PROFitting frOM 'COUNTERVAILING' POWER*

An Effect of Government Control

William James ADAMS
University of Michigan, Ann Arbor, MI 48109, USA

David E.M. SAPPINGTON
Bell Communications Research

Final version received September 1987

We demonstrate that a firm subject to government regulation might expect an increase in profit upon creation of an independent monopoly upstream. Such a monopoly serves to increase the expected cost of the regulated firm. As a result, even regulators who behave socially optimally will decrease the frequency with which they audit reports of high production-cost. The reduction in governmental investigation permits the firm to increase the rents it derives from its superior information about cost. The possible implications of our model for a theory of 'political limit pricing' are also briefly mentioned.

1. Introduction

When the prices of its inputs rise, a firm's profit falls. In competitive markets, this basic intuition is usually confirmed. Where market power exists, however, intuition can mislead. For example, as Williamson (1968) shows, an increase in the wage rate can protect capital-intensive incumbents from the potential competition of labor-intensive rivals. Salop and Scheffman (1983) have also demonstrated the importance of raising rivals' costs when attempting to erect barriers to new competition. Such attempts could involve increases in the cost of the firm itself.

In this article, we develop a complementary explanation of the profitability of raising input prices. Our explanation stresses the impact of governmental regulation on the firm's behavior. For a variety of reasons, governments are often prepared to intervene in industries to limit the gap between price and (some measure of) cost. Intervention occurs in our model when its expected social benefit exceeds its social cost. In this setting, an increase in the firm's expected costs can reduce the expected social benefit of government regula-

*The views expressed in this paper are not necessarily those of Bell Communications Research. We are grateful to the Editorial Board and anonymous referees for helpful comments.
tion; and the reduced intervention that results can increase the profit of the firm. In other words, from the firm's perspective, the direct reduction in profit that ensues from higher expected input prices can be more than offset by the increase in profit that results from less stringent government control. This possibility is established in section 3.1.

Although we cannot claim that our simple model is sufficiently rich to analyze industries of realistic complexity, we do believe our analysis can inform studies of cost control in regulated industries. Furthermore, the same basic insight might apply to industries under the threat of potential government control. 3 We shall return to this point, albeit briefly, in section 4. The analysis begins in section 2 with the benchmark case of an unregulated monopolist.

2. Unregulated environment

In this section, we examine a monopolist whose behavior is not subject to governmental regulation. We do so to clarify the impact of regulation per se in section 3. The cost function of the monopolist is $C(Q) = F + cQ$, where $F \geq 0$ and $c > 0$ are parameters, and $Q$ is the level of output. Demand for the firm's product is given by $Q(p)$, where $p$ is the unit price, and $dQ/dp < 0$.

Initially, because of residual uncertainty about input prices, the monopolist's costs are not known perfectly. We capture this uncertainty simply by allowing the parameter $c$ to take on one of two values, $c_1$ or $c_2$, where $c_2 > c_1$. The probability that $c = c_1$ is $\phi_1$, where $\sum_{i=1}^{2} \phi_i = 1$. The greater the value of $\phi_2$, the greater the expected cost of production.

The cost uncertainty is resolved for the monopolist before he selects the price for his product. Knowing $F$, $c$, and $Q(p)$, he chooses $p$ so as to maximize profit. We let $\Pi_u = [p_i - c_i]Q(p_i) - F$ represent the maximum profit the firm can earn in an unregulated environment with marginal cost $c_i$. $p_i$ denotes the price that secures the profit level $\Pi_i$ when marginal cost is $c_i$.

