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SOLUTIONS OF THE DIFFUSION EQUATION IN CONES AND WEDGES

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Abstract—The neutron diffusion equation has been solved for cones and finite wedges. Criticality conditions are derived linking the dimensions of the cones and wedges to the nuclear properties of the system. The fundamental mode flux is evaluated numerically for a number of cases. We have also discovered an interesting minimum in the leakage rate as the cone or wedge angle passes through a certain value for a fixed volume. Some practical implications of this result are pointed out.

The exact solutions derived have been used as benchmarks to assess the accuracy of the finite element code TRIPAC in R–Z and X–Y–Z geometries for a four-group criticality problem in cones and wedges. Excellent agreement is found between the analytical results and the code for both eigenvalues and fluxes.

1. INTRODUCTION

Solutions of the diffusion equation in nonstandard geometries are of intrinsic interest but also of practical value since the results may be used to check the accuracy of computer codes. With this in mind we present some solutions of the equation (Glastone and Edlund, 1953):

$$\nabla^2\phi + B^2\phi = 0, \tag{1}$$

subject to zero flux on the boundary for nonreentrant cones and wedges.

Our solutions are given analytically and we present the flux distribution and the critical condition in terms of the physical dimensions of the cone or wedge. These results are employed to obtain a comparison with results from the finite element code TRIPAC and lend further support to its accuracy.

In addition to obtaining analytical solutions, we are able to show that, for a given buckling, the critical cone volume and corresponding surface area exhibit a minimum for a certain value of the cone angle. Similarly, for a fixed critical volume, the buckling has a minimum at a particular cone angle. Analogous behaviour arises for wedges.

2. SOLUTION OF THE DIFFUSION EQUATION FOR A BARE CONE

2.1. General solution and eigenvalues

Following the usual procedure (Morse and Feshbach, 1953), we can write the Helmholtz equation (called the diffusion equation in reactor physics) in spherical coordinates as:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right) + B^2\phi = 0, \tag{2}$$

where we note that $\phi = \phi(r,\theta)$, θ being the angle measured with respect to the z-axis. There is no dependence on the azimuthal angle due to symmetry.

The boundary conditions are:

(a) the flux is everywhere finite, (3)

(b) $\phi(r,\theta_0) = 0$, (4)

(c) $\phi(R_0,\theta) = 0$, (5)

where R_0 is the cone radius and θ_0 is its half-angle. We restrict $\theta_0 < \pi/2$ because we wish to avoid reentrant conditions. However, values of $\theta_0 > \pi/2$ could have physical significance if it is assumed that the volume contained within the reentrant line-of-sight region is black to neutrons.

Using the standard procedure of separation of variables, we can write the solution to equation (2) which satisfies condition (3) as:

$$\phi(r,\theta) = \frac{A}{\sqrt{r}}J_{\nu+1/2}(Br)P_\nu(\cos\theta), \tag{6}$$

where A is an arbitrary constant and ν is a parameter to be determined. $J_{\nu+1/2}(Z)$ is a Bessel function and $P_\nu(\cos\theta)$ is a Legendre polynomial.

Boundary condition (4) leads to:

$$P_\nu(\cos\theta_0) = 0; \text{ all } \nu \geqslant 0. \tag{7}$$

The roots of this equation lead to the appropriate value of ν. In fact unless ν is an integer there is an infinite number of solutions, but we are interested only in the smallest root since this corresponds to the fundamental mode. The precise roots of equation (7) require some effort to obtain but a good first approximation arises from the following asymptotic representation, viz. (Abramowitz and Stegun, 1965):

$$P_\nu(\cos\theta) \sim \left[\frac{2}{\pi\sin\theta}\right]^{1/2}\frac{\Gamma(\nu+1)}{\Gamma(\nu+\frac{3}{2})} \times \sin\left\{(\nu+\tfrac{1}{2})\theta+\frac{\pi}{4}\right\}+O(\nu^{-3/2}), \tag{8}$$

where $\nu \geqslant 1$ and θ is confined to the interval $(\varepsilon, \pi-\varepsilon)$ where ε is a small number. The greater the value of θ, the more accurate is equation (8).

Neglecting the term of $O(\nu^{-3/2})$ and applying condition (7) we find

$$(\nu+\tfrac{1}{2})\theta_0+\frac{\pi}{4} = n\pi, \quad n = 0, 1, 2, \ldots \tag{9}$$

The fundamental mode arises when $n = 1$ ($n = 0$ corresponds to negative ν) and thus:

$$\nu = \frac{3\pi}{4\theta_0}-\frac{1}{2}. \tag{10}$$

By comparison with an exact calculation, it may be shown that this formula is accurate to better than 2% even for $\theta_0 = 1.8°$. It becomes progressively more accurate as θ_0 increases being exact for $\theta_0 = \pi/2$.

