η, η' MIXING AND ANOMALIES

R. AKHOURY and J.-M. FRÈRE [1,2]
Randall Laboratory of Physics, University of Michigan, Ann Arbor, MI 48109, USA

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We discuss the effects of η, η' mixing and of the gluon anomaly in the decays $\eta\to 3\pi^0$, $\eta'\to 3\pi^0$, $\psi\to\eta\gamma$ and $\psi\to\eta'\gamma$. These effects are particularly important for $\eta'\to 3\pi^0$ in reducing the total rate by about two orders of magnitude from the naive estimate. The decay rates are calculated for various values of the η, η' mixing angle (θ) and a reasonable fit with experiment is obtained for $-20°\leqslant\theta\leqslant -16°$.

1. Introduction

The pseudoscalar meson mass spectrum is believed to provide strong evidence for the gluon anomaly and the topological charge. In the limit of exact SU(3), the pions, kaons and the η are degenerate in mass and the η' is heavier on account of the gluon anomaly and the topological charge. In the real world of broken SU(3), the η and η' can mix with each other and the large η–π mass difference arises from two *related* symmetry breaking effects: the large m_s–m_u, m_d mass difference and the η, η' mixing. This mixing between the η and the η' gives an enhancement of the η mass through the gluon anomaly and the nonvanishing topological charge $\langle 0|G\tilde{G}|\eta\rangle$ for the η, analogously for the η'. The mixing between the η and η' is characterized by a mixing angle θ and an analysis based on the Gell-Mann–Okubo quadratic mass formula for broken SU(3) gave an estimate of $\theta\approx -10°$. A more recent analysis [1] allows a value closer to $-20°$. With a view to further testing the above picture, in this paper we will consider decays involving pseudoscalar mesons that are sensitive to the mixing angle, to the gluon anomaly and to the non-vanishing topological charges $\langle 0|G\tilde{G}|\eta\rangle$, $\langle 0|G\tilde{G}|\eta'\rangle$ of the physical η and η' respectively.

In particular, the processes we consider are $\eta\to 3\pi^0$, $\eta'\to 3\pi^0$ and the branching ratio $(\psi\to\eta\gamma)/(\psi\to\eta'\gamma)$. Using orthogonality conditions we relate the topological charges $\langle 0|G\tilde{G}|\eta\rangle$, $\langle 0|G\tilde{G}|\eta'\rangle$ to the mixing angle θ, f_0 the decay constant of the singlet current and f_8 that of the current corresponding to the eighth component of the octet. The matrix elements for $\eta\to 3\pi^0$ and $\eta'\to 3\pi^0$ can then be expressed, using standard current-algebra techniques [2], in terms of f_π, f_0, f_8, θ and $m_\eta(m_{\eta'})$. We proceed to determine these constants as follows: f_π can be related using PCAC and the electromagnetic anomaly to the decay $\pi^0\to 2\gamma$, and f_0, f_8 can be expressed in terms of the rates for $\eta\to 2\gamma$, $\eta'\to 2\gamma$ and θ. The rates for $\eta\to 3\pi^0$, and $\eta'\to 3\pi^0$ are determined for a range of values of θ and we conclude that present experimental evidence seems to favour $-20°\leqslant\theta\leqslant -16°$. Further confirmation comes from the branching ratio $(\psi\to\eta\gamma)/(\psi\to\eta'\gamma)$ following the model of ref. [3]. A striking feature to come out of our analysis is the effect of the mixing in the process $\eta'\to 3\pi^0$. If one ignores the anomaly in the singlet current and the η_0, η_8 mixing and computes $\eta'\to 3\pi^0$ along the lines of ref. [2], a decay rate about two orders of magnitude higher than the experimental value is obtained. Incorporating these effects give a rate in qualitative agreement with experiment. As discussed below $\eta'\to 3\pi^0$ essentially goes through the quark content of the η' and

[1] Chercheur Qualifié, Fonds National de la Recherche Scientifique, Belgium.
[2] Postal address: ULB, CP 225, Campus Plaine, B-1050 Brussels, Belgium.