In the unregulated environment, the firm's expected profit before production costs are known perfectly is denoted $E\Pi_u = \phi_1 \Pi_1 + \phi_2 \Pi_2$. Thus, $E\Pi_u$ is

1 Let us be clear from the outset as to why we assume the government acts to promote economic efficiency. Certainly there are alternative theories of government behavior. For example, those based on the government's inability to formulate policies consistent with its goals, and those based on the government's capture by those it regulates. Under such theories, it would not be surprising to find that a regulated firm can increase its profit by increasing the costs of its operation. In contrast, when government is inspired solely by economic efficiency, the profitability to a regulated firm of increasing its expected cost is not obvious. In short, we choose an objective for government that does not facilitate unduly our intended demonstration.

2 For example, the argument developed here explains how the international petroleum corporations could conceivably have gained from the creation of OPEC, even had the corporations owned no crude, and even had OPEC not served to stabilize oligopolistic agreements downstream.

3 The superscript $u$ denotes the 'unregulated' environment.
simply a weighted average of the firm's profits under the two possible cost realizations. Since \( \Pi_1 > \Pi_2 \), expected profit declines as the probability of high cost rises. Hence, in an unregulated environment, the profit-maximizing firm will never strictly prefer the creation of an independent monopoly upstream if the sole effect of the upstream monopolist is to increase expected downstream production costs, i.e., to increase \( \phi_2 \). This is how we model the creation of a monopoly upstream.

3. Regulated environment

Now suppose the monopolist of section 2 is subject to government regulation. In particular, suppose there is a regulator who is charged with setting the price the firm can charge. Regulation is a nontrivial exercise here because the regulator is not perfectly informed about the monopolist's cost structure. At the time when the terms of the regulatory policy are announced, the regulator knows the distribution of costs (i.e., \( c = c_i \) with probability \( \phi_i \), \( i = 1, 2 \)), but not the actual realization of \( c \). This is in contrast to the monopolist who has learned the level of marginal cost by this time. However, the regulator can discover the firm's costs through an audit that requires an expenditure, \( I (> 0) \).

In addition to deciding when to audit the firm, the regulator sets the price \( (p) \) the firm can charge for its output and the magnitude of taxes \( (T) \) the firm must pay. \( (T < 0 \) implies that the firm receives a subsidy). \(^{5}\) We assume the regulator can commit to carry out the terms of any announced regulatory policy. The policy announced by the regulator stipulates the probability \( (a_i) \) that a report by the firm that marginal cost is \( c_i \) will be investigated (i.e., audited at cost \( I \)). \(^{6}\) The policy also specifies allowed prices based upon the firm's cost report and the outcome of the investigation, if one is conducted. \( p_i \) is the allowed price if costs are reported to be \( c_i \) and no investigation of the report follows. \( p_{ij} \) is the allowed price if \( c_i \) is reported, and an audit reveals that costs are actually \( c_j \). Associated lump sum taxes are given by \( T_i \) and \( T_{ij} \), respectively. The regulatory policy is designed to maximize expected consumers' surplus. \(^{7}\)

\(^{4}\)An alternative approach would be to allow the probability of determining the firm's true costs to be a strictly increasing function of auditing expenditures. This and other extensions of our model are noted in section 4.

\(^{5}\)Introducing lump sum taxes and transfers (or two-part tariffs) as policy instruments simplifies the analysis. These instruments allow marginal cost prices to serve as the relevant benchmark, as subsidies can then ensure the profitability of the regulated enterprise. However, allowing the regulator to tax and subsidise is not necessary to establish our major conclusion.

\(^{6}\)Related models in which the regulator can audit cost reports are those of Baron (1984), Baron and Besanko (1984), and Demski et al. (1987).

\(^{7}\)The basic insights recorded below are unaltered if the regulator's objective is to maximize the expected value of a weighted average of consumers' surplus and profits, provided the former is afforded more weight than the latter. The special case we consider is chosen for expositional simplicity.
There are two major constraints on the regulatory policy. First, the expected profit of the firm must be non-negative. This ensures that the profit-maximizing monopolist will find it rational to continue operating. Second, the realized profit of the firm must not fall below the level \((B < 0)\) the firm could secure by declaring bankruptcy. This prevents the regulator from establishing such an immense penalty for a false cost report that the probability, and hence expected cost, of auditing necessary to ensure truthful cost reports approach zero. Effectively, this assumption enforces the asymmetry of information between firm and regulator.