The flux can now be written as:

$$\phi(r, \theta) = \frac{A}{\sqrt{r}} J_{\frac{3\pi}{4\theta_0}}(Br)\, P_{\frac{3\pi}{4\theta_0}-\frac{1}{2}}(\cos\theta). \tag{11}$$

Finally, using equation (5), we have:

$$J_{\frac{3\pi}{4\theta_0}}(BR_0) = 0, \tag{12}$$

which yields the critical condition between B and the physical dimensions of the cone.

The roots of equation (12) have been found from tables of Bessel functions (Abramowitz and Stegun, 1965) and are listed in Table 1a. There is an infinite number of such roots for a given θ_0 but we are only concerned with the one that corresponds to a non-negative eigenfunction. A useful empirical relation between BR_0 and θ_0 is found to be:

$$BR_0 = 3.07250\theta_0^{-0.9415}+2.5076. \tag{13}$$

Table 1a. Zeros of Bessel function of first kind of order $3\pi/4\theta_0$

$3\pi/4\theta_0$	θ_0	(BR_0) exact	(BR_0) approx.
3/2	$\pi/2$	4.493409	4.516
2	$3\pi/8$	5.135620	5.141
5/2	$3\pi/10$	5.763459	5.756
3	$\pi/4$	6.380160	6.365
7/2	$3\pi/14$	6.987932	6.967
9/2	$\pi/6$	8.182561	8.158
11/2	$3\pi/22$	9.355812	9.333
6	$\pi/8$	9.936110	9.915
7	$3\pi/28$	11.08637	11.072
8	$3\pi/32$	12.22509	12.220
25/2	$3\pi/50$	17.250455	17.291
39/2	$3\pi/78$	24.878005	24.979

The accuracy of this relationship is better than 0.5% throughout the range of values of θ_0 shown in Table 1.

As an indicator of the overall accuracy of equations (10) and (13) we note that an accurate determination of BR_0 for $\theta_0 = 1.8°$ leads to $BR_0 = 80.53$, whilst the approximation yields $BR_0 = 82.39$, an error of 2.3%. As θ_0 increases, this error progressively becomes smaller, e.g. at $\theta_0 = 13.22°$, the error is 2.1% and at $\theta_0 = 54.73°$ it is -0.7%. Thus our empirical result would appear to be a useful approximation for bare cones.

In some calculations, it is more practical to select an integer value of ν and find the corresponding value of θ_0 from the equation:

$$P_\nu(\cos\theta_0) = 0.$$

Such roots are simply the Gauss–Legendre quadrature points and are known very accurately. With an integer value of ν it is easy to find the roots of

$$J_{\nu+1/2}(BR_0),$$

from the tables in Abramowitz and Stegun (1965). We shall adopt this procedure in our comparison with the finite element method to be described below and give some illustrative results in Table 1b.

Table 1b. Exact roots and half-angles for cone

ν	BR_0	θ_0
1	4.493409	90
2	5.763459	54.73561
3	6.987932	39.23152
4	7.58834	30.55559
5	8.77148	25.01733
6	9.93611	21.17690
7	11.08637	18.35785
8	12.22509	16.20078

2.2. Optimum size of cone

Let us assume that we have a cone of fixed buckling, i.e. fixed net leakage, and require to know how the volume and surface area vary with θ_0.

The volume of a cone is given by the expression:

$$V = \frac{2\pi R_0^3}{3}(1-\cos\theta_0). \tag{14}$$

Using the empirical relationship (13) for R_0 as a function of θ_0 (remember that B is fixed), we find that the volume is:

$$V(\theta_0) = \frac{2\pi}{3B^2}(\alpha\theta_0^{-\beta}+\gamma)^3(1-\cos\theta_0) \tag{15}$$

where α, β and γ are given in (13).

It is readily shown that $V(\theta_0)$ has a minimum at $\theta_0 = 0.5259$ (30.132°). Similarly, the surface area of the cone which is given by:

$$S(\theta_0) = 2\pi R_0^2(1-\cos\theta_0)+\pi R_0^2\sin\theta_0 \tag{16}$$

also has a minimum at $\theta_0 = 0.559$ (32°). Both $V(\theta_0)$ and $S(\theta_0)$ are shown in Fig. 1 as functions of θ_0.

Let us now consider a cone of constant volume and examine the buckling as a function of θ_0. Using the earlier formulae we readily find that:

$$B^2(\theta_0) = \left(\frac{2\pi}{3V}\right)^{2/3}(\alpha\theta_0^{-\beta}+\gamma)^2(1-\cos\theta_0)^{2/3}. \tag{17}$$

This is illustrated graphically in Fig. 2 and exhibits a minimum at $\theta_0 \simeq \pi/6$. Clearly there is a critical value of cone angle which minimizes the leakage. The possibility of using such a phenomenon as a reactor control device should be noted.

