

Redesign of Marine Structures by Perturbation

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ABSTRACT

Finite element (FE) methods are extensively used for analysis of static and dynamic behavior of marine structures. Often predicted response is unacceptable from the point of view of design or operation. Improvement of response then, becomes a design goal which can be achieved by redesign or reduction of operational threshold. Traditional trial and error techniques using FE methods make redesign expensive and are often inconclusive. In this paper a perturbation-based method is developed to solve redesign problems with both static and modal dynamic objectives using data only from the FE analysis of the baseline structure. Code RESTRUCT implements this method and functions as postprocessor to general or special purpose FE codes. Several simple numerical applications are used to illustrate the efficiency of this redesign method and how it can be used to resolve conflicts caused by incompatible redesign requirements. A 192-degree-of-freedom tower with repeated eigenvalues is redesigned subject to frequency and displacement constraints. Finally the impact of perturbation-based redesign on marine structural design is discussed.

Key words: Redesign, marine structures, perturbation, static and modal dynamic objectives, finite elements.

NOTATION

$b_{u_i}, b_{\omega_i}, b_{\phi_{ki}}$	Bounds on $\Delta u_i, \Delta \omega_i, \Delta \phi_{ki}$, respectively
c_{ij}	Admixture coefficient c_{ij} ; participation of the j th mode to changes in the i th mode
D_i, D_i'	Inner diameter of tubular element of baseline and objective structures
D_o, D_o'	Outer diameter of tubular element of baseline and objective structures
E	Young's modulus
$\{f\}$	Nodal force vector
f_j	j th component of vector $\{f\}$
$[k], [k']$	Stiffness matrix of baseline and objective structures
$[k_e], [k'_e]$	Baseline and objective stiffness matrices of element or group of elements related to property e
$[\bar{K}], [\bar{K}']$	Generalized stiffness matrix of baseline and objective structures
l	Index denoting quantities in increment l
$[m], [m']$	Mass matrix of baseline and objective structures
$[m_e], [m'_e]$	Baseline and objective mass matrix of the element or group of elements related to property e
M_i, M_i'	Generalized mass of the i th mode of baseline and objective structures
$[\bar{M}], [\bar{M}']$	Generalized mass matrix of baseline and objective structures
n	Number of degrees of freedom in structural model
n_r	Number of modes of the baseline structure used in redesign
N	Total number of increments in predictor-corrector algorithm
p	Number of design variables
q	Increment number in predictor-corrector algorithm; $q = 1, 2, \dots, N$.
$\{Q'\}$	Transformed displacement vector
r_u, r_ω, r_ϕ	Incremental changes defined by eqns (34), (35), (36)
S	Total number of redesign constraints
S_b	Total number of upper and lower bound inequality constraints imposed on α_s
S_e	Number of equality redesign constraints
S_i	Number of inequality redesign constraints
S_u	Number of displacement redesign constraints
S_ω	Number of frequency redesign constraints

S_ϕ	Number of modal node redesign constraints
$\{u\}, \{u'\}$	Nodal displacement vector of baseline and objective structures
$[\]^T, \{ \ }^T$	Denote transpose of a matrix and vector, respectively
$[\]^{-1}$	Denotes inverse of a square matrix
\diagdown	Denotes a diagonal matrix

Greek symbols

α_e	Fractional change to element or group property e
δ	Prefix denoting incremental change to structural quantities
Δ	Prefix denoting total change to structural quantities
$[\Delta k]$	Change to stiffness matrix $[k]$
$[\Delta k_e]$	Change to matrix $[k_e]$
$[\Delta m]$	Change to mass matrix $[m]$
$[\Delta m_e]$	Change to matrix $[m_e]$
$\{\Delta u\}$	Change to the nodal displacement vector
$\{\Delta \phi\}$	Matrix of the mode shape vector changes
$\Delta(\omega_i^2)$	Change to the i th eigenvalue
$[\phi], [\phi']$	Matrix of mode shape vectors of baseline and objective structures
$\{\psi\}_i, \{\psi'\}_i$	i th mode shape of baseline and objective structures
ϕ_{ki}	k th degree of freedom of i th baseline mode shape
ω_i, ω'_i	i th natural frequency of baseline and objective structures

1 INTRODUCTION

Perturbation-based redesign methods were first developed in linear form by Stetson *et al.*¹⁻⁴ and were further improved by Sandström *et al.*^{5,6} Linear perturbation methods are applicable to problems where small structural changes are allowed and small changes in undesirable response particulars (static or modal dynamic) are adequate. Nonlinear perturbation methods allowing for large changes in response particulars and structural properties were developed by Kim *et al.*⁷⁻⁹ and Bernitsas, Hoff *et al.*¹⁰⁻¹⁷ Kim *et al.*⁷ have used a penalty method to satisfy modal dynamic objectives in the redesign process. This method is theoretically exact but a desired solution can be achieved only for small-scale structures due to inaccuracy of the nonlinear numerical search technique used. For larger scale structures they used dynamic condensation which introduces an additional source of numerical error but reduces the prohibitively high cost of the standard methods.^{8,9} In previous work the authors have developed algorithms for solution of the nonlinear perturbation redesign

problem for only static¹⁶ or only modal dynamic¹¹⁻¹⁴ objectives. Extensive reviews and comparison of perturbation methods are provided in Ref. 13. The model developed in Ref. 12 was recently extended by Gans¹⁸ to include centrifugal and Coriolis effects. In this work a nonlinear perturbation redesign algorithm is developed for solving redesign problems with simultaneous static and modal dynamic objectives.

Perturbation-based redesign does not produce globally optimal design. Actually redesign problems may or may not be formulated as optimization problems depending on the number of constraints and allowable changes (see Section 4). If a redesign problem results in an optimization problem, computer code RESTRUCT (REdesign of STRUCTures), which implements the algorithm developed in this work, will produce an optimal design in the vicinity of the baseline structure and within the domain defined by the redesign constraints (design goals). In that sense the method developed in this work is related to another type of research, namely design optimization based on sensitivity.¹⁹⁻²¹ These methods produce optimal design. They are efficient when analytical expressions of the sensitivity parameters are available. These parameters are valid within small structural changes. The perturbation-based redesign methods developed in this work are not limited by the size of the structural changes. The problem of repeated eigenvalues can also be handled in this work, as illustrated by the offshore tower application.

In the analysis and design process, diagrammatically shown in Fig. 1, the designer usually generates a FE model for his structure and runs a general or special purpose FE program to analyze the structure's static and/or dynamic response. After identifying possible undesirable static and/or dynamic response characteristics — e.g. vibrations unfavorable to fatigue life, large amplitude vibrations, large static stresses — he has to develop a strategy for modifying his baseline structure in order to eliminate or alleviate such undesirable response. Thus he defines the goals of his design effort which become constraints in the redesign process. The method developed in this work allows the designer to modify modal properties and/or static displacements to achieve his design goals. That is, this method allows the designer to place constraints on displacement and modal properties in redesign. For this he has to specify which substructures or elements may be allowed to change. RESTRUCT will produce an objective design, if one exists. If the designer's goals, that is the redesign constraints, are incompatible, an objective design will not exist. The designer may then accept the solution produced by RESTRUCT which compromises conflicting constraints,

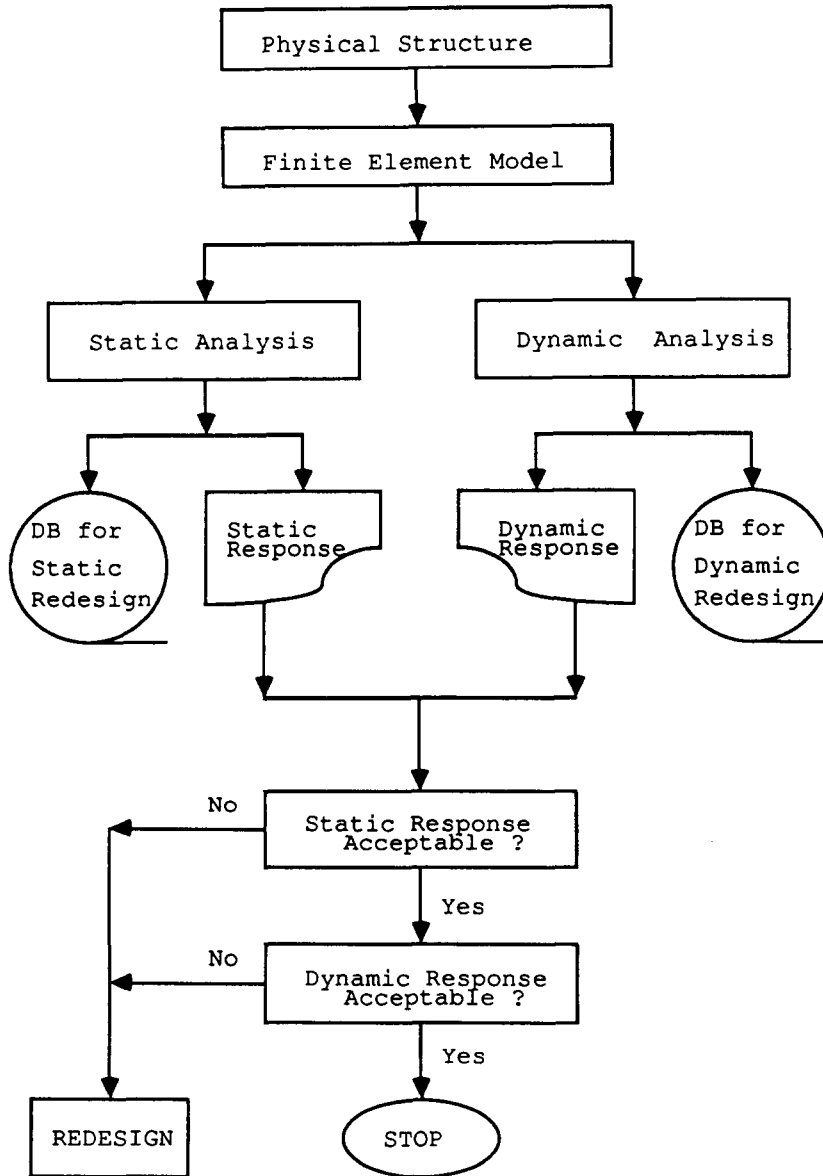


Fig. 1. Block diagram for structural analysis and design.

or he must redefine the redesign problem by changing constraints and/or allowable changes (redesign variables). In any event RESTRUCT uses only results from a single FE program run, that for the baseline structure.