To characterize the regulator’s problem formally, we focus on that equilibrium in which the firm is induced to truthfully report its cost parameter to the regulator. The Revelation Principle [e.g., Myerson (1979)] ensures there is no loss of generality in doing so. Thus, the regulator’s problem \((RP)\) is:

\[
\begin{align*}
\text{Maximize} & \sum_{p, T, a} \phi_i \left( a_i [CS(p_{ui}) + T_{ui} - I] + [1 - a_i] [CS(p_i) + T_i] \right) \\
\text{subject to} & \\
\end{align*}
\]

\[
\begin{align*}
a_i \Pi^l(c_i|c_i) + [1 - a_i] \Pi^{NI}(c_i|c_i) & \geq 0, \quad i = 1, 2, \quad (IR_i) \\
\end{align*}
\]

\[
\begin{align*}
a_i \Pi^l(c_i|c_i) + [1 - a_i] \Pi^{NI}(c_i|c_i) & \geq a_j \Pi^l(c_j|c_i) + [1 - a_j] \Pi^{NI}(c_j|c_i), \\
\end{align*}
\]

\[
\begin{align*}
\Pi^l(c_j|c_i) & \geq B, \quad \forall i, j = 1, 2, \quad (B^l_i) \\
\Pi^{NI}(c_i|c_i) & \geq B, \quad i = 1, 2, \quad (B^{NI}_i)
\end{align*}
\]

where \(\Pi^l(c_j|c_i) = [p_{ji} - c_i]Q(p_{ji}) - F - T_{ji}\), and \(\Pi^{NI}(c_j|c_i) = [p_j - c_i]Q(p_j) - F - T_j\).

The individual rationality \((IR)\) constraint ensures the firm with marginal cost \(c_i\) a non-negative level of expected profits. The truth-telling \((TT_{ij})\) constraints guarantee that the firm’s expected profits are greater when it truthfully reports its cost parameter \(c_i\) than when it claims \(c_j\). \((B^l_i)\) and \((B^{NI}_i)\) denote the aforementioned bankruptcy constraints when investigation does
W.J. Adams and D.E.M. Sappington, Profitability from 'countervailing' power

occur, and when it doesn't respectively. If the regulator shared the firm's cost information, he would implement the first-best outcome, in which (a) price and marginal cost are equated, i.e., $p_i = c_i$, $i = 1, 2$; (b) there is no investigation of the firm's costs (since they are already known to the regulator), i.e., $a_i = 0$, $i = 1, 2$; and (c) the firm earns no rents, i.e., $\Pi^{NI}(c_i | c_i) = 0$, $i = 1, 2$.

Absent perfect knowledge of the firm's costs, the regulator will have to sacrifice some resources to deter the firm from exaggerating its costs. Deterrence can be effected in two ways. The direct approach consists of investigating a claim by the firm that $c = c_2$. This strategy entails a cost $I$ to the regulator. The indirect strategy of deterrence consists of responding to a statement of $c = c_2$ by setting $p_2 > c_2$ and adjusting the lump sum tax on the firm to avoid any profit increase. The resulting decrease in output below the first-best level ($Q(p_2) < Q(c_2)$) serves to reduce the profit enjoyed by the monopolist with actual cost $c_1$ if he reports $c = c_2$ and is not audited. Thus, the output reduction limits the gains to the firm of an unaudited cost exaggeration. This indirect approach to limiting the firm's incentive to exaggerate cost entails no investigating expenses; but, in comparison with marginal cost pricing, does entail a reduction in consumers' surplus.

There are circumstances under which the regulator will employ only the indirect method of deterrence. The optimal manner in which to do so is recorded in Lemma 1. The proofs of Lemma 1 and all subsequent findings are in the appendix.