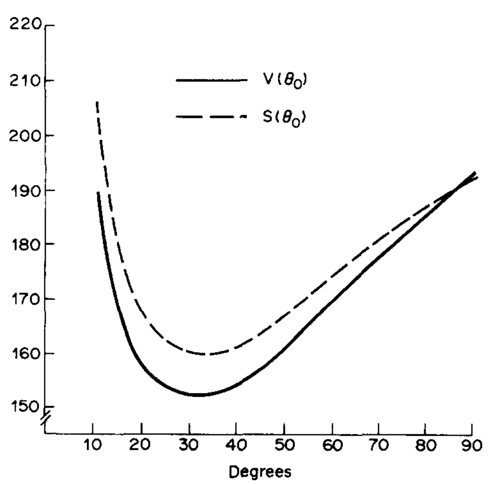

Fig. 1. The variation of the critical volume and surface area with cone half-angle θ_0; the ordinate is in arbitrary units.

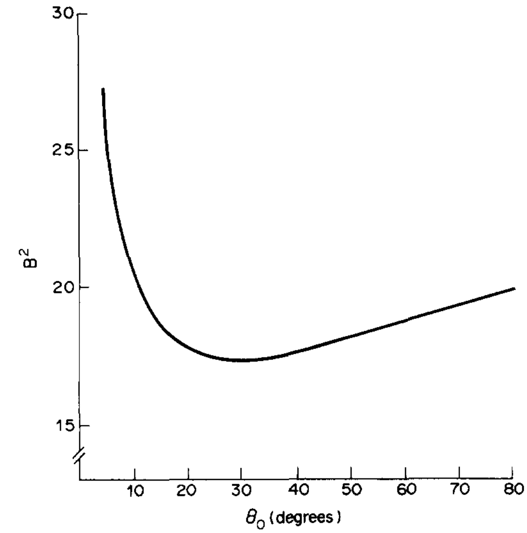

Fig. 2. The variation of the buckling with cone half-angle, θ_0, for a fixed volume; the ordinate is in arbitrary units.

2.3. Flux distribution in a cone

To illustrate the flux shape in the cone, we consider the following parameters: $\theta_0 = \pi/6$ (30°) and hence $BR_0 = 8.182561$, from Table 1. We also note that $v = 4$ and thus the flux distribution is from equation (6):

$$\phi(r,\theta) = \left(\frac{\pi}{2Br}\right)^{1/2} J_{9/2}(Br)P_4(\cos\theta). \tag{18}$$

The flux is given in Table 2 for $\theta = 0$ for various values of the ratio r/R_0. We note that a maximum arises at $r = 0.685R_0$. It is also interesting to examine the angular variation of the flux which follows the function $P_4(\cos\theta)$. The values of $P_4(\cos\theta)$ for a range of

Table 2. Flux along central axis of cone, $\theta_0 = \pi/6$, $BR_0 = 8.18$

r/R_0	$\phi(r, 0)$
1	0
0.917	0.0793
0.856	0.1327
0.733	0.1968
0.685	0.2015
0.672	0.2008
0.611	0.1870
0.489	0.1249
0.367	0.05615
0.244	0.01408
0.122	1.011×10^{-3}
0.0611	6.539×10^{-5}
0.0	0.0

Table 3. Angular flux across cone

θ°	$P_4(\cos\theta)$
0.0	1.0
5	0.9623
10	0.8532
20	0.4750
25	0.2465
30	0.0234
30.55559	0.0

θ are shown in Table 3. The flux is not precisely zero at $\theta = 30^{\circ}$ due to the approximate formula for ν of equation (10). In fact, it is zero at the smallest root of $P_4(\cos\theta) = 0$, i.e. $\theta = 30.55559^{\circ}$. The fluxes are illustrated graphically in Fig. 3.

3. SOLUTION OF THE DIFFUSION EQUATION FOR A BARE, FINITE WEDGE

3.1. General solution and eigenvalues

The diffusion equation for a wedge of finite height is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}+\frac{\partial^2\phi}{\partial z^2}+B^2\phi = 0 \quad (19)$$

where $\phi = \phi(r,\theta,z)$.

