordingly, two different schemes were adopted to represent the constraint forces/moments that appear on the right-hand side of Equations (22) through (26). Specifically, the forces were determined from kinematic expressions which state, in effect, that the acceleration at a coupling point is the same for both the lead and the trailing units of the coupling. The moments, on the other hand, were calculated as a function of the angular displacement of the lead and trailing elements of a given coupling mechanism.

Four particular coupling mechanisms were of interest. These mechanisms are diagrammed in Figure D.8. Note that the fifth wheel and the inverted fifth wheel arrangement permit the lead and the trailing units to yaw and pitch with respect to one another, but are stiff in roll. The kingpin-type connection permits only yaw motions. In the case of the pintle hook connection, the trailing unit is permitted to roll, bounce, yaw, and pitch with respect to the lead unit, the only constraint being that which requires the lateral position of the lead coupler to be the same as the lateral position of the forward end of the drawbar.

Let us consider, first, the procedure by which the unknown constraint forces can be determined on the basis of the kinematic conditions which must be satisfied. If the set of k second-order differential equations of motion corresponding to n sprung masses and m unsprung masses are written in matrix notation (recall that k = 5n + 2m), we obtain:

$$M \ddot{x} = \ddot{y} + N \dot{f}_{c}$$
(42)

where

M is the inertia matrix of size k×k
is a vector of the first derivative of the motion variables of size k:
[(v_i, w_i, r_i, q_i, p_j) i=1,...,n, (p_{ui}, z_{ui}) i=1,...,m
j is a vector of size k, which is a function of x, x and the dimension of the vehicle
f_c is a vector of j unknown constraint forces and moments
N is a matrix of size k×k which is a function of vehicle dimensions and x.



CONVENTIONAL FIFTH WHEEL



INVERTED FIFTH WHEEL



KING PIN



PINTLE HOOK

Figure D.8. Schematic diagrams of four coupling mechanisms commonly used on commercial vehicles.

The kinematic constraints that exist at the various connecting points, when written as a set of acceleration constraint equations, are of the form:

$$C \ddot{\vec{x}} = \vec{d}$$
(43)

where

$$\vec{x} = M^{-1}\vec{y} + M^{-1}N\vec{f}_{c}$$
 (44)

On substituting Equation (44) in (43), we get

$$C M^{-1} \dot{y} + C M^{-1} N \dot{f}_{c} = \dot{d}$$
 (45)

which, on being solved for the constraint forces, yields:

$$\vec{f}_{c} = [C M^{-1}N]^{-1} \{\vec{d} - C M^{-1}\vec{y}\}$$
 (46)

The set of differential equations given by Equation (42) can now be solved by substituting Equation (46) back into (42). Upon doing so, we obtain:

$$\ddot{\vec{x}} = M^{-1}\vec{y} + M^{-1}N[C M^{-1}N]^{-1} \{\vec{d} - C M^{-1}\vec{y}\}$$
(47)

Since all the terms in the right-hand side of (47) are known, Equation (47) can be integrated by any standard integration subroutine. Each of the four connections considered here are single-point constraints. Specifically, there is a point C common to both the lead and the trailing units, about which articulation takes place. (See, for example, Figure D.9.) The required equations of constraint are obtained by equating the acceleration of point C on the lead unit to the acceleration of the same point on the trailing unit.

With reference to Figure D.9, the acceleration of point C on the lead unit is seen to be:



Figure D.9. A single point constraint in which the articulation takes place about point C.

$$\vec{a}_{c} = [\vec{u}_{s_{1}} + q_{s_{1}}w_{s_{1}} - r_{s_{1}}v_{s_{1}} + \dot{q}_{s_{1}}z_{c_{1}} - x_{c_{1}}q_{s_{1}}^{2} + p_{s_{1}}r_{s_{1}}z_{c_{1}}$$

$$- x_{c_{1}}r_{s_{1}}^{2}]^{\vec{i}}s_{1} + [\dot{v}_{s_{1}} + u_{s_{1}}r_{s_{1}} - p_{s_{1}}w_{s_{1}} - \dot{p}_{s_{1}}z_{c_{1}} + x_{c_{1}}\dot{r}_{s_{1}}$$

$$+ z_{c_{1}}q_{s_{1}}r_{s_{1}} + z_{c_{1}}q_{s_{1}}r_{s_{1}} + x_{c_{1}}q_{s_{1}}r_{s_{1}}]^{\vec{j}}s_{1} + [\dot{w}_{s_{1}} + p_{s_{1}}v_{s_{1}}$$

$$- q_{s_{1}}u_{s_{1}} - x_{c_{1}}\dot{q}_{s_{1}} - p_{s_{1}}^{2}z_{c_{1}} + x_{c_{1}}r_{s_{1}}p_{s_{1}} - z_{c_{1}}q_{s_{1}}^{2}]^{\vec{k}}s_{1}$$

$$= a_{1}\vec{i}_{s_{1}} + b_{1}\vec{j}_{s_{1}} + c_{1}\vec{k}_{s_{1}} \qquad (48)$$

The acceleration of the same point in terms of the motion variables of the trailing unit is:

$$\dot{\bar{a}}_{c} = [\dot{\bar{u}}_{s_{2}} + q_{s_{2}}w_{s_{2}} - r_{s_{2}}v_{s_{2}} + \dot{q}_{s_{2}}z_{c_{2}} - x_{c_{2}}q_{s_{2}}^{2} + p_{s_{2}}r_{s_{2}}z_{c_{2}} - x_{c_{2}}q_{s_{2}}^{2} + p_{s_{2}}r_{s_{2}}z_{c_{2}} + x_{c_{2}}\dot{r}_{s_{2}} - x_{c_{2}}r_{s_{2}}z_{c_{2}} + x_{c_{2}}\dot{r}_{s_{2}}z_{c_{2}} - q_{s_{2}}u_{s_{2}} - x_{c_{2}}\dot{q}_{s_{2}}z_{c_{2}} + x_{c_{2}}r_{s_{2}}p_{s_{2}}z_{c_{2}} + x_{c_{2}}r_{s_{2}}p_{s_{2}}z_{c_{2}} - z_{c_{2}}q_{s_{2}}^{2}]\dot{\bar{k}}_{s_{2}} - x_{c_{2}}\dot{q}_{s_{2}}z_{c_{2}} + x_{c_{2}}r_{s_{2}}p_{s_{2}}z_{c_{2}} - z_{c_{2}}q_{s_{2}}^{2}]\dot{\bar{k}}_{s_{2}} - a_{s_{2}}\dot{\bar{k}}_{s_{2}} - a_{s_{2}}\dot{\bar{k}}_{s_{2}} + b_{2}\dot{\bar{j}}_{s_{2}}z_{c_{2}} + c_{2}\dot{\bar{k}}_{s_{2}}$$

Equations (48) and (49) can be equated to each other after transforming the coordinate system of the lead unit to that of the trailing unit, or vice versa.

Referring to Equation (7), we note that:

$$\begin{cases} \vec{i}_n \\ \vec{j}_n \\ \vec{k}_n \end{cases} = [A_{ij}]_1 \{ \vec{i}_{s_1}, \vec{j}_{s_1}, \vec{k}_{s_1} \}^T$$
(50)

But

$$\begin{cases} \vec{i}_{s_{2}} \\ \vec{j}_{s_{2}} \\ \vec{k}_{s_{2}} \end{cases} = [A_{ij}]_{2}^{T} \{\vec{i}_{n}, \vec{j}_{n}, \vec{k}_{n}\}^{T}$$

$$(51)$$

Upon combining Equations (50) and (51), we get:

$$\begin{pmatrix} \vec{i}_{s_{2}} \\ \vec{j}_{s_{2}} \\ \vec{k}_{s_{2}} \end{pmatrix} = [A_{ij}]_{2}^{T} [A_{ij}]_{1}^{T} (\vec{i}_{s_{1}}, \vec{j}_{s_{1}}, \vec{k}_{s_{1}})^{T} = [T_{ij}]^{T} (\vec{i}_{s_{1}}, \vec{j}_{s_{1}}, \vec{k}_{s_{1}})^{T}$$

$$(52)$$

where:

$$\begin{split} T_{11} &= \cos(\psi_{s_2}^{-}\psi_{s_1}) \\ T_{12} &= \sin(\psi_{s_2}^{-}\psi_{s_1})\cos\phi_{s_1}^{-} - \theta_{s_2}^{-}\sin\phi_{s_1}^{+} + \sin\phi_{s_1}^{-}\theta_{s_1}^{-}\cos(\psi_{s_2}^{-}\psi_{s_1})) \\ T_{13} &= -\sin(\psi_{s_2}^{-}\psi_{s_1})\sin\phi_{s_1}^{-} - \theta_{s_2}^{-}\cos\phi_{s_1}^{+} + \cos\phi_{s_1}^{-}\theta_{s_1}^{-}\cos(\psi_{s_2}^{-}\psi_{s_1})) \\ T_{21} &= -\cos\phi_{s_2}^{-}\sin(\psi_{s_2}^{-}\psi_{s_1}) - \theta_{s_1}^{-}\sin\phi_{s_2}^{+} + \sin\phi_{s_2}^{-}\theta_{s_2}^{-}\cos(\psi_{s_2}^{-}\psi_{s_1})) \\ T_{22} &= \cos\phi_{s_1}^{-}\cos\phi_{s_2}^{-}\cos(\psi_{s_2}^{-}\psi_{s_1}) + \sin\phi_{s_1}^{-}\sin\phi_{s_2}^{-} \\ &\quad - \sin\phi_{s_1}^{-}\theta_{s_1}^{-}\cos\phi_{s_2}^{-}\sin(\psi_{s_2}^{-}\psi_{s_1}) \\ + \sin\phi_{s_2}^{-}\theta_{s_2}^{-}\cos\phi_{s_1}^{-}\sin(\psi_{s_2}^{-}\psi_{s_1}) \\ T_{23} &= -\sin\phi_{s_1}^{-}\cos\phi_{s_2}^{-}\cos(\psi_{s_2}^{-}\psi_{s_1}) + \cos\phi_{s_1}^{-}\sin\phi_{s_2}^{-} \\ &\quad - \cos\phi_{s_1}^{-}\cos\phi_{s_2}^{-}\theta_{s_1}^{-}\sin(\psi_{s_2}^{-}\psi_{s_1}) \\ - \sin\phi_{s_1}^{-}\sin\phi_{s_2}^{-}\theta_{s_2}^{-}\sin(\psi_{s_2}^{-}\psi_{s_1}) \\ T_{31} &= \sin\phi_{s_2}^{-}\sin(\psi_{s_2}^{-}\psi_{s_1}) - \cos\phi_{s_2}^{-}\theta_{s_1}^{-} + \cos\phi_{s_2}^{-}\theta_{s_2}^{-}\cos(\psi_{s_2}^{-}\psi_{s_1}) \end{split}$$

$$T_{32} = -\cos \phi_{s_{1}} \sin \phi_{s_{2}} \cos(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{2}} \sin \phi_{s_{1}} + \sin \phi_{s_{1}} \sin \phi_{s_{2}} \theta_{s_{1}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{1}} \cos \phi_{s_{2}} \theta_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{1}} \cos \phi_{s_{2}} \theta_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{1}} \cos \phi_{s_{2}} \theta_{s_{2}} + \cos \phi_{s_{1}} \sin \phi_{s_{2}} \theta_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{1}} \cos \phi_{s_{2}} \theta_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{1}} \cos \phi_{s_{2}} \theta_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{1}} \cos \phi_{s_{2}} \theta_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{1}} \cos \phi_{s_{2}} \theta_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos \phi_{s_{1}} \cos \phi_{s_{2}} \theta_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}})$$

$$(53)$$

After the transformation is applied to Equation (51), one obtains the following constraint equations:

$$b_2 \dot{j}_{s_2} = (a_1 T_{21} + b_1 T_{22} + c_1 T_{23}) \dot{j}_{s_2}$$
 (54)

$$c_2 \vec{k}_{s_2} = (a_1 T_{31} + b_1 T_{32} + c_1 T_{33}) \vec{k}_{s_2}$$
 (55)

Note that Equations (54) and (55) are equivalent to Equation (43) above, since the quantities, a_1 , b_1 , and c_1 , etc., are accelerations. Equations (54) and (55) serve to determine the lateral and vertical forces acting at the coupling point C. In the case of the pintle hook connection, only Equation (54) is required since a constraint force cannot exist in the vertical direction.

D.4.1 <u>Roll and Pitch Moments for a Conventional Fifth</u> <u>Wheel Connection</u>. Figure D.10 presents both the side and rear views of a conventional fifth wheel arrangement. It is observed that the conventional fifth wheel arrangement permits free rotational motions of the trailing unit along the pitch axis, \vec{j}_{S1} , of the lead unit, and along the yaw axis, \vec{k}_{S2} , of the trailing unit. When the two units are in line, the pitch axis, \vec{j}_{S2} , of the trailing unit coincides with the \vec{j}_{S1} axis. Therefore, when the relative yaw angle is zero,



Figure D .. 10. Conventional fifth wheel arrangement.

the trailing unit is free to <u>pitch</u> with respect to the lead unit. When the relative yaw angle between the two units reaches 90 degrees, the roll axis, \vec{i}_{s_2} , of the trailing unit coincides with the pitch axis, \vec{j}_{s_1} , making the trailing unit free to <u>roll</u> with respect to the lead unit.