The problem of redesign by perturbation is defined in Section 2 in the general case where the redesign process constraints are both static and

modal dynamic. These constraints are in general in an implicit nonlinear form and must be modified before they can be used to solve for the unknown structural changes. In Section 3 the redesign algorithm for modification of simultaneously applied static and modal dynamic constraints is developed, and briefly compared to algorithms for static or dynamic redesign developed in previous work.^{11, 14, 16} In Section 4 the algorithms used for computation of structural changes are explained. Computer implementation of the redesign algorithm is presented in Section 5. The developed computer code serves as postprocessor to FE codes and produces the objective design at a small cost compared to the cost of trial and error methods. Intricacies of redesign related to conflicting design objectives (redesign constraints) and redesign of structures with repeated eigenvalues are illustrated and resolved in Section 6 using numerical applications. In the concluding section the impact of the perturbation-based redesign method developed in this work on marine structural design is assessed.

2 REDESIGN BY PERTURBATION

According to the rationale for structural analysis and design shown diagrammatically in Fig. 1, a marine structure is first analyzed by a FE code. Then redesign is performed as necessary using the discretized structural FE model. In this section the problem of redesign of the discretized structure by perturbation is defined. Three different problems can be identified depending on whether design goals — that is redesign constraints — are static, modal dynamic or both. In previous work, as explained in the Introduction, the authors have defined the problems of static¹⁶ and dynamic^{11, 14} redesign by perturbation and developed solution algorithms. In this section the integrated static/dynamic redesign problem is defined.

First the static part of the integrated problem is defined. Let the equation of static equilibrium of the baseline discretized structure be

$$[k]_{n \times n} \{u\}_{n \times 1} = \{f\}_{n \times 1} \quad (1)$$

where $[k]$ is the global stiffness matrix, $\{u\}$ is the nodal displacement vector, and $\{f\}$ is the nodal force vector. Equation (1) can be solved for the displacement vector $\{u\}$ which can be used subsequently for computation of stresses. If either static displacements or stresses of this baseline structure violate design constraints the structure must be redesigned. Assume that a structure which satisfies all design constraints — and differs from the basic structure only in structural or material

properties of elements and not in location of nodes — exists. Let quantities related to this objective structure be primed. Then the governing equation of the objective structure is

$$[k'] \{u'\} = \{f\} \quad (2)$$

where $[k']$ is the objective stiffness matrix, $\{u'\}$ is the objective displacement vector, and the same nodal force vector is assumed as in the baseline model. To relate the baseline and objective models the following perturbation relations are used:

$$[k'] = [k] + [\Delta k] \quad (3)$$

$$\{u'\} = \{u\} + \{\Delta u\} \quad (4)$$

where $[\Delta k]$ is the change in the stiffness matrix, and $[\Delta u]$ is the change in the displacement vector. Applying eqns (3) and (4) to eqn (2) the general static perturbation equation governing the manner in which the structure may change is developed:

$$[\Delta k] \{u\} + [\Delta k] \{\Delta u\} + [k] \{\Delta u\} = \{0\} \quad (5)$$

Equation (5) also can be written as

$$\{\Delta u\} = -([k] + [\Delta k])^{-1} [\Delta k] \{u\} \quad (6)$$

Further assume that p properties of elements or groups of elements are allowed to change and let α_e be the fractional change in property e . More than one property may be changed in each element or group of elements. Then expressing the change in the global stiffness matrix as sum of all changes we get

$$[\Delta k] = \sum_{e=1}^p [\Delta k_e] = \sum_{e=1}^p [k_e] \alpha_e \quad (7)$$

where several $[k_e] \alpha_e$ terms may refer to the same element but different properties like bending, torsion, stretching, etc. For example, for a beam element we may have four change properties; that is, bending in two directions, torsion and axial tension. Inserting in eqn (6) the expression for $[\Delta k]$ from eqn (7) we derive the following expression for the change to displacements between baseline and objective structures:

$$\{\Delta u\} = - \sum_{e=1}^p ([k] + \sum_{e=1}^p [k_e] \alpha_e)^{-1} [k_e] \{u\} \alpha_e \quad (8)$$

In redesign the designer is allowed to place constraints on displacement changes to achieve his design goals. These may be related directly to

changes to displacements or indirectly to reduction of stress levels. Selecting out of eqn (8) only those degrees of freedom for which constraints on displacement changes are imposed we get a system of simultaneous nonlinear implicit equations and/or inequalities in α_e . Due to their complexity they must first be modified using the algorithm developed in Section 3 before they can be used to produce the values of α_e s by employing the solvers presented in Section 4.

Next the dynamic part of the integrated problem is studied. In the redesign process we also allow the designer to change modal characteristics of the baseline structure. Thus, a technique similar to the one presented above for static constraints is used below in order to develop perturbation equations for modal constraints. Let the equation of dynamic equilibrium for the vibration of the baseline discretized structure be

$$([k] - \omega_j^2 [m]) \{\psi\}_j = \{0\} \text{ for } j = 1, 2, \dots, n \quad (9)$$

where $\{\psi\}_j$ is the j th mode shape and ω_j is the corresponding natural frequency defined as the j th eigenvalue of eqn (10):

$$\det ([k] - \omega^2 [m]) = 0 \quad (10)$$

$[m]$ is the $n \times n$ global mass matrix which may include added mass, and damping may be included only in the Rayleigh form by contributing to $[k]$ and $[m]$. Combining eqns (9) into matrix form, we have

$$[k] [\phi] = [m] [\phi] [\omega^2] \quad (11)$$

where $[\phi] = [\{\psi\}_1, \{\psi\}_2, \dots, \{\psi\}_n]$, and $[\omega^2]$ is the diagonal matrix of the eigenvalues. Premultiplying eqn (11) by $[\phi]^T$ we obtain the uncoupled modal equations in the following matrix form:

$$[\bar{K}] = [\bar{M}] [\omega^2] \quad (12)$$

where $[K]$ is the generalized stiffness matrix

$$[\bar{K}] = [\bar{K}_{ii}] = [\phi]^T [k] [\phi] \quad (13)$$

and $[M]$ is the generalized mass matrix

$$[\bar{M}] = [\bar{M}_{ii}] = [\phi]^T [m] [\phi] \quad (14)$$

The counterpart equations for the objective structure are

$$([k'] - \omega_j'^2 [m']) \{\psi'\}_j = \{0\} \text{ for } j = 1, 2, \dots, n \quad (15)$$

$$\det ([k'] - \omega'^2 [m']) = 0 \quad (16)$$

$$[k'] [\phi'] = [m'] [\phi'] [\omega'^2] \quad (17)$$

$$[\bar{K}'] = [\bar{M}'] [\omega'^2] \quad (18)$$

$$[\bar{K}'_{-}] = [\phi']^T [k'] [\phi'] \quad (19)$$

$$[\bar{M}'_{-}] = [\phi']^T [m'] [\phi'] \quad (20)$$

Further we introduce the following three perturbation relations between the baseline and the objective structure:

$$[m'] = [m] + [\Delta m] \quad (21)$$

$$[\bar{\omega}'^2_{-}] = [\bar{\omega}^2_{-}] + [\bar{\Delta}(\omega^2)_{-}] \quad (22)$$

$$[\phi'] = [\phi] + [\Delta \phi] \quad (23)$$

where $[\Delta m]$, $[\bar{\Delta}(\omega^2)_{-}]$ and $[\Delta \phi]$ are changes to mass matrix, eigenvalues and eigenmodes, respectively. Allowing changes to the mass matrix we get the counterpart of eqn (7):

$$[\Delta m] = \sum_{e=1}^p [\Delta m_e] = \sum_{e=1}^p [m_e] \alpha_e \quad (24)$$

Inserting eqns (19) and (20) in eqn (18) and using perturbation relations (3) and (21) the general modal perturbation equation is derived as

$$\begin{aligned} & [\phi']^T [\Delta k] [\phi'] - [\phi']^T [\Delta m] [\phi'] [\bar{\omega}'^2_{-}] \\ & = [\phi']^T [m] [\phi'] [\bar{\omega}'^2_{-}] - [\phi']^T [k] [\phi'] \end{aligned} \quad (25)$$

This equation can be decomposed into the following n^2 scalar equations:

$$\begin{aligned} & \{\psi'\}_j^T [\Delta k] \{\psi'\}_i - \omega_i'^2 \{\psi'\}_j^T [\Delta m] \{\psi'\}_i = \{\psi'\}_j^T [m] \{\psi'\}_i \omega_i'^2 \\ & - \{\psi'\}_j^T [k] \{\psi'\}_i \end{aligned} \quad (26)$$

for $i, j = 1, 2, \dots, n$. Equations (26) can be expressed in terms of the structural changes α_e s using eqns (7) and (24) as

$$\begin{aligned} & \sum_{e=1}^p (\{\psi'\}_j^T [k_e] \{\psi'\}_i - \omega_i'^2 \{\psi'\}_j^T [m_e] \{\psi'\}_i) \alpha_e \\ & = \{\psi'\}_j^T [m] \{\psi'\}_i \omega_i'^2 - \{\psi'\}_j^T [k] \{\psi'\}_i \end{aligned} \quad (27)$$

for $i, j = 1, 2, \dots, n$. Equation (27) is the counterpart of eqn (8). In the redesign process the designer is also allowed to place redesign constraints on natural frequencies and normal modes. These constraints along with eqn (27) provide a system of simultaneous nonlinear implicit equations and/or inequalities in α_e $e = 1, 2, \dots, p$. These are very complex and must also be modified like their static counterparts using the algorithms developed in Section 3.

In general, in redesign, both static and modal dynamic constraints are imposed on the objective design. Solution algorithms used in the general redesign problem are presented in Sections 3 and 4. If only static constraints are imposed the problem will be reduced to a static redesign problem which has been solved in Ref. 16. If only modal dynamic constraints are imposed the general redesign problem will be reduced to a dynamic redesign problem, which has been solved in Refs 11 and 14. The algorithm developed in this work, besides being able to simultaneously handle both static and modal dynamic constraints, is more accurate, faster and efficient. This is particularly the case in the manipulation of static redesign constraints which are modified as explained in the next section using a modal expansion.