We assume that one face of the wedge is in the plane $\theta = 0$ and the other in the plane $\theta = \theta_0$. The wedge is

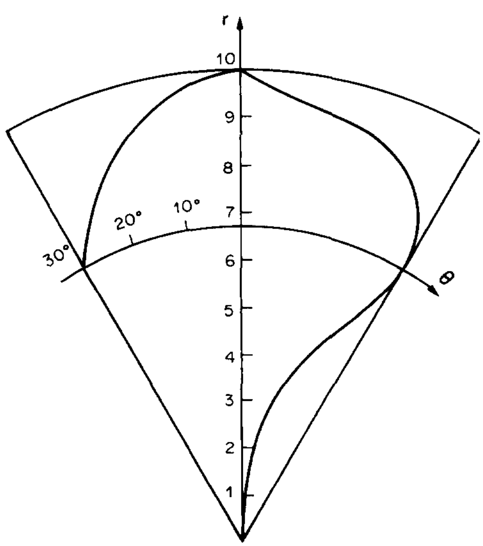

Fig. 3. Flux distribution inside a cone with $\theta_0 = \pi/6$, $R_0 = 10$. The curve on the right hand side of the figure represents the radial flux distribution and that on the left the angular distribution; actual magnitudes are arbitrary.

of radius R_0 and height H. The zero flux boundary conditions are:

$$\text{(a)} \quad \phi(r,0,z) = 0, \quad (20)$$

$$\text{(b)} \quad \phi(r,\theta_0,z) = 0, \quad (21)$$

$$\text{(c)} \quad \phi(R_0,\theta,z) = 0, \quad (22)$$

$$\text{(d)} \quad \phi(0,\theta,z) = 0, \quad (23)$$

$$\text{(e)} \quad \phi(r,\theta,0) = 0, \quad (24)$$

$$\text{(f)} \quad \phi(r,\theta,H) = 0. \quad (25)$$

Using separation of variables, we find that the solution can be written:

$$\phi(r,\theta,z) = CJ_{\mu_1}(B_r r)\sin(\mu_1\theta)\sin(B_z z), \quad (26)$$

where

$$\mu_1 = \pi/\theta_0 \quad (27)$$

and

$$B_z^2 = \left(\frac{\pi}{H}\right)^2 = B^2 - B_r^2. \quad (28)$$

B_r^2 is the radial buckling and is found from condition (23) viz.

$$J_{\pi/\theta_0}(B_r R_0) = 0. \quad (29)$$

Thus we have a relation between the physical dimensions of the wedge and its nuclear properties, B^2. From tables of zeros of Bessel functions we can obtain Table 4. Again, for convenience in calculation, an empirical relationship between $B_r R_0$ and θ_0 has been obtained thus:

$$B_r R_0 = 4.04888\theta_0^{-0.9378}+2.50. \quad (30)$$

Table 4. Zeros of Besssel function of first kind of order, π/θ_0

π/θ_0	θ_0	$(B_r R_0)$ exact	$(B_r R_0)$ approx.
1	π	3.83171	3.884
3.2	$2\pi/3$	4.493409	4.524
2	$\pi/2$	5.135620	5.151
5/2	$2\pi/5$	5.763459	5.768
3	$\pi/3$	6.380160	6.377
7/2	$2\pi/7$	6.987932	6.981
4	$\pi/14$	7.58834	7.578
9/2	$2\pi/9$	8.182561	8.171
5	$\pi/5$	8.77148	8.760
11/2	$2\pi/11$	9.355812	9.346
6	$\pi/6$	9.936110	9.928
13/2	$2\pi/13$	10.512835	10.507
7	$\pi/7$	11.08637	11.083
15/2	$2\pi/15$	11.657032	11.657
8	$\pi/8$	12.22509	12.228
25/2	$2\pi/25$	17.250455	17.284
39/2	$2\pi/39$	24.878005	24.933

The accuracy of the expression may be judged from Table 4. For $\theta_0 < 120°$ it is better than 0.7%.

3.2. Optimum size of wedge

As in the case of the cone, we look for an angle which minimizes the buckling for a given wedge volume. Using equation (30) and the volume of the wedge, we get:

$$B_r^2(\theta_0) = \frac{H}{2V}\theta_0(a\theta_0^{-b}+c)^2, \tag{31}$$

where a, b and c are defined by equation (30). As Fig. 4 shows, the radial buckling has a minimum and by differentiation of equation (31) we find:

$$(\theta_0)_{\min} = \left\{\frac{a(2b-1)}{c}\right\}^{1/b} = 83.15°. \tag{32}$$

3.3. Flux distribution in a wedge

For $\theta_0 = \pi/4$, we find that the flux distribution can be written:

$$\phi(r,\theta,z) = J_4(B_r r)\sin(4\theta)\sin\left(\frac{\pi z}{H}\right) \tag{33}$$

where $B_z = \pi/H$ and $B_r R_0 = 7.58834$.

The maximum value of ϕ along the radial direction arises when:

$$\frac{\mathrm{d}J_4(B_r r)}{\mathrm{d}r} = 0, \tag{34}$$

which leads to $B_r r_{\max} = 5.31755$, i.e. $r_{\max} = 0.70075R_0$. Of course, in the axial direction, the maximum is at $z = H/2$ and in the θ-direction at $\theta_0/2$.