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our analysis indicates that the topological charge of the η' as measured by $\langle 0|G\tilde{G}|\eta'\rangle$ is much greater than the corresponding quark contribution $\langle 0|2im_s\bar{s}\gamma_5 s|\eta'\rangle$.

2. Evaluation of $\langle 0|G\tilde{G}|\eta\rangle$ and $\langle 0|G\tilde{G}|\eta'\rangle$

We begin our analysis by relating the topological charges of η and η' to θ, f_0, f_8. Let $|\eta_8\rangle$ and $|\eta_0\rangle$ represent respectively the pure octet and singlet states. Then the physical $|\eta\rangle$ and $|\eta'\rangle$ states which are orthogonal are related by

$$|\eta\rangle = \cos\theta\,|\eta_8\rangle - \sin\theta\,|\eta_0\rangle\,, \quad |\eta'\rangle = \sin\theta\,|\eta_8\rangle + \cos\theta\,|\eta_0\rangle\,, \tag{1}$$

according to standard convention. A^0_μ and A^8_μ are the axial currents corresponding to the singlet and eighth component of the octet, i.e.,

$$A^8_\mu = \frac{1}{2\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)\,, \quad A^0_\mu = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)\,. \tag{2, 3}$$

Ignoring for the moment, the electromagnetic anomaly, the above currents are not conserved in the presence of quark mass terms, further, the divergence of A^0_μ has a gluon anomaly

$$\partial_\mu A^8_\mu = \frac{1}{2\sqrt{3}}(2im_u\bar{u}\gamma_5 u + 2im_d\bar{d}\gamma_5 d - 4im_s\bar{s}\gamma_5 s)\,, \tag{4}$$

$$\partial_\mu A^0_\mu = \frac{1}{\sqrt{6}}(2im_u\bar{u}\gamma_5 u + 2im_d\bar{d}\gamma_5 d + 2im_s\bar{s}\gamma_5 s) + \frac{1}{\sqrt{6}}\frac{3\alpha_s}{4\pi}G\tilde{G}\,, \tag{5}$$

with

$$G^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + gf^{abc}B_{\mu b}B_{\nu c}\,, \quad \tilde{G}^{\mu\eta}_a = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}G_{a\alpha\beta}\,.$$

The nonzero couplings of the divergence of these currents to the physical nonets are defined through the relations (note that we are not assuming PCAC for the η')

$$\langle 0|\partial_\mu A^8_\mu|\eta\rangle = f_8\cos\theta\frac{m^2_\eta}{(2\pi)^{3/2}}\,, \quad \langle 0|\partial_\mu A^0_\mu|\eta\rangle = -f_0\sin\theta\frac{m^2_\eta}{(2\pi)^{3/2}}\,,$$

$$\langle 0|\partial_\mu A^0_\mu|\eta'\rangle = f_0\cos\theta\frac{m^2_{\eta'}}{(2\pi)^{3/2}}\,, \quad \langle 0|\partial_\mu A^8_\mu|\eta'\rangle = f_8\sin\theta\frac{m^2_{\eta'}}{(2\pi)^{3/2}}\,, \tag{6}$$

where f_0, f_8 are respectively the decay constants corresponding to the singlet and the eighth component of the octet. Equivalently, we have

$$\langle 0|\frac{A^8_\mu}{f_8}\cos\theta - \frac{A^0_\mu}{f_0}\sin\theta|\eta\rangle = \frac{ip_\mu}{(2\pi)^{3/2}}\,, \quad \langle 0|\frac{A^8_\mu}{f_8}\sin\theta + \frac{A^0_\mu}{f_0}\cos\theta\,|\eta'\rangle = \frac{ip_\mu}{(2\pi)^{3/2}}\,, \tag{7, 8}$$

where p_μ are the momenta of the η or η'. Consider the following orthogonality conditions that follow from (6):

$$-ip_\mu\langle 0|\frac{A^8_\mu}{f_8}\cos\theta - \frac{A^0_\mu}{f_0}\sin\theta\,|\eta'\rangle = 0\,, \quad -ip_\mu\langle 0|\frac{A^8_\mu}{f_8}\sin\theta + \frac{A^0_\mu}{f_0}\cos\theta\,|\eta\rangle = 0\,. \tag{9, 10}$$

