NEUTRINO MAGNETIC MOMENT IN THE $SU(3)_L \times U(1)$ MODEL

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We discuss the prospect and difficulties of having a neutrino with a large magnetic moment and a *naturally* small mass in the $SU(3)_L \times U(1)$ model.

Over the last few years, there has been a considerable interest in the neutrino magnetic moment, μ_{ν_e} in an attempt to understand [1] the solar neutrino problem [2]. The required value of μ_{ν_e} is of the order of $(10^{-10}\text{–}10^{-11})\mu_B$ ($\mu_B = e/2m_e$ is the Bohr magneton). However, among other things [3,4], this scenario suffers from a theoretical difficulty associated with the compatibility of a large μ_{ν_e} with a naturally small neutrino mass. Recently, the first step in attempting to solve this problem has been taken [5]. The idea is to realize an $SU(2)_\nu$ symmetry suggested originally by Voloshin [6] in viable theoretical models. An interesting feature of this symmetry is that even a massless neutrino is allowed to have a magnetic moment. However, this particular model, as pointed out by its authors, cannot solve the naturalness problem completely. Instead, it only makes the problem less severe. While most of the interesting features of the model have been carefully analyzed in ref. [5], the question of how less severe the naturalness problem actually is has not been fully addressed. Since for the first time in many years we finally have an explicit model that appears to have the desired feature, it is of interest to see more carefully (1) why the naturalness problem still remains, and (2) to what extent such a problem becomes less severe in comparison with the old approaches [7,8].

The gauge group of the model is $SU(3)_L \times U(1)_X$. Following ref. [5], it contains both the standard $SU(2)_L \times U(1)_Y$ and the $SU(2)_\nu$ symmetries as its subgroups. The particle content of the method is summarized in table 1, from which the quarks and the mirror fermions are ignored for simplicity (mirror fermions are required to cancel the anomaly). Also, scalars which couple to the quarks are neglected in our discussion. Including these fields will not change our conclusion qualitatively. To have the desired phenomenology, one finds [5] that it is necessary to have

$$\langle \eta^0 \rangle = \langle (T_{2,3})^0_{12,22} \rangle = 0 \,. \tag{1}$$

The lepton Yukawa interaction of the model is given by

$$\sum_{a=1}^{3} \lambda'_a \psi_a e^c_a \phi + f\psi_1 \psi_3 \eta^+ + f' \psi_1 e^c_3 \eta$$
$$+ g_2 \psi_2 \psi_2 T_2 + g_3 \psi_3 \psi_3 T_3 + \text{h.c.} \tag{2}$$

The assumption of only seven terms in eq. (2) among the ten fields ($\psi_{1,2,3}$, $e^c_{1,2,3}$, η, $T_{2,3}$, ϕ) requires three $U(1)$ symmetries. Evidently, one of them is gauged $U(1)_X$, and the rest of the two are global $U(1)$ symmetries which can be regarded as the "muon-" and the "tau-lepton" numbers, which will be denoted by L_2 and L_3, respectively. Notice that L_2 and L_3 are not the usual lepton numbers. The usual lepton numbers do not commute with the gauge group in this model. Rather, they are two global symmetries which transform, separately, on the second and the third generation lepton triplets. Both L_2 and L_3 will be broken spontaneously once $\langle (T_{2,3})^0_{11} \rangle \neq 0$. One can, of course, equally break $L_{2,3}$ explicitly by introducing, for instance, soft terms in the Higgs potential, if so desired.

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Table 1
Particle content of the model. The vector bosons, quarks and mirror fermions are not presented in the table for simplicity.

