MATRIX ELEMENTS IN THE Sp(6) ⊃ U(3) BRANCH OF THE S, D FERMION PAIR MODEL

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Abstract: General expressions are given for the matrix elements of single-nucleon creation and annihilation operators for one-body and pair operators coupled to arbitrary i-space spin, in the Sp(6) ⊃ U(3) basis of the S, D pair algebra. These should facilitate calculations for rotational nuclei in the fermion dynamic symmetry model.

1. Introduction

In recent years the Ginocchio S, D fermion pair algebra [1]) has been proposed as a fermion dynamic symmetry model [2-5]) which can serve as a microscopic model for collective excitations in nuclei throughout the periodic chart [6-8]). In this model the normal-parity single-particle states of the shell model are reclassified in terms of a pseudo-orbital angular momentum (k) and pseudo spin (i), with $k + i = j$. The so-called k-active version of the model, with k limited to $k = 1$, leads to a Sp(6) symmetry with a Sp(6) ⊃ U(3) branch which can lead to strongly rotational spectra, particularly in the actinide region, where both the normal ($-$) parity valence protons with $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$, (modeled by $k = 1$; $i = \frac{1}{2}, \frac{7}{2}$), and the normal ($+$) parity neutrons with $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$; (with $k = 1$; $i = \frac{3}{2}, \frac{9}{2}$) may lead to collective states of good Sp(6) symmetry. Very recently, it has also been shown that the new vector coherent state techniques [9-11]) are tailor-made to construct the matrix representations of all branches of the Ginocchio S, D pair algebra [12-14]). The matrix elements of the group generators are therefore known. For the Sp(6) ⊃ U(3) branch and states of low generalized S, D-pair seniority or "heritage", u, particularly for $u = 0, 1, 2$, matrix elements of the group generators have been given essentially in analytic form [14,15]). This has made possible detailed studies of model hamiltonians built from the group

generators [16]). It is one of the great advantages of the fermion dynamic symmetry model that, unlike the interacting boson model of Arima and Iachello, it is firmly based on the nuclear shell model. There is therefore no necessity to build model hamiltonians in terms of the group generators. The $Sp(6) \supset U(3)$ basis states can be used as a collective basis for the most general shell model interactions, and transition probabilities can be calculated in terms of the true electromagnetic moment operators, rather than model quadrupole moment operators similar to those used in the interacting boson model. For such calculations, however, it becomes necessary to evaluate $Sp(6) \supset U(3)$ matrix elements of operators lying outside the $Sp(6)$ algebra, in particular operators coupled to a total i-spin other than zero.

Very recently, vector coherent state theory has been generalized to include the coherent state realizations of simple operators lying outside the group algebra [17]). It is the purpose of this contribution to apply this generalization of vector coherent state theory to evaluate the $Sp(6) \supset U(3)$ matrix elements of operators lying outside the $Sp(6)$ algebra. In particular, matrix elements of single-nucleon creation and annihilation operators can lead to general cfp expressions. Matrix elements are also given for the most general one-body unit tensors, coupled to arbitrary k and i-space angular momenta, together with the corresponding matrix elements of nucleon pair-creation and annihilation operators. These should facilitate calculations in the $Sp(6) \supset U(3)$ basis of the fermion dynamic symmetry model. Such matrix elements are needed, in particular, to calculate the single-particle energy contributions, single-particle transfer strengths, and $B(E2)$ values as well as other electromagnetic transition probabilities. The fermion dynamic symmetry model assumes that the low-lying collective states of nuclei are dominated by states of low "heritage", u. Matrix elements of operators coupled to arbitrary k-space and i-space spherical tensor character should make it possible to test the goodness of this assumption. An assessment can now be made of the importance of higher heritage admixtures using realistic effective interactions.