It is assumed that frictional couples which exist along the \vec{j}_{s_1} and \vec{k}_{s_2} directions are sufficiently small that they can be neglected. Thus, the only constraining moment that can act on the lead unit is a roll moment along the \vec{i}_{s_1} direction. The roll

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compliance which exists both in the tractor and trailer structures and in the coupling device is lumped to constitute the torsional stiffness, K_1 , shown in Figure D.11. A second set of axes $(\vec{i}_{S1}, \vec{j}_{S1}, \vec{k}_{S1})$ affixed to the fifth wheel are also defined in Figure C.11. This axis system has the same yaw and pitch angles as those of the lead unit, but has a different roll angle, ϕ_{S1}^{i} . The



Figure D.11. Representation of the conventional fifth wheel arrangement in the yaw/roll model.

difference in the roll angle $(\phi_{S_{1}}^{*}-\phi_{S_{1}})$ represents the structural compliance. The roll moment acting through the fifth wheel can therefore be expressed as:

$$M_{x_{1}} = K_{1}(\phi'_{s_{1}} - \phi_{s_{1}})$$
(56)

The construction of the fifth wheel is such that the pitch axis, $\vec{j}_{s|}$, is always perpendicular to the yaw axis, \vec{k}_{s2} . In terms of unit vectors, this condition can be written as:

$$\dot{j}'_{s_1} \cdot \dot{k}_{s_2} = 0$$
 (57)

Both $\vec{j}_{\$1}$ and $\vec{k}_{\$2}$ can be expressed in terms of the inertial unit vector $(\vec{i}_n, \vec{j}_n, \vec{k}_n)$ using the transform equation (9). Upon doing so, carrying out the dot product and solving for $\phi_{\$1}^{i}$, we find that Equation (56) can be expressed as

$$M_{x_{1}} = K_{x_{1}} \left\{ \tan^{-1} \left[\frac{\sin\phi_{s_{2}} \cos(\psi_{s_{2}} - \psi_{s_{1}}) - \theta_{s_{2}} \cos\phi_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}})}{\theta_{s_{1}} \sin\phi_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos\phi_{s_{2}}} \right] - \phi_{s_{1}} \right\}$$
(58)

The constraining moments acting on the trailing unit are

$$M_{x_2} = -M_{x_1}T_{11}$$
(59)

and

$$M_{y_2} = -M_{x_1}T_{21}$$
(60)

where T_{11} and T_{21} are defined in Equation (53).

D.4.2 <u>Roll and Pitch Moments for an Inverted Fifth Wheel</u> <u>Arrangement</u>. The inverted fifth wheel is an arrangement in which the lower and upper halves of a conventional fifth wheel coupling are reversed. The inverted fifth wheel arrangement is shown in Figure D.12.

The coupler permits free rotational motion of the trailing unit along the pitch axis, j_{S_2} , of the trailing unit and the yaw axis, k_{S_1} , of the lead unit. Unlike the conventional fifth wheel arrangement, the pitch axis of the inverted coupler is always lined up with the pitch axis of the trailer for all values of articulation angles. The inverted fifth wheel coupling can therefore transmit a roll-resisting moment from the lead unit to the trailing unit for all values of the relative yaw angle between the lead and the trailing



units. In the case of the inverted fifth wheel, the structural compliance in roll is modeled by a torsional spring of stiffness K_{X_1} , oriented along the \vec{i}_{s_2} axis of the trailing unit. Upon carrying out the derivation, we get:

$$M_{x_{2}} = K_{x_{1}} \left\{ \tan^{-1} \left[\frac{\sin\phi_{s_{1}} \cos(\psi_{s_{1}} - \psi_{s_{2}}) - \theta_{s_{1}} \cos\phi_{s_{1}} \sin(\psi_{s_{1}} - \psi_{s_{2}})}{\theta_{s_{2}} \sin\phi_{s_{1}} \sin(\psi_{s_{1}} - \psi_{s_{2}}) + \cos\phi_{s_{1}}} \right] - \phi_{s_{2}} \right\}$$
(61)

The roll and pitch moment acting on the lead unit are given by

$$M_{x_{1}} = -M_{x_{2}}T_{11}$$
(62)

$$M_{y_1} = -M_{x_2}T_{12}$$
(63)

where T_{11} and T_{12} are once again defined in Equation (53).

D.4.3 <u>Roll and Pitch Moments for a Kingpin-Type Connection</u>. In a kingpin-type arrangement, only yaw motion is permitted between the lead and the trailing units. Hence, constraining moments act in both the pitch and yaw directions. The structural compliance is therefore represented by torsional springs, K_{X1} and K_{y1} , along the pitch and roll axes. Shown in Figure D.13 is an axis system $(\vec{1}_{S1}, \vec{1}_{S1}, \vec{k}_{S1})$ which has the same yaw angle, ψ_{S1} , as the lead unit, but different roll and pitch angles, ϕ_{S1}^{i} and θ_{S1}^{i} , respectively. The axis system is so oriented that the k_{S1}^{i} axis is parallel to the \vec{k}_{S2} axis of the trailing unit. Therefore, the vector equations

$$\vec{i}_{s_1} \cdot \vec{k}_{s_2} = 0$$
 (64)

and

$$\dot{j}_{s_1} \cdot \dot{k}_{s_2} = 0$$
 (65)



Figure D.13. Representation of the kingpin-type connection in the yaw/roll model.

have to be satisfied. Equation (64) yields the pitch angle

$$\theta'_{s_1} = \theta_{s_2} \cos(\psi_{s_2} - \psi_{s_1}) + \tan \phi_{s_2} \sin(\psi_{s_2} - \psi_{s_1})$$
(66)

Therefore

$$M_{y_{1}} = K_{y_{1}} (\theta_{s_{1}} - \theta_{s_{1}})$$

= $K_{y_{1}} [\theta_{s_{2}} \cos(\psi_{s_{2}} - \psi_{s_{1}}) + \tan \phi_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) - \theta_{s_{1}}]$ (67)

Equation (65) yields a result which is identical to (58), therefore

$$M_{x_{1}} = K_{x_{1}} \left[\tan^{-1} \left(\frac{\sin\phi_{s_{2}} \cos(\psi_{s_{2}} - \psi_{s_{1}}) - \theta_{s_{2}} \cos\phi_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}})}{\theta_{s_{1}}^{\dagger} \sin\phi_{s_{2}} \sin(\psi_{s_{2}} - \psi_{s_{1}}) + \cos\phi_{s_{2}}} \right) - \phi_{s_{1}} \right]$$
(68)

The constraining moments, $\rm M_{X2}$ and $\rm M_{Y2},$ which act on the trailing unit are now given by

$$M_{x_2} = -M_{x_1}T_{11} - M_{y_1}T_{12}$$
(69)

and

$$M_{y_2} = -M_{x_1}T_{21} - M_{y_1}T_{22}$$
(70)

where T_{11} , T_{12} , T_{21} , and T_{22} are once again defined in Equation (53).

D.5 Forces and Moments at the Tire-Road Interface

In order to obtain a high degree of accuracy in predicting roll/yaw behavior, the computer code utilizes measured tire data for computing the lateral forces and aligning moments generated at the tire-road interface. If the sideslip angle and the vertical load acting on a tire are known, the lateral force and aligning moment can be computed by a linear interpolation of the tabulated tire data. Expressions for the sideslip angle and the vertical load at the tireroad interface will now be derived in terms of the velocities and displacements of the sprung and unsprung masses.

D.5.1 <u>Sideslip Angles</u>. Let us first express the sideslip angle at the tire-road interface in terms of the body-fixed velocities of the sprung mass and the axle. The sideslip angle at the j-th tire on axle i is given by the expression:

$$\alpha_{ij} = \tan^{-1} \left(v_{axle_i} / u_{tire_{ij}} \right) - \delta_i$$
 (71)

where

$$v_{axle_i} = [v_s - z_{R_i} p_s] \cos \phi_s + x_u r_s / \cos \phi_s - p_u HR_i \cos \phi_u e_i$$
(72)

$$u_{tire_{i1}} = u_s + (T_i + A_i)r_s$$
 (73)

$$u_{\text{tire}_{12}} = u_s + T_i r_s \tag{74}$$

$$u_{\text{tire}_{i3}} = u_s - T_i r_s \tag{75}$$

$$u_{tire_{i4}} = u_s - (T_i + A_i)r_s$$
 (76)

(Note that the term δ_i in Equation (71) represents the angle made by the wheel plane with respect to the longitudinal axis of the sprung mass coordinate system.)

D.5.2 <u>Vertical Loads</u>. The vertical compliance in the tires is modeled by linear springs, KT_{ij} . Therefore, if the vertical deflection, Δ_{ij} , at the tire is known, the vertical tire load, $F_{Z_{ij}}$, can be calculated from the expression:

$$F_{z_{ij}} = KT_{ij}^{\Delta}ij$$
(77)

The vertical deflection of the tires can be expressed in terms of the deflection of the sprung and unsprung masses. The deflection of the outer left tire on axle i is given by the equation:

$$\Delta_{i1} = \Delta_{i0} + \Delta z_{s} - z_{R_{i}}(1 - \cos \phi_{s}) + z_{u_{i}} \cos \phi_{u_{i}} - z_{u_{i0}} - (T_{i} + A_{i}) \sin \phi_{u_{i}} - x_{u_{i}} \theta_{s}$$
(78)

where

$$\Delta z_s$$
 is the vertical deflection of the sprung mass c.g.
along the inertial axis k_n
 $\Delta z_s = 0.0$ at time t = 0.0
 z_u_{10} is the vertical distance between the roll center,
 R_i , and the axle c.g. at time t = 0.0
 Δ_{10} is the static deflection of the tires at time
t = 0.0.

The deflection of the other three tires on axle i is given by:

$$\Delta_{i2} = \Delta_{i1} + A_i \sin \phi_{u_i}$$
 (79)

$$\Delta_{i3} = \Delta_{i2} + 2T_i \sin \phi_{u_i}$$
(80)

$$\Delta_{i4} = \Delta_{i3} + A_i \sin \phi_{u_i}$$
 (81)

Equations (78) through (81) yield the vertical load on a given tire which, in combination with Equation (71), permits one to perform a linear interpolation of tabulated tire data to determine the value of lateral force and aligning moment corresponding to the prevailing load and slip angle.

APPENDIX E

VEHICLE PARAMETER SETS

58

CEYENT MIYER 4 AXL'S

DE SPRUNG MASSES	11		
TOTAL # OF AXLES	11	2	
GPOSS VENICLE VEICHT	11	66000.00	1.Fs .
FCTUARP VELOCITY	14	55.00	н. Р. И
OPSH LOOP STEER INPUT			
STEFRING GEAR RATIO			= 7

ΟΡΕΝ Ι.ΟΟΡ STEER ΙΝΡυτ σεσσσσσσσσσσοςοσο		
STEFRING GEAR RATIO	**	25.60
STEERING STIFFNETS (IK-LP/DEC)	в	5000.00
TIS ROP STIFFRESS (IM.LP/CFC)	- 25	00.0003
HECHARICAL TRAIL (TR)	11	1.10
DE POINTS IN SICEP TAPLE	~ "	
TTME 211EE BREEF		
0.0		
1.00 2.00 2.00 2.00		

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CEMENT MIXER 4 AYLOS

UNIT **)** 1 \$\$\$\$\$\$\$\$\$

OF AXLES ON THIS UNIT = N
 WFIGHT OF SPRUNG MASS = €0400.00 L9.