Using (4), (5) and neglecting the up and down quark masses in comparison to the strange quark mass, we get

$$\frac{\langle 0|(3\alpha_s/4\pi)G\tilde{G}|\eta'\rangle}{\langle 0|2im_s\bar{s}\gamma_5 s|\eta'\rangle} = -\sqrt{2}\,\frac{(1/f_8)\cos\theta + (1/\sqrt{2}f_0)\sin\theta}{(1/f_0)\sin\theta}\,, \tag{11}$$

and

$$\frac{\langle 0|(3\alpha_s/4\pi)G\tilde{G}|\eta\rangle}{\langle 0|2im_s\bar{s}\gamma_5 s|\eta\rangle}=-\frac{(1/f_0)\cos\theta-(\sqrt{2}/f_8)\sin\theta}{(1/f_0)\cos\theta}. \tag{12}$$

From (7) and (8) we get after using (4), (5), (11) and (12),

$$\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta\rangle=\frac{\sqrt{3}}{(2\pi)^{3/2}}m_\eta^2(\cos\theta f_8-\sqrt{2}\sin\theta f_0), \tag{13}$$

$$\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta'\rangle=\frac{\sqrt{6}}{(2\pi)^{3/2}}m_{\eta'}^2\left(\cos\theta f_0+\frac{\sin\theta f_8}{\sqrt{2}}\right). \tag{14}$$

Eq. (14) gives the correct expression in the chiral limit and $\theta=0$. We would like to remark here that SU(3) breaking by m_s-m_u, m_d mass difference and η_0, η_8 mixing are related effects. In the approximation that we are using, $(m_{u,d}\ll m_s)$, in order to go to the symmetry limit one must consider both $m_s\to 0$, $\theta\to 0$ together. For comparison with other estimates of these quantities we note that at $\theta\approx -18°$,

$$(2\pi)^{3/2}\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta\rangle=2.5f_\pi m_{\eta'}^2(2\pi)^{3/2}, \quad \langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta'\rangle=2f_\pi m_{\eta'}^2 \quad (f_\pi\approx 93\text{ MeV}),$$

which is remarkably close but slightly higher than the estimate in ref. [3] and within the limits of the estimate in ref. [4] (after taking into account different normalizations). We emphasize that the relations (13) and (14) are derived without assuming η' to be a Goldstone boson.

3. $\eta\to 3\pi^0$ and $\eta'\to 3\pi^0$

Next, we will use the results derived above in the calculation of the decays $\eta\to 3\pi^0$. The calculation of $\eta\to 3\pi^0$ has been performed using PCAC and current algebra but without allowing for η, η' mixing in ref. [2] and we will follow it closely up to a point. The PCAC definition of the π^0 field will be taken as

$$\phi_{\pi^0}=\frac{1}{2f_\pi m_\pi^2}\partial_\mu(\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d).$$

The amplitudes for these decays are written respectively as

$$F(\eta\to 3\pi^0)=\frac{i}{(2\pi)^2}a_0^\eta(k_1,k_2,k_3), \quad F(\eta'\to 3\pi^0)=\frac{i}{(2\pi)^2}a_0^{\eta'}(k_1,k_2,k_3),$$

where, the k are the momenta of the pions. a_0^η, $a_0^{\eta'}$ may be written as

$$a_0^{\eta,\eta'}(k_1,k_2,k_3)=(k_1^2+k_2^2+k_3^2-2m_\pi^2)c_0^{\eta,\eta'}(p^2),$$

where $c_0^{\eta,\eta'}(p^2)$ is free from Adler zeros and can be evaluated using standard current-algebra techniques. The on-shell decay rates are