	Particles	$(SU(3)_L, U(1)_X)$
lepton-triplet	$\psi_{aL}=\begin{pmatrix}\nu^c\\ \nu\\ e^-\end{pmatrix}_{aL}$ $(a=1,2,3)$	$(3,-\frac{1}{3})$
lepton-singlet	e^c_{aL} $(a=1,2,3)$	$(1,1)$
scalar-sextets	$T_{2,3}=\begin{pmatrix}T^0_{11} & T^0_{12} & T^+_{13}\\ T^0_{12} & T^0_{22} & T^+_{23}\\ T^+_{13} & T^+_{23} & T^{++}_{33}\end{pmatrix}_{2,3}$	$(\bar{6},\frac{2}{3})$
scalar-triplets	$\phi=\begin{pmatrix}\phi^-_1\\ \phi^-_2\\ \phi^0\end{pmatrix}$, $\eta=\begin{pmatrix}\eta^-_1\\ \eta^-_2\\ \eta^0\end{pmatrix}$	$(\bar{3},-\frac{2}{3})$, $(\bar{3},-\frac{2}{3})$

A conserved electron-lepton number, L_e, is required if ν_e is to be a Dirac particle. It is already seen that L_e cannot be a global symmetry of the model. Such a lepton number, if it exists, must therefore be a linear combination of the generators of the gauge and the global U(1) symmetries described above. Furthermore, the assignments of L_e must be such that fields which get vacuum expectation values have $L_e=0$. With this constraint, we found that, in the fundamental representation of $SU(3)_L$, L_e is

$$L_e=L_2+L_3+\tfrac{1}{2}\sqrt{3}\,\lambda_8+\tfrac{1}{2}\lambda_3-Q\,. \quad (3)$$

Here Q is the charge operator of the model

$$Q=X+\tfrac{1}{2}\lambda_3-\lambda_8/2\sqrt{3}\,, \quad (4)$$

and

$$\lambda_3=\begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix},\quad \lambda_8=\frac{1}{\sqrt{3}}\begin{pmatrix}-2&0&0\\0&1&0\\0&0&1\end{pmatrix}, \quad (5)$$

are the diagonal generators of $SU(3)_L$ with the last two columns and rows corresponding to the standard $SU(2)_L$. The assignments of L_e according to eq. (3) are summarized in table 2, from which we see that the second and the third generation leptons have the same L_e structure and so do the two sextets Higgs fields. By contrast, the scalar-triplets ϕ and η have different L_e structure. It is easy to show that the conservation of L_e, $\Delta L_e=0$, follows because $\Delta Q=0$ (the conservation of charge) and $\Delta(L_2+L_3+\frac{1}{2}\sqrt{3}\,\lambda_8+\frac{1}{2}\lambda_3)=0$. We point out that our L_e assignments for the different fields do not agree with that in ref. [5], where the same L_e structure for the first and the third generations of leptons were suggested. We find, in fact, that an L_e assignment consistent with that of ref. [5] requires

Table 2
The assignments of L_e. Here $\psi_{2,3}$ and $T_{2,3}$ carry an $L_{2,3}$ charge +1 and −2, respectively.

	$L_e=L_2+L_3+\frac{1}{2}\sqrt{3}\,\lambda_8+\frac{1}{2}\lambda_3-Q$
lepton-triplets	$\begin{pmatrix}-1\\+1\\+1\end{pmatrix}_{1L}$, $\begin{pmatrix}0\\+2\\+2\end{pmatrix}_{2L,3L}$
lepton-singlets	$(-1)_{1L}$, $(-2)_{2L,3L}$
scalar-sextets	$\begin{pmatrix}0&-2&-2\\-2&-4&-4\\-2&-4&-4\end{pmatrix}_{2,3}$
scalar-triplets	$\begin{pmatrix}+2\\0\\0\end{pmatrix}_\phi$, $\begin{pmatrix}+3\\+1\\+1\end{pmatrix}_\eta$

$$L_e=L_2+\tfrac{1}{2}\sqrt{3}\,\lambda_8+\tfrac{1}{2}\lambda_3-Q\,. \quad (6)$$

In that case, $(T_3)^0_{11}$ has to carry L_e by two units, and consequently L_e cannot be a conserved quantum number because $\langle (T_3)^0_{11} \rangle \neq 0$. Finally, the baryon number of the model is defined as usual. It is represented by a conserved global U(1) symmetry.