ROLL ROMENT OF INFETIA OF ELIUMS MASS = 100090.00 LF.IN.SECA02

PITCH MORENT OF INFETIA OF STRUNG MASS = 1250900.00 [P.IM.SEC+02

YET HOMENT OF INSETIA OF STRUNG MASS = 125000.00 L9.IN.SCC++2

HEIGHT OF SPRUNG MASS CC AFOVE GROWNE = 70.90 IACHES

	AXLE 1 1 Coscocco	LXLE 2 000000000	AXLE 8 3 070007000	AXIE e a	AXLE B GGGGGGGGGG	*****	000000000000000000000000000000000000000	****
LPAP ON EACH EXLE (LB.)	18000.03	19000.000	19030.00	17006-09				
AXLF VEICHT (LP.)	1509.00	2300.00	2370.00	1500.00				
AVLF POLL 4.1 (LB.14.SEC**2)	9700.00	3700.00	4540.00	4500.09				
X DIST FROM SP MASS CG (IP)	205.33	-15.24	-68.23	-131.00				
HETCHT OF AXLE C.G. ABOVE SPOUND (THCHES)	52.70	00-02	20.00	20.00				
HETCHT OF ROLL CENTER APOVE SPOUND (INCHED)	22+50	29+00	27.05	19-09				
HELF SPEING STACTIG (11)	16.20	19.60	19.00	19.00				
HILE TPACK - INVER TIRES (IN)	37.00	00°1i	17.00	36.09				
(UAL TIFE SPACING (IN)	C•J	0.0	0.0	C•0				
STIFFAESS OF FACH TIRE (LPZIN)	8000.63	900.9008	80.00.00	500C+09				
POLL STREP COFFEICIENT	C•J	ن• 0	0*0	0°J				
AUX FOLL STIFFESS (THLEASE)	c•J	0"3	0.0	Ū• J				
- ALLING CONTON EFFCTION - BFF STATION (AL)	250-00	7.5.00	759-00	7*1.5				
VITCOUS EAMPLIC, FLP SERVIC ATTOOR EAMPLIC, FLP SERVIC	د • ب	ر•0	0.0	а• J				
SEFINS TAPLE .	1	۴.	-	<i>c</i> .				
COPNERING FORCE TARLE .	1	Ţ		ľ				
FLIGHING TOFCUE TABLE .	ł		-	-				

SPRTNG TAPLE # 1 \$\$\$\$\$\$

RELECT ION NCHES	-5,00	5.00	8	relection nches	-10.00	0.0	19.00		NELECTION HCMES	-1.75	-0.75	0.0	•
FORCT LR	-1 0000 - 00	10000-00	SPRTMG TAMLE seesee oseecee	FOPCE I LB I	00.00011-	0.0	1 0000 00	SEPTIC TAPLE . Ocoado cacada	EOPTE D Le	- 75,00 - 0.0	c •0	0.7	

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COPRERING FORCE TABLE | 1

LATFRAL FCRCE VS. SLIP ANGLL

6.00	3600.00	n640.00	5500.00
u. co	3589.00	3260.00	3900 . 00
0C • E	2040.00	2560.00	00°001E
2.00	1440.00	1840.00	2200.60
1.00	780.00	960.00	1200.00
0-0	60.0003	8000.10	10000.00

ALICHING TORQUE TAPLE # 1

ALIGHING TOPQUE VS. SLIP APGLE

6.00	99.0006	11604.00	13752.00
u.c0	01.444.00	03,3619	03.426
00°E	5100.96	6376.30	7752.00
2.00	3500.00	4536.00	10.0642
1.90	00* nnšl	2400.00	3000-00
0.0	£000.00	PP00.00	00.0001

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5 -axle-cement-miyer				
I OF SPRUNG MASSES	0	1		
TOTAL # OF AXLES	н	ŝ		
GROSS VEHICLE VEIGHT	н	6960	00.00	. 81
FORWARD VELOCITY	H	ŗ	5.00	H. P. H
OPEN LOOP STEER INPUT ************************				
STEERING GEAR RATIO				= 25.00
STEERING STIFFNESS (IN	.LP/	reg)		= 25000.00
TIE ROD STIFFHESS (IN.	1.P./D	EC)	и	25000.00
MECHANICAL TRAIL (IN)			Ħ	1.00
DE POINTS IN STEER T	APLE			
TIME STEER SEC D 0.0 0.0 1.00 25.00 20.00 255.00	ING	53 133H		

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5-AXLE-CEMENT-MIXER

UNIT # 1

OF AXLES ON THIS UNIT = 5 WEIGHT OF SPRUNG MASS = 61000.00 LR. ROLL MOMENT OF INERTIA OF SPFUNG MASS = 100000.00 LB.IN.SEC**2 PITCH MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2 YAW MOMENT OF INERTIA OF SPRUNG MASS = 600000.00 LB.IN.SEC**2 HEIGHT OF SPRUNG MASS CG ABOVE GROUND = 71.70 INCHES

LOAD ON EACH AXLE (LB.)	18000.00	9600.00	18000.00	18000.00	6000.00
AXLE WEIGHT (I.P.)	1500.00	1500.00	2300.00	2300.00	1000.00
AYLF ROLL H.I (LB.111.SEC**2)	3700.00	3700.00	4500.00	4500.00	3000.00
X DIST FROM SP MASS CG (IN)	140.70	14.90	-30.10	-80.10	-143.10
HEIGHT OF AXLE C.G. ABOVE Ground (inches)	21.00	20.00	29.00	20.00	15.00
HEIGHT OF ROLL CENTER ABOVE Ground (Inches)	22.00	22.00	29.00	29.00	17.00
HELE SPRING FRACING (IN)	16.00	19.00	19.00	19.00	19.00
HELE TRACK - INNER TIRES (IN)	49.50	42.00	29.00	27.00	36.00
DUAL TIPE SPACING (TN)	e.,	0.0	13.00	13.00	0.0
STIFFNESS OF FACH TIRE (LB/IN)	8000.00	5000.00	5000.00	5000+00	5000.00
POLL STEER COPFFICIENT	0.0	0.0	0.0	0.0	0.0
AUX POLL STIFFNESS (IN.LE/DEG)	0.0	100000.00	0.0	0.0	10000.00
SERING COULOPE ERICTION - PER SERING (LP)	250.00	250.00	750,00	750.00	250.00
VISCOUS CAMPING PER SPRING (LP+STC/IN)	o.⊓	0.0	0.0	0.0	0.0
SEPING TABLE .	1	2	3	٦.	2
COPNERING FORCE TABLE #	1	2	2	2	2
ALIGHING TOPOUE TABLE .	1	2	2	2	2

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SPRIAG TAFLE 1
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 CORCE
 DEFLECTION

 LB
 TACNES

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 SPRING TABLE 1
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 SPRING TABLE 2
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 -10.00

 1 0000.00
 -10.00

 1 0000.00
 10.00

-

STRTHG TABLE (3 ****** ******* FOPCE DEFLECTION LB INCHES -7500.00 -1.75 0.0 -0.75 0.0 0.0

1.00

7500.00

CORPERING FORCE TABLE # 1 ACCORDON BOOMD ACCORDING LATFRAL FORCE VS. SLIP ANGLL

6.00	3600.00	4640.00	5500.00
4.00	2580.00	3280.00	00°006E
00°E	2040.00	2560.00	3100.00
2.00	1440.00	1840.00	2200.00
1.00	7A0.00	960.00	1200.00
0.0	6000.000	A000.00	10000.00

CORNTRING FORCE TAPLE # 2 coccosco scoss coscosses LATEPAL FORCE VS. SLIP ANGLL

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	6.00	1440.00	2400-00	3360.00
	00 ° N	1020.00	1720.00	2400.00
	3.00	820.00	1360.00	1920.00
ACLL	2.00	580.00	960.00	1360.00
VS. SLIP A	1.00	300-00	520.00	720.00
IFFAL FURLE	0.0	2000-00	000.000	P000.00

ALICHING TORQUE TARLE | 1 00000000 000000 00000000 ALICHING TORQUE VS. SLIP AMGLE

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00	00	00	00
		n196.	1756.
3.00	5100.00	6396.00	7752.00
2.00	3690.00	8596 . 00	00.3948
1.00	00° un61	2400.00	00.0016
0.0	6000.003	P000.00	1 0000 .00

6.00

9000.00 11604.00 13752.00

6.00	1728.00	2880.00	00.5604
11.50	1224.00	0」、หงาวะ	7880.00
06-6	00.000	1632.00	00.0CES
2.00	00*969	1152.00	1632.00
1.00	360.00	624 . 00	26a.00
0.0	2000.00	000 000 0	00.000

CEMENT MIXER 4 AXLES- TAG STFERING

# OF SPRUNG MASSES	н	1	
TOTAL # OF AXLES	11	1	
GROSS VEHICLE WEIGHT	ti	68000.00	LB.
FORWARD VELOCITY	11	55.00	н.р.н
OPEN LOOP STEFR INPUT			
STEERING GEAR RATIO			"

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STEERING GEAR RATIO	*1	25.00
STEERING STIFFNESS (IN.LB/DEC)	H	25000.00
TIE ROD STIFFNESS (IN.LR/DEG)	u	25000.00
MECHANICAL TRAIL (IN)	H	1.00
I OF POINTS IN STEER TABLE	11	E
TIME STEERING WHEEL SEC DEGREES		
1.00 20.00 6.00 20.00		

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CEMENT MIXER 4 AXLES- TAC STRERING

UNIT # 1 000000000

1 OF AXLES ON THIS UNIT = 4

WEIGHT OF SPRUNG MASS = 60400.00 LB.

PITCH MOMENT OF INEFTIA OF SPRUNG MASS = 1250000.00 LB.IN.SEC**2 ROLL MOMENT OF INERTIA OF SPPUNG MASS = 100000.00 LB.IN.SEC**2 YAW MCHENT OF INERTIA OF SFRUNG MASS = 1250000.00 LD.IN.SEC++2

70.90 INCHES HEIGHT OF SPRUNG MASS CG APOVE GROUND =

	AXLE 8 1 000000500	AXLE 2 444444444	AXLE # 3 44444444	AXLE 4 ******	AXLE vototta	******	*****	****
LOAD ON EACH AXLE (LB.)	18000.00	19000.00	19000.00	12000.00				
AXLE WEIGHT (LB.)	1500.00	2300.00	2300.00	1500.00				
AXLE ROLL M.I (LB.IN.SEC**2)	3700.00	3700.00	4500.00	4500.00				
X DIST FROM SP MASS CG (IN)	205.31	-15.24	-68.28	-192.72				
HEICHT OF AXLE C.G. ABOVE GPOUND (INCHES)	21.00	20-00	20.00	20-00				
HEIGHT OF ROLL CENTER ABOVE Ground (Inches)	22-00	29.00	00°62	29.00				
HALF SPRING SPACING (IN)	16.00	19.00	19.00	19.00				
HILF TRACK - INNER TIRES (IN)	37.00	37 . 00	37.00	36.00				
DUAL TIRE SPACING (IN)	0.0	0.0	0.0	0.0				
STIFFNESS OF FACH TIRE (LB/IN)	8000.00	000 000	8000.00	5000.00				
ROLL STEER COCFFICIENT	0.0	0.0	0.0	0.0				
AUX ROLL STIFFNESS (IN.LD/DEG)	0.0	0.0	0.0	0.0				
SFRING COULOME FRICTION - PER SPRING (LE)	250.00	750.00	750.00	750.00				
VISCOUS DAMPING PER SPRING (LB.SEC/IN)	0.0	0.0	0.0	0.0				
SPEING TABLE 0	1	(T)	Ē	2				
CORNERING FORCE TABLE \$	1	*1	*1	2				
FLIGNING TORQUE TABLE #	T	Ł	1	7				

-68

SPRING TABLE	♥ 1 \$*
FORCE LB	DEFLECTION INCHES
-10000.00	-5.00
10000.00	5.00
SPRING TABLE ****** *****	₿ 2 \$*
FORCE LB	DEFLECTION INCHES
-10000.00	-10.00
0.0	0.0
10000.00	10.00

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SPRING TABLE # 3

FORCE LB	DEFLECTION INCHES
-7500.00	-1.75
0.0	-0.75
0.0	0.0
7500.00	1.00

CORNERING FORCE TABLE | 1

LATERAL FORCE VS. SLIP ANCLL

0-0	1.00	2.00	3°00	4 . 00	6.00
60.00.00	780.00	1440.00	2040.00	2580.00	3600.00
8000.00	00.096	1940.00	2560.00	3280.00	4640.00
1000.00	1200.00	2200.00	3100.00	00°006E	5500.00
CORVERING FOR	(CE TABLE # 2				
		L			
LATEPAL FORCE	: VS. SLIP AN	1151			
0*0	1.00	2.00	00°E	tt - 00	6 • 00
	C - C	0-0	0.0	0.0	0.0

•

0.0	1.00	2.00	00°E	4.00	6.00
2000.00	0.0	0.0	0.0	0.0	0.0
4000.00	0.0	0.0	0.0	0.0	0.0
6000.00	0.0	0.0	0.0	0.0	0.0

ALIGNING TORQUE TABLE | 2 ******** ******

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ALICNING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	00 ° E	4.00	6.00
2000.00	0.0	0.0	0.0	0.0	0.0
000.000	0.0	0.0	0.0	0.0	0.0
6000.00	0.0	0.0	0.0	0.0	0.0

ALIGHING TORQUE TABLE # 1 ********

ALICHING TORQUE VS. SLIP ANGLE

6 • 00	000.0006	11604.00	13752.00
4.00	00 * ###9	0196.00	9756.00
0C*E	5100.00	6396.00	7752.00
2.00	3600.00	4596.00	5496.00
1.00	1944.00	2400.00	3000-00
0.0	6000.00	8000.00	10000.00