**Lemma 1** When $a_2 = 0$ in the solution to (RP):

(i) \[ p_2 = c_2 + \left( \phi_1 / \phi_2 \right) [c_2 - c_1] \]; and

(ii) \[ \Pi^{NI}(c_1 | c_1) = \Pi^{NI}(c_2 | c_1) > \Pi^{NI}(c_2 | c_2) = 0. \]

If the firm's report of $c_2$ is never investigated directly, the firm will earn a profit if actual cost is low ($c_1$). This profit will be smaller, though, the more likely is marginal cost to be $c_1$. With $\phi_1$ close to unity and $\phi_2$ close to zero,

---

8The only bankruptcy constraint not implied by this formulation of (RP) is $\Pi^{NI}(c_1 | c_2) \geq B$. As the proof of Proposition 1 (below) makes clear, if the firm's marginal cost is $c_2$, it will strictly prefer to report as such rather than claim $c = c_1$. Hence, in equilibrium, this additional bankruptcy constraint will not be violated in the solution to (RP).

9In equating first-best and marginal cost prices, we abstract from distortions that may arise in taxing consumers to raise the revenues to cover any fixed cost of production.

10In Lemma 1 and throughout, we assume without essential loss of generality that $Q(c_1 + (\phi_1 / \phi_2) [c_2 - c_1]) > 0$. 

---
the expected loss in consumers’ surplus that results from setting a high value for \( p_2 \) is small relative to the expected reduction in the firm’s profit.

When the costs of conditional investigation are not too large, the regulator will supplement the indirect form of deterrence with the direct form. The conditions under which auditing is optimal are recorded in Lemma 2.

**Lemma 2.** \( a_2 > 0 \) in the solution to (RP) when \( CS(c_2) - CS(c_2 + (\phi_1/\phi_2)[c_2 - c_1]) > 1 \). \( a_2 = 0 \) when the direction of the inequality is reversed.

Selective investigation of reports of high cost allows the regulator to more closely approximate first-best pricing. Provided the marginal benefit of lowering \( p_2 \) toward \( c_2 \) exceeds the marginal cost of auditing, the optimal regulatory policy will set \( a_2 > 0 \). In fact, when investigation of reported high costs is optimal, it will be conducted to the point where the firm’s expected profit is reduced to its first-best level. This observation is recorded as property (iv) in Proposition 1.

**Proposition 1.** When \( CS(c_2) - CS(c_2 + (\phi_1/\phi_2)[c_2 - c_1]) > 1 \), the solution to (RP) has the following properties:

(i) \( p_1 = c_1, p_{21} = c_1, p_{22} = c_2 \);

(ii) \( CS(c_2) - CS(p_2) = I, \) and \( p_2 = c_2 + \alpha(\phi_1/\phi_2)[c_2 - c_1] \) where \( \alpha \in (0, 1) \);

(iii) \( a_1 = 0, 0 < a_2 < 1 \);

(iv) \( \Pi^{NI}(c_1, c_1) = a_2 \Pi^I(c_2 | c_2) + [1 - a_2] \Pi^{NI}(c_2 | c_2) = 0 \);

(v) \( \Pi^I(c_2 | c_1) = B \).

Whenever the firm reports low cost, and whenever the regulator audits cost, price and marginal cost are equated. If subsidies were not feasible, prices in excess of marginal cost \( (p_1 > c_1) \) would be necessary (with \( F > 0 \)) to ensure non-negative profits to the low cost firm. By allowing subsidies in our model, we are able to distinguish most clearly between the two roles that prices play: controlling the firm’s output level and providing revenue to the firm. It is the former role that is of greatest interest here. Notice that taxes are readily calculated once profit levels (contingent on cost realizations) and prices are specified, as they are in Proposition 1. For example, \( T_{\mu} = -\Pi^I(c_1 | c_1) + [p_{\mu} - c_1]Q(p_{\mu}) - F \).
cost is not investigated. Auditing is made most effective by reducing the actual profit of an audited liar to the bankruptcy level, as reported in property (v).