The strategy for the evaluation of matrix elements is that common to all seniority schemes. The matrix elements for the n-particle system are converted to those of the u- or $(u+1)$- or $(u+2)$-particle system. For sufficiently low u, the latter are then reduced to simple standard shell-model calculations. Vector coherent state (VCS) theory is tailor-made for this purpose since the VCS realization of a general operator factors this operator into two parts in a coupled basis, an "intrinsic" operator acting only in the u-particle subspace and a "collective" operator acting in the subspace of favored S, D-pair excitations, similar to the "spin" and "orbital" operators in an SLJ basis. Sect. 2 gives the needed VCS realization of the basic unit operators of this investigation as well as the u-space reduced matrix elements of the intrinsic operators introduced by this realization. These are summarized through a catalogue of intrinsic operator reduced matrix elements in table 3. This catalogue converts the needed intrinsic operator matrix elements to the corresponding standard shell-model matrix elements connecting states of heritage, u, to states of $u' = u, u \pm 1$,

or $u \pm 2$. Very specific examples of these so-called starting matrix elements are given in an appendix. In sect. 3 the k-space SU(3), i-space SU(2) reduced matrix elements of the single-nucleon creation and annihilation operators for the n-particle system are then reduced to those for the u-particle system by standard SU(3), SU(2) recoupling techniques. Very specific results for u and $u' \leq 2$ are tabulated in terms of Sp(6) ⊃ U(3) reduced Wigner coefficients in tables 5–11. Sect. 4 gives the reduction of the n-particle matrix elements for the nucleon pair-creation (annihilation) operators and the one-body unit tensors or multipole operators. The needed n-particle matrix elements are collected in a catalogue of general matrix elements in table 4. These matrix elements are expressed in terms of SU(3) recoupling coefficients which are readily available through the code of Draayer and Akiyama [18]) and the starting matrix elements illustrated by the simple examples of the appendix. In some simple special cases, of particular interest for fermion dynamic symmetry model calculations, the needed SU(3) coefficients can be given in analytic form making it possible to give simple analytic formulae for the needed matrix elements. This is illustrated with some examples in sect. 4.

2. The vector coherent state realizations

In the S, D fermon pair model the single-nucleon creation operators, $a_{jm}^\dagger$, for the normal-parity partners of a major oscillator shell are given in terms of pseudo angular momenta k and i, with $k + i = j$:

$$a_{jm}^\dagger = \sum_{m_k m_i} \langle k m_k i m_i | j m \rangle b_{k m_k i m_i}^\dagger . \tag{1}$$

For the Sp(6) k-active version of the model, $k = 1$, and it is useful to define cartesian components, $b_{a i m_i}^\dagger$, where $a = x, y, z$, and

$$b_{1 \pm 1 i m_i}^\dagger = \mp \sqrt{\tfrac{1}{2}} (b_{x i m_i}^\dagger \pm i b_{y i m_i}^\dagger) , \qquad b_{1 0 i m_i}^\dagger = b_{z i m_i}^\dagger . \tag{2}$$

The Sp(6) algebra is then generated by the pair creation operators

$$A_{ab}^\dagger = A_{ba}^\dagger = \sum_{i, m_i} (-1)^{i - m_i} b_{a i m_i}^\dagger b_{b i - m_i}^\dagger , \tag{3}$$

(with $a, b = x, y$, or z), the corresponding pair annihilation operators, $A_{ab} = (A_{ab}^\dagger)^\dagger$, and the one-body operators, C_{ab}, coupled to total i-spin of zero with a coherent sum over all i, [see eqs. (2)–(6) of ref. [14]]. The vector coherent state, (VCS), is built in terms of the six complex variables $z_{ab} (= z_{ba})$

$$|z\rangle = \exp\left(\tfrac{1}{2} \sum_{ab} z_{ab}^* A_{ab}^\dagger\right) |[\sigma]\alpha\rangle , \tag{4}$$

where $[\sigma] \equiv [\sigma_1 \sigma_2 \sigma_3]$ is the "intrinsic" U(3) symmetry of the u nucleons entirely free of the favored S, D pairs of the model, where the generalized seniority u (or "heritage") is given by $u = \sigma_1 + \sigma_2 + \sigma_3$. In eq. (4), α stands for any set of convenient