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3-AXLE EMPTY DUMP TRUCK

OF SPRUNG MASSES	ŀ	3		
TOTAL # OF AXLES	h	E		
GROSS VENICLE WEIGHT	Ð	27000.00	LB	
FORWARD VELOCITY	#1	55.00	ŗ	Н• 4
OPEN LOOP STEER JUPUT *********************				
STEFRING GEAR RATIO			H	2
STEERING STIFFNESS (IN.	LP/	DEC)	н	25.00
TIT ROD STIFFNESS (IN.)	8 / D	501	н Н	5000

•

STEFRING GEAR RATIO	= 25.	8
STEERING STIFFNESS (IN.LP/D	0EC) = 25000°	ę
TIE ROD STIFFNESS (IN.LA/DE	EG) = 25000.0	c
MECHANICAL TRAIL (IN)	= 1.0	с
OF POINTS IN STEEP TARLE	m H	
TIME STEEPING W	MF EL	
SEC DEGREE 0.0	S	
1.00 25.00 20.00 225.00		

						000000000 00000																	
						10000 0000																	
		00.00 LP.IN.SFC**2	00.00 LB.IN.SEC**2	.00 L8.IN.SEC**2	0.00 THCHES	(1.5 e •••••••••																	
T 8 1 000000	0 L.A.	MASS = 2000	MASS = 4000	ASS = 40000	= 01100	AXLE 1 3 A) 000000000 0x	0200-00	2300.00	4500.00	-110.70	19.00	29.00	19.00	29.50	13.00	5000.00	0.0	0.0	500.00	J * 0	~	l	-
001 ***	= 21200.0	A OF SPPUNG	IA OF STRUNG	OF SPRING M	CC ABOVE GR	AXLE 0 2 00000000	00.0028	2300-00	4500.00	-66.70	19.00	29.00	19.00	29.50	13.00	£000 • 00	0.0	0.0	500.00	0.0	2	ſ	Ŧ
INN SIHI NO	SPRUNG HASS	T OF INEPTIN	NT OF INCRT	OF INFRTIA	SPRUNG MASS	AXI.E 1 000000000	10000-00	1200.00	00-0016	126.30	01-61	23-00	17.00	40.50	0.0	5000.00	0.0	U-J	200-002	G • J	ſ	l	1
B OF AXLES	UEICHI OF	ROLL NOMEN'	PITCH MOME	YAY MOHENT	HEICHT OF		LEAD ON FACH AXLE (L.B.)	AYLF WEICHT (LR.)	AYLF FOLL M. J (LB. IM. SFC**2)	Y DIST FROM SP MASS CG (IN)	HFICHT OF AXLE C.G. ABOVE GPCUND (TNCHES)	HETCHT OF POLL CENTER ABOVE CPOUND (TRCHES)	WHILE SPRING SPACING (IN)	HPLF TRACK - THNER TIGES (IH)	CPAL TIPE SPACING (IN)	STIFFNESS OF FACH TIRE (LEVIN)	POLL STFER COEFFICIENT	AUY FOLL STIFFUESS (IN.LP/PFG)	EFRIME COULOPE EPICTION - PER SPRIME (LE)	VIECCUS LAMEINC PER SPRING (LA.SEC/IN)	SELLES TARLE .	COPYFRING FORCE TARLE .	FLIGHING TOPQUE TARLE .

3-AXLE EMPTY DUMP TPUCK

<pre>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</pre>	-5+00	5.00	<pre> 2 DEFLECTION INCHES</pre>	-10.00	-1.00	0.0	10.00
SIRTNG TABLE 444444 44444 FORCE LB	-6000.00	600.000	SPRING TABLE 44444 44044 Force LB	-20000-00	0.0	0.0	6 0000 • 0 0

0.0	1.00	2.00	3.00	4.00	6.00
2000.00	300.00	580.00	820.00	1020.00	1440.00
00.0000	520.00	360.00	1360.00	1720.00	2400.00
r009.00	720.00	1360.00	1920.00	2400+00	3360.00

6.00	1728.00	2880.00	00.5604
0J * U	1224.00	00*1902	2480.00
3.00	00.480	00.5131	2304.00
2.00	00.762	11 . 2 . 00	03.5531
1.00	00°09E	624.00	F64.00
0.0	2000.00	00.000	A000.00

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5-AXLE-DIRT-TRUCK		
DOF SPRUNG MASSES		
TOTAL # OF AXLES	11 I	
GROSS VEHICLE WEIGHT	= 70070.00	LB.
FORMARD VELOCITY	= 55.00	н. Р. н
OFSN LOOP STEER INPUT *************************		
STEERING GEAR RATIN		= 25.00
STEERING STIFFNESS (IN	.LB/DFG)	= 25000.00
TIE ROP STIFFRESS (IN.)	LB/DEG)	= 25000.00
HECHANICAL TRAIL (IN)		= 1.00
OF POINTS IN STEER TO	ABLE	
TIME STEER	INC WHEEL	
	chero.	
1.00 25.00		
0244 27 DD427		

S TATES	NI SIHI NO S	UN 11 = 5 #3	11 8 11 \$\$\$\$\$\$					
VEICHT OF	SPRUNG MASS	- 60900.	00 LB.					
LOLL MOMEN	IT OF INERTI	A OF SPRUNG	: MASS = 10	0 000 ° 000 FE	•IN.SEC##2			
PITCH NOME	INT OF INERT	TA OF SPRUM	IG MASS = 6	00000.00	.B.IN.SECot2			
YA4 HCMENT	OF INERTIA	OF SPRUNG	MASS = 600	000.000 Ln.	IN.SEC##2			
HEIGHT OF	SPRUNG MASS	CC RPOVE C	= UNI:04:	78.30 IN	ICHES			
	AXLE # 1 000000000	AXLE 2 000000000	AXLE 4 3 000000000	AX1.E 8 4 00000000	AXLE 0 5 00000000	AXLF 8 ¢¢¢¢¢¢¢¢	*****	****
LOAP ON EACH AXLE (LB.)	18000.00	13000.00	13000.00	1 3000.00	13000.00			
AYLF VEIGHT (LD.)	1500.90	1500.00	2300.00	2300-00	1500.00			
AXLF FOLL M.I (LB.IN.SEC**2)	3700.00	00.007E	4500.00	4500.00	3700.00			
X DIST FROM SP MASS CG (IN)	137.80	21.80	-24.20	-78.20	-124.20			
HEIGHT OF AXLE C.G. ABOVE Gpound (Inches)	21.00	20.00	20.00	20.00	20.00			
HFIGHT OF RJLL CENTER AROVE Gpound (Inches)	22.00	22.00	29.00	29.00	22.00			
WELF SPPING SPACING (IN)	16.00	19.00	19.00	19.00	19.00			
HELE TPACK - INNER TIRES (14)	40.25	29.00	29.00	29.00	29.00			
DUAL TIPE SPACING (IN)	0° U	13.00	13.00	13.00	13.00			
STIFFNESS OF FACH TIRE (LE/IN)	00008	5000.00	5000.00	¢000.00	5000-00			
POLL STEER COLFEICIENT	6.0	0° Û	0.0	0°U	0.0			
AUX POLL STIFFNESS (IN.L.P.PEG)	G•v	100000.00	0.0	0.0	100000.00			
SFRING COULOPE FRICTION - PFP SPPING (LP)	250-00	250.00	750.00	750.00	250.00			
VISCOUS DAMPING PER SPRING (RF-SEC/14)	0.0	0.0	0.0	0.0	0.0			
SPETUS TARLE .	ſ	•	ć	2	e			
COPREPING FONCE TRALE	ſ	~	2	۲	2			
PLIGHING TOFCUE TABLE	I	2	ć	~	۲			

5-AXLE-DIRT-TRUCK

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DEFLECTION THCHES SPRIFIC TAPLE 0 2 ******* ******* FOPCF DEFLECTION LB INCHES -1.75 7.13 -5.00 7.25 -0.75 0.0 1.00 SPRING TARLE 0 1 000000 0000000 FORCE DEFLEC LD TNCHES -10000.00 14250.00 4 0000 ° 00 -7500.00 7500.00 0.0 0.0 .

SEPTHG TABLE # 3 ****** ******* FOPCF CFEECTION LP INCHES -1000.00 -10.00 1000.00 10.00

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6.00	3600 . 00	4640.00	5500.00
4.00	2580.00	3280.00	00*006E
3.00	2040.00	2550.00	3100.00
2.00	1440.00	1940.00	2200.00
1.00	780.00	00°095	1200.00
0.0	6000.000	8000.00	1 0000 .00

CORNERING FORCE TAPLE 1 2 concessor corrected Lateral force vs. SLIP Angle

	6 • 00	1440.00	2400.00	00°09EE
	00.4	1020.00	1720.00	2400.00
	3.00	e 20.00	1360.00	1920.00
7744	2.00	590.00	960.00	1360.00
	1.00	300.00	520.00	720.00
	0.0	2000-00	000.000	A000.000

ALICHING TOPQUE TABLE # 1 ******** ****** ********** ALICHING TOPQUE VS. SLIP ANGLE

	-				
11604.00	8196.00	6376.00	a596.00	2400.00	PC00-00
000.000	6444.00	5190.00	3600.00	00° an <u>ó</u> I	£000.00
6.00	a. no	3.00	2.00	1.00	0.0
				•	

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0.0	1.00	2.00	3.00	0 .	6.00
2000.00	360.00	00-363	994.00	1224.00	1728.00
00.000#	624.00	1152.00	00.2631	206 H + 00	2880.00
000.000	864.00	1632-00	2374.00	2800-00	00.25.00

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τοαντααπουσταστήτα από το ποτοστάσο ΦΠΡΕΓΤΙΟΝΑL RESTCHSE SIMULATIONΦ πααστάστροσοσασσήτερη στοστάσοσο

3-AXLE-DUMPSTER

SPRUNG HASSES	IJ	1	
UF AXLES	11	* 1,	
ENICLE VETCHT	11	E8000-00	LB.
VELOCITY	4	55.00	M. P. I

FORMARD VELOCITY =	55.00	M. P. H
open Loop steer input 203034988888888888888		
STEERING GEAR RATIO		= 25.00
STEERING STIFFNESS (IN.LP/DEC	6	= 25000 . 00
TIE ROD STIFFNESS (TH.LP./PFG)	H	25000 .00
MECHANICAL TRAIL (TN)	14	1.00
OF POINTS IN STEEP INPLE		f =
TIME STFFPINC WH	EL	

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SILL FLUE OF	DEGPEES	0° ú	25.00	225.00
1 1 11	550	0.0	1.00	20.00

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UNIT | 1 700000000

ΛΧΙΕ # σειφορείας φοροσόφο φασφορικο φοροσοκο κοικροσφο FITCH POHENT OF INFRITA OF STRUNG MASS = 331790.00 LB.IN.SEC0#2 ROLL MOMENT OF INERTIA OF SPRUNG MASS = 100856.00 LB.IN.SEC3#2 YAU MOMENT OF INERTIA OF SPRUNG MASS = 331799.00 LB.IN.SEC402 RI.40 INCHES -HEIGHT OF SPRUNG MASS CG APOVE GROUND = 51900.00 LA. m • OF AYLES ON THIS UNIT = **WEIGHT OF SPRUNG MASS =**

ć	~	1	ALIGHING TOPCUE TAPLE
2	~	L	COP4FR185 FOPCE TARLE .
2	~	1	SFFIRE DATA DATA
0.0	0.0	0°u	VISCOS PAMEING PER SEPTING (LB+SEC/TH)
150.00	750.00	250-00	- NELLONE ENTERION - FER SPEING (LP)
0.0	0.0	0.0	AUY FOLL STIFFNESS (IN.LR/PEG)
0.0	0.0	0.0	FOLL STEER COFFEICTENT
5000-00	£000 *00	9000-00	STIFFHESS OF FACH TIRE (LEZIR)
13.00	13.00	6-6	("AL TIPE SPACING (IP)
29.0(29.00	40.25	HELE TPACE - INHER TIRES (IN)
19.0(19.00	16.00	HELE SPEING SPACING (IN)
29.0(29.00	22.00	HETCHT OF POLL CENTER ABOVE GPOUND (TNCHES)
20.00	20.00	21.00	HETCHT OF AXLE C.G. ABOVE GPCUND (THCHES)
-90-06-	-30.00	00.011	X BIST FROM SP MASS CG (IM)
4500.00	#500.00	3700.00	AYLF FOLL M.T (LR.IN.SEC042)
2300.00	2300.00	1500.00	AYLF UEIGHT (LB.)
20000.00	20000.00	19000-00	LOAP ON FACH AXLE (LB.)
AXLE I Coccocc	AXLE 2 00000000	AXLE 1 000000000	

<pre>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</pre>	-5+00	5.00	2 b\$ DFFLECTION INCHES	-2.75	-0 - 75	0.0	2.00
SPRTNG TAPLE 000000 00000 FORCE LP	-1 0000 -00	1 0000 .00	SPRJHG TAPLE 646440 000001 FORCF LP	-15000.00	0.0	0.0	15000.00