3 ALGORITHM TO MODIFY REDESIGN CONSTRAINTS

Redesign constraints imposed on $\{\Delta u\}$, $[\sim\omega'^2]$ and $\{\psi'\}_j$ ($j = 1, 2, \dots, n$) may be in the following equality or inequality form

$$u'_i = u_i + \Delta u_i \begin{matrix} \leq \\ \geq \end{matrix} b_{u_i} \quad i = 1, 2, \dots, s_u \quad (28)$$

$$\omega'^2_i = \omega_i^2 + \Delta\omega_i^2 \begin{matrix} \leq \\ \geq \end{matrix} b_{\omega_i} \quad i = 1, 2, \dots, s_\omega \quad (29)$$

INCREMENT l

PROBLEM DEFINITION: Eqns: (88), (89), (90), (91), (92)
 Unknowns: α_{e_i} , $e = 1, 2, \dots, p$
 Constraints: $S = S_\omega + S_\phi + 2p$

INPUT: $b_{\omega_i}^{(l)}$, $i = 1, 2, \dots, S_\omega$
 $b_{\phi_{kl}}^{(l)}$, $(k, i) \in J(1, 2, \dots, S_\phi)$
 $[k_e]_l$, $[m_e]_l$, $[\phi]_l$, $[\sim\omega'^2]_l$

SOLVER: Solve for α_{e_i} , $e = 1, 2, \dots, p$ (Section 4)

PREDICTION: Compute $[c]$ from eqn (62)
 Compute $[\delta\phi]_l$ from eqn (59)

OUTPUT: Compute $[\phi']_l = [\phi]_l + [\delta\phi]_l$

Fig. 2. Predictor phase problem.

$$\phi'_{ki} = \phi_{ki} + \Delta\phi_{ki} \begin{matrix} \leq \\ \equiv \\ \geq \end{matrix} b_{\phi_{ki}} \quad (k, i) \in J(1, 2, \dots, S_\phi) \quad (30)$$

where b_{u_i} , b_{ω_i} , and $b_{\phi_{ki}}$ are quantities specified by the designer and S_u , S_ω and S_ϕ are the number of constraints imposed on elements of $\{\Delta u\}$, $[\Delta\omega^2]$ and $[\Delta\phi]$, respectively. In all cases the expressions provided for Δu_i by eqn (8) and for $\omega_i'^2$ and ϕ'_{ki} by eqn (27) must be modified before being used in the redesign process. These expressions are in a nonlinear implicit form which can be altered efficiently by the incremental predictor-corrector scheme explained below and diagrammatically shown in Figs 2 and 3.

In the predictor and corrector phases certain linear approximations are used. This implies that imposed constraints on structural response as defined by eqns (28), (29) and (30), as well as expected fractional changes α_e s, are small. If they are small they can be applied in one incremental step. If they are not small they must be applied in N small increments as follows. Let u'_{i_l} , $\omega_i'^2$, ϕ'_{ki_l} be redesign goals (desirable values) of i th component of displacement vector, i th eigenvalue, and k th component of i th eigenvector, respectively, in increment l , with $l = 1, 2, \dots, N$. Then we have

$$u'_{i_l} \quad i = 1, 2, \dots, S_u \quad u'_{i_N} = u'_{i_l} \quad (31)$$

$$\omega_i'^2 \quad i = 1, 2, \dots, S_\omega \quad \omega_i'^2_N = \omega_i'^2 \quad (32)$$

$$\phi'_{ki_l} \quad (k, i) \in J(1, 2, \dots, S_\phi) \quad \phi'_{ki_N} = \phi'_{ki_l} \quad (33)$$

INCREMENT l

PROBLEM DEFINITION: Eqns: (93), (94), (95), (96), (97)
 Unknowns: $\alpha_e, e = 1, 2, \dots, p$
 Constraints: $S = S_u + n_r S_\omega + 2p$

INPUT: $b_{u_i}^{(l)}, i = 1, 2, \dots, S_u$
 $b_{\omega_i}^{(l)}, i = 1, 2, \dots, S_\omega$
 $[k_e]_l, [m_e]_l, [\Delta\omega_i^2]_l, [\phi']_l$ from predictor phase

SOLVER: Solve for $\alpha_e, e = 1, 2, \dots, p$ (Section 4)

CORRECTION: Compute $[k']_l$ from eqns (3) and (7)
 Compute $[m']_l$ from eqns (21) and (24)
 Compute $[\Delta\omega_i'^2]_l$ using Rayleigh's quotient
 Compute $\{u'\}_l$ from eqn (80)

Fig. 3. Corrector phase problem.

Further we assume that the above intermediate values form geometric sequences defined by eqns (34)–(36):

$$u'_{i_l} = u'_{i_{l-1}} (1 + r_u) \quad (34)$$

$$\omega'^2_{i_l} = \omega'^2_{i_{l-1}} (1 + r_\omega) \quad (35)$$

$$\phi'_{ki_l} = \phi'_{ki_{l-1}} (1 + r_\phi) \quad (36)$$

where $u'_{i_0} = u_i$, $\omega'^2_{i_0} = \omega_i^2$ and $\phi'_{ki_0} = \phi_{ki}$.

Combining eqns (31) and (34), (32) and (35), (33) and (36) we get respectively eqns (37), (38) and (39):

$$(1 + r_u)^N = \frac{u'_i}{u_i} \quad (37)$$

$$(1 + r_\omega)^N = \frac{\omega'^2_i}{\omega_i^2} \quad (38)$$

$$(1 + r_\phi)^N = \frac{\phi'_{ki}}{\phi_{ki}} \quad (39)$$

To ascertain small errors in the linear approximation used in the predictor phase, $|r_u|$, $|r_\omega|$ and $|r_\phi|$ are set $\leq r$, where in practice $r = 0.07$. This inequality along with eqns (37), (38) and (39) yields the following three lower bounds for N :

$$N \geq \frac{\ln(u'_i/u_i)}{\ln(1 + r)} = \frac{\ln(b_{u_i}/u_i)}{\ln(1 + r)} \quad (40)$$

$$N \geq \frac{\ln(\omega'^2_i/\omega_i^2)}{\ln(1 + r)} = \frac{\ln(b_{\omega_i}/\omega_i^2)}{\ln(1 + r)} \quad (41)$$

$$N \geq \frac{\ln(\phi'_{ki}/\phi_{ki})}{\ln(1 + r)} = \frac{\ln(b_{\phi_{ki}}/\phi_{ki})}{\ln(1 + r)} \quad (42)$$

N is the smallest integer which satisfies these three inequalities. Then r_u , r_ω and r_ϕ can be defined using eqns (37), (38) and (39), respectively.

The following symbols are also defined. At each increment l the values achieved by the redesign algorithm are, in general, not equal to the goals, u'_{i_l} , $\omega'^2_{i_l}$, ϕ'_{ki_l} . Instead, values which are used as baseline values for the next increment ($l + 1$) are achieved. These are denoted by $u_{i_{l+1}}$, $\omega^2_{i_{l+1}}$ and $\phi_{ki_{l+1}}$. For the first increment we have $u_{i_1} = u_i$, $\omega^2_{i_1} = \omega_i^2$ and $\phi_{ki_1} = \phi_{ki}$.

Let δ denote difference between values of a quantity in any two sequential steps in the incremental algorithm. Then, the following differences can be defined:

$$\{\delta u\}_l = \{u'\}_l - \{u\}_l \quad (43)$$

$$[\delta \omega^2]_l = [\omega'^2]_l - [\omega^2]_l \quad (44)$$

$$[\delta \phi]_l = [\phi']_l - [\phi]_l \quad (45)$$

which implies that the modal vectors satisfy the following relation:

$$\begin{aligned} \{\delta \psi\}_{j_l} &= \{\psi'\}_{j_l} - \{\psi\}_{j_l} \\ [\delta k]_l &= [k']_l - [k]_l \end{aligned} \quad (46)$$

$$[\delta m]_l = [m']_l - [m]_l \quad (47)$$

Further, the counterparts of the bounds provided by eqns (28), (29) and (30) must be defined for each increment l . That is, $b_{u_i}^{(l)}$, $b_{\omega_i}^{(l)}$ and $b_{\phi_{ki}}^{(l)}$ must be defined in the following equality or inequality forms of the redesign constraints at increment l :

$$u'_{i_l} = u_{i_l} + \delta u_{i_l} \begin{cases} \leq \\ \geq \end{cases} b_{u_i}^{(l)} \quad i = 1, 2, \dots, S_u \quad (48)$$

$$\omega_i'^2 = \omega_i^2 + \delta \omega_i^2 \begin{cases} \leq \\ \geq \end{cases} b_{\omega_i}^{(l)} \quad i = 1, 2, \dots, S_\omega \quad (49)$$

$$\phi'_{ki} = \phi_{ki} + \delta \phi_{ki} \begin{cases} \leq \\ \geq \end{cases} b_{\phi_{ki}}^{(l)} \quad (k, i) \in J(1, 2, \dots, S_\phi) \quad (50)$$

These bounds can be defined using equations similar to (37), (38) and (39). Thus

$$b_{u_i}^{(l)} = u_i (1 + r_u)^l \quad i = 1, 2, \dots, S_u \quad (51)$$

$$b_{\omega_i}^{(l)} = \omega_i^2 (1 + r_\omega)^l \quad i = 1, 2, \dots, S_\omega \quad (52)$$

$$b_{\phi_{ki}}^{(l)} = \phi_{ki} (1 + r_\phi)^l \quad (k, i) \in J(1, 2, \dots, S_\phi) \quad (53)$$

Finally, bounds should be imposed on the allowable structural changes α_e s. Two kinds of bounds are used. First, for practical purposes

$$\alpha_e^- \leq \alpha_e \leq \alpha_e^+ \quad e = 1, 2, \dots, p \quad (54)$$

where α_e^- and α_e^+ are specified lower and upper bounds for α_e and obviously $\alpha_e^- > -1$. Second, at each increment l in order to ascertain validity of the linear approximations used in both predictor and corrector phases the following bounds are imposed:

$$-0.1 \leq \alpha_{e_l} \leq 0.1 \quad \text{for } l = 1, 2, \dots, N \text{ and } e = 1, 2, \dots, p \quad (55)$$

The bounds in (55) are directly applied to the allowable structural changes at each increment. Those in (54), however, are applied to the global α_e s and must be modified to reveal bounds applicable to the incremental α_{e_l} s. Let

$$1 + \alpha_e = \prod_{l=1}^N (1 + \alpha_{e_l}) \quad (56)$$

Then (54) and (56) yield

$$1 + \alpha_e^- \leq \prod_{l=1}^N (1 + \alpha_{e_l}) \leq 1 + \alpha_e^+ \quad e = 1, 2, \dots, p \quad (57)$$

This suggests use of the following bounds on the incremental α_{e_l} s:

$$\frac{1 + \alpha_e^-}{\prod_{q=1}^{l-1} (1 + \alpha_{e_q})} \leq 1 + \alpha_{e_l} \leq \frac{1 + \alpha_e^+}{\prod_{q=1}^{l-1} (1 + \alpha_{e_q})} \quad (58)$$

where $\prod_{q=1}^0 (1 + \alpha_{e_q}) = 1$. Of the two upper and two lower bounds imposed

on α_{e_l} by (55) and (58) the dominant (stricter) ones are used.