Two features of the regulatory equilibrium are most important. First, the selectively audited firm will earn fewer rents than will the firm that is never audited. [Recall property (ii) of Lemma 1 and property (iv) of Proposition 1.] Second, the probability of auditing declines to zero as the probability of high cost rises (see Lemma 2). These observations lead to our main finding, which is summarized in Proposition 2. This proposition describes the impact of changes in the probability of high cost on the expected profit of the firm. Expected profit (Ell) is defined prior to the time at which the firm learns its cost. Thus, \( EII = \sum_{i=1}^{2} \phi_i \left( a_i \Pi^H(c_i | c_i) + \left[ 1 - a_i \right] \Pi^H(c_i | c_i) \right) \). As expected profits are discontinuous in \( \phi_2 \) at the point where \( CS(c_2) - CS(c_2 + \left( \frac{\phi_2}{\phi_1} \right) [c_2 - c_1]) = 1 \), the proposition is stated not in terms of derivatives, but rather in terms of discrete changes (\( \Delta \)) in \( \phi_2 \).

Proposition 2. Provided the inequality in Proposition 1 holds, \( \left( \frac{\Delta EII}{\Delta \phi_2} \right)_{\phi_2 = 1} \geq 0 \) in the solution to (RP), where the inequality is strict in some relevant ranges.\(^\text{12}\)

The implication of Proposition 2 is that a regulated monopolist might value the formation of 'countervailing power' upstream. To the extent that such countervailing power serves to increase the probability of high cost, it can also serve to reduce the probability that the regulator will investigate claims of high cost. This reduction occurs for two reasons. First, as \( \phi_2 \) increases, a report of \( c_2 \) is more likely to be true. Second, the price (\( p_2 \)) that obtains following an unaudited report of high cost tends to fall as the expected cost of the firm rises. For both reasons, the direct expected benefits of auditing (i.e., reducing price to marginal cost) fall as the probability of high cost rises. The diminished frequency of auditing, in turn, can enhance the expected profit of the downstream monopolist.

Two additional observations regarding Proposition 2 are appropriate. First, the result does not depend on erroneous beliefs by the regulator. Throughout, the regulator knows the true probabilities associated with \( c_1 \) and \( c_2 \). The expected cost of the monopolist does indeed rise; yet so does his expected profit. Second, it is readily verified that the firm's profit can be strictly enhanced even if the creation of monopoly upstream reduces the variance of the distribution of \( c \). Thus, although the value of upstream\(^\text{12}\)

\(^{12}\) To be more precise, define the function \( F(\phi_2) = CS(c_2) - CS(c_2 + (\phi_2 / \phi_1) [c_2 - c_1]) - 1 \). Also, define the sets \( S^+ = \{ \phi_2 | F(\phi_2) > 0 \} \) and \( S^- = \{ \phi_2 | F(\phi_2) \leq 0 \} \). Then the firm's expected profits are strictly greater when \( \phi_2 \in S^- \) than when \( \phi_2 \in S^+ \).
monopoly to the downstream monopolist does depend on governmental uncertainty regarding cost, it does not depend on the uncertainty becoming more pronounced as the upstream monopoly is created.

4. Discussion

The model analyzed here demonstrates that a profit-maximizing regulated firm might welcome increases in its expected costs. Its desire for such increases does not depend on the distortions in factor shares that arise from socially suboptimal regulation [as in Averch and Johnson (1962)]. Nor does it depend on a desire to disadvantage the firm's rivals [as in Williamson (1968), Salop and Scheffman (1983), or Krattenmaker and Salop (1986)]. Rather, it stems from government control over prices and asymmetries in information regarding the firm's costs. Increases in expected input prices can reduce the optimal frequency of government investigation of allegedly high costs, leading to greater profit for the regulated firm.