$$\Gamma(\eta\to 3\pi^0)=\frac{3}{2}\frac{\delta_\eta}{\sqrt{3}(3456\pi^2)}\frac{(m_\eta-3m_\pi)^2}{m_\eta}|m_\pi^2 c_0^\eta(m_\eta^2)|^2, \tag{15}$$

$$\Gamma(\eta'\to 3\pi^0)=\frac{3}{2}\frac{\delta_{\eta'}}{\sqrt{3}(3456\pi^2)}\frac{(m_{\eta'}-3m_\pi)^2}{m_{\eta'}}|m_\pi^2 c_0^{\eta'}(m_{\eta'}^2)|^2, \tag{16}$$

δ_η and $\delta_{\eta'}$ are factors incorporating the relativistic corrections to the phase space. (Numerically one finds $\delta_\eta=0.86$, $\delta_{\eta'}=0.69$.)

From ref. [5] we have

$$C_0^{\eta} = (2\pi)^{3/2} \frac{1}{f_\pi^3 m_\pi^2} \langle 0| m_d \bar{d}(0)\gamma_5 d(0) - m_u \bar{u}\gamma_5 u(0)|\eta\rangle , \tag{17}$$

$$C_0^{\eta'} = (2\pi)^{3/2} \frac{1}{f_\pi^3 m_\pi^2} \langle 0| m_d \bar{d}\gamma_5 d(0) - m_u \bar{u}\gamma_5 u(0)|\eta'\rangle . \tag{18}$$

To estimate these matrix elements, we use the quark contents of $|\eta_0\rangle$ and $|\eta_8\rangle$ together with (1). Then,

$$\langle 0|\bar{u}\gamma_5 u|\eta\rangle = \langle 0|\bar{d}\gamma_5 d|\eta\rangle = \frac{+\mathrm{i}}{2m_s} \langle 0|2\mathrm{i}m_s\bar{s}\gamma_5 s|\eta\rangle \frac{1}{2}\left(\frac{\cos\theta - \sqrt{2}\sin\theta}{\cos\theta + (1/\sqrt{2})\sin\theta}\right), \tag{19}$$

$$\langle 0|\bar{u}\gamma_5 u|\eta'\rangle = \langle 0|\bar{d}\gamma_5 d|\eta'\rangle = \frac{-\mathrm{i}}{2m_s} \langle 0|2\mathrm{i}m_s\bar{s}\gamma_5 s|\eta'\rangle \left(\frac{\cos\theta + (1/\sqrt{2})\sin\theta}{\cos\theta - \sqrt{2}\sin\theta}\right). \tag{20}$$

Then, using the above results and (11)–(14), we obtain

$$m_\pi^2 C_0^{\eta} = \frac{-\sqrt{3}}{4}\mathrm{i}\left(\frac{m_d - m_u}{m_s}\right)\cos\theta \frac{f_8}{f_\pi^3}\left(\frac{\cos\theta - \sqrt{2}\sin\theta}{\cos\theta + (1/\sqrt{2})\sin\theta}\right) m_\eta^2 , \tag{21}$$

$$m_\pi^2 C_0^{\eta'} = \frac{\sqrt{3}}{2}\mathrm{i}\left(\frac{m_d - m_u}{m_s}\right)\sin\theta \frac{f_8}{f_\pi^3}\left(\frac{\cos\theta + (1/\sqrt{2})\sin\theta}{\cos\theta - \sqrt{2}\sin\theta}\right) m_{\eta'}^2 . \tag{22}$$

Note that we have *not reduced* the η or η' from the initial state using PCAC, but have instead used (11)–(14) to evaluate the matrix elements $\langle 0|2\mathrm{i}m_s\bar{s}\gamma_5 s|\eta'\rangle$ and $\langle 0|2\mathrm{i}m_s\bar{s}\gamma_5 s|\eta\rangle$. This latter procedure therefore *does not* assume the η and η' to be Goldstone bosons. It is for this reason that we derived (11)–(14) earlier.