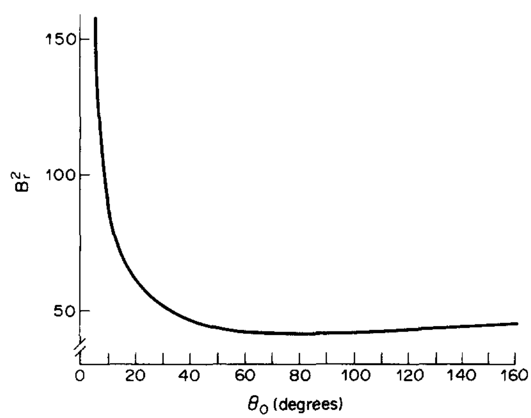

Fig. 4. Variation of radial buckling B_r^2 with wedge angle θ_0 for constant volume; the ordinate is in arbitrary units.

4. NUMERICAL VERIFICATION OF CODE AND THEORY

To verify the theoretical relationships derived in the previous sections, particular examples of the cone and wedge configurations are solved by means of a multigroup finite element transport code called TRIPAC. This code is a development of two earlier codes, namely FELICIT (Wood and Williams, 1984; Wood, 1986) and EVENT (De Oliveira, 1986, 1987). A feature of TRIPAC, as of its precursors, is its ability to handle complex geometries, and the two configurations considered in this paper provide an interesting test of this aspect of the code's capabilities. The numerical solutions that we give also provide useful benchmarks for proving other techniques and computer codes for which the claim of geometrical flexibility is also advanced. In this paper we are concerned with diffusion theory, i.e. P_1/P_1 solutions, but of course TRIPAC is also capable of providing higher-order transport solutions, namely, P_n/P_l.

Two cases of the cone are considered, and two of the wedge. Four-group linear anisotropic scattering data is used for the homogeneous systems. The data is based on the simple HTGR core data used by White and Frank (1987). Our data is listed in Appendix A; the corresponding value of materials buckling is $B_{\max}^2 = 1.9701604 \times 10^{-2}$. In the computations, iterations are continued until the eigenvalue (k_{eff}) and eigenvector have converged, respectively, to 10^{-6} and 10^{-5}. Also the eigenvector is normalized to unit fission neutrons produced in the system, i.e.

$$\frac{1}{k_{\text{eff}}}\int\int \mathrm{d}E\,\mathrm{d}v\,\bar{\nu}\Sigma_f\phi(E,\mathbf{r}) = 1.$$

In the case of the wedge, the eigenvalue is actually normalized to 1/4 of the total volume—this is explained below. It is perhaps worth mentioning that this particular data has the fundamental and next lower eigenvalue close together (White and Frank, 1987). Thus, unless a power iteration convergence acceleration scheme is employed in the code, a large number of iterations may be necessary to produce the accuracy we quote in TRIPAC, outer iteration acceleration is achieved by Chebyshev extrapolation.

4.1. The cone

The cone is solved in R–Z geometry using 81 quadratic elements. The element mesh employed is illustrated in Fig. 5. The exact and TRIPAC results are compared in Tables 5 and 6. Clearly, the agreement is excellent. The flux distribution along the z-axis of the smaller cone is shown in Fig. 6.

Figure 7 shows the variation of critical volume of a cone for fixed buckling as a function of the cone