Our model is deliberately narrow in scope. It is designed to make a single point simply, rather than to permit satisfactory analysis of an industry of realistic complexity. Nevertheless, it is important to recognize that the intuition underlying the model is quite robust. For example, although generalizations of the firm's cost structure and the the regulator's auditing technology might diminish the drama of our findings, the reported propensity for increases in expected input prices is likely to persist.13 The same basic insights are also likely to characterize a corresponding dynamic model of regulation. Although the regulator's optimal strategy in this setting will generally depend on the entire history of cost reports and updated beliefs about likely future costs, the inverse relationship between expected costs and auditing frequency will persist.

We also believe that the basic conclusion of our analysis is relevant in settings where regulation is depicted in potential terms. In this version of the story, the firm knows that an excessive discrepancy between observed price and externally estimated cost will result in governmental inquiry with positive probability, possibly leading to formal regulation. The firm would choose its price not to prevent entry of new sellers, but rather to prevent entry of government into the pricing process, presuming the firm expects such entry by the government to be effective in limiting profits. In effect, an increase in expected input price could raise the maximum price the firm can

13With more general cost structures and auditing technologies, it will not necessarily be the case that when investigation occurs, it will be so extensive as to eliminate the firm's rents, whatever its actual costs. Therefore, the reduced probability of audit must be weighed against the increased probability that actual operating costs are higher (and operating profits are correspondingly lower absent an audit, as they generally will be). The key point is that there are these two effects of higher anticipated input prices, and not just the latter effect.
charge without provoking government action. Our model thus might inform a theory of 'political limit pricing'.14,15

Before applying this model to actual industries, we would embellish it in many ways. One of these, involving the treatment of uncertainty, merits discussion here. Our model puts the firm in a position of full information and the regulator in a position of substantial uncertainty. Two important features of reality are thereby ignored. First, most industries are oligopolies rather than monopolies. Unless collusion is perfect, the existence of multiple producers provides the government with natural benchmarks against which to compare individual reports of input prices. As a result, even without costly investigation, the government might possess good information regarding any particular firm's cost.

Second, in practice the firm cannot be so certain of the consequences of its actions. For example, regulatory responses to increases in expected cost cannot be predicted with certainty. In particular, neither the regulator's objectives nor his ability to execute an announced policy can be taken for granted by the firm.16

Similarly, the firm cannot be certain that creation of 'countervailing' power upstream entails nothing more than stochastically higher cost. Thus, a more complete model would include more details of realistic interactions among regulated firms, their suppliers, and their regulators.

14Braff and Miller (1961) have applied this intuition to the automobile and steel industries of the 1950s. The argument might also be relevant to the petroleum industry of the 1970s. In particular, the argument is consistent with the claim that formation of OPEC enhanced the profits of international petroleum corporations by reducing the threat of government regulation. When regulation did occur, it basically followed the framework described in our model: Despite the information contained in prices on spot and futures markets, despite sometime regulation of the domestic price of crude, and despite information generated during operation of the entitlements program, government regulators were not well informed about the prices paid by specific refiners for delivered crude—especially when the seller was located abroad. In practice, attorneys at the administrative enforcement division of the Department of Energy (DoE) followed a three-step procedure while monitoring refiner prices: First, they examined invoices relating to purchases of crude. If the prices appeared abnormally high, the refiner would be audited in depth. When audits revealed attempts to overstate prices paid, DoE would attempt to recover overcharges on petroleum products from distorting refiners. As of March, 1986, DoE expected to collect $7.69 billion on overcharges for crude oil and refined products combined. ('Lawyers for Diverse Interests Haggle in Private Over a $4 Billion Kitty of Oil-Price Overcharges', Wall Street Journal, March 6, 1986, p. 42.)

15A related but distinct model is that of Fudenberg and Tirole (1986). In their model, the incumbent firm is interested in deterring entry by potential competitors rather than by the government. The incumbent gains when potential entrants acquire information that leads them to believe the industry is less profitable than originally anticipated.