Putting everything together,

$$\Gamma(\eta\to 3\pi^0) = \frac{3\sqrt{3}}{32}\frac{\delta_\eta}{3456\pi^2}(m_\eta - 3m_\pi)^2\left(\frac{m_d - m_u}{m_s}\right)^2 \frac{m_\eta^3}{f_\pi^6} f_8^2 \left[\left(\frac{\cos\theta - \sqrt{2}\sin\theta}{\cos\theta + (1/\sqrt{2})\sin\theta}\right)^2 \cos^2\theta\right], \tag{23}$$

$$\Gamma(\eta'\to 3\pi^0) = \frac{3\sqrt{3}}{4}\frac{\delta_{\eta'}}{3456\pi^2}(m_{\eta'} - 3m_\pi)^2\left(\frac{m_d - m_u}{m_s}\right)^2 \frac{m_{\eta'}^3}{f_\pi^6} f_8^2 \left[\left(\frac{\cos\theta + (1/\sqrt{2})\sin\theta}{\cos\theta - \sqrt{2}\sin\theta}\right)^2 \left(\frac{\sin^2\theta}{2}\right)\right]. \tag{24}$$

Here we would like to remark that the decay rate of the $\eta\to 3\pi^0$ given in (23), which includes the effect of the anomaly and mixing, modifies the result of the previous analysis [2] not including these effects, by the factor given in square brackets in (23). This modification for $\theta\approx -18°$ is about a factor of 3. More remarkable is the expression for the decay $\eta'\to 3\pi^0$. If the η' were treated as a Goldstone boson and if the effect of the anomaly and mixing were neglected then one would get the expression (24) without the last factor in square brackets. The decay $\eta'\to 3\pi^0$ would be much too large. The suppression factor in square brackets at $\theta\approx -18°$ is about 10^{-2} which brings down the naive estimate closer to the experimental value (see fig. 3). This is because in the above analysis $\eta'\to 3\pi^0$ goes through the quark content of η', and in the presence of the anomaly and η_0, η_8 mixing, the ratio of $\langle 0|2\mathrm{i}m_s\bar{s}\gamma_5 s|\eta'\rangle$ to $\langle 0|(3\alpha_s/4\pi)G\tilde{G}|\eta'\rangle$ is very small (≈ 0.2 at $\theta\approx -18°$) as seen from (11). If the η' were considered a pure glue state then it could decay to $3\pi^0$ via the pentagon anomaly. However this contribution is zero as shown in ref. [6]. We would like to emphasize that in the calculation of $\eta'\to 3\pi^0$ we are not treating the η' as a Goldstone boson.

4. $(\psi\to\eta'\gamma)/(\psi\to\eta\gamma)$

A model for this branching ratio has been considered in ref. [3]. The satisfactory feature of this model is that estimates made here are independent of any on-shell extrapolation for the η and η'. This latter statement is not true for the $\eta\to3\pi^0$ and $\eta'\to3\pi^0$ analyses made in the previous section. From ref. [3]

$$\frac{\Gamma(\psi\to\eta'\gamma)}{\Gamma(\psi\to\eta\gamma)}=\frac{|\langle0|G\tilde{G}|\eta'\rangle|^2}{|\langle0|G\tilde{G}|\eta\rangle|^2}\left(\frac{P_{\eta'}}{P_\eta}\right)^3,$$

P_η, $P_{\eta'}$ are phase space factors, $P_{\eta'}/P_\eta=1/1.07$.

From (13) and (14) we obtain

$$\frac{\Gamma(\psi\to\eta'\gamma)}{\Gamma(\psi\to\eta\gamma)}=\frac{1}{(1.07)^3}\left(\frac{m_{\eta'}^2}{m_\eta^2}\right)^2 2\,\frac{[(1/f_8)\cos\theta+(1/\sqrt{2}f_0)\sin\theta]^2}{[(1/f_0)\cos\theta-(\sqrt{2}/f_8)\sin\theta]^2}. \tag{25}$$