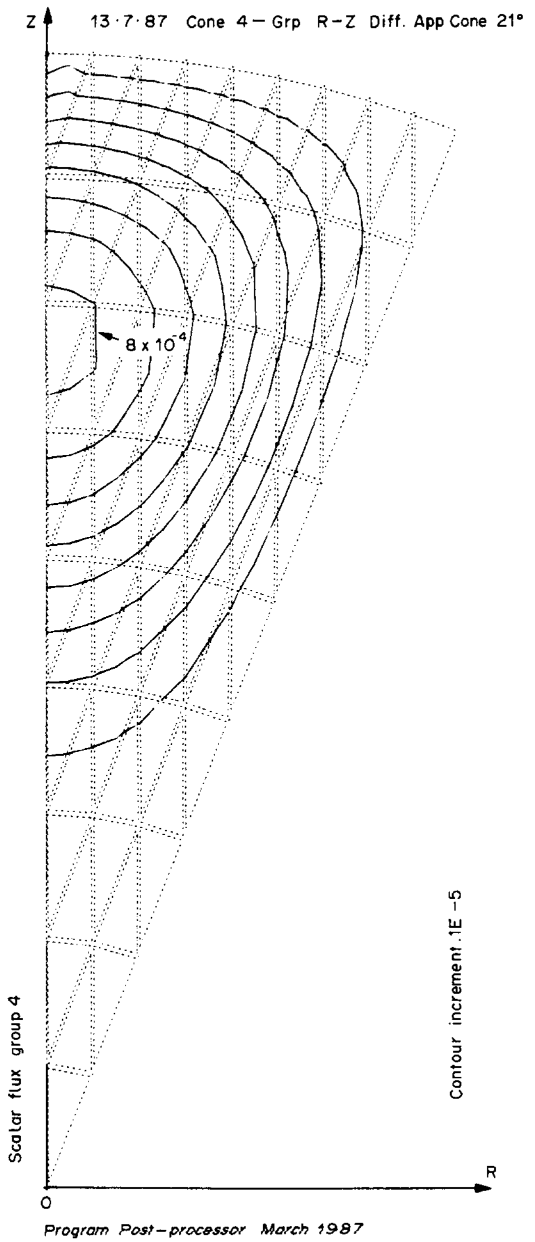

Fig. 5. Flux contours for group 4 neutrons in 21° cone. Contour increment in 1×10^{-4}.

half-angle. This is a four-group calculation using the data in Appendix A; it confirms the analytical result in Section 2.2 and indicates an optimum half-angle of about 30°.

4.2. The wedge

This problem is solved in X–Y–Z geometry. The basic element in the X–Y plane is an arbitrarily ori-

Table 5. Comparison of results for critical cones

	Critical dimensions		Exact					TRIPAC			
Case	Radius	θ_0 (degrees)	$\bar{\Phi}_1 \times 10^5$	$\bar{\Phi}_2 \times 10^5$	$\bar{\Phi}_3 \times 10^6$	$\bar{\Phi}_4 \times 10^5$	k_{eff}	$\bar{\Phi}_1 \times 10^5$	$\bar{\Phi}_2 \times 10^5$	$\bar{\Phi}_3 \times 10^6$	$\bar{\Phi}_4 \times 10^5$
a	410.6127	54.73561	0.12244	0.28434	0.43620	0.21030	1.000027	0.12244	0.28434	0.43621	0.21031
b	748.9780	21.17690	0.12627	0.29323	0.44984	0.21688	0.999988	0.12627	0.29323	0.44984	0.21688

Table 6. Comparison of critical flux distributions in 54° cone, group 4 neutrons

r	z	TRIPAC × 10^5	EXACT × 10^5
0.0	390.082	0.1324	0.1294
0.0	307.960	0.6317	0.6271
0.0	287.429	0.7155	0.7116
0.0	246.368	0.7991	0.7970
0.0	184.776	0.7031	0.7020
0.0	123.184	0.4173	0.4173
0.0	61.592	0.1234	0.1231
95.777	248.827	0.6213	0.6208
126.706	160.352	0.3215	0.3215
152.234	336.739	0.1960	0.1969

Note: cone is solved by TRIPAC in R–Z geometry.

ented triangle. Again, a quadratic triangle is used. The subdivision of the X–Y plane is shown in Fig. 8. These triangles form the cross sections of prisms whose axes are parallel to the z-axis. Each prism so formed is further subdivided into 3 tetrahedra. Thus the basic element in space is, in this case, a quadratic tetrahedron (with 10 nodes). Because of symmetry, the problem need only be solved in a 1/4 wedge by taking the origin to be at the mid-height of the wedge and exploiting symmetry boundary conditions along appropriate axes. As with the cone, the boundary condition on the surface of the wedge is zero flux.

The exact and TRIPAC results are compared in Tables 7 and 8. In TRIPAC, 735 quadratic tetrahedra are used for each wedge. An isometric view of the flux in a critical wedge is shown in Fig. 9. For the wedge, as for the cone, the agreement between theory and TRIPAC results is excellent.

Appendix B indicates how the volume-averaged fluxes are calculated for cones and wedges.

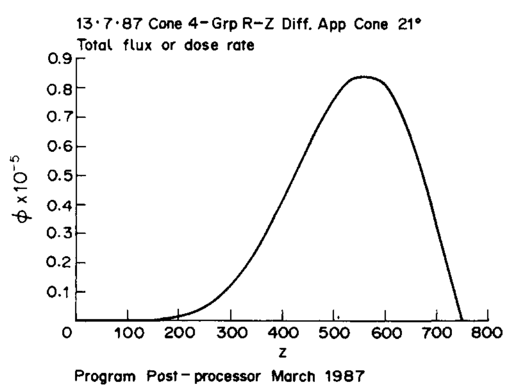

Fig. 6. Variation of flux along z-axis for 21° cone (group 4 neutrons).