16A third aspect of our depiction of uncertainty that warrants additional investigation is the presumed auditing technology. If a 'successful' audit did not always result in perfect knowledge of the firm's costs, the firm would be able to command greater rents from its private information.
Appendix

Proof of Proposition 1 and Lemma 2. Let \( \lambda_i, \lambda_{ij}, \gamma_{ij}, \) and \( \gamma_i, \) represent the Lagrange multipliers associated with constraints \((IR_i), (TT_j), (B_{ij}^I),\) and \((B_{ij}^N),\) respectively. Let \( \xi_i \) be the multiplier associated with the restriction \( a_i \leq 1.\) Also, let \( L_v \) denote the partial derivative of the Lagrangian function associated with \((RP)\) with respect to variable \( v.\) Then the necessary conditions for a solution to \((RP)\) are readily shown to include the following:

\[
\begin{align*}
L_{T_i} &= a_i[\lambda_i + \lambda_{ij} + \gamma_i - \phi_i] = 0, & i = 1, 2; j \neq i, \\
L_{T_j} &= a_j[\lambda_j - \gamma_j] = 0, & i, j = 1, 2; j \neq i, \\
L_{T_{ij}} &= [1 - a_i]\{\phi_i + \lambda_{ij} - [\lambda_i + \lambda_{ij} + \gamma_i]\} = 0, & i = 1, 2; j \neq i, \\
L_{p_{ij}} &= a_i[\phi_i(p_i - c_i)Q'(p_i)] = 0, & i = 1, 2.
\end{align*}
\]

Also, using \((A.3)\) and noting that the derivative of \(CS(p)\) is given by \(-Q(p),\) we have

\[
L_{p_i} = [1 - a_i]\{(\lambda_i + \lambda_{ij} + \gamma_i)[p_i - c_i] - \lambda_{ij}[p_i - c_j]\} = 0, & j \neq i, \quad i = 1, 2, \\
\]

and

\[
L_{a_i} = \phi_i\{CS(p_i) - I + T_{ij} - CS(p_i) - T_i\} + \lambda_{ij}\{T_{ij} + [p_i - c_j]Q(p_i) - T_i\} - \xi_i \leq 0, & j \neq i, \quad i = 1, 2.
\]

We now derive the solution to \((RP)\) assuming \( \lambda_{21} = \gamma_1 = \gamma_{11} = 0.\) It is straightforward to verify that the resulting solution does not violate any of the \((TT_{ij}), (B_{ij}^I),\) or \((B_{ij}^N)\) constraints.

Since \( \lambda_{21} = 0, p_1 = c_1 \) from \((A.5).\) And from \((A.4), p_{22} = c_2 \) and \( p_{11} = c_1.\) The condition corresponding to \( L_{p_{11}} = 0 \) reveals the level of \( p_{21} \) is arbitrary, so we can set \( p_{21} = c_1,\) completing the proof of property \((i)\) of the Proposition.

Now suppose \( \lambda_{12} = 0.\) Then from \((A.1)\) and \((A.3), \lambda_1 = \phi_1.\) Hence, \( a_2 > 0,\) since if \( a_2 = 0, \) then from \((TT_{ij}), (IR_2), a_1, II'(c_1\{c_1\} + [1 - a_1] II_N(c_1\{c_1\}) \geq II_N(c_2\{c_1\}) \geq \Omega_N(c_2\{c_1\} > \Omega_N(c_2\{c_1\}) \geq 0,\) contradicting \( \lambda_1 > 0.\) If \( a_2 \in (0, 1), \) then \( \xi_2 = 0.\) Also, \( \gamma_2 = \gamma_{22} \) from \((A.1)\) and \((A.3).\) Because \( B < 0, (IR_2)\) implies \( \gamma_2 = \gamma_{22} = 0.\) Hence, \( \lambda_2 = \phi_2 \) from \((A.1).\) Furthermore, from \((A.5), p_2 = c_2.\) Hence, \((A.6)\) implies \( -\phi_2 I = 0,\) which is a contradiction. Thus, if \( a_2 < 1, \lambda_{12} > 0.\)