U(3) subgroup labels. The σ_i also label the Sp(6) representations

$$(\Omega_1\Omega_2\Omega_3) = (\tfrac{1}{3}\Omega - \sigma_3, \tfrac{1}{3}\Omega - \sigma_2, \tfrac{1}{3}\Omega - \sigma_1), \qquad \text{with } \Omega = 3\sum_i (i + \tfrac{1}{2}). \qquad (5)$$

In the VCS method, state vectors are mapped into z-space functional realizatons

$$|\Psi\rangle \to \Psi_{[\sigma]\alpha}(z) = \langle[\sigma]\alpha| e^{z\cdot A}|\Psi\rangle, \qquad \text{with } (z\cdot A) = \tfrac{1}{2}\sum_{ab} z_{ab}A_{ab}, \qquad (6)$$

and operators O are mapped into their z-space realizations, $\Gamma(O)$,

$$O|\Psi\rangle \to \langle[\sigma]\alpha| e^{z\cdot A} O\, e^{-z\cdot A}\, e^{z\cdot A}|\Psi\rangle = \langle[\sigma]\alpha|\Gamma(O)\, e^{z\cdot A}|\Psi\rangle$$

$$= \langle[\sigma]\alpha|\{O + [z\cdot A,\, O] + \tfrac{1}{2}[[z\cdot A,\,[z\cdot A,\, O]]] + \cdots\}\, e^{z\cdot A}|\Psi\rangle. \qquad (7)$$

The z-space realizations of the Sp(6) generators are given in eq. (12) of ref. [14]. We now want to generalize the VCS method to include operators O outside the algebra, in particular the single-nucleon operators $b^\dagger$, b, and the one-body operators $[b^\dagger \times b]$, as well as the pair operators $[b^\dagger \times b^\dagger]$ and $[b \times b]$ where the latter are not restricted to i-space angular momenta of zero but are coupled to arbitrary i-space angular momenta. These operators can be organized into Sp(6) $\supset$ U(3) irreducible tensors $T^{(\Omega_1\Omega_2\Omega_3)}_{[\omega]\alpha,IM_I}$ as shown in table 1, where $[\omega] \equiv [\omega_1\omega_2\omega_3]$ is the U(3) irreducible representation with subgroup labels α. The single-nucleon $b^\dagger$, b transform according to the 6-dimensional Sp(6) representation, $(\Omega_1\Omega_2\Omega_3) = (100)$, where for fixed i, m_i the subgroup label α could take on the three values $\alpha \equiv a = x, y$, or z. Alternatively, $\alpha \equiv 1m_k$ in a U(3) $\supset$ SO(3) basis, (see table 1 which exhibits both the k-space SU(3) $\supset$ SO(3) and the i-space SU(2) irreducible tensor character of $b^\dagger$ and b).

TABLE 1

The basic tensors $T^{(\Omega_1\Omega_2\Omega_3)}_{[\omega]\alpha,IM_I}$

(ω_0)	$(\Omega_1\Omega_2\Omega_3) = (100) -$ operators
(10)	$b^\dagger_{\alpha im_i} = (b^\dagger)^{(10)}_{\alpha, im_i}$
(01)	$-b_{\alpha i - m_i}(-1)^{i+m_i+x} = -(\tilde{b})^{(01)}_{\tilde{\alpha}, im_i}$

(ω)	$(\Omega_1\Omega_2\Omega_3) = (200) -$ operators
(20)	$\sqrt{\tfrac{1}{2}}\, A^\dagger(ii')^{(20)}_{KM_K,IM_I} = \sqrt{\tfrac{1}{2}}\, [b^\dagger_i \times b^\dagger_{i'}]^{(20)}_{KM_K,IM_I} \qquad (K=0,2)$
(00)	$-P(ii')^{(s)(\omega_0)}_{KM_K,IM_I} = -\tfrac{1}{2}\{[b^\dagger_i \times \tilde{b}_{i'}]^{(\omega_0)}_{KM_K,IM_I} - (-1)^{i+i'-I}[b^\dagger_{i'} \times \tilde{b}_i]^{(\omega_0)}_{KM_K,IM_I}\}$
(11)	$K=0$ for $(\omega_0)=(00)$, $\qquad K=1,2$ for $(\omega_0)=(11)$
(02)	$\sqrt{\tfrac{1}{2}}\, A(ii')^{(02)}_{KM_K,IM_I} = \sqrt{\tfrac{1}{2}}\, [\tilde{b}_i \times \tilde{b}_{i'}]^{(02)}_{KM_K,IM_I} \qquad (K=0,2)$