Table 7. Comparison and results for critical wedges

Case	Critical dimensions			Exact					TRIPAC			
	Radius	Total height	θ_0	$\bar{\phi}_1 \times 10^5$	$\bar{\phi}_2 \times 10^5$	$\bar{\phi}_3 \times 10^6$	$\bar{\phi}_4 \times 10^5$	k_{eff}	$\bar{\phi}_1 \times 10^5$	$\bar{\phi}_2 \times 10^5$	$\bar{\phi}_3 \times 10^6$	$\bar{\phi}_4 \times 10^5$
a	600	282.4044	90°	0.37589	0.87291	0.13391	0.64562	0.99981	0.37587	0.87281	0.13389	0.64549
b	600	516.0318	45°	0.41142	0.95542	0.14657	0.70665	0.99980	0.41140	0.95530	0.14655	0.70650

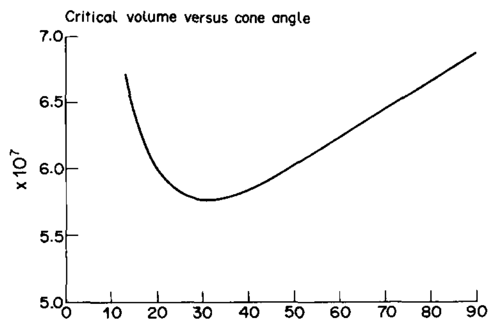

Fig. 7. Variation of critical volume of cone with half-angle θ_0 for fixed buckling using four-group data.

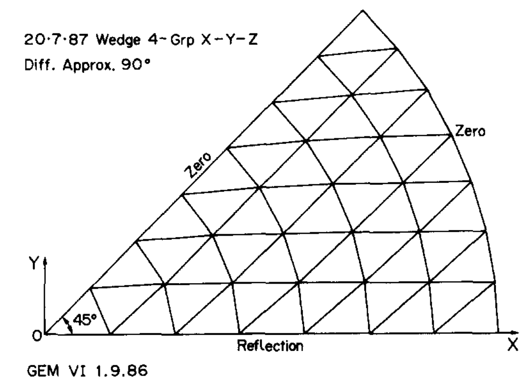

Fig. 8. Subdivision of X–Y plane into quadratic triangles for 90° wedge. Because of symmetry, only 1/2 of X–Y plane need be solved and 1/4 of volume of wedge.

5. CONCLUSIONS AND SUMMARY

Some exact solutions of the diffusion equation in cones and wedges have been obtained leading to critical conditions and fundamental mode flux distributions. We have observed that the volume and surface area of cones and wedges go through a minimum at a certain value of cone or wedge angle for a fixed buckling. Similarly, the buckling has a minimum for fixed volume at a critical angle. The latter condition implies a minimum leakage condition and has implications for reactor control and storage of fissile material.

We have also used these exact solutions to assess the accuracy of the TRIPAC finite element code. A