It is straightforward to show [5] that the dominant contributions to the neutrino mass, m_{ν_e}, and the neutrino magnetic moment, μ_{ν_e}, come from diagrams with virtual $\eta^+_{1,2}$ exchanges:

$$m_{\nu_e} = \frac{ff'}{16\pi^2} m_\tau \ln \frac{m^2_{\eta 1}}{m^2_{\eta 2}} , \quad (7)$$

$$\mu_{\nu_e} = \frac{ff' em_\tau}{16\pi^2} \left(\frac{1}{m^2_{\eta 1}} \ln \frac{m^2_{\eta 1}}{m^2_\tau} + \frac{1}{m^2_{\eta 2}} \ln \frac{m^2_{\eta 2}}{m^2_\tau} \right) , \quad (8)$$

where $\eta_{1,2}$ are the charged components of the η triplet with masses m_{η_1}, $m_{\eta_2} \gg m_\tau$. From eqs. (7) and (8) one sees that m_{ν_e} can be made very small while μ_{ν_e} maintaining large provided η_1 and η_2 are very degenerate in mass. The essence of the model is that in the limit the original gauge symmetries are exact, one has $m_{\eta_1} = m_{\eta_2}$. As a consequence, $m_{\nu_e} = 0$ but $\mu_{\nu_e} \neq 0$.

Unfortunately, nature does not seem to have such a symmetry. The original $SU(3)_L \times U(1)_Y$ has to be broken down to $SU(2)_L \times U(1)_Y$ at an energy scale at least of the order 1 TeV and, consequently, m_{ν_e} is necessarily not zero. Assuming $m_{\nu_e} \lesssim 20$ eV as would be required from experiments, one finds [5] that in order to have $\mu_{\nu_e} \sim 10^{-11} \mu_B$, it is necessary to require

$$\frac{\Delta m^2_\eta}{m^2_\eta} \lesssim 10^{-4}\text{–}10^{-5} , \quad (9)$$

where $\Delta m^2_\eta = |m^2_{\eta 1} - m^2_{\eta 2}|$ and $m_\eta \gtrsim 1$ TeV is the average of $m^2_{\eta 1}$ and $m^2_{\eta 2}$. Eq. (9) turns out to be the naturalness condition of the model.

The reason that this model cannot solve the naturalness problem completely is because condition (9) cannot be realized in the present model in a natural way. In fact, since η_1 belongs to $SU(3)_L$, it should, therefore, have a mass of the order of the $SU(3)_L$ breaking scale. On the other hand, η_2 is part of the $SU(2)_L$ doublet and thus its mass should be of the order of 100 GeV. Consequently, $\Delta m_\eta \sim m^2_{\eta 1}$ and hence $\Delta m_\eta / m_\eta \sim O(1)$ which is about 10^{-4}–10^{-5} orders of magnitude too big. Thus, condition (9) can only be realized in the present model by means of fine-tunings. Indeed, ignoring a small contribution from the gauge invariant quartic coupling

$$f^{abc} f_{a'b'c} \phi_a \eta_b \phi^{*a'} \eta^{*b'} , \quad (9')$$

where f is the $SU(3)_L$ structure constant and a, a', b, b', $c = 1, 2, 3$ are the $SU(3)_L$ indices, we find that one way of satisfying condition (9) is to tune the coefficients in the coupling

$$\lambda_2 (T_2\eta)^\dagger (T_2\eta) + \lambda_3 (T_3\eta)^\dagger (T_3\eta) + \tilde{\lambda}_2 (T_2^\dagger \eta)^\dagger (T_2^\dagger \eta) + \tilde{\lambda}_3 (T_3^\dagger \eta)^\dagger (T_3^\dagger \eta) , \quad (10)$$

of the Higgs potential to be 10^{-4}–10^{-5} orders of magnitude smaller than the coefficients in the coupling

$$(\eta^\dagger \eta) [\lambda_{22} \mathrm{Tr}(T_2^\dagger T_2) + \lambda_{33} \mathrm{Tr}(T_3^\dagger T_3)] . \quad (11)$$

Such a tuning is required because eq. (10) only contributes to m_{η_1} and hence to Δm^2_η, whereas eq. (11) contributes equally to m_{η_1} and m_{η_2} and thus to m^2_η. Including the quartic term in eq. (9') requires an additional moderate fine-tuning of the order 10^{-2} on its coupling constant. Alternatively, we may adjust the parameters of the model in such a way that contributions to Δm^2_η from different terms, say in eq. (10), cancel almost exactly among themselves. Still, one sees from eq. (10) that in this case a tuning approximately of the order of 10^{-4}–10^{-5} is required.