In the following description of predictor and corrector phases, in each increment l , index l is omitted in order to simplify notation. It should also be emphasized that operators Δ and δ have identical functions, the only difference being that the former indicates global and possibly large change while the latter indicates incremental small change. Thus all equations involving Δ (e.g. eqns (1) and (4)) are valid for δ as well.

Predictor phase

In the predictor phase eqn (27) is linearized following the method originally proposed by Stetson¹⁻⁴ and later extended by Sandström.^{5,6} According to this method changes in eigenvectors are expressed as

$$[\delta\phi] = [\phi] [c]^T \quad (59)$$

where $[c]$ is the matrix of admixture coefficients. It is assumed that $c_{ii} = 0$ and that the rest of the coefficients are small. Numerically it is more convenient to work in the space spanned by the eigenvectors $\{\psi\}_i$, $i = 1, 2, \dots, n$ transformed by $[c]$ rather than the eigenvectors themselves.

Applying eqns (59), (22) and (23) for changes to eigenvectors, mass matrix and eigenvalues to eqn (26) and neglecting terms of the order of δ^2 , δ^3 and δ^4 we get the first order perturbation equations as

$$\left. \begin{aligned} \{\psi\}_j^T [\delta k] \{\psi\}_i - \{\psi\}_j^T [\delta m] \{\psi\}_i \omega_i^2 &= M_i \delta \omega_i^2 & \text{for } i = j \\ &= M_j c_{ij} (\omega_i^2 - \omega_j^2) & \text{for } i \neq j \end{aligned} \right\} \quad (60)$$

These equations are derived assuming that the neglected terms are of higher order. This is correct only for small changes which is the case in each increment. For $i = j$ eqn (60) gives the following expression for changes to eigenvalues:

$$\delta\omega_i^2 = \frac{1}{M_i} (\{\psi\}_i^T [\delta k] \{\psi\}_i - \{\psi\}_i^T [\delta m] \{\psi\}_i \omega_i^2) \quad (61)$$

For $i \neq j$ eqn (60) gives the admixture coefficients c_{ij} :

$$c_{ij} = \frac{1}{M_j(\omega_i^2 - \omega_j^2)} (\{\psi\}_j^T [\delta k] \{\psi\}_i - \omega_i^2 \{\psi\}_j^T [\delta m] \{\psi\}_i) \quad (62)$$

Combining eqns (59) and (60) we can express the change to the k th degree of freedom of the i th mode as

$$\delta\phi_{ki} = \sum_{j=1, j \neq i}^n \left[\frac{\phi_{kj}}{M_j(\omega_i^2 - \omega_j^2)} (\{\psi\}_j^T [\delta k] \{\psi\}_i - \omega_i^2 \{\psi\}_j^T [\delta m] \{\psi\}_i) \right] \quad (63)$$

Introducing eqns (7) and (24) in (61) and (63) the following relations for $\delta\omega_i^2$ and $\delta\phi_{ki}$ are derived in terms of the required structural changes α_e , $e = 1, 2, \dots, p$:

$$\delta\omega_i^2 = \frac{1}{M_i} \left[\sum_{e=1}^p (\{\psi\}_i^T [k_e] \{\psi\}_i - \omega_i^2 \{\psi\}_i^T [m_e] \{\psi\}_i) \alpha_e \right] \quad (64)$$

$$\delta\phi_{ki} = \sum_{e=1}^p \left[\sum_{j=1, j \neq i}^n \frac{\phi_{kj}}{M_j(\omega_i^2 - \omega_j^2)} (\{\psi\}_j^T [k_e] \{\psi\}_i - \omega_i^2 \{\psi\}_j^T [m_e] \{\psi\}_i) \right] \alpha_e \quad (65)$$

where n_r is the number of modes of the baseline structure used in redesign. Obviously n_r has to be high enough to achieve an accurate approximation of $\delta\phi_{ki}$. The predictor phase algorithm is summarized in Fig. 2.

In the predictor phase only dynamic redesign constraints are used to compute a first order approximation to the structural changes α_e s. These constraints may be in the form of equalities or inequalities as indicated in eqns (28)–(30). Depending on the number p of unknown α_e s and the number of equality and/or inequality constraints ($S_\omega + S_\phi$), several different solution algorithms may be used as explained in Section 4.

There is one subcase in the above algorithm which requires special attention — specifically the case of repeated eigenvalues. If, say, $\omega_i = \omega_j$, c_{ij} cannot be defined by equation (62). This problem of repeated eigenvalues is not new. It instigates similar complications in sensitivity-based optimal design methods.¹⁹⁻²¹ In this work the problem of repeated

eigenvalues is resolved as follows. It is assumed that the designer would like his objective structure to possess similar properties and therefore preserve any eigenvalue repetition. Then it is expected that change to mode j will not depend on mode i just as it does not depend on mode j itself. In the definition of the admixture coefficient matrix by equation (59) it was assumed that $c_{ii} = 0$. Similarly we can impose the restriction on $[c]$ that $c_{ij} = 0$ for $\omega_i = \omega_j$. Other elements in $[c]$ may also be zero. These can be computed by RESTRUCT as in the offshore tower application in Section 6. Coefficients that are zero or nearly zero are purged from the system of nonlinear constraints by RESTRUCT to increase the speed and accuracy of solution.

Corrector phase

In the corrector phase a first order approximation to the objective eigenvectors is computed using eqns (65) and (23). These approximate eigenvectors of the objective structure are subsequently used in eqn (27) to provide better approximation to the elemental changes α_e s. Specifically we get ($n_r \times S_\omega$) number of redesign constraints of the following form:

$$\begin{aligned} \sum_{e=1}^P (\{\psi'\}_j^T [k_e] \{\psi'\}_i - \omega_i'^2 \{\psi'\}_j^T [m_e] \{\psi'\}_i) \alpha_e \\ = \{\psi'\}_j^T [m] \{\psi'\}_i \omega_i'^2 - \{\psi'\}_j^T [k] \{\psi'\}_i \quad j = 1, 2, \dots, n_r \end{aligned}$$

or

$$\begin{aligned} \delta \omega_i^2 = -\omega_i^2 + \frac{1}{\{\psi'\}_j^T [m] \{\psi'\}_i} \times \\ \left[\left\{ \sum_{e=1}^P (\{\psi'\}_j^T [k_e] \{\psi'\}_i - \omega_i'^2 \{\psi'\}_j^T [m_e] \{\psi'\}_i) \alpha_e \right\} \right. \\ \left. + \{\psi'\}_j^T [k] \{\psi'\}_i \right] \quad j = 1, 2, \dots, n_r \quad (66) \end{aligned}$$

At this point the static redesign constraints must be introduced and taken into account in computing the α_e values. For this purpose eqn (8) is modified using the following approach.

Let $\{Q'\}$ be defined by eqn (67):

$$\{u'\} = [\phi'] \{Q'\} \quad (67)$$

where $\{Q'\}$ is the transformed displacement vector. Application of eqn (67) to (2) gives

$$[k'] [\phi'] \{Q'\} = \{f\} \quad (68)$$

Premultiplying eqn (68) by $[\phi']^T$ gives

$$[\phi']^T [k'] [\phi'] \{Q'\} = [\phi']^T \{f\} \quad (69)$$

Since $[\phi']^T [k'] [\phi'] = [\bar{K}'_{ij}]$, eqn (69) becomes

$$[\bar{K}'_{ij}] \{Q'\} = [\phi']^T \{f\} \quad (70)$$

$$\{Q'\} = [\bar{K}'_{ij}]^{-1} [\phi']^T \{f\} \quad (71)$$

Since $[\bar{K}'_{ij}]^{-1} = [1/K'_{ij}]$, eqn (71) becomes

$$\{Q'\} = [1/K'_{ij}] [\phi']^T \{f\} \quad (72)$$

Combination of eqns (72) and (67) yields

$$\{u'\} = \begin{bmatrix} [\phi'] & [1/K'_{ij}] [\phi']^T \{f\} \\ n \times 1 & n \times n_r & n_r \times n_r & n_r \times n & n \times 1 \end{bmatrix} \quad (73)$$

Thus the components of vector $\{u'\}$ can be expressed as

$$u'_i = \sum_{m=1}^{n_r} \left[\frac{\phi'_{im}}{K'_m} \sum_{j=1}^n \phi'_{jm} f_j \right] \quad (74)$$

To calculate K'_m , eqns (3), (7) and (19) are combined to produce

$$\begin{aligned} [\bar{K}'_{ij}] &= [\phi']^T \left([k] + \sum_{e=1}^p [k_e] \alpha_e \right) [\phi'] \\ &= [\phi']^T [k] [\phi'] + \sum_{e=1}^p \alpha_e [\phi']^T [k_e] [\phi'] \end{aligned} \quad (75)$$

or

$$K'_m = \{\psi'\}_m^T [k] \{\psi'\}_m + \sum_{e=1}^p \alpha_e \{\psi'\}_m^T [k_e] \{\psi'\}_m \quad m = 1, 2, \dots, n_r \quad (76)$$

Combination of eqns (76) and (74) yields

$$u'_i = \sum_{m=1}^{n_r} \left[\frac{\phi'_{im}}{\{\psi'\}_m^T [k] \{\psi'\}_m + \sum_{e=1}^p \alpha_e \{\psi'\}_m^T [k_e] \{\psi'\}_m} \left(\sum_{j=1}^n \phi'_{jm} f_j \right) \right] \quad (77)$$

Equation (77) shows that the displacement vector of the objective system can be expressed in terms of eigenvectors of the objective system,

the stiffness matrix of baseline structure, the fractional changes of stiffness properties and the applied force vector. Let

$$A_m = \sum_{j=1}^n \phi'_{jm} f_j, B_m = \{\psi'\}_m^T [k] \{\psi'\}_m \text{ and } C_{me} = \{\psi'\}_m^T [k_e] \{\psi'\}_m \quad (78)$$

Then eqn (77) becomes

$$u'_i = \sum_{m=1}^{n_r} \frac{\phi'_{im} A_m}{B_m + \sum_{e=1}^p C_{me} \alpha_e} = \sum_{m=1}^{n_r} \frac{1}{1 + \frac{1}{B_m} \sum_{e=1}^p C_{me} \alpha_e} \frac{\phi'_{im} A_m}{B_m} \quad (79)$$

If at each step we assume small changes from the previous step, B_m will be much greater than $\sum_{e=1}^p C_{me} \alpha_e$. Therefore we can linearize eqn (79) as follows:

$$\begin{aligned} u'_i &\approx \sum_{m=1}^{n_r} \left[\frac{\phi'_{im} A_m}{B_m} \left(1 - \frac{1}{B_m} \sum_{e=1}^p C_{me} \alpha_e \right) \right] \\ &= \sum_{m=1}^{n_r} \frac{\phi'_{im} A_m}{B_m} - \sum_{m=1}^{n_r} \left[\frac{\phi'_{im} A_m}{B_m^2} \left(\sum_{e=1}^p C_{me} \alpha_e \right) \right] \end{aligned} \quad (80)$$

or

$$\Delta u_i = -u_i + \sum_{m=1}^{n_r} \frac{\phi'_{im} A_m}{B_m} - \sum_{e=1}^p \left(\sum_{m=1}^{n_r} \frac{\phi'_{im} A_m}{B_m^2} C_{me} \right) \alpha_e \quad (81)$$

In the corrector phase of the redesign process the static redesign equality or inequality constraints are introduced. Let S_u be the total number of static constraints. Depending on the number p of unknown α_e s and the total number of static and/or dynamic equality and/or inequality redesign constraints ($n_r \times S_\omega + S_u$) several different solution algorithms may be used as explained in Section 4.