(ω)	$(\Omega_1\Omega_2\Omega_3) = (110) -$ operators
(01)	$\sqrt{\tfrac{1}{2}}\, A^\dagger(ii')^{(01)}_{KM_K,IM_I} = \sqrt{\tfrac{1}{2}}\, [b^\dagger_i \times b^\dagger_{i'}]^{(01)}_{KM_K,IM_I} \qquad (K=1)$
(11)	$-P(ii')^{(a)(11)}_{KM_K,IM_I} = -\tfrac{1}{2}\{[b^\dagger_i \times \tilde{b}_{i'}]^{(11)}_{KM_K,IM_I} + (-1)^{i+i'-I}[b^\dagger_{i'} \times \tilde{b}_i]^{(11)}_{KM_K,IM_I}\}$
(10)	$\sqrt{\tfrac{1}{2}}\, A(ii')^{(10)}_{KM_K,IM_I} = \sqrt{\tfrac{1}{2}}\, [\tilde{b}_i \times \tilde{b}_{i'}]^{(10)}_{KM_K,IM_I} \qquad (K=1)$

Throughout the paper the SU(3) labels $(\lambda_\omega \mu_\omega) \equiv (\omega_1 - \omega_2, \omega_2 - \omega_3)$ will often be abbreviated by (ω), in round parentheses, whereas the U(3) labels $[\omega_1 \omega_2 \omega_3]$ will be abbreviated by $[\omega]$, in square brackets. Note also that a k-space SU(3), i-space SU(2) double tensor has the conjugation property

$$(T^{(\omega)I}_{\alpha M_I})^\dagger = T^{(\tilde\omega)I}_{\tilde\alpha - M_I}(-1)^{\chi(\omega,\alpha)}(-1)^{I-M_I} , \qquad (8)$$

where $(\tilde\omega) = (\mu_\omega \lambda_\omega)$, $\tilde\alpha = K, -M_K$. The SU(3) conjugation phase factor, $\chi(\omega, \alpha)$, is somewhat dependent on phase conventions and on the specific choice of subgroup label α. However, this phase factor drops out of all final expressions for SU(3) reduced matrix elements.

The one-body and pair creation (annihilation) operators with antisymmetric i-space coupling belong to the 21-dimensional Sp(6) representation (200), while those with symmetric i-space coupling belong to the 14-dimensional Sp(6) representation (110). The coupled pair creation operators are defined through

$$A^\dagger(ii')^{(\omega_p)}_{KM_KIM_I} = [b^\dagger_i \times b^\dagger_{i'}]^{(\omega_p)}_{KM_KIM_I} = [b^\dagger_i \times b^\dagger_{i'}]_{KM_KIM_I}\langle(10)1;(10)1||(\omega_p)K\rangle ,$$

$$[b^\dagger_i \times b^\dagger_{i'}]_{KM_KIM_I} = \sum_{m_k m_{k'}} \sum_{m_i m_{i'}} \langle 1m_k 1m_{k'}|KM_K\rangle\langle im_i i'm'_i|IM_I\rangle b^\dagger_{1m_k i m_i}b^\dagger_{1m_{k'} i' m_{i'}} , \qquad (9a)$$

where $K = 0, 2$ for the SU(3) symmetric representation $(\omega_p) = (\omega_s) = (20)$, while $K = 1$ for the SU(3) antisymmetric representation $(\omega_p) = (\omega_a) = (01)$. Note the identity