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6 • 00	3600.00	4640.00	5500.00
4.00	2580.00	3280.00	00.0000
3.00	2040.00	2560.00	3100.00
2.00	1440.00	1940.00	2200.00
1.00	780.00	00*090	1200.00
0.0	6000.00	8000°00	10000.00

	6.00	1440.00	2400.00	3360.00
	u. 00	1020.00	1720.00	2400.00
	3.00	820.00	1360.00	1920.00
MGLL	2.00	580.00	760.00	1350.00
ATTC .CA	1.00	300.00	520.00	120.00
THERE LOURS	0.0	2000-00	00.0004	00.0008

6.00	9000.00	11604.00	13752.00
4.00	6444.00	00.3919	7756-00
3.00	5100.00	6376.00	1752.00
2.00	3600.00	4596.00	5a76 , CO
1.00	1944 . 00	2400.00	3000.00
0.0	00,0003	A000.00	1 ^000 . 00

ALIGHING TOFOUE TABLE • 2 99999900 999999 ngengggg ALIGHING TOPOUE VS. SLIP ANGLE

	00-1	2.00	00.6	UJ . H	6.00
0.0	160.00	00*363	00.1160	1724.00	1728.00
	P64.00	15.12.00	00.4631	101 H 101	70.12.00 10.12.00

οσοσεφοροσοσοσοροιας ΣΙΡΗΙΑΙΟΟ ΦΟΙΡΕCTΙΟΝΑL RESTORSE ΣΙΡΗΙΑΤΙΟΝΟ οσοσοσοροσοσοσοσοσοσο

6-AXLE TRACTOR -SEMI (DUMP-SEMITRAILER)

0 OF SPPUNG MASSES = 2 Total 0 of axles = 6

GROSS VEHICLE VEIGHT	H	60100.00	LP.			
FORWARP VELOCITY	ti	55.00	M. P. H			
			DISTANCE AVEAD CF SPRUNG MASS C.C. (JUCHES)	HEIGHT RELOU SPRUNG PASS C.G. (IMCHES)	POLL STIFFNESS (IN.LA/DFG)	TYPE OF CONSTRAINT
TINU NO	•	1	-109.70	-6.00	88.69999	1
ARTICOLATION FI F I ORI T	•	2	111.30	35.00		
TYPE CF CONSTRAIMT : 01 02 03 04		TONVENTIONAL PERTED STH PITTLE HOOK	, STH UNEFL I WHEFL STD JR POLL & PITC	£		
OFEN LOOP STEER JUPUT 00000000000000000000000000000000000						
STEERING GEAR RATIO		•	25.00			
STEFRING STIFFNESS (IN.L	P/1	- (J)	25000.00			
TIE RCP STIFFNESS (TN.LP	105	= (ມ	25000 .00			
MECHANICAL TRAIL (11)		ð1	1.00			

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a of points in stefp terle

STEEPING VIEEL DIGPEES 0.0 25.00 225.00

TTME 5FC 0.0 1.00 20.00

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			1. IN. SEC##2	B.IN.SEC##2	IN.SEC**2	CHES	*****																	
			10000.00 LF	65000.00 1	1000.000 LR.	44.00 IN	AXI.E 0 00000000																	
RAILER) IT 8 1 ФФФФФФФ		00 LB.	MASS = 2	G MASS =	MASS = 65	Round =	AXLE 8 3 000000000	14661.00	2300.00	4500.00	-152.70	20.00	29.00	19.00	29.00	13.00	5000.00	0.0	0.0	200.00	0.0	2	2	2
TIM32-4400) tu **	IT = 3	= 10000.	A OF SPRUNG	IA OF SPRUN	OF SPRUNG	CC ABOVE G	AXLE 1 2 00000000	11661.00	2300.00	4500.00	-102.70	20.00	29.00	19.00	29.00	13.00	1000.00	0.0	0.0	500.00	0° J	2	د	~
JTOR -SEMI	NU ZHI NO	SPRUNG MASS	r of Inerti	IT OF INERT	OF INFETTA	SPRUNC MASS	AX1.E 1 0000000000	11778.00	1500.00	00.0004	06.04	20-00	22.00	17.00	40°52	0.0	P000-00	u • 0	0.0	250+00	· · ·	-	-	1
6-AYLE TRA	I OF AXLES	UEIGHT OF	ROLL MOMENT	PITCH MOMEI	YAU MOMENT	HEIGHT OF		TOAL ON EACH AXLE (LB.)	AYLF WEIGHT (LN.)	AYLE ROLL H. I (LP. IN. SFC++2)	X DIST FPOM SF MASS CG (IN)	HFICHT CF AXLE C.G. ADOVE Gpound (Inches)	HEICHT OF POLL CENTER ARCVE GPCUND (INCHES)	WLF SPRING SPACING (IN)	HELE TPACK - INNER TIRES (IP)	UNT TIPE SPACING (IN)	STIFFNESS OF FACH TIRE (LE/IM)	FOLL STEEN COFFEICIENT	AUY POIL STIFFNESS (IN.LP/PEG)	CERTING COULOUP EFICTION - PEP SEPING (LP)	VICCOUS CAMPLAC PEP SEALAG VICCOUS CAMPLAC PEP SEALAG	SPERS TARLE .	COPAERING FORCE TANLE	PLIGNING TOPCUE TAME (

		.IN.SEC##2	B.IN.SEC¢¢2	IN.SEC++2	CHES	*****																	
		0000.00 LP	000000000	000.00 LB.	85.00 IN	AX1.E 0 00000000																	
RATLEq) IT 0 2 0000000	00 LA.	MASS = 0	G MASS = 4	MASS = 400	ROUND =	AXLF 1 6 000000000	13000.00	1500.00	4000.00	-122.70	20.00	29.00	19.00	29.00	13.00	2000.00	0.0	0.0	501.00	J.C	•	~	~
E = II	= 59500.	A OF SPRUNG	IA OF SERUM	OF SFRUNG	CC ABOVE G	AXLE 1 5 \$\$\$\$\$\$\$\$	13000.00	1500.00	4000.00	-80.70	20.00	29.00	19.00	29.00	13.00	5000.00	0.0	0.0	00.002	0.0	P.	ć	د
CTOR -SEMI ON THIS UN	SPRUNG MASS	T OF INERTI	NT OF INERT	OF INFRTIA	SPRUNG NASS	ΑΧLΕ Ι 4 ΦΦΦΦΦΦΦΦΦΦ	1 3000 .00	1500.00	4000.00	-30.70	20.00	29+00	19.00	29.70	13.00	5009.00	0.0	0.0	500.00	0°0	e	2	۲
6-AXLE TRA 1 OF AYLES	HEIGHT OF	ROLL ROLL	PITCH HOME	YAU MCMENT	HEIGHT OF		LOAP ON EACH AXLE (LB.)	AYLF VEIGHT (LB.)	AYLE ROLL M.I (LP.IN.SEC002)	X DIST FPOM SP MASS CG (11)	HFICHT OF AXIF C.G. AROVE Gfound (Inches)	HFICHT CF RJLL CFHTER AROVE Gfound (Inches)	HELF SPPING STACING (IN)	HFLF TRACK - INNER TIRES (IN)	DUAL TIFE SPACING (IN)	STIFFNESS OF EACH TIRE (LEVIN)	FOLI STEEP COFFICIENT	AUY FOLL STIFFRESS (IN.LN/DEC)	SFRING COULOPP FRICTION - PFR SPRING (LP)	VIEGOUS DAMPING PEP SPRING (RESECTIN)	I STRIFT	COP4ERING ECPCE TARLE D	ALIGNING TOPCUE TARLE

● 1 →◆ DEFLECTION TACHES	-10.00	10.00	<pre> 2 DEFLECTION INCHES</pre>	-10.00	-1.00	0.0	10.00	● 3 DEFLECTION INCHES	-10.00	-1.00	0.0	10.00
SPRTING TABLE ****** ***** FOPCE LB	-15000.00	15000-00	SPRTNG TABLE ************************************	-2000.00	0 -0	0.0	5 0000 • 0 0	SFRING TABLE 600000 00000 FORF Lb	00°0000E-	0*0	0-0	75000.00

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6.00	3600.00	4640.00	5500.00
00 ° tı	2580.00	3280.00	00.0095
00°E	2040.00	2560.00	3100.00
2.00	1440.00	1940.00	2200.00
1.00	780.00	960.00	1200.00
0.0	£000.00	P000.00	1 0000 .00

. CORPERING FORCE TAPLE 2 000000000 00000 000000000 LATFRAL FCRCE VS. SLIP ANG

	6.00	1440.0D	2400.00	3360.00
	00 ° H	1020.00	1720.00	2400-00
	9°°6	820.00	1360.00	1920.00
110	2.00	580.00	960.00	1360.00
VS. SLIP AF	1.00	00°00E	520.00	720.00
IT HAL FURCE	0.0	2000.00	000.000	000-0004

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00*4	6444.00	0196.00	9756.00
3.00	5100.00	6376.00	7752.00
2.00	00°009E	4536.00	54 95.00
1.00	1944.00	2400.00	3000-00
0.0	000.000	P000-00	1 0000.00

6.00

9000.0006

11604.00 13752.00

r . 0

u. no 6. no	1224.00 1728.00	7064.00 2480.00	2080-00 00 00
3.00	00.486	1632.00	00.000
2-00	00-969	11-2.60	1612.60
1.00	360.00	624.00	964.00
0.0	00.0075	00.0034	B000.000

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CWT_TOATIED 4 NU NU N 1 0110 à Ģ ¢ --

Э-АХГЕ ЕМРТҮ ООИР ТРИС	К + ЕОИЛОҮ + ЭАСК – Н	OE SUML-TRAILER			
4 OF SPRUNG MASSES	= 2				
TOTAL A OF AXLES	9 =				
GROSS VENICLE VEIGHT	= 45900.00	LP.			
FORWARD VELOCITY	= 55.00	H.P.H			
		DISTANCE AHEAD CF SPRUNG MASS C.G. (INCHES)	HEIGHT RELOV SPRUNG PASS C.G. (INCHES)	ROLL STIFFNESS (IN.LB/DEG)	TYPE OF CONSTRAINT
NO LO	IT 0 1	-154.00	19.00	0.0	7
	IT • 2	192.00	30 ° 00		
TYPE OF CONSTRAINT :	01 COPVENTIONA 02 INVERTED 57 03 PIPTLE HOOK 04 FING PIN(AI	L 5TR VHFFL 4 VHCEL 6TD IV POLL & PITCI	£		
OPEN LOOP STEER INPUT sessessessessessessessesses					
STEERING GEAR RATIO		= 25.00			
STEERING STIFFNESS (IN	(DEC)	= 25000.00			
TIE ROD STIFFNESS (10.	ER/DFG) =	25000 .00			
MECHARICAL TRAIL (12)	Η	1.00			
t of points in stefa J	APLE	6 #			
TIME STFEA 5EC D 0.0 0.0 1.00 25.00 27.00 275.00	LIAC WHEEL Argpers				

		₿•IN•SECª¢2	LP.IN.SEC**2	• IN. SEC## 2	NCHES	****																	
11 -TRAILFR		1 00.0000	00000.00	000.000 LB	50.00 II	AXLE # 030000000																	
ACK-NOE 5CH IT 0 1 	00 LA.	MASS = 2	G MASS = 4	MASS = 400	Round =	AXLE 1 3 000000000	9800.00	2300.00	4500.00	-110.70	19.00	29-00	19.00	29.50	13.00	5000.00	0.0	0.0	500.00	J.C	~	-	•
CK+LOUBOY+3 UN TT = 3	= 21200.	A OF SPPUNG	IA OF SFRUN	OF SPRUNG	CG ABOVE G	AXLE 2 00000000	9800.00	2300.00	4500.00	-69.70	19.00	29.00	19.00	29.50	13.00	00.0002	0°J	0.0	500.00	0.0	2	F	·
IY DUMP IRU	SPRUNG MASS	C OF INERTI	IT OF INERT	OF INERTIA	SPRUNG MASS	AXI.E # 1 2003030300	94 00 . 00	1200.00	3700.00	126.30	19.90	23.70	17.00	40.50	0.0	5000.00	0°0	u°û	200.00	0.0	1	1	-
3-AXLE EMP. P OF AXLES	VEIGHT OF	ROLL MOMENT	PITCH MOMER	YAU POMENT	HEIGHT OF 2		LUAL ON EACH AXLE (LB.)	AYLE VEIGHT (18.)	AYLF POLL M.I (LB.IN.SFC**2)	X DIST FROM SF MASS CG (IN)	HFIGHT OF AXLE C.G. ABOVE GPCUND (INCHES)	HFIGHT CF ROLL CENTER AROVE GPOUND (INCHES)	HPLF SPRING SPACING (IN)	HLLE TPACK - INNER TIRES (JH)	DUAL TIRE SPACING (IN)	STIFFNESS OF FACH TIRE (LPZIN)	POLL STFEP COFFEICIENT	AUX FOIL STIFFRESS (IN.LR/PEG)	SEPING COULOMP EPICTION - PEP SPRING (LD)	VISCEUS DAMPING PEP SFRING (LP+SEC/IN)	SEFING TAPLE .	COPNERING FORCE TABLE 1	ALISHING TOFCUE TANLS