At the end of each increment $\{u'\}$ and $[\omega'^2]$ are computed using eqns (80) and (66). These values are used along with the values of $[\phi']$ — computed in the predictor phase — at the beginning of the next increment as baseline values. The corrector phase algorithm is summarized in Fig. 3.

4 COMPUTATION OF STRUCTURAL CHANGES

The problem of redesign by perturbation as defined in Section 2 consists of constraint equations (28), (29) and (30), the dynamic perturbation equation (27) and the static perturbation equation (8). That is, the problem of redesign by perturbation is reduced to problem P1.

P1: Redesign by perturbation

$$u'_i = u_i + \Delta u_i \begin{matrix} \leq \\ \geq \end{matrix} b_{u_i} \quad i = 1, 2, \dots, S_u \quad (82)$$

$$\omega_i'^2 = \omega_i^2 + \Delta \omega_i^2 \begin{matrix} \leq \\ \geq \end{matrix} b_{\omega_i} \quad i = 1, 2, \dots, S_\omega \quad (83)$$

$$\phi'_{ki} = \phi_{ki} + \Delta \phi_{ki} \begin{matrix} \leq \\ \geq \end{matrix} b_{\phi_{ki}} \quad (k, i) \in J(1, 2, \dots, S_\phi) \quad (84)$$

$$\begin{aligned} & \sum_{e=1}^p (\{\psi'_j\}^T [k_e] \{\psi'_i\} - \omega_i'^2 \{\psi'_j\}^T [m_e] \{\psi'_i\}) \alpha_e \\ & = \{\psi'_j\}^T [m] \{\psi'_i\} \omega_i'^2 - \{\psi'_j\}^T [k] \{\psi'_i\} \quad i, j = 1, 2, \dots, n \end{aligned} \quad (85)$$

$$\{\delta u\} = - \sum_{e=1}^p \left([k] + \sum_{e=1}^p [k_e] \alpha_e \right)^{-1} [k_e] \{u\} \alpha_e \quad (86)$$

$$\alpha_e^- < \alpha_e < \alpha_e^+ \quad e = 1, 2, \dots, p \quad (87)$$

In problem P1 the unknowns are α_e , $e = 1, 2, \dots, p$.

As explained in Section 2 this problem is very complicated because of the nonlinear implicit form of expressions (85) and (86). The incremental predictor-corrector scheme developed in Section 3 is used to apply b_{u_i} , b_{ω_i} and $b_{\phi_{ki}}$ incrementally and simplify expressions (85) and (86). At each increment l ($l = 1, 2, \dots, N$) two different problems arise; problem P2 appears in the predictor phase and problem P3 in the corrector phase. Both are defined below.

P2: Predictor phase problem (increment l , $l = 1, 2, \dots, N$)

$$\omega_i'^2 = \omega_i^2 + \delta \omega_i^2 \begin{matrix} \leq \\ \geq \end{matrix} b_{\omega_i}^{(l)} \quad i = 1, 2, \dots, S_\omega \quad (88)$$

$$\phi'_{ki} = \phi_{ki} + \delta \phi_{ki} \begin{matrix} \leq \\ \geq \end{matrix} b_{\phi_{ki}}^{(l)} \quad (k, i) \in J(1, 2, \dots, S_\phi) \quad (89)$$

where

$$\delta\omega_{ii}^2 = \left(\frac{1}{M_i} \left[\sum_{e=1}^p (\{\psi\}_i^T [k_e] \{\psi\}_i - \omega_i^2 \{\psi\}_i^T [m_e] \{\psi\}_i) \alpha_e \right] \right)_l \quad (90)$$

and

$$\delta\phi_{ki} = \left(\sum_{e=1}^p \left[\sum_{j=1, j \neq i}^{n_r} \frac{\phi_{kj}}{M_j (\omega_i^2 - \omega_j^2)} (\{\psi\}_j^T [k_e] \{\psi\}_i - \omega_i^2 \{\psi\}_j^T [m_e] \{\psi\}_i) \right] \alpha_e \right)_l \quad (91)$$

$$\max \left\{ (1 - 0 \cdot 1), \frac{1 + \alpha_e^-}{\prod_{q=1}^{l-1} (1 + \alpha_{e_q})} \right\} \leq 1 + \alpha_{e_l} \leq \min \left\{ (1 + 0 \cdot 1), \frac{1 + \alpha_e^+}{\prod_{q=1}^{l-1} (1 + \alpha_{e_q})} \right\} \quad (92)$$

$$e = 1, 2, \dots, p$$

In problem P2 the unknowns are α_{e_l} , $e = 1, 2, \dots, p$, and the total number of constraints is $S = S_\omega + S_\phi + 2p$. This problem and its solution algorithm are presented in Fig. 2.

P3: Corrector phase problem (increment l , $l = 1, 2, \dots, N$)

$$u'_{ii} = u_{ii} + \delta u_{ii} \leq b_{u_i}^{(l)} \quad i = 1, 2, \dots, S_u \quad (93)$$

$$\omega_i'^2 = \omega_{ii}^2 + \delta\omega_{ii}^2 \leq b_{\omega_i}^{(l)} \quad i = 1, 2, \dots, S_\omega \quad (94)$$

where

$$\delta\omega_{ii}^2 = -\omega_{ii}^2 + \left(\frac{1}{\{\psi'\}_j^T [m] \{\psi'\}_i} \times \left[\sum_{e=1}^p \left\{ \{\psi'\}_j^T [k_e] \{\psi'\}_i - \omega_i'^2 \{\psi'\}_j^T [m_e] \{\psi'\}_i \alpha_e + \{\psi'\}_j^T [k] \{\psi'\}_i \right\} \right] \right)_l \quad j = 1, 2, \dots, n_r \quad (95)$$

and

$$\delta u_{ii} + u_{ii} = u_{ii}' = \left(\sum_{m=1}^{n_r} \frac{\phi'_{im} A_m}{B_m} - \sum_{e=1}^p \left(\sum_{m=1}^{n_r} \frac{\phi'_{im} A_m}{B_m^2} C_{me} \right) \alpha_e \right)_l \quad (96)$$

$$\max \left\{ (1 - 0.1), \frac{1 + \alpha_e^-}{\prod_{q=1}^{l-1} (1 + \alpha_{e_q})} \right\} \leq 1 + \alpha_e \leq \min \left\{ (1 + 0.1), \frac{1 + \alpha_e^+}{\prod_{q=1}^{l-1} (1 + \alpha_{e_q})} \right\}$$

$$e = 1, 2, \dots, p \quad (97)$$

In problem P3 the unknowns are again α_{e_i} , $e = 1, 2, \dots, p$, and the total number of constraints is $S = S_u + n_r S_\omega + 2p$. This problem and its solution algorithm are summarized in Fig. 3.

Problems P2 and P3 are solved at each increment. Their form is similar and the number of unknowns is in both equal to p . They differ only in the number of equality and inequality constraints. Due to their similarity problems P2 and P3 can be solved for the unknown α_{e_i} s using the same solvers which are described below.

Let S_e and S_i be the number of equality and inequality constraints, respectively. Further let S_b be the number of upper and lower bound inequality constraints imposed on α_{e_i} s. Depending on the relation between S_e , S_i , S_b and p the following problems are identified.

- (i) $p > S_e$: In this case an optimization problem with p design variables, S_e equality constraints and $(S_i + S_b)$ inequality constraints must be solved.
- (iia) $p = S_e$ and $S_i = S_b = 0$: In this case a system of simultaneous linear equations must be solved.
- (iiβ) $p = S_e$ and $(S_i + S_b) > 0$: In this case the problem is in general overconstrained. To solve it, first the linear solver of case (iia) is used and then the $(S_i + S_b)$ inequality constraints are checked. If they are satisfied the solution will be accepted; if not the problem will be solved approximately by the minimum error algorithm in (iii).
- (iii) $p < S_e$: In this case the minimum error algorithm is used to approximately satisfy all equality and inequality constraints.

The solvers used in the above problems are briefly described below.

Optimization solvers

In case (i) above the problem in general has infinite solutions and an optimization criterion must be used to select one solution. Then the problem becomes an optimization problem with p unknowns, S_e linear equality constraints, S_i linear inequality constraints and $S_b = 2p$ simple bound constraints. Depending on the type of criterion used, one of the following three solvers may be employed:

- (i) The Simplex linear programming algorithm²² for a linear criterion.
- (ii) A quadratic programming algorithm^{23, 24} for a quadratic criterion like the following minimum change criterion:

$$\min \left(\sum_{e=1}^p \alpha_{e_i}^2 \right) \quad (98)$$

which is most often used in applications.^{11, 12, 14, 16}

- (iii) A conjugate gradient nonlinear algorithm²⁵ combined with an external penalty technique²⁶ for a nonlinear criterion of order higher than quadratic. In this case a nonlinear programming method based on sequential quadratic programming is also used.²⁷

The purpose of using so many different optimization algorithms at the expense of a significantly larger code is the following. The constraint equations in their present form are linear. The objective is in general quadratic but may be linear or nonlinear of order higher than quadratic. Algorithms that take advantage of specific properties of the problem in hand are dedicated algorithms and consequently faster and more accurate. We have found after many applications that it is preferable to expand RESTRUCT to include the Simplex algorithm and a quadratic algorithm based on Simplex, rather than solving all problems using the more general — and therefore not dedicated — nonlinear algorithms mentioned in (iii) above.

Linear equation solver

In case (ii α) explained above the problem is reduced to a set of simultaneous linear equations which are solved by the classical Cholesky factorization method.