$$[b^\dagger_i \times b^\dagger_{i'}]_{KM_K,IM_I} \equiv \tfrac{1}{2}\{[b^\dagger_i \times b^\dagger_{i'}]_{KM_K,IM_I} \mp [b^\dagger_{i'} \times b^\dagger_i]_{KM_K,IM_I}(-1)^{i+i'-I}\} , \qquad (9b)$$

for $K = 0, 2$ (or $K = 1$) upper (lower) signs, respectively; and note that I is automatically restricted to even (odd) integer values for $K = 0, 2$ (or $K = 1$) in the special case $i = i'$. Note also that the SU(3) $\supset$ SO(3) Wigner coefficient, the double-barred coefficient of eq. (9a) has absolute value of unity for the trivial couplings of eq. (9a). The coupled pair annihilation operators are defined similarly by

$$A(ii')^{(\tilde\omega_p)}_{KM_K,IM_I} = [\tilde b_i \times \tilde b_{i'}]^{(\tilde\omega_p)}_{KM_K,IM_I}$$

$$= (A^\dagger(ii')^{(\omega_p)}_{K-M_K,I-M_I})^\dagger(-1)^{1+\hat\omega_p+K-M_K+I-M_I} . \qquad (9c)$$

The SU(3) phase factor, designated by $\hat\omega$, arises in many conjugation and SU(3)-coupling reordering transformations. It is defined through

$$(-1)^{\hat\omega} \equiv (-1)^{\lambda_\omega + \mu_\omega} = (-1)^{\omega_1 - \omega_3} . \qquad (10)$$

Finally, the one-body unit tensors or multipole operators are defined through

$$P(ii')^{(p)(\omega_0)}_{KM_K,IM_I} = \tfrac{1}{2}\{[b^\dagger_i \times \tilde b_{i'}]^{(\omega_0)}_{KM_K,IM_I} \mp (-1)^{i+i'-I}[b^\dagger_{i'} \times \tilde b_i]^{(\omega_0)}_{KM_K,IM_I}\} \qquad (11)$$

where the operators with $(p) = (s)$, (upper sign) and $(\omega_0) = (00)$ or (11), belong to the family of Sp(6) (200)-operators, whereas those with $(p) = (a)$, (lower sign), and $(\omega_0) = (11)$ only belong to the Sp(6) (110)-operators. The symbol s(a) denotes that they are symmetric (antisymmetric) in the k-space.

The z-space realizations of the above operators can now be constructed by the applications of eq. (7). The z-space realizations $\Gamma(O)$ in eq. (7) consist of two types of operators, z-space polynomials or "collective operators", and fermion operators such as $b^\dagger$, b, $A(ii')$, etc. The latter commute with the z-space polynomials and are to be interpreted through their left actions on the vacuum or "intrinsic" states with quantum numbers $[\sigma]\alpha$. Following the standard VCS notation these "intrinsic" operators are denoted by $\mathbb{b}^\dagger$, $\mathbb{b}$, $\mathbb{A}(ii)$, etc. Since intrinsic operators such as $\mathbb{b}^\dagger$, $\mathbb{b}$, $\mathbb{A}(ii)$, ... are defined through their left actions on intrinsic states, they are to be interpreted as follows: They must always be commuted through to the left in a matrix element so that they can act on the intrinsic state. Eq. (7) gives the following $\Gamma(O)$:

$$\Gamma(\tilde{b}_{aim_i}) = \tilde{\mathbb{b}}_{aim_i} , \tag{12a}$$

$$\Gamma(b^\dagger_{aim_i}) = \mathbb{b}^\dagger_{aim_i} - 2[\tilde{\mathbb{b}}_i^{(01)} \times Z^{(20)}(z)]_a^{(10)} , \tag{12b}$$

$$\Gamma(A(ii')_{\alpha,IM_I}^{(\tilde{\omega}_p)}) = \mathbb{A}(ii')_{\alpha,IM_I}^{(\tilde{\omega}_p)} , \tag{12c}$$