3-AXLE EMI	DAT AMNA YTAN	CK+LOUBOY+B	ACK-NOE SEM IT # 2	I -TPAILFR				
• OF AXLES	NU SIHL NO S	IT = 3	****					
UEICHT OF	SPRUNC MASS	= 18000.	00 I.H.					
ROLL MOMEN	NT OF INERTI	A OF SPPUNG	MASS = 2	0000.00 LP.	.IN. SECoa2			
FITCH MON	ENT OF INCRT	IA OF SFRUN	C MASS = 4	0 00 00 TH	9.IN.SECot2			
YAU HOHENI	T OF INERTIA	OF SFRUNG	MASS = 400	000.00 LA.I	[N. SEC¢¢2			
HEICHT OF	SPRUNC HASS	CG APOVE G	FOIND =	61.00 INC	CHES			
	AXI.E 1 4 oooooooo	AXLE 0 5 640460640	AXLE 1 6 070040000	AXLE . 000000000	*****	******	*****	*****
LOAD ON EACH AXLE (LB.)	5633.00	2633.00	5633.00					
AYLF WEIGHT (IB.)	00.006	30.00	300.00					
AYLE FOLL M. I (LB. IN.SFC**2)	1000.00	1000.00	1000.00					
X DIST FROM SP MASS CG (14)	00.6	-24.00	-57.00					
HFICHT OF AXLE C.G. ABOVE GPCUND (INCHES)	13.50	13.50	13.50					
HFIGHT OF ROLL CENTEP AROVE GFOUND (INCHES)	24.00	24.00	24.00					
HELF SPRING STACING (IN)	24.00	24.00	24.00					
HELF TRACY - INNER TIRES (IN)	01.3E (36.00	36.00					
WAL TIPE SPACING (IN)	0°0	0.0	0.0					
(HIZAT) BULL HOLE OF EVEN LIVE (TEZIN)) 2500.00	2500.00	2500.00					
POLL STEER COFFEICIERT	0-0	0.0	0.0					
AUY FOLL STIFFIESS (IN.LP.PPC)	0.0 (0.0	0.0					
SFAING COULOME FPICTION - PFP SPAING (LB)	100-00	100.00	100.00					
(NT232+1) (NT232+1)	0°ù	0.0	0.0					
erring trate o	Ē	Ē	Ē					
COPNERING FORCE TAILS .	~	~	~					
ALIGNIEG TOPQUE TARLE #	2	~	2					

E 9 1 *** PEFLECTION TNCHES	-5.00	5.00	F 9 2 660 DFFLECTION INCHES	-10.00	-1.00	0.0	10.00	E # 3 *** DEFLECTION	-10.00	-1.00	0.0	10.00
SPR I HIG TABLI ******* ***** FOR CE LB	- 6000 - 00	6000.00	SPRING TAPL \$\$\$\$\$ FORCE	-20000-00	0.0	0.0	6 0 0 0 0 • 0 0	SPRTNG TABLI SPRTNG TABLI Sobood Dadd Force	00.00001-	0.0	0.0	25000.00

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6.00	1 440 . 00	2400.00	00°09EE
4 ° CO	1020.00	1720.00	2400.00
3°00	820.00	1360.00	1920.00
2.00	580.00	00.036	1360.00
1.00	00°00E	520.00	720.00
0.0	2000.00	4000.000 A	A000.00

1490.00	910.00	500.00	260.00	2800.00
1200.00	750.00	420.00	225+00	2050.90
840.00	530.00	00-01E	150.00	1335.00
9.00	n . 00	2.00	1.00	0.0
		ANGLL	VS. SLIP	TFRAL FORCE

ALICHING TOPOUE TAPLE 1 •••••••• •••••• ••••••• Aliching Torque VS. Slip Angle

6.00	1728.00	2860.00	4032.00
4 - 00	1224.00	2064.00	2889.00
3,00	984 .00	1632.00	2304.00
2.00	596.00	1152.00	1632.00
1.00	360.00	624.00	90 ° 194
0.0	2000-00	000.000	R000.00

ALLENING TOPQUE TAFLE | 2 00000000 000000 0000000 ALLENING TOPQUE VS. SLIP AUCLE

0-0	1.00	2.00	4 n 0	ć
00*5661	140.00	298.00	00.27E	. N 24 N .
2050.00	324.00	00.522	756.00	- U Û L
2400.00	n56.00	924.00	1216.00	1736.

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5-AXLE TRACTOR-SEMITRAILER (CAR HAULER, STINGER HITCH)

• OF SPRUNG MASSES = 2 Total • Of Axles = 5

TOTAL & OF MALES	۱.	n				
GROSS VEHICLE VEIGHT	61	66000.00	tb.			
FORWARD VELOCITY	11	55.00	н. Р. н			
			DISTANCE ANEAD Of Sprung Mass C.C. (Inches)	HEIGHT BELOU SPRUNG MASS C.G. (INCHES)	POLL STIFFNESS (IN.LR/DEG)	TYPE OF CONSTRAINT
IND NO	1T e	1	-132.60	10.00	88.69996	T
THO HO	IT •	2	250.PO	65 • 00		
TYPE OF CONSTRAINT : 0 0 0		CHVENTIONAL NVERTED STH THTLE HOOK THG PIN (RIG	5711 RHSEL Uneel To In Roll & Pitch	£		
OFEN LOOP STEEP INPUT UTEN LOOP STEEP						
STEERING GEAR RATIO		N	25.00			
STEERING STIFFNESS (IV.	J/J1.	FC) =	25000.00			
TIE ROD STIFFNESS (14.1.	P.DE	» ت	25000.00			

1.90

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I OF POINTS IN STEER TEPLE

MECHANICAL TRAIL (11)

STFEPJUC WHEL pfGFEF5 0.0 25.00 225.00

1146 5FC 0.0 1.00 29.00

5-AXLE TRA	ACTOR-SEMITR	AILFR (CAR UN	HANLER, STI	NGEP HITCH)			
# OF AXLES	NU THIS UN	11 = 3 ¢8	*****				
UEICHT OF	SPRUNG MASS	= 27000.	00 L.B.				
IDNU TIOU	NT OF INERTL	A OF SPHUNG	4 = SSAH	0000.00 LB.IN.SEC+4	2		
FITCH MOHE	ENT OF INERT	IA CF STRUN	C MASS = 1	50000.00 LB.IN.SEC	202		
YAU HOMENI	r of Infrtia	OF SPPUNG	MASS = 150	000.00 LB.IN.SEC**2			
HEIGHT OF	SPRUNG MASS	CC APOVE C	POUND =	60.00 INCHES			
	AXLE 1 *********	AXLE 1 2 ¢¢¢¢¢¢¢¢¢	AXLE 1 3 *********	AXIE 8 001000000 00000000	*******	0000000000	\$\$\$\$\$\$\$
LOAP ON EACH AXLE (LB.)	12000.00	1000.00	14000.00				
AXLF VEIGHT (LD.)	1200.00	2300.00	2300.00				
AVLF FOLL M. J (LR. IN.SFC**2)	3509.00	4500.00	4500.00				
X DIST FROM SP MASS CG (IN)	95.20	-61.80	-107.80				
HEICHT OF AXLE C.G. ABOVE GPCUND (INCHES)	20.00	20.00	29-00				
HE ICHT OF ROLL CENTER ABOVE GPOUND (INCHES)	22.90	29.00	29.00				
HELF SFRING SPACING (14)	17.00	19.00	19.00				
HALF TRACK - THNER TIRES (IM)	B0.25	29.00	29.00				
FUAL TIPE SPACING (IN)	0.0	13.00	13.00				
STIFFUESS OF FACH TIRE (LB/IN)	5000.00	2010-00	5000-00				
HOLL STEER COFFEICIENT	0.0	0° U	0.0				
AUY POLL STIFFESS (IN.LP/PEC)	0.0	0.0	0.0				
e coulom fre spend (LP) - served (LP)	250.00	500.00	200-00				·
VIECOUS FAMPING PER SPRING (LP.SEC/IN)	0°ŭ	0.0	0-0				
SPETES TAPLE .	1	2	2				
COPREPLING FORCE TABLE 0	ſ	F	1				
ALIGNING TOPCUE TARLE	1	Ļ	1				

		2	542									•											
AR HAULER, STINGER HITCH) Unit 0 2 \$\$\$\$\$\$\$\$\$	00.00 I.H.	UNG MASS = 100000.00 LB.IN.SEC**	RUNG MASS = 1000000.00 LA.IN.SEC	HG MASS = 1000000.00 LB.IN.SEC+03	E GROUND = A5.00 INCHES	5 ΑΧLE (Φο οσόσσοφου σωποσιοφο οσφοσοσι	00	00	00	20	00	00	00	00	00	00	0	0	00	c			
NILFR (CF 11T = 2	: 3020	A OF SPFL	IL OF SFF	OF SPPU	CC APOVE	AXLE coopooc	13000-0	1500.0	3500.0	-116.2	20.0	29.0	19.0	29.0	0.61	5000-0	0.0	0.0	500.0	0.0	•	-	ſ
CTOR-SENITH ON THIS W	SPRUNG NASS	L OF INFFI	NT OF THERI	OF INERTIA	SPRUNG HASS	AXLE # 4	1 3000.00	1500.00	3500.00	-42.20	20-00	39.00	19.00	29.00	13.00	0000-0005	0.0	u° J	500.00	0 • 0	Ē	1	ľ
5-AXLE TRA 0 of AXLES	UFIGHT OF	ROLL MOHEN	PITCH NOME	YA4 MOHENT	HEIGHI OF		LOAD ON EACH AXLE (LB.)	AYLF WEIGHT (LB.)	AYLF ROLL M.J (LB. IN.SEC002)	X DIST FPOM SP MASS CG (IN)	HEICHT OF PXLE C.G. ABOVE GPCUND (INCHES)	HETCHT CE POLL CENTER AROVE Ground (Thches)	6 HELF SPPING STACING (IN)	HELF TPACK - JUNER TIRES (IN)	PUAL TIRE SPACING (IN)	STIFFUESS OF FACH TIRE (LE/IN)	POLL STEER COFFETCIENT	AUY FOLL STIFFRESS (IN.LN/PEG)	SFRING COULOM EPICTION - FFP SPRING (LP)	VITCOUS LAMPING PER SERTHG (LP+SEC/TH)	SPEINS TAPLE .	COPAFRING FORCE TAPLE .	ALIGNING TOPQUE TABLE

· •

♦ 1 •• PEFLECTION INCHES	-10.00	10.00	2 \$\$ \$fecteriou	PETECUTOR 1NCHES -10+00	-1.00	0.0	10.00	m •• 1	DEFLECTION INCHES
SPRTMG JARLE 000000 000000 FORCE LB	-12000.00	12000.00	SERING TABLE Second scond	LP -2 0000.00	0.0	c •0	5 0000 - 00	SPRING TAPLE	FORCE LB

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96

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-10.00 -1.00 0.0 10.00

0.0

75000.00

6 • 00	1440.00	2400.00	3360.00
u. 00	1020.00	1720.00	2400-00
3.00	820.00	1360.00	1920.00
2.00	580.00	960.00	1360.00
1.00	300.00	520.00	720.00
0.0	2000-0002	4000-000	R000.00

	6.00	1728.00	2880.00	4032.00
	00 ° U	1224.00	2064.00	2880.00
	3.00	00.486	1632.00	2304.00
ANCI.F	2.00	00*969	1152.00	1632.00
VS. SLIP	1.00	360.00	624.00	€64.00
LI CHING TORQUE	0.0	2000.00	4 000 - 0 0	F000 • 0 0

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cDIFECTIONAL RESPONSE SIMULATIONe
coccereterneete

5-AXLE TRUCK-SEMITRAILER (DROMEDARY)

ØF SPPUNG MASSES = 2
 TOTAL | ØF AXLES = 5
 GROSS VENTCLE VETCHT = 75228.000 LB.
 FORWARD VELOCITY = 55.00 M.F.H

TYPE OF CONSTRAINT -ROLL STIFFNESS (IN.LB/DEC) 9996999.88 HEIGHT PELOU SPRUNG PASS C.G. (INCHES) 23.00 41.00 CONVENTIONAL STH WHEFL TRVERTED STM WHEEL PINTLE HOOK FING PIN (RIGID IN ROLL & PITCH) DISTANCE AHEAD OF SPRUNG MASS C.G. (INCHES) -153.60 238.00 ON UNIT 0 2 ARTICULATION PT | 1 5005 0000 OFEN LOOP STEER INFUT TYPE OF CONSTRAINT :

25.00 25000.00 1.70 25000.00 с н H H н H STEEPING WHEEL DECREES STEERING STIFFNESS (IN.LP/DFC) TIE ROP STIFFRESS (IN.LP/DFG) I OF POINTS IN STEER TAPLE 0.0 25.00 YECHANICAL TRAIL (TN) STEEPING GEAR RATIO 0.0 1.00 20.00 TIME SEC