Minimum error solver

This case arises in (ii β) and (iii) above. The algorithm works as follows.²⁸ The S_i inequality constraints are considered as equalities and the S_b bound constraints are ignored. The resulting problem has ($S_e + S_i$) equalities and p unknowns and is represented by eqn (99):

$$\begin{array}{l} [A] \{x\} = \{b\} \\ S \times p \quad p \times 1 \quad S \times 1 \end{array} \quad (99)$$

Since $S > p$ eqn (99) has no solution. Premultiplying by $[A]^T$ and solving for $\{x\}$ we get

$$\{x\} = ([A]^T [A])^{-1} [A]^T \{b\} \quad (100)$$

This approximate solution $\{x\}$ minimizes the norm

$$\| \{b\} - [A] \{x\} \| \quad (101)$$

5 COMPUTER IMPLEMENTATION

Most problems in structural analysis and design of marine structures are large-scale problems that can be solved only by general or special purpose finite element codes. For a redesign tool of such structures to be

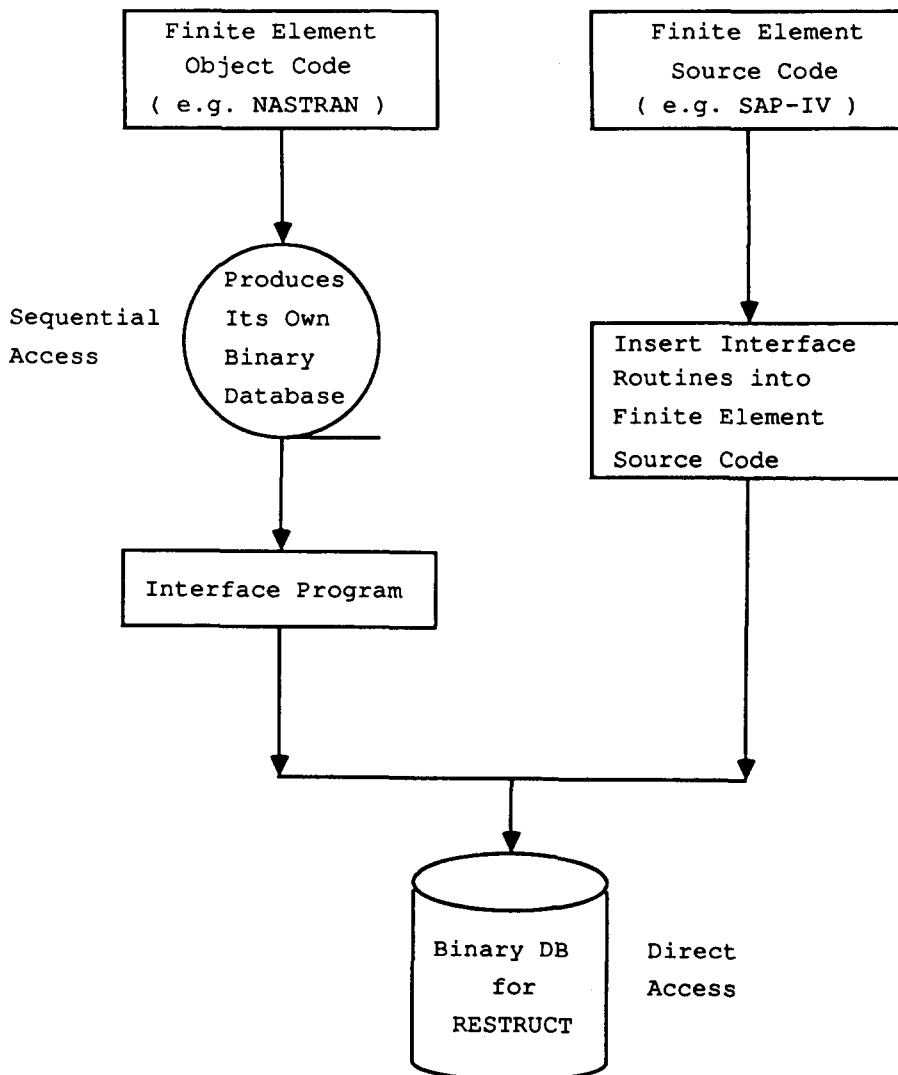


Fig. 4. Block diagram for production of database.

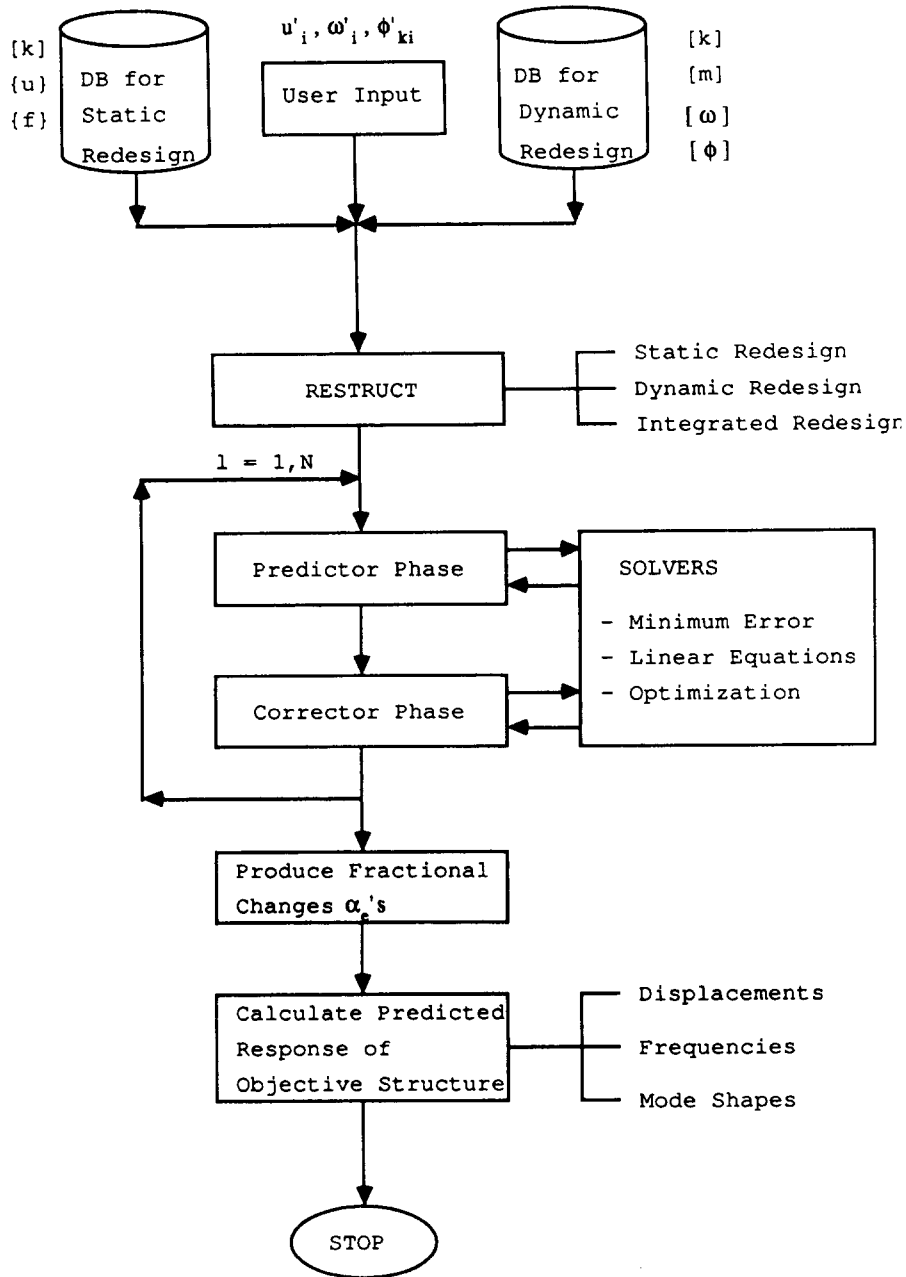


Fig. 5. Block diagram for redesign procedure (RESTRUCT).

of practical use it must serve as a postprocessor to such codes. Computer code **RESTRUCT** (about 19 500 Fortran commands) works as a postprocessor to general or special purpose FE programs. Thus it provides a high accuracy, fully automated structural redesign tool which can be used for solution of practical problems.

Some key ideas for computer implementation are diagrammatically summarized in Figs 1, 4 and 5. Figure 1 shows the relation between structural analysis and redesign. Any FE code may be used to compute the structural response of a given structure and fill in the redesign database. If response is not satisfactory, redesign must be performed using the database produced in the baseline analysis step.

Figure 4 shows the way the database for redesign is produced. If a FE code produces its own type of database the latter will be changed to produce the **RESTRUCT** database through an interface program. Such is the case when **NASTRAN** is used for the baseline structure. **NASTRAN** produces a database in binary code which can be read using a dedicated language (**DMAP**) and transformed to the **RESTRUCT** database which is also in binary code. This process is necessary because the source code of **NASTRAN** is not available. If the source code of the general or special purpose FE program, which is used for the static and/or dynamic analysis of the baseline structure is available, interface routines will be appended to its source code to produce the **RESTRUCT** database.

Figure 5 shows the redesign procedure implemented in **RESTRUCT**. It has three options, one for static, one for dynamic and one for static/dynamic (integrated) redesign. The latter uses data stored in the **RESTRUCT** database by both static and dynamic analyses runs. Following reading of the database and the user supplied input, **RESTRUCT** constructs static and/or dynamic constraint equations using the algorithm developed in Section 3. The resulting constraint problem is then formulated and solved for the required structural changes according to the methods explained in Section 4. Finally, **RESTRUCT** uses the produced structural changes to compute displacements, frequencies and eigenmodes of the objective structure.

6 REDESIGN APPLICATIONS

Several numerical applications are used in this section to test the applicability and efficiency of **RESTRUCT** as well as some practical aspects related to conflicting redesign requirements, number of modes

used in modal expansions, repeated eigenvalues and size of the FE structural model. Two structures are used, a simple 5-element clamped-hinged beam and a 192-d.o.f. offshore tower.

Beam problem

The simple 5-element clamped-hinged beam shown in Fig. 6 is first used to illustrate some of the basic features of RESTRUCT and the developed perturbation-based redesign methodology. Analysis of the baseline structure by NASTRAN produced a first frequency $F_1 = \omega_1/2\pi = 29.14$ Hz and a static displacement for the fourth node $d_4 = 7.09$ mm. Eight cases are analyzed for various values of the number of eigenmodes used in redesign, the objective values of d_4 and F_1 , and the number p of allowable structural changes. Results are shown in Tables 1–8. By comparing results in these tables the following conclusions can be drawn:

- (i) Trying to satisfy both static and dynamic constraints by changing only the beam stiffness — as in Cases 2, 4, 6 — does not produce good results. Results improve significantly when the cross-sectional area is also allowed to change, as in Cases 1, 3, 5. This is so because change in $A_i, i = 1, 2, \dots, 5$, results in mass change. It is, in general, impossible to satisfy both dynamic and static constraints by changing only structural stiffness. Change to mass is also needed.
- (ii) In Cases 1, 2, 3 and 4, F_1 and d_4 are assigned objective values that are in some sense compatible, that is $F_1' > F_1$ and $d_4' < d_4$. Both indicate that the objective structure is stiffer than the baseline structure. Thus, results in Cases 2 and 4, where only the beam stiffness and not the mass (area) is allowed to change, are reasonable.