$$\Gamma(P(ii')_{\alpha,IM_I}^{(p)(\omega_0)}) = \mathbb{P}(ii')_{\alpha,IM_I}^{(p)(\omega_0)} + c_{(\omega_0)}^{(p)}[\mathbb{A}(ii')_I^{(\tilde{\omega}_p)} \times Z^{(20)}(z)]_{\alpha,IM_I}^{(\omega_0)} , \tag{12d}$$

$$\Gamma(A^\dagger(ii')_{\alpha,IM_I}^{(\omega_p)}) = \mathbb{A}^\dagger(ii')_{\alpha,IM_I}^{(\omega_p)} + \sum_{(\omega_0)} f_{(\omega_0)}^{(p)}[\mathbb{P}(ii')_I^{(p)(\omega_0)} \times Z^{(20)}(z)]_{\alpha,IM_I}^{(\omega_p)}$$

$$+ \sum_{(n)} g_{(n)}^{(p)}[\mathbb{A}(ii')_I^{(\tilde{\omega}_p)} \times Z^{(n)}(z)]_{\alpha,IM_I}^{(\omega_p)}$$

$$+ \delta_{(\omega_p)(20)}\delta_{ii'}\delta_{I0}\sqrt{2i+1}\, Z_\alpha^{(20)}(z) , \tag{12e}$$

where the coefficients are given in table 2.

In eqs. (12a)–(12e), α is any convenient SU(3) subgroup label, such as the appropriate KM_K. As in eqs. (9) and (11) square brackets denote both SU(3) coupling and i-space angular momentum coupling. In eqs. (12a)–(12e) however, a right to left coupling order is used. This simplifies phases in the VCS construction and will be used *henceforth*. [Note, however, that the coupled operators, such as $[b_i^\dagger \times b_i^\dagger]$, are defined in terms of their standard left to right coupling order, see eq. (9a).] The z-space polynomials are the polynomials used to construct the orthonor-

TABLE 2

Coefficients used in the equations for the z-space realizations $\Gamma(O)$ of the intrinsic states

(p)	(ω_p)	$(\tilde{\omega}_p)$	(ω_0)	$c_{(\omega_0)}^{(p)}$		$f_{(\omega_0)}^{(p)}$		$g_{(n)}^{(p)}$	
				$c_{(00)}^{(p)}$	$c_{(11)}^{(p)}$	$f_{(00)}^{(p)}$	$f_{(11)}^{(p)}$	$g_{(40)}^{(p)}$	$g_{(02)}^{(p)}$
(s)	(20)	(02)	$(00),(11)$	-2	$-\sqrt{\frac{5}{2}}$	$-\sqrt{\frac{8}{3}}$	$-\sqrt{\frac{40}{3}}$	$2\sqrt{5}$	$-\sqrt{2}$
(a)	(01)	(10)	(11)		$+\sqrt{\frac{3}{2}}$		$+4$		$2\sqrt{3}$

mal Bargmann state vectors

$$[Z^{[n_1 n_2 n_3]}(z) \times |[\sigma_1 \sigma_2 \sigma_3]])_\alpha^{[\omega_1 \omega_2 \omega_3]\rho} \equiv |[[n] \times [\sigma]][\omega]\rho, \alpha) \equiv |\phi_{[n]\rho}) , \qquad (13)$$

where the polynomials of degree $n_1 + n_2 + n_3$, with n_i all *even* integers, carry the "collective" U(3) quantum numbers $[n_1 n_2 n_3]$ which are coupled with the "intrinsic" $[\sigma_1 \sigma_2 \sigma_3]$, (right to left coupling order), to resultant U(3) states $[\omega_1 \omega_2 \omega_3]$ with outer multiplicity label ρ, when needed. In eqs. (12a)-(12e) the $Z^{(n)}(z)$ have been labeled by their equivalent SU(3) quantum numbers $(n) \equiv (\lambda_n \mu_n) = (n_1 - n_2, n_2 - n_3)$. For their full construction, including their normalization, see e.g. p. 81 of ref. [11]).