		.P.IN.SECa¢2	LB.IN.SEC**2	J.IN.SEC##2	riches	. 000000000000																	
		50000.00	150000.00	20000.00 LE	75.00 1	3 AYLE 0 00000000	0	Ō	•	6		c			-								
0ARY) 117 8 1 00000000	.00 LB.	3 MASS =	4G MASS =	MASS = 1	ROUND =	AXLE I ecenerati	17076.00	2300-00	4500.01	-133.6(20.0(29.0(19.00	29.00	13.00	2000.00	0.0	0.0	500.00	с•с	2	1	L
LER (PROHE) UU AIT = 3	: 25000	A OF SPPUN	IN OF SFRUI	OF SPRING	CC APOVE	AXLE 1 2 \$\$\$\$\$\$\$\$\$	17076.00	2300.00	4500.00	1.60	20.00	29.00	19.00	29.00	13.00	£000 °00	0.0	0.0	200-005	0.0	2	-	-
CK-SEMITRAI ON THIS UN	SPRUNG MASS	T OF INERTI	NT OF INFRT	OF INERTIA	SPRUNC KASS	AXI.E 1 1 447447444	9076.00	1200.00	3500.00	161.40	29.00	52 .00	17.00	40°52	u • 0	5000.00	0.0	0.0	250.00	· · J	1	1	1
5-AXLE TRU 0 OF AYLES	UEIGHT OF	ROLL ROMEN	PITCH MOME	YAU POHENT	HEIGHT OF		TUVD ON EVCH VXTE (TB*)	AYLE VEIGHT (LB.)	AYLF POLL M.I (LB.IN.SEC**2)	X DIST FROM SP MASS CG (IN)	HEICHT OF AXIE C.G. ABOVE GPCUND (INCHES)	HEICHT CE ROLL CENTER AROVE Geound (Inches)	HELF SPRING STACING (IM)	HELF TPACK - INNER TIRES (IN)	DUAL THE SPACING (IN)	STIFFRESS OF FACH TIRE (LE/IN)	ROLI STEFA COFFEICIENT	AUY POLL STIFFRESS (IN.LB/PEC)	SPRING COULONP FRICTION - PER SPRING (LB)	VISCOUS PAMPING PEP SPRING (LA-SEC/IN)	SEPTRE TARLE .	COPNERING FORCE TABLE 0	ALIGNING TOFQUE TAPLE •

	1. IN. SEC⇔⇔ 2	.B.IN.SEC⇔⇔2	·IN.SEC¢#2	ICHES	00000000 00000000 00000000 00000000																	
	0000.000 LI	00.00000	000.00 LB	11 00°E6	4444444																	
00 LB.	MASS = 10	3 MASS = 10	1855 = 1000	= UNNO?	AXLE 4 ********																	
= 41428.(OF SPRUNG	A CF SFRUNG	OF SPRING 1	CG APOVE GI	AXLE # 5 ¢¢¢¢¢¢¢¢	16000.00	1500.00	3500.00	-128.00	20.00	29.00	19.00	29.00	13.00	5000.00	0.0	0.0	200.00	0 ° 0	e		ſ
PRUNG MASS	OF INERTIA	T OF INFRTI	OF INERTIA	PRUNG MASS	AXLE # 4 ¢¢¢¢¢¢¢¢	16000.00	1500.00	3500.00	-76.00	20.00	29.00	19.00	29+00	13.00	5000.00	0.0	6.0	500.00	u•0	Ē	1	ı
NEIGHT OF S	ROLL MOMENT	PITCH MOMEN	YAU HOMENT	HEIGHT OF S		ON FACH AXLE (LB.)	VEIGHT (LB.)	ROLL M.J (LB.IN.SFC**2)	ST FROM SP HASS CG (111)	IT OF AXLE C.G. ABOVE Ground (Inches)	IT OF ROLL CENTER APOVE Gfcund (Inches)	SPRING STACING (IN)	TPACK - INNER TIRES (IN)	TIRE SPECING (IN)	IESS OF EACH TIRE (LE/IN)	STFER COFFEICIENT	UL STIFFNESS (IN.LN/DEG)	IC COULORP FRICTION - PEP SPRING (LP))S LAMPINC PEP SPRING (LP+SEC/IN)	: TAPLE .	IFRING FORCE TABLE 0	INING TOPQUE TAMLE 1

5-AXLE TRUCK-SEMITRAILER (DROMEDARY) UNIT 9 2 *********

• OF AXLES ON THIS UNIT = 2

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DEFLECTIO	0 -10-00	0 10.00	BLE # 2 ##### DEFLECTIO INCHES	0 -10.00	-1-00	0.0	0 10.00	ALE 9 3 00000 DFFLECTIO	0 -10-00	-1-00	0.0	0 10.00
FORCF LB	-12000.0	1 2000 .0	SPRING TA 0000000 000 FORCE LB	-2 0000 -01	0-0	0.0	5 0000 -01	SPRING TA 598705 TA 608655 66 FORCE LA	U* 0000 E -	0.0	0.0	75000.0

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10.00

CORPERING FORCE TABLE # 1 ********* ***** ********** Latepal fonce vs. slip Angle

1440.00 2400.00 3360.00	1029.00 1720.00 2400.00	820.00 1360.00 1920.00	580.00 960.00 1360.00	300.00 520.00 720.00	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
2400.00	1720.00	1360.00	960.00	520.00	00
1 uu0 . 00	1020.00	820.00	580.00	300°00	00
6.00	4.00	3.00	2.00	1.00	6

ALICHING TOPQUE TABLE | 1 00000000 000000 00000000 ALICHING TORQUE VS. SLIP ANGL

	6.00	1728.00	2880.00	4032.00
	4.00	1224.00	2064.00	2880.00
	3.00	984.00	1632.00	2304.00
AHGLE	2.00	696.00	1152.00	1632.00
VS. SLIP	1.00	360.00	E24.00	864.00
LICHING TORQUE	0.0	2000-00	00.0004	A000.00

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5AXLE-CALIFORNIA-TRUCK-FULLTFAILER

DE SPRUNG MASSES	6		-			
TOTAL OF AXLES		n.	£			
GROSS VEHICLE VETO	THO	ม	P0000.00	LB.		
FORWARD VELOCITY	,	н	55.00	М.Р.Н		
				DISTANCE AHEAD CF SPRUNG MASS C.G. (INCHES)	HEIGHT RELOW SPRUNG MASS C.G. (INCHES)	ROLL STIFFNESS (IN.LR/DEG)
	TINU NO	•		-163.00	26.10	0*0
U IA NOTIVICATION	TINU PC		~	148.00	-1.00	
	TINU RC	•	~	0.0	-1.00	89,999,88
U ANTICATVILA	TINU NC	•		114.40	00 - hE	
TYPE OF CONSTPAINT	r : 01 03 04	8422	DRVENTTONAL VVERTED 5TI LHTLE HOOK LHC PIN (RTC	L STILVHEEL H NHEEL SID IN ROIL & PITC	£	
OPEN LCOF STEER II aaaaaaaaaaaaaaa	oe e e Lii dh					
STEERING GEAR RATI	12			= 25.00		
STEERING STIFFNESS	5 (JN.LI	P/PF	();	- 25000.00		
TIE ROP STIFFMESS	(TN-LB,	/DF(:	22000 .00		
HECHANICAL TRAIL	(1:1)		u	1.00		
• OF POINTS IN STE	EER TAPI	LE				
TYHE SEC 7.0 1.00 20.00	5TEFRIN DECI DECI DECI DECI DECI DECI DECI DECI		THE			

TYPE OF CONSTRAINT

m

3

		5EC¢¢2	.SEC¢¢2	EC¢¢2																			
		3640.00 LB.IN.	04000.00 LB.IN	000.00 LB.IN.SI	71.10 INCHES	ΛΧΙΕ φ ΦΦΦΦΦΦΦΦΦΦΦΦ																	
ER IT 8 1 000000	00 TB.	MASS = 5	G MASS = 9	HASS = 904	ROUND =	AXLE 3 00000000	15750.00	2330.00	4500.00	-86.00	20.60	29.00	19.00	29.00	1 3.00	5000.00	0.0	0.0	500.00	0 • 0	2	1	
(-FULLTPAIL) UN ET = 3	= 36040.	A OF SPPUNG	IA OF SFRUM	OF SPRUNG	CG AROVE G	AXLE 8 2 000000000	15750.00	00.0552	4500.00	00.46-	20.60	29.00	19.00	29.00	13.00	5000.00	0.0	0.0	500.00	0.0	2	1	
ORNIA-TRUCH ON THIS UNI	PRINC MASS	OF INERTIA	IT OF INCRT	OF INERTIA	EPRUNG MASS	AXLE 1 1 000000000	10500.00	1300.00	00.00TF	175.00	20.60	22.00	16.00	40.25	0.0	5000.00	6 • v	0.0	259.00	0.0	1	L	
5AXLE-CALIF 9 OF AXLES	NEIGHT OF 5	ROLL MOMENT	PITCH MOMEN	YAU MOMENT	HEIGHT OF 5		LPAP ON EACH AXLE (LB.)	AXLF VEICHT (LP.)	AYLF POLL M.T (LB.IN.SEC**2)	X DIST FROM SP MASS CG (IN)	HEICHT OF AXIF C.G. ABOVE Gpound (Inches)	HEICHT CE POIL CENTER ABOVE Ground (Inches)	HELF SPPING SPACING (IN)	H/LF TRACK - INNEP TIRFS (IV)	UPAL TIRE SPACING (IN)	STIFF4ESS OF FACH TIRE (LB/IN)	ROLL STEEN COFFEICIENT	AUX POLL STIFFNESS (IN.LR/PEG)	STRING COULOMP ERICTION - Len Spring (Le)	VISCOUS DAMPING PER SERING (LR-SEC/TH)	SIFTE'S TAPLE .	COPRERING FORCE TANLE .	

5AXLE-CALI	FORNIA-TPUCK-FULLTRAILER
I OF AXLES	ON THIS UNIT = 1
VEICHT OF	SPRURIC MASS = 965.00 LA.
NGHON TION	T OF INERTIA OF SPFUNG MASS = 1900.00 LB.IN.SEC++2
PITCH MOME	NT OF INERTIA OF SFRUNG MASS = 2560.00 LB.IN.SEC**2
YAU HOMENT	OF INEETIA OF STRUNG MASS = 2560.00 LB.IN.SEC**2
HEIGHT OF	SPRING MASS CC APOVE CROIND = 44.00 INCHES
	ΑΧΙΕ # 4 ΑΧΙΕ # σεθεσίος σοσερικό στοεθίος ποσορισος σεροθοφό θεστρότο τοτουτικά πιατουστο
LCAP ON EACH AXLE (LB.)	1900.00
AXLF VEIGHT (LB.)	1500.00
AXLF FOLL H.I (LB.IN.SEC++2)	4100.00
X DIST FROM SP MASS CG (IM)	0.0
HFICHT OF AXLE C.S. ABOVE GPCUND (INCHES)	20.60
NETCHT CE POLL CENTER ABOVE Gecund (Inches)	29-00
HELF SPEING SFACING (IN)	13°00
HULF TPACY - INHER TIRES (IN)	29.30
UPAL TIPE SPACING (IN)	13.00
STIFFAESS OF FACH TIRE (LB/IN)	5000.00
POLI STFFA COUFFICIENT	0.0
AUY FOIL STIFFIESS (IN.LA/DEC)	۲۰ ۵
STRIMS COULDER FRICTION - FUP SPAING (LB)	500.00
VISCOUS LAMEIRO PEP SPRING (LP.SSCZIN)	
SFFTPG TAPLE 1	
CORVEPTING FORCE TABLE	
PLIGHING TOPOUE TAMES	1

5AXLE-CALIFORHIA-TRUCK-FULLTRAILER UNIT # 3 ********

I OF AXLES ON THIS UNIT = 1

UEIGHT OF SPRUNG MASS = 34035.00 LB.