If \( a_2 = 1, \) then it is immediate that properties \((a)\) and \((b)\) of the first-best outcome will characterize the solution to \((RP),\) and \( \lambda_{12} = 0.\) Hence, from \((A.6)\) and \((A.1), -\phi_2 I + \gamma_{22}[T_{22} - T_2] - \xi_2 = 0.\) Thus, \( \gamma_{22} > 0.\) But since \( a_2 = 1 \) and \( B < 0,\) we have a contradiction of \((IR_2).\) Thus, the solution to \((RP)\) has \( a_2 < 1,\) so that \( \lambda_{12} > 0.\) And from \((A.2), \gamma_{12} > 0.\)
Now, suppose \( a_2 > 0 \). Then, noting that \( \lambda_1 + \lambda_{12} = \phi_1 \) from (A.1), (A.6) states \( -\phi_1 I - \xi_1 = 0 \), which is a contradiction. Hence, property (iii) of Proposition 1 is proved.

Next, if follows from (A.1), (A.2) and (A.3) that \( \gamma_2 = \lambda_{12} = \gamma_{12} > 0 \), \( \lambda_1 + \lambda_{12} = \phi_1 \), and \( \gamma_{22} = 0 \). Property (v) of the Proposition follows from \( \gamma_{12} > 0 \), and the expression for \( p_2 \) in property (ii) follows from (A.6). Also, from (A.6), for \( a_2 \in (0, 1) \), and with \( p_2 = c_2 + (\lambda_{12}/\phi_2)[c_2 - c_1] \)

\[
CS(c_2) - CS(p_2) = I. \tag{A.7}
\]

If \( a_2 = 0 \), then \( \lambda_1 = 0 \), \( \lambda_{12} = \phi_1 \) and \( \lambda_2 = 1 \) from (A.3) and so \( p_2 = c_2 + (\phi_1/\phi_2)[c_2 - c_1] \) from (A.5). Hence, rearranging (A.6) reveals \( CS(c_1) - CS(c_2 + (\phi_1/\phi_2)[c_2 - c_1]) \leq I \). Thus, if \( CS(c_2) - CS(c_2 + (\phi_1/\phi_2)[c_2 - c_1]) > I \), \( a_2 > 0 \). Conversely, suppose \( a_2 > 0 \). Then, since \( p_2 = c_2 + (\lambda_{12}/\phi_2)[c_2 - c_1] \) and \( \lambda_{12} + \lambda_1 = \phi_1 \), (A.7) reveals \( I = CS(c_2) - CS(p_2) \leq CS(c_2) - CS(c_2 + (\phi_1/\phi_2)[c_2 - c_1]) \). Therefore, if \( CS(c_2) - CS(c_2 + (\phi_1/\phi_2)[c_2 - c_1]) < I \), \( a_2 = 0 \). Hence Lemma 2 is proved.

Finally, note that the solution to (RP) can only have \( \lambda_1 = 0 \) with \( a_2 > 0 \) if the razor’s edge condition given by (A.7) with \( \lambda_{12} = \phi_1 \) holds. This condition is ruled out by assumption in Proposition 1. Thus, with \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \), property (iv) of the Proposition holds.

**Proof of Lemma 1.** Using the notation developed in the proof of Proposition 1, suppose \( a_2 = 0 \). Then it is immediate that \( \lambda_1 = 0 \), so that from (A.3), \( \lambda_{12} = \phi_1 \) and \( \lambda_2 + \gamma_2 = 1 \). Property (ii) of the Lemma follows from \( \lambda_{12} > 0 \), and property (i) now follows immediately by rearranging (A.5).

**References**