5. f_π, f_0, f_8 in terms of $\Gamma(\pi^0\to2\gamma)$, $\Gamma(\eta'\to2\gamma)$, $\Gamma(\eta\to2\gamma)$

f_π can be expressed in terms of $\Gamma(\pi^0\to2\gamma)$ through

$$f_\pi^2=\frac{1}{\Gamma(\pi^0\to2\gamma)}\left(\frac{\alpha}{2\pi}\right)^2\frac{m_\pi^3}{16\pi},$$

and f_0 and f_8 can be expressed in terms of θ, $\Gamma(\eta\to2\gamma)$, $\Gamma(\eta'\to2\gamma)$ f_π and $\Gamma(\pi^0\to2\gamma)$ as follows:

$$\frac{1}{f_8}=\frac{\sqrt{3}}{f_\pi}\left[\cos\theta\left(\frac{\Gamma(\eta\to2\gamma)}{\Gamma(\pi^0-2\gamma)}\right)^{1/2}\left(\frac{m_\pi}{m_\eta}\right)^{3/2}+\sin\theta\left(\frac{\Gamma(\eta'\to2\gamma)}{\Gamma(\pi^0\to2\gamma)}\right)^{1/2}\left(\frac{m_\pi}{m_{\eta'}}\right)^{3/2}\right], \tag{26}$$

$$\frac{1}{f_0}=\frac{\sqrt{3}}{2\sqrt{2}f_\pi}\left[-\sin\theta\left(\frac{\Gamma(\eta\to2\gamma)}{\Gamma(\pi^0-2\gamma)}\right)^{1/2}\left(\frac{m_\pi}{m_\eta}\right)^{3/2}+\cos\theta\left(\frac{\Gamma(\eta\to2\gamma)}{\Gamma(\pi^0\to2\gamma)}\right)^{1/2}\left(\frac{m_\pi}{m_{\eta'}}\right)^{3/2}\right]. \tag{27}$$

6. Numerical results

In order to make numerical estimates we fit f_π, f_0, f_8 from experimentally known data for $\Gamma(\pi^0\to2\gamma)$, $\Gamma(\eta\to2\gamma)$, $\Gamma(\eta'\to2\gamma)$ for a range of values of θ from $-10°$ to $-30°$. We have used the following experimental data. For the radiative decay rates we take the world averages from ref. [7],

$$\Gamma(\pi^0\to2\gamma)=0.00729\pm0.00019\ \text{keV},\quad \Gamma(\eta\to2\gamma)=0.524\pm0.031\ \text{keV},\quad \Gamma(\eta'\to2\gamma)=4.25\pm0.19\ \text{keV}.$$

For example at $\theta=-18°$, $f_0=1.11\times93$ MeV, $f_8=1.07\times93$ MeV. For the decays $\eta\to3\pi^0$, $\eta'\to3\pi^0$, $\mathrm{BR}(\psi\to\eta'\gamma)/(\psi\to\eta\gamma)$, we have plotted our results from (23), (24), and (25) in figs. 1–3 respectively. [The values of f_0, f_8 are taken for each θ from the above-mentioned fit and substituted in (23), (24) or (25) to obtain a plot of Γ versus θ for the three above-mentioned processes.] Solid curves are our estimates, dashed curves reflect the theoretical uncertainty which arises mostly from $(m_d-m_u)/m_s=0.027\pm0.005$. For comparison, we have plotted the current experimental values (dot-dashed lines) and their 1σ deviations (dashed lines). We have used the following experimental data from ref. [8]:

$$\mathrm{BR}\left(\frac{\psi\to\eta'\gamma}{\psi\to\eta\gamma}\right)=4.88\pm0.75, \tag{28}$$

$$\Gamma(\eta\to3\pi^0)=0.335\pm0.05\ \text{keV},\quad \Gamma(\eta'\to3\pi^0)=0.408\pm0.110\ \text{keV}. \tag{29, 30}$$

The last value is based on the branching ratio $\Gamma(\eta'\to3\pi^0)/\Gamma(\eta'\to\eta\pi\pi)$ from ref. [9]. More recent data indicate a lower value for $\Gamma(\eta'\to3\pi^0)=0.28\pm0.06$ keV [5].