In taking matrix elements it will be useful to indicate specifically whether matrix elements are to be calculated through their z-space integrations or in standard Hilbert space form. For this reason, the z-space state vector of eq. (13) has been written with a round parenthesis, $|\ldots)$. The corresponding state vector, written in standard Hilbert space notation with angular carets, $|\ldots\rangle$,

$$[Z^{[n_1 n_2 n_3]}(A^\dagger) \times |[\sigma_1 \sigma_2 \sigma_3]\rangle]_\alpha^{[\omega_1 \omega_2 \omega_3]\rho} \equiv |[[n] \times [\sigma]][\omega]\rho, \alpha\rangle \equiv |\phi_{[n]\rho}\rangle , \qquad (14)$$

is constructed through the successive action of the fermion operators $A^\dagger$ on the vacuum or "intrinsic" state with particle number, u. In eq. (14) the operator $A_{ab}^\dagger$ has replaced the z_{ab} in the polynomial $Z^{[n]}$. The states of eq. (13) form an orthonormal set in z-space.

$$(\phi_{[n']\rho'}|\phi_{[n]\rho}) = \delta_{[n][n']}\delta_{\rho\rho'} . \qquad (15)$$

The states of eq. (14), however, form a nonorthonormal set, including redundant states for some $[\sigma][\omega]$ combinations. The overlap of the states (14) are given by the $(KK^\dagger)$ matrix which is central to VCS theory

$$\langle\phi_{[n']\rho'}|\phi_{[n]\rho}\rangle = (KK^\dagger)_{[n']\rho',[n]\rho} . \qquad (16)$$

Note that the hermitian matrix $(KK^\dagger)$ was chosen to be real in all earlier VCS applications, see e.g. ref. [14]) where the $(KK^\dagger)$ are real symmetric matrices. In the VCS technique, K is also interpreted as the operator which converts the nonunitary realization, $\Gamma(O)$, of an operator O into a unitary realization, $\gamma(O)$,

$$\gamma(O) = K^{-1}\Gamma(O)K . \qquad (17)$$

To define K^{-1} it is necessary to convert the $(KK^\dagger)$ matrices to diagonal form via the unitary matrix, U

$$U(KK^\dagger)U^\dagger = \lambda , \qquad (18)$$

with $\lambda_{\nu\nu'} = \delta_{\nu\nu'}\lambda_\nu$. Note that zero eigenvalues of $(KK^\dagger)$ immediately signal the presence of Pauli-forbidden states. The Pauli-allowed states can be designated by the new quantum number $\nu = 1, 2, \ldots$ corresponding to the states with nonzero eigenvalues λ_ν. (This quantum number was generally labeled by $i = 1, 2, \ldots$ in earlier VCS references. The notation has been changed to avoid confusion with the angular

momentum quantum number i.) Eq. (18) leads to

$$(K)_{[n]\rho,\nu} = (U^\dagger)_{[n]\rho,\nu}\sqrt{\lambda_\nu} \, . \tag{19}$$

For Pauli-allowed states, with $\lambda_\nu \neq 0$,

$$(K^{-1})_{\nu,[n]\rho} = \frac{1}{\sqrt{\lambda_\nu}} \, U_{\nu,[n]\rho} \, . \tag{20}$$

The states with the new quantum number ν, corresponding to nonzero eigenvalues λ_ν, form the orthonormal set

$$|[\sigma][\omega]\nu, \alpha\rangle = \sum_{[n]\rho} (K^{-1})^*_{\nu,[n]\rho}[Z^{[n_1 n_2 n_3]}(A^\dagger) \times |[\sigma_1\sigma_2\sigma_3]\rangle]_\alpha^{[\omega_1\omega_2\omega_3]\rho} \, , \tag{21}$$

where the orthonormality follows from eqs. (16), (19) and (20). It may also be useful to define a new orthonormal set through a linear combination of the z-space set of eq. (13), through

$$|[\sigma][\omega]\nu, \alpha) = \sum_{[n]\rho} |\phi_{[n]\rho})(U^\dagger)_{[n]\rho,\nu} \, . \tag{22}$$