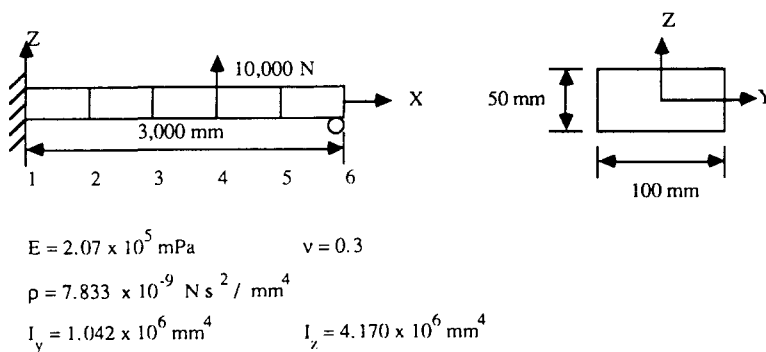


Fig. 6. Five element clamped-hinged beam model.

TABLE 1
Redesign of a Clamped-Hinged 5-Element Beam: Case 1

	Baseline	Constraints	Predicted	Reanalyzed	Error	
Frequency (Hz)	29.14	40.0	39.98	39.94	0.15%	
Displacement (mm)	7.09	3.0	3.017	3.083	2.77%	
<i>Element:</i>						
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	
$I_y = 1.042 \times 10^6$	1.3941	1.7075	1.3451	1.1491	1.5519	
α_c	2.495×10^6	2.821×10^6	2.444×10^6	2.239×10^6	2.659×10^6	
I'_y	1.6494	0.6597	0.0051	0.0954	1.1057	
α_c	13 247	8299	5026	5477	10 529	
A'						
<i>Prediction</i>						
Solver	S_c	S_i	S_b	S_e	S_i	S_b
Optimum	1	0	10	4	0	10
				<i>Number of modes used</i>		
				n_r	N	
				3	12	

TABLE 3
Redesign of a Clamped-Hinged 5-Element Beam: Case 3

	Baseline	Constraints	Predicted	Reanalyzed	Error
Frequency (Hz)	29.14	40.0	40.01	39.90	0.25%
Displacement (mm)	7.09	4.0	4.014	4.13	3.25%
<i>Element:</i>					
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
$I_y = 1.042 \times 10^6$	0.7664	1.2487	0.7792	0.5767	1.0545
I_y'	1.841×10^6	2.343×10^6	1.854×10^6	1.643×10^6	2.141×10^6
α_e	1.2181	0.2792	-0.2699	-0.1961	0.6890
A'	11 091	6396	3651	4020	8445
<i>Prediction</i>					
Solver	S_c	S_b	S_i	S_b	S_i
Optimum	1	10	0	10	0
<i>Correction</i>					
<i>Number of modes used</i>					
n_r					
3					
<i>Number of steps</i>					
N					
10					

TABLE 4
Redesign of a Clamped-Hinged 5-Element Beam: Case 4

	Baseline	Constraints	Predicted	Reanalyzed	Error
Frequency (Hz)	29.14	40.0	39.20	38.72	3.20%
Displacement (mm)	7.09	4.0	3.922	3.995	1.25%
<i>Element:</i>					
	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>
$I_y = 1.042 \times 10^6$	1.0845	0.0440	0.6759	0.8743	0.1110
I'_y	2.172×10^6	1.088×10^6	1.746×10^6	1.953×10^6	1.158×10^6
<i>Prediction</i>					
	<i>Correction</i>		<i>Number of modes used</i>		<i>Number of steps N</i>
Solver	S_c	S_b	S_i	S_b	n_r
Optimum	1	5	0	5	3
		4	0	5	10

TABLE 5
Redesign of a Clamped-Hinged 5-Element Beam: Case 5

	Baseline	Constraints	Predicted	Reanalyzed	Error
Frequency (Hz)	29.14	40.0	40.09	40.08	0.20%
Displacement (mm)	7.09	8.0	8.00	8.012	0.15%
<hr/>					
Element:	1	2	3	4	5
$I_y = 1.042 \times 10^6$	α_c	-0.3189	-0.1109	-0.0860	-0.2425
	I'_y	0.710×10^6	0.926×10^6	0.952×10^6	0.789×10^6
$A = 5000$	α_c	-0.6059	-0.5131	-0.5940	-0.2807
	A'	1971	2435	2030	3597
<hr/>					
	Prediction		Correction		Number of steps N
Solver	S_c	S_b	S_i	S_b	n_r
Optimum	1	10	0	10	8
					10

TABLE 6
Redesign of a Clamped-Hinged 5-Element Beam: Case 6

	Baseline	Constraints	Predicted	Reanalyzed	Error
Frequency (Hz)	29.14	40.0	30.45	30.39	24.03%
Displacement (mm)	7.09	8.0	6.44	6.51	18.63%
<i>Element:</i>					
	<i>I</i>	2	3	4	5
$I_y = 1.042 \times 10^6$	a_e	-0.0313	0.2456	-0.0053	-0.1300
	I_y'	1.009×10^6	1.298×10^6	1.036×10^6	0.907×10^6
<i>Prediction</i>					
Solver	S_e	S_i	S_b	S_c	S_r
Optimum	1	0	5	4	3
					<i>Number of modes used</i>
					<i>N</i>
					10

TABLE 7
Redesign of a Clamped-Hinged 5-Element Beam: Case 7

	Baseline	Constraints	Predicted	Reanalyzed	Error
Frequency (Hz)	29.14	40.0	40.09	40.00	0.00%
Displacement (mm)	7.09	8.0	8.00	8.26	3.25%
<hr/>					
Element:	1	2	3	4	5
$I_y = 1.042 \times 10^6$	α_c	-0.4171	-0.2664	-0.1698	-0.1578
	I'_y	0.607×10^6	0.764×10^6	0.865×10^6	0.878×10^6
$A = 5000$	α_c	-0.3055	-0.6735	-0.6589	0.1962
	A'	3473	1633	1706	5981
<hr/>					
	Prediction		Correction		Number of steps N
Solver	S_c	S_i	S_b	S_i	n_r
Optimum	1	0	10	0	5
				S_b	10
					10

TABLE 8
Redesign of a Clamped-Hinged 5-Element Beam: Case 8

	Baseline	Constraints	Predicted	Reanalyzed	Error	
Frequency (Hz)	29.14	40.0	40.09	39.10	2.25%	
Displacement (mm)	7.09	8.0	8.00	8.73	9.13%	
<i>Element:</i>						
	1	2	3	4	5	
$I_y = 1.042 \times 10^6$	α_i	0.9450	-0.0752	-0.2957	0.4160	
	I_y'	2.027×10^6	0.964×10^6	0.734×10^6	1.475×10^6	
$A = 5000$	α_c	-0.3090	-0.6899	-0.6485	0.1220	
	A'	3455	1551	1758	5610	
<i>Prediction</i>						
Solver	S_e	S_c	S_i	S_b	S_r	Number of steps N
Optimum	1	4	0	10	3	10
					n_r	

- (iii) On the contrary in Case 6 where $F_1' > F_1$ and $d_4' > d_4$ the first constraint requires the objective structure to be stiffer while the second less stiff. Allowing only change in stiffness shows that RESTRUCT cannot satisfy both constraints and produces a minimum error solution. These two constraints are incompatible in this case.
- (iv) Comparing Cases 5, 7, 8 it can be seen that the accuracy of the redesign algorithm reduces with the number of eigenmodes used in the modal series. This is expected since only three modes, as in Case 8, are not expected to contain most of the structure's modal energy.

Offshore tower problem

The 8-legged offshore tower shown in Fig. 7 is 229.5 ft high[†] and operates in 150 ft water depth. The tower at the base is 125 ft square and tapers linearly to 75 ft square at the deck. The FE model of the tower is composed of 104 circular tubular beam elements and has 192 degrees of freedom. Loading on the tower is due to:

- (i) 200 long ton[†] deck load which is applied to the structure as uniformly distributed loads at the deck nodal points.
- (ii) Wave hydrodynamic forces calculated for a design wave of 600 ft length and 20 ft height using Morison's equation. The wave propagates in the x direction.
- (iii) Wind current in the x direction with linear velocity profile of 2 knots at the mean free surface waterline and zero at the sea bed.

Static and dynamic analyses by NASTRAN produced (i) maximum displacement in the x -direction at node 11 (shown in Fig. 7) equal to 2.2739 in,[†] and (ii) a repeated first natural frequency $\omega_1 = \omega_2 = 4.695$ rad/s ($F_1 = F_2 = 0.7472$ Hz). Our redesign goals are to raise the first natural frequency above 5.3401 rad/s (0.85 Hz) and reduce static displacement at node 11. Two redesign problems are solved. In the first the displacement upperbound is set equal to 1.86 in and in the second equal to 1.71 in.

For redesign purposes the tower is subdivided into 6 substructures described in Table 9. Two variables are allowed to change in each substructure. Specifically for bending and torsional stiffness, changes to area moment of inertia are allowed; for mass and axial stiffness, changes to area are allowed. Let I, J and A be the moment of inertia, polar moment

[†] 1 in = 25.4 mm; 1 ft = 0.3048 m; 1 long ton = 1.016 05 tonne.