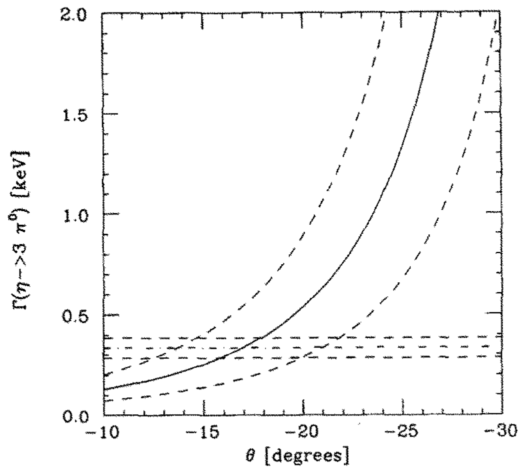

Fig. 1. For a discussion of the error bars, see the text.

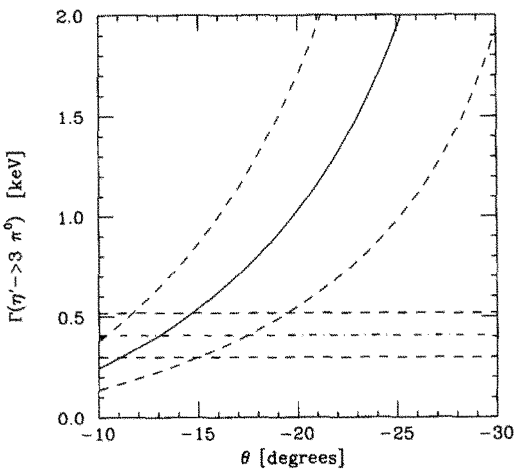

Fig. 2. See fig. 1.

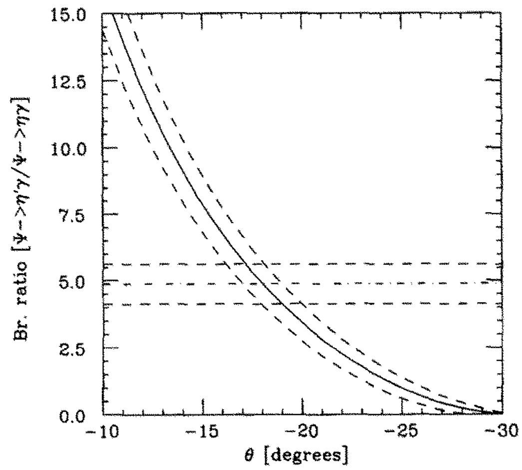

Fig. 3. See fig. 1.

7. Conclusions

The theoretical uncertainties represented in figs. 2, 3 do not take into account the effect of the on-shell extrapolation for the pions; further uncertainties associated with taking the η, η' on-shell arise in the determination of f_0, f_8. To obtain an estimate of this error, one could write down a dispersion relation subtracted at zero momentum where our results are exact. From an analysis of the intermediate states that can occur we estimate that these uncertainties could be as high as (20–30)%. Note that if we were using a standard SU(3) chiral lagrangian analysis then higher order corrections to the $\eta \to 3\pi$ rate would contribute not only towards the off-shell extrapolation but also to terms responsible for η–η' mixing. We have incorporated the effect of the latter phenomenologically in our analysis. taking the values (28) and (29), (30) we see that the fit to the three processes is good for values of $-20° \leqslant \theta \leqslant -16°$. This value is comparable to that found in refs. [1,10], although different physical mechanisms have been invoked.

The most recent value of $\eta' \to 3\pi^0$ [5] would however, disagree at the 1σ level with the above range of θ. This might indicate that explicit mixing between η' and pseudoscalar glueball states is needed. Such would indeed further depress the $3\pi^0$ decay rate while the radiative decay rate could be maintained through a change in f_0. This conclusion seems however premature in view of the theoretical uncertainties involved.

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