 ROLL MOMENT OF INERTIA OF SPRUNG MASS =
 11725.00
 Lh.IN.SEC002

 PITCH MOMENT OF INFRIIA OF SFRUNG MASS =
 546000.00
 Lh.IN.SEC002

YAW HOMENT OF INERTIA OF SFRUNG MASS = 546000.00 LB.IN.SFC002

HEIGHT OF SPRUNG MASS CG APOVE GROUND = 79.00 INCHES

ΑΧΙΕΙ 5 ΑΧΙΕ 9 Φερατατός σθοροφτός αροσορτος σοσοροτος εκεροροφές φοροροφός τος τουτουτος

LOAD ON EACH AXLE (LB.)	19000.00
AYLE VEIGHT (LB.)	1500.00
AYLF ROLL M. I (LB.IN.SFC**2)	4100.00
X DIST EPON SP HASS CG (IN)	-100.19
HFICHT CF AXLE C.G. ABOVE GFOUND (INCHES)	29.60
HFICHT CF ROLL CENTER ABOVE GFONND (INCHES)	23,00
HELE SPRING SPACING (IN)	19.00
HELE TPACY - THNER TIRES (14)	29.00
CUAL TIPE SPACING (IN)	00.51
STIFFLESS OF FACH TIRE (LP/IN)	2000.00
FOLL STEER COEFFICIENT	د.،
AUX FOLL STIFFRESS (IN.ER/DEG)	0.0
- NCLINE ERICIS	500.00
VISCOUS FAMEING PER SPRING (RP-SSC/IR)	(* ù
STELIS TARLE 0	£
COPNERING FORCE TABLE	1
PLISHING TOPOUE TABLE 0	1

■ 1 ⇒ ⊕ DEFLECTION THCIES	-5.00	5.00	7 4 4	DEFLECTION INCHES	-2.50	-1.50	0.0	1.00	~ ~ ~	DEFLECTION	-3.25	-1.00	0.0	0.25	0.50	1.00	1.75
SPRING TAPLE 0000000 00000 FORCE LB	-7500.00	7500.00		FORCE LB	- 4 000 • 0 0	0.0	0.0	6000.00	Juri Duirde Juri Duirde	E OR TE L B	-1 000 -00	ú•0	0.0	1100.00	00°002€	00-0000	2 0000 - 00

CORPERING FORCE TABLE | 1 000000000 00000 00000000 Lateral fonce vs. Slip Angll

10.00	1767.00	00.1716	4182.00	u 8 6 1 . 0 0	5056.00	
1.00	1502.00	2612.00	3378.00	3049.00	4020.00	
5.00	1221.00	2123.00	2711.00	3072.00	3182.00	
4.00	1018.00	1770.00	2259.00	2593.00	2674.00	
00°E	A 24.00	1421.00	1908.00	2032.00	2194.00	
1.00	356 . 00	580.00	701.00	767.00	784.00	
0.0	2000-00	000°00v	600.000	P000.000	00.0009	

ALICHING TORQUE TABLE | 1 0000000 000000 0000000 ALICHING TORQUE VS. SLIP ANGLE

10.00	468.00	1896.00	00.0996	5676.00	6780.00
7.00	732.00	00.8262	4476.00	6744.00	7800.00
5.00	672.00	2268.00	a 24 A. 00	6384 ° 00	7464.00
0 0 * 1 1	552.00	1894.00	3588.00	5508.00	6376.00
9.00	528.00	1716.00	00.2616	4644.00	54 24 .00
1.00	372.00	00°096	1560.00	2148.00	2400.00
0.0	2000-00	00.000#	6000.00	R000.000	000.000

•

5-axle dirt truck + 6-axle full trailer

m н DF SPRUMG MASSES

= 11 TOTAL 1 OF AXLES

GROSS VEHTCLE VETCH	Ë	= 148000.00	LB.			
FORWARD VELOCITY	4	- 55.00	М.Р.Н			
			DISTANCE AVEAD Of SPRUNG MASS C.G. (INCHFS)	HEIGHT FELOU Sprung Mass C.G. (Inches)	ROLL STIFFNESS (IN.LR/DEG)	TYPE OF CONSTRAINT
10 • - TA HOTTENDITAR	TIMU	• 1	-154.60	51 + 30	0.0	E
LO	UNIT	• 2	00.261	9.00		
	UNIT	• 2	0 * 0	00 • 6-	88.99999	I
NO 7 1 T.I NOITVINITIN	UNIT		72.00	40.00		
TYPE OF CONSTPAINT		CONVENTIONAI Inverted Sti Pirtle Hook Firg Pin (Rig	. STR UNCEL 1 UNCEL 31D IN ROLL & PITC	£		
OPEN LUOP STEER JNP ***************	TU ••					
STEERING GEAR RATID	_	·	- 25,00			
STEFRING STIFFNESS	(1N-LI	, /PFC)	- 25000.00			
TIS ROD STIFFNESS (JU.LP.	/DFG) =	25000.00			
HECHANICAL TRAIL (I	5	iu	1.00			
• DF POINTS IN STEE	IVVI d		E .			
TIME 51 5FC 0 0.0 2 1.00 25 275	FFR1N NF CI 00 00 00 00	LINEL 1946 1947				

5-axle dirt	truck + 6	-axle full UN	trailer IT 6 1 spoceo					
I DE AXLES	NU THIS NU	IT = 5						
VEIGHT OF S	PRUNG MASS	= 60900.	00 LB.					
ROLL MOHENT	OF INERTI	A OF SPPUNG	. MASS = 10	0000-000 FF	9. IN. SEC##2			
PITCH MOMEN	IT OF INCRT	IA OF SFRUN	IC MASS = 6	00000.00	B.IN.SEC##1	•		
УАИ ИСНЕНТ	OF INERTIA	OF SPRING	MASS = 600	000.00 LR.	IN.SEC##2			
HEIGHT OF S	PRUNG MASS	CC APOVE G	ROUND =	78.30 IN	ICHES			
	AXLE 1 00000000	AXLE # 2 \$#\$\$\$\$\$\$	AXLE 4 3 4444444	AX1.E # 4 *********	AXI.E 0 5 00000000	AXLF. 8 000000000	00000000000000000000000000000000000000	*****
(18.)	19000.00	13000.00	13000.00	13000.00	13000.00			
	1500.00	1500.00	2300.00	2300-00	1500.00			
•. IN • 5EC ** 2)	3700.00	00.00TE	4500.00	4500.00	3700.00			
ASS C3 (IN)	137.80	21.60	-24.20	-7A.20	-124.20			
.G. ABOVE D (INCHES)	21.00	20.00	20.00	20.00	20.00			
ENTER ABCVE D (TNCHES)	22-90	22.00	29.00	29.00	22.00			
1HG (1H)	16.00	19.00	19.00	19.00	19.00			
ER TIRFS (IN)	40.25	29.00	29.00	29.00	29.00			
c (11)	0°0	13.00	13.00	13.00	13.00			
TIRE (LP/IN)	00°°008	5010.00	5000.00	5000.00	5000.00			
ICIENT	0.0	0.0	0.0	0.0	0.0			
(D30/41.41) S	0.0	100000.00	0.0	0.0	100000.00			
ATCTION - Pring (LP)	250.00	250.00	150.00	750.00	250-00			
R SPRING •-SEC/TH)	0.0	0.0	0.0	0.0	0.0			
	T	•	2	2	£			
1 JULE 1	1	2	2	2	~			
TAALS	1	2	2	~	2			

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5-axle dirt truck + 6-axle full trailer UNIT 2 ******* • OF AMLES ON THIS UNIT = 3 WEIGHT OF SPRUNG MASS = 2500.00 LR. **BOLL MOMENT OF INERTIA OF SPRUNG MASS =** 4000.00 LP.IN.SEC**2 PITCH MOMENT OF INEPTIA OF STRUNG MASS = 8000.00 LB.IN.SEC**2 YAW HOMENT OF INERTIA OF SPRUNG MASS = 8000.00 LP.IN.SEC##2 HEIGHT OF SPRUNG MASS CG APOVE GROUND = 36.00 INCHES AXLE | 6 AXLE | 7 AXLE | 8 AMLE | LOAD ON EACH AXLE (LB.) 13417.00 13417.00 13417.00 AXLF WEIGHT (LP.) 1500.00 1500.00 1500.00 AXLE BOIL M.T (LB.IN.SEC##2) 3700.00 3700.00 3700.00 X DIST FROM SP MASS CG (IN) 42.00 0.0 -42.00 HEICHT OF AXLE C.G. ABOVE 20.00 20.00 20.00 GFOUND (INCHES) HEIGHT OF POLL CENTER ABOVE 22.00 22.00 22.00 GPOUND (INCHES) HELE SPEING SPACING (IN) 19.00 19.00 19.00 HELE TRACK - THNEP TIRES (IN) 27.00 29.00 29.00 DUAL TIPE SPACING (IN) 13.00 13.00 13.00 STIFFNESS OF FACH TIRE (LE/IN) 5000.00 5000.00 5000.00 ROLL STEER COEFFICIENT 0.0 0.0 0.0 AUY FOLL STIFFHESS (IN.LB/DEG) 0.0 0.0 0.0 STRING COULDN' FPTCTION -750.00 750.00 750.00 PFP SPRING (LP) VISCOUS DAMEING PEP SPRING C.O 0.0 9.0 (LP. 55C/IN) SPEING TAPLE # 2 2 2 COENERING EOPCE TABLE . 2 2 2 FLIGHING TOPQUE TABLE . 2 2 2

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			IN. SECor2	IN.SEC#02	N. SEC++2	HES	****																	
			000.00 LP.	0000.00 1.8	1.01 00.00	85.00 INC	AXI.E . 000000000																	
trailer (T 0 3 202020		00 LB.	MASS = 8(3 MASS = 80	1ASS = 8000	ROUND =	AXLE # 11 00000000	12583.00	1500.00	3700.00	-114.00	20.00	22.00	19.00	29.00	1 3.00	5000.00	0.0	0.0	750.00	0.0	2	~	2
-axle full 1 Uni east	(T = 3	= 66500.0	A OF SPEUNG	LA CF SFRUNG	OF SPRING P	CG APOVE GF	AXLE 8 10 ********	12583.00	1500.00	00.00TE	-72.00	20-00	22.00	19.00	29.00	00.61	5000 . 00	0.0	0.0	750.00	0.0	2	~	2
truck + 6-	ON THIS UNI	PRUNG MASS	OF INERTIA	IT OF INERT	OF INERTIA	PRUNC MASS	AXLE 1 9 ¢¢¢¢¢¢¢¢	12583.00	1500.00	9700.00	- 30.00	20.00	22.00	19.00	29.00	13.90	5000-00	0.0	0.0	750.00	6°0	~	2	2
5-axle dirt	• DF AXLES	VEIGHT OF S	ROLL MOHENT	PITCH MOMEN	YAU HOMENT	HEIGHT OF S		LOAP ON EACH AXLE (LB.)	AXLF VEICHT (LP.)	AXLF ROLL M. I (LB. II. SEC++2)	X DIST FPOM SP MASS CG (111)	HFICHT OF AXLE C.C. ABOVE GFOUND (INCHES)	HEICHT CE POIL CENTER ABOVE Gecund (Inches)	HELF SPRING SPACING (IN)	HALF TRACK - INNEP TIRES (III)	DUAL TIPE SPACING (IN)	STIFFNESS OF FACH TIRE (LD/IH)	POLL STEER COFFFICIENT	AUY POLL STIFFHESS (IN.L9/DEG)	SFRING COULOVE ERICTION - PEP SPPING (LB)	VISCOUS DAMPING PER SPAING (HE-SEC/14)	SEFING TAPLE .	COPHERING FORCE TAMLE	ILIGHING TOFQUE TAME .

analaise sister sature 1 and 14

• 1 • • • • • • • • •	-5.00	7.13	7.25	■ 2 DEFLECTION INCHES	-1.75	-0-75	0.0	1.00	● 3 DEFLECTION
SPP 146 TABLE ####################################	-10000-00	14250.00	4 0000 ° 00	SPRING TARLE 000000 000000 FORCE LB	-7500.00	0.0	0.0	7500.00	SPRTNG TAPLE SSRTNG TAPLE SSRESS S30000

-10.00 10.00

-1 0000 .00 1 0000 .00

6.00	3600 . 00	4640.00	5500.00			6.00	1440.00	2400.00	3360.00	
00 - 11	2580.00	3280.00	3900 . 00			41.00	1020.00	1720.00	2000.00	
00°E	2040.00	2560 .0 0	00°001E			3.00	020.00	1360.00	1920.00	
2.00	1440.00	1940.00	2200.00		110	2.00	580.00	760.00	1369.00	
1.00	780.00	00°096	1200.00	CE TABLE 0 2 00 0000000000	VS. SLIP AN	1.00	300.005	520.00	720.00	
0.0	(000°00)	P000-00	10000.00	CORNERING FOR	LATEPAL FORCE	0.0	2000-00	00.0004	A000.00	

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ALLCHING TORJUE TABLE 0 1 00000000 0000000 00000000 ALLCHING TORQUE VS. SLIP ANGLE

0.0	1.00	2.00	00°E	00 ° ti	6.00
100.000	1944.00	36.00.00	5100.00	644 n. 00	0000.000
P 0 0 0 - 4 0	2n00 .00	8596.00	6376.00	00.79618	11604.00
1 0000 .00	00°0'00	00-3643	7752.00	7756.00	1 3752.00

ALLCHING TOFQUE TAPLE • 2 00000000 000000 0000000 ALLCHING TOPQUE VS. SLIP ANGLF

0.0	1.00	2.00	3.00	00 ° b	6.00
2000.00	360.00	696.00	00.116	1724.00	1728.00
000.000 #	624.00	11-2.00	1632.00	0J.#335	2080.00
P000.00	864.00	1532.00	2374,00	2000.00	00.2104