Evidently, all such tunings are not natural in the technical sense because, for instance, setting $\lambda_{2,3}$, $\tilde{\lambda}_{2,3} = 0$ does not enlarge the symmetry of the model. For example, even if one chooses λ_2, $\lambda_3 = 0$ at tree level, divergent radiative corrections to λ_2 and λ_3 will arise through one-loop diagrams connecting the quartic coupling

$$(\phi^\dagger \eta)^\dagger (\phi^\dagger \eta) \quad (11a)$$

to terms

$$(T_{2,3}\phi)^\dagger (T_{2,3}\phi) \quad (11b)$$

in the Higgs potential. This implies that in order to satisfy condition (9), the arbitrary fine-tunings described above have to be carried out through all orders.

That the model makes the naturalness problem less severe may be viewed as follows: First, unlike all the other models, it has a symmetry which, if unbroken, allows a massless neutrino to have a magnetic moment. We believe that if there is a model, which is able to solve the long standing naturalness problem at all, it must at least have this feature. From this point of view, it is reasonable to conclude that the model at least has the desired (if not the correct) limit. Second, the degree of fine-tuning becomes somewhat less severe in comparison with some other models [7–9]. To be more specific, we consider μ_{ν_e} in $SU(2)_L \times U(1)_Y$ models, where one finds [10] that μ_{ν_e} is directly proportional to m_{ν_e}:

$$\mu_{\nu_e} = 6\times10^{-18}\left(\frac{m_{\nu_e}}{20\,\mathrm{eV}}\right)\mu_B\,. \qquad (12)$$

Such a proportionality is natural in the technical sense because the limit $m_{\nu_e}\to0$ and $\mu_{\nu_e}\to0$ corresponds to the usual chiral symmetry. For that reason, we may regard eq. (12) as the natural solution of μ_{ν_e} in general [#1]. By contrast, in models discussed in refs. [7–9], one introduces new physics [7,8] to make additional contributions to μ_{ν_e} be proportional to a heavy mass m_{eff} with $m_{\mathrm{eff}}\sim 1$–10 MeV [9]. Furthermore, in contrast to the $SU(3)_L\times U(1)_X$ model where the required fine-tuning is introduced in the Higgs potential, in these models it takes place among the different mass terms in the Yukawa interaction. The degree of fine-tuning of these models can therefore be measured directly in terms of the natural solution of the $SU(2)_L\times U(1)_Y$ model [see eq. (12)],

$$\frac{m_{\nu_e}}{m_{\mathrm{eff}}}\sim10^{-6}\text{–}10^{-7}\,, \qquad (13)$$

which is probably about 10^2–10^3 times worse than that suggested by eq. (9).

In summary, we have studied the naturalness problem of having ν_e with a large μ_{ν_e} and small m_{ν_e} in the $SU(3)_L\times U(1)_X$ model. Although this model has many interesting features, it still cannot solve the naturalness problem completely. We have shown explicitly that, in order to generate a sufficiently large μ_{ν_e}, one has to fine-tune the parameters in the Higgs potential so that m_{ν_e} can remain small. We believe that the necessity of fine-tuning in this model arises because the original $SU(2)_\nu$ symmetry, which is precisely invented to prevent the fine-tuning, has to be broken at an energy scale higher than that for generating μ_{ν_e}. As a result, the would be of interest $SU(2)_\nu$ symmetry becomes more or less irrelevant for the discussion of the naturalness problem. Finally, we add that we have ignored through our discussions scalar fields which do not couple to the lepton sector for simplicity. It is easy to show that including them will not change our conclusion qualitatively.

[#1] A natural solution like eq. (12) with a slightly larger coefficient can, in principle, be obtained in extended models if, for instance, the model has scalars with masses smaller than M_W. The enhancement on μ_{ν_e} is typically of the order of $(M_W/M_H)^2$, where M_H is the Higgs boson mass. However, if μ_{ν_e} is generated from one-loop diagrams, the scalar fields have to carry charge and hence $(M_W/M_H)^2\lesssim10$.

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