Note that the operators K or K^{-1} acting on the members of this set are given through the eigenvalues, λ_ν,

$$K|[\sigma][\omega]\nu, \alpha) = \sqrt{\lambda_\nu}\,|[\sigma][\omega]\nu, \alpha) \, ;$$

$$([\sigma][\omega]\nu, \alpha|K^{-1} = ([\sigma][\omega]\nu, \alpha|\frac{1}{\sqrt{\lambda_\nu}} \, . \tag{23}$$

The matrix element of an operator O in the orthonormal basis $|[\sigma][\omega]\nu, \alpha\rangle$ can then be transcribed to the z-space matrix element if we use the unitary realization of the operator, $\gamma(O)$, and the orthonormal z-space states $|[\sigma][\omega]\nu, \alpha)$. The reduced matrix element relation is

$$\langle[\sigma'][\omega']\nu'; I'\|O^{[\omega_0]I_0}\|[\sigma][\omega]\nu; I\rangle_{\rho_0}$$

$$= ([\sigma'][\omega']\nu'; I'\|\gamma(O)^{[\omega_0]I_0}\|[\sigma][\omega]\nu; I)_{\rho_0}$$

$$= \sum_{[n]\rho}\sum_{[n']\rho'} (K^{-1}([\sigma'][\omega']))_{\nu',[n']\rho'}$$

$$\times ([[n']\times[\sigma']][\omega']\rho'; I'\|\Gamma(O)^{[\omega_0]I_0}\|[[n]\times[\sigma]][\omega]\rho; I)_{\rho_0} K([\sigma][\omega])_{[n]\rho,\nu} \, , \tag{24}$$

where we have used eqs. (17), (22), and (23), and where the dependence on $[\sigma]$ and $[\omega]$ is shown explicitly in the K matrix elements. In eq. (24) a triple bar denotes the fact that the reduced matrix element is reduced with respect to both the k-space SU(3)-coupling and the i-space angular momentum coupling. Such reduced matrix elements are defined *without* $[2I'+1]^{-1/2}$ and SU(3) dimensional factors.

$$\langle[\sigma'][\omega']\nu'; I'\|T^{[\omega_0]I_0}\|[\sigma][\omega]\nu; I\rangle_{\rho_0}$$

$$= \langle[\sigma'][\omega']\nu'\alpha'; I'M'_I|[T^{[\omega_0]I_0}\times|[\sigma][\omega]\nu; I)]_{\alpha';M'_I}^{[\omega']\rho_0; I'} \, , \tag{25}$$

where the square bracket denotes both the U(3) coupling $[\omega] \times [\omega_0] \to [\omega']$ and the
i-space angular momentum coupling $I \times I_0 \to I'$. The multiplicity label ρ_0 is rarely
needed. The special case $(\omega_0) = (11)$, $(\omega') = (\omega)$, is one of the few cases treated in
this investigation in which the coupling $(\omega) \times (11)$ leads to a two-fold SU(3) multi-
plicity. In such a case the full matrix element will involve a sum over ρ_0

$$\langle [\sigma'][\omega']\nu'\alpha_{\omega'}; I'M'_I | O_{\alpha_0 M_{I_0}}^{[\omega_0]I_0} | [\sigma][\omega]\nu\alpha_\omega; IM_I \rangle$$

$$= \sum_{\rho_0} \langle [\sigma'][\omega']\nu'; I' \| O^{[\omega_0]I_0} \| [\sigma][\omega]\nu; I \rangle_{\rho_0}$$

$$\times \langle [\omega]\alpha_\omega; [\omega_0]\alpha_0 | [\omega']\alpha_{\omega'} \rangle_{\rho_0} \langle IM_I I_0 M_{I_0} | I'M'_I \rangle . \tag{26}$$

In the applications to the fermion dynamic