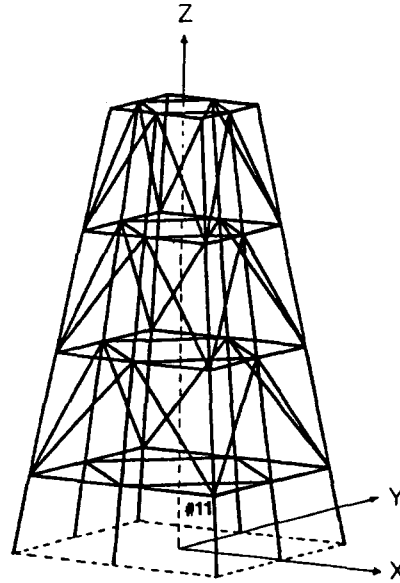


Fig. 7. Offshore tower with 8 legs and cross bracings.

of inertia and area respectively. Then we have

$$I = \frac{\pi(D_o^4 - D_i^4)}{64} \quad (102)$$

$$J = \frac{\pi(D_o^4 - D_i^4)}{32} = 2I \quad (103)$$

$$A = \frac{\pi(D_o^2 - D_i^2)}{4} \quad (104)$$

where D_o , D_i are the outer and inner tubular element diameters respectively. Let I' , J' and A' be the corresponding quantities of the objective structure. Then α_I and α_A , the allowable fractional changes to I and A respectively, are defined by the following equations

$$I' = I(1 + \alpha_I) \quad (105)$$

$$J' = J(1 + \alpha_I) \quad (106)$$

$$A' = A(1 + \alpha_A) \quad (107)$$

The total number of allowable changes is $p = 12$, that is fractional changes α_I and α_A are allowed in each structural group. From α_I and α_A , D'_o and D'_i can be computed using equations (108) and (109):

$$D'_o = \sqrt{(a^2 + b)/2a} \quad (108)$$

$$D_i' = \sqrt{(b - a^2)/2a} \quad (109)$$

where

$$a = (1 + \alpha_A) (D_o^2 - D_i^2) \quad (110)$$

$$b = (1 + \alpha_I) (D_o^4 - D_i^4) \quad (111)$$

The six substructures for the offshore tower, their redesign variables, and D_o and D_i for each substructure are shown in Table 9.

For the two redesign problems considered the results are summarized in Table 10 and evaluated in Table 11. The results in Table 10 show that substructures 1, 4 and 5 must be altered significantly in order to achieve the designer's goals. The remaining substructures are only slightly modified in both problems. In both problems the number of design variables p (=12) is greater than the number of equations in both predictor and corrector phases. The optimality criterion used to select the optimal redesign is that of minimum change between baseline and objective structures. An interesting byproduct of Redesign 1 is the reduction of the offshore tower weight, that is, by increasing the stiffness

TABLE 9
Offshore Tower Substructures, Redesign Variables, and Dimensions

Substructure number	Design variable α_e	Substructure description	D_o (in) ^a	D_i (in) ^a	Number of elements
1	α_1 (α_I) α_2 (α_A)	Legs below first bracing	30	29	8
2	α_3 (α_I) α_4 (α_A)	Legs between first and second bracing	24	23	8
3	α_5 (α_I) α_6 (α_A)	Legs above second bracing	24	23	16
4	α_7 (α_I) α_8 (α_A)	Horizontal bracing	19	18-25	32
5	α_9 (α_I) α_{10} (α_A)	Horizontal cross bracing	20	19-25	16
6	α_{11} (α_I) α_{12} (α_A)	Vertical cross bracing	24	23-25	24

^a1 in = 25.4 mm.

TABLE 10
Redesign of Offshore Tower

Substructure number	Redesign 1			Redesign 2		
	α_1	α_A	D'_o (in) ^a	α_1	α_A	D'_o (in) ^a
1	0.3032	-0.0167	34.391	0.4535	0.0094	35.823
2	0.0520	-0.0398	25.058	0.0643	0.0032	24.693
3	0.0022	-0.0443	24.532	0.0028	-0.0016	24.050
4	0.1748	-0.0032	20.565	0.1606	0.0493	19.962
5	-0.1248	-0.1636	20.383	-0.1053	-0.1147	20.060
6	0.0223	-0.0816	25.253	0.0271	0.0156	24.137

^a1 in = 25.4 mm.

TABLE 11
Evaluation of Results of Offshore Tower Redesign

	Redesign 1		Redesign 2			
	Redesign constraints (1)	Reanalysis by NASTRAN (2)	Error $\frac{ (2)-(1) }{(1)}$	Redesign constraints (1)	Reanalysis by NASTRAN (2)	Error $\frac{ (2)-(1) }{(1)}$
F_1 (Hz)	0.7472	0.8468	0.4%	0.8500	0.8511	0.1%
$(d_x)_{11}$ (in) ^a	2.2739	1.8714	0.6%	1.7100	1.7355	1.5%

^a 1 in = 25.4 mm.

and decreasing the mass of the tower the design goals of increasing the first natural frequency and decreasing the maximum displacement are achieved. If further reduction of the tower weight is desirable, a minimum weight optimality criterion should be used in place of the minimum structural change criterion used in this application. In Redesign 2 the maximum displacement reduction requirement is stricter, resulting in higher stiffness increase and minor change to the structural mass. In both redesign problems the following data are relevant:

- (i) Number of incremental steps $N = 4$.
- (ii) Number of redesign variables $\alpha_e, p = 12$.
- (iii) Number of used modes in modal expansions $n_r = 10$.
- (iv) CPU time for static analysis running NASTRAN on IBM 3090-400 is 5831 ms.
- (v) CPU time for dynamic analysis running NASTRAN on IBM 3090-400 is 16565 ms.

TABLE 12
CPU Time (ms) for Redesign of Offshore Tower, Running RESTRUCT on IBM 3090-400

Operation	Increment N				Total
	1	2	3	4	
<i>Predictor Phase</i>					
(i) Generation of linear dynamic equations	510	526	507	507	2050
(ii) Solution by optimization	33	34	33	33	133
(iii) Admixture coefficients	52 759	53 271	52 536	52 582	211 148
<i>Corrector Phase</i>					
(iv) Generation of nonlinear dynamic equations	16 112	16 136	16 008	16 010	64 266
(v) Generation of nonlinear static equations	7416	7400	7379	7340	29 535
(vi) Solution by optimization	50	52	43	45	190
<i>Totals</i>	76 880	77 419	76 506	76 517	307 322
Database operations					5385
Miscellaneous operations					25 223
Overall total					337 930

Table 12 shows a breakdown by operation and increment number of the CPU time required by RESTRUCT to run on IBM 3090-400. The most time consuming operation of RESTRUCT is that of computing the admixture coefficients at the end of the predictor phase in each increment. The overall redesign time is about 15 times the CPU time required for static and dynamic analysis by NASTRAN. Factors affecting the total cost of redesign by RESTRUCT are (i) the number of modes used, n_r , in modal expansions, (ii) the number of incremental steps N , (iii) the number of redesign constraints and variables, and (iv) the number of elements in the FE model.

The offshore tower has repeated eigenvalues because its geometry and structural properties do not change by a 90° rotation about the z-axis. As shown in Fig. 7 the z-axis is the axis of symmetry of the tower because it is the intersection of its two planes of symmetry. Equation (62) shows that when $\omega_i = \omega_j$, admixture coefficient c_{ij} is not defined. As explained in Section 3, c_{ij} can be set equal to zero. Then, when the static and dynamic equations are formulated in the corrector phase, the system of these equations is first purged of non-dominant coefficients to accelerate solution.

The simple redesign beam problems studied show that RESTRUCT can be used by designers to gain experience in redesign at an affordable cost. The observations made on the redesign of the clamped-hinged beam are obvious for such a simple and small-scale structure. For larger-scale structures and combinations of constraints whose effect on the objective structure is not so obvious RESTRUCT provides a unique tool for efficient redesign and identification of incompatible constraints. Finally, it should be mentioned that earlier versions of RESTRUCT have been used to redesign larger-scale structures with only static¹⁶ or only dynamic^{5,6,11-13} constraints. In these earlier versions different algorithms were used which could handle only static or only dynamic constraints. The largest structure redesigned so far is a 1253-d.o.f. microcomputer disk drive¹² with one frequency constraint. The integrated static/dynamic redesign algorithm developed in this work is faster and more efficient in database operations and accuracy in addition to being able to handle both static and dynamic redesign constraints.

7 CONCLUSIONS AND IMPACT ON STRUCTURAL DESIGN

A direct redesign method based on perturbation has been developed and shown to be an attractive alternative to trial and error redesign

techniques. Objectives in the design process become constraints in redesign. The algorithm developed and implemented in code RESTRUCT can handle static displacement, frequency and eigenmode constraints. RESTRUCT serves as a postprocessor to general and special purpose FE codes and can thus be used for redesign of large-scale structures. Further, it allows for large structural changes and can satisfy constraints which represent large changes in response properties. The cost of redesign depends on the number of degrees of freedom of the FE model, the number of redesign constraints, the size of the desirable changes to response particulars, the desired accuracy of the results and, most important, on compatibility of redesign constraints. The latter is obviously an important factor even in simple trial and error approaches. In any event perturbation-based redesign is conclusive — in the sense that it will produce an optimal or a minimum error design — and inexpensive compared to trial and error techniques. Several numerical applications were used to illustrate the method and look into problems of compatibility of redesign constraints, number of eigenmodes in modal expansions, repeated eigenvalues, and equality and inequality redesign constraints.

The perturbation-based redesign method developed in this work and implemented in RESTRUCT is expected to have impact on several aspects of design of marine structures. First of all, RESTRUCT being a postprocessor to general or special purpose FE programs automates redesign, reduces the required man-hours in data preparation, and takes the guesswork out of the redesign process. In addition it informs the designer of possible incompatibilities in the redesign constraints, can handle structures with repeated eigenvalues, and can produce a local optimal redesign in the vicinity of the baseline structure. Thus the designer may look forward to redesign as a means of producing a feasible design or simply improving his baseline structure within the original design constraints.

Redesign techniques have other important applications in structural design which the authors are currently pursuing. Specifically, the four following problems can be recast as redesign problems and are problems that attract significant research effort in structural reliability.

- (i) *Model correlation:* Mathematical models imperfectly reflect reality. The error in modelling is in general unknown. Computer (numerical) models, like FE models, provide approximations to mathematical models. Thus computer models may poorly represent the physical structure and may therefore poorly predict the response of structures. This is particularly true for

marine structures for which the manufacturing process is not very accurate. The model correlation problem is not so severe in aerospace structures or in general structural components which are manufactured more accurately.

- (ii) *Identification of failure mode:* The number of possible modes of failure of even simple structures is too large to permit enumeration. In structural reliability this is a challenging problem that currently prohibits application of sophisticated probabilistic techniques to large scale structures like marine structures. RESTRUCT can be used to identify dominant (most probable) failure modes according to some optimality criterion like (a) minimum structural change, (b) minimum strain energy change, or (c) minimum weight change between baseline and objective structures. In this case redesign constraints are used to establish bounds of structural failure.
- (iii) *Structural redundancy:* Large-scale structures are indeterminate to a high degree. Redundancy towards certain loads and environmental conditions is built into structures. The degree of redundancy can be computed by RESTRUCT by finding the difference between baseline and damaged structures in the process of identification of failure modes.
- (iv) *Redundancy in deteriorated structures:* Structures age and deteriorate in time. For example corrosion and fatigue may cause failure of structural members. RESTRUCT can be used to identify failure modes and redundancy of aged or deteriorated structures. This process may be carried out sequentially till structural collapse.

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