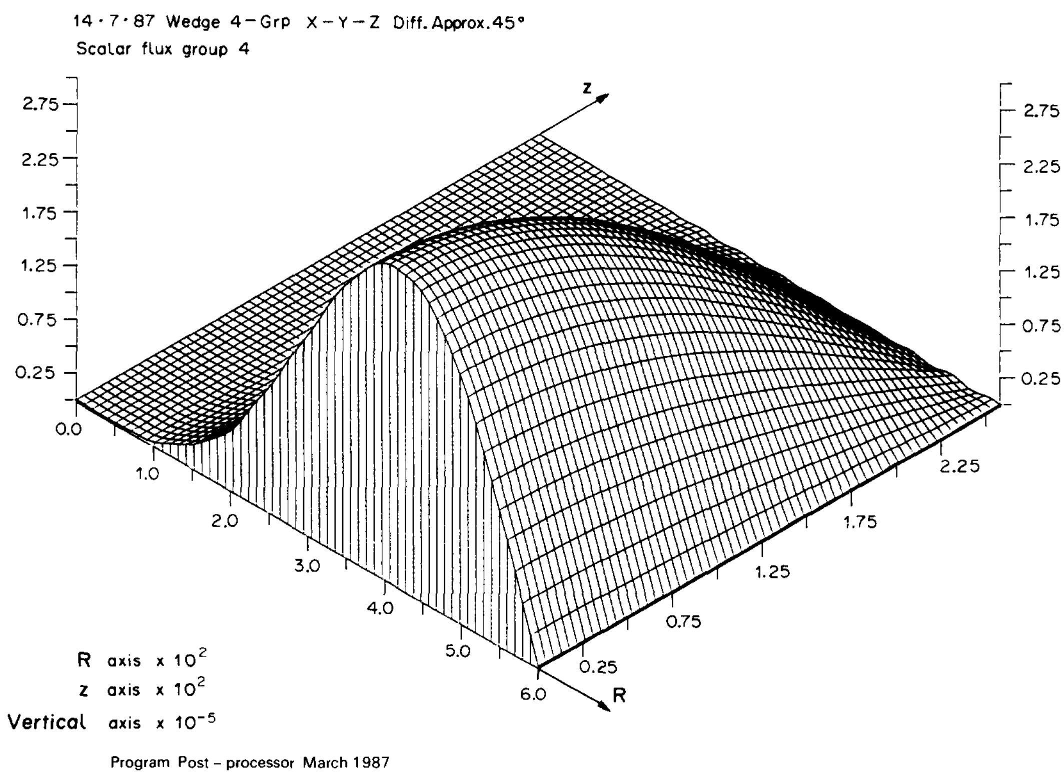

Fig. 9. Isometric view of flux in 45° wedge, for the plane $Y = 0$ (group 4 neutrons).

Table 8. Comparison of critical flux distributions in 90° wedge, group 4 neutrons

x	y	z	TRIPAC $\times 10^5$	EXACT $\times 10^5$
85.714	0.0	0.0	0.3324	0.3370
171.429	0.0	0.0	1.1782	1.1737
257.143	0.0	0.0	2.0833	2.0712
342.857	0.0	0.0	2.5521	2.5390
428.571	0.0	0.0	2.2798	2.2733
514.286	0.0	0.0	1.3032	1.3072
190.536	97.0872	70.6011	0.6853	0.6826
414.445	99.740	84.7213	1.1970	1.1958
355.528	308.062	0.0	0.2642	0.2666
300.917	160.844	70.6011	0.9993	0.9964

Note: wedge is solved by TRIPAC in X–Y–Z geometry.

four-group bare reactor criticality problem is solved for a variety of wedges and cones. Both eigenvalues and critical fluxes are shown to be in excellent agreement with the analytical results. Cones and wedges are a sensitive test of any numerical procedure and lend further confidence in the accuracy of the TRIPAC code.

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APPENDIX A

For each energy group i:

$$\Sigma \qquad \Sigma_a \qquad \bar{\nu}\Sigma_f$$

$$\Sigma_{s0}(E_i \to E_1) \quad \Sigma_{s0}(E_i \to E_2)$$

$$\Sigma_{s1}(E_i \to E_1) \quad \Sigma_{s1}(E_i \to E_2)$$

Note: diffusion coefficient $D = 1/3\Sigma_{tr}$ where $\Sigma_{tr} = \Sigma - \Sigma_{s1}$.

Four-group data

0.14184397E+00	0.844000E−04	0.784000E−04	
0.12945957E+00	0.12300000E−01	0.00000000E+00	0.00000000E+00
0.012945957E+00	0.0	0.0	0.0
0.27777778E+00	0.145100E−02	0.488000E−03	
0.00000000E+00	0.27255678E+00	0.37700000E−02	0.00000000E+00
0.0	0.027255678E+00	0.0	0.0
0.28248588E+00	0.121900E−02	0.132000E−02	
0.00000000E+00	0.00000000E+00	0.25816688E+00	0.23100000E−01
0.0	0.0	0.025816688E+00	0.0
0.28985507E+00	0.454000E−02	0.678000E−02	
0.00000000E+00	0.00000000E+00	0.00000000E+00	0.28531507E+00
0.0	0.0	0.0	0.028531507E+00

Normalized fission spectrum 0.9675 0.0325 0.0 0.0

APPENDIX B

Average Fluxes

In order to obtain average fluxes it is necessary to calculate:

$$\bar{\phi} = \frac{1}{V}\int_V \mathrm{d}\mathbf{r}\,\phi(\mathbf{r}).$$

For the cone with:

$$\phi(r,\theta) = \frac{1}{\sqrt{r}} J_{\nu+1/2}(B_r)P_r(\cos\theta),$$

this expression becomes:

$$\bar{\phi} = \frac{3}{R_c^3(1-\cos\theta_0)}\int_0^{R_c} \mathrm{d}r\, r^{3/2}J_{\nu+1/2}(B_r)$$

$$\times \int_0^{\theta_0} \mathrm{d}\theta \sin\theta\, P_\nu \cos\theta).$$

For integer values of ν, it is easy to carry out the θ integration and although the integration over the Bessel function can be represented by an infinite sum, it is more convenient to evaluate this directly by quadratures.

For wedges we have:

$$\phi(r,\theta,z) = J_{\mu_1}(B_r r)\sin(\mu_1\theta)\sin(B_z z),$$

whence

$$\bar{\phi} = \frac{8}{\pi^2(B_r R_0)^2}\int_0^{B_r R_0} \mathrm{d}x\, x J_{\mu_1}(x),$$

where

$$\mu_1 = \pi/\theta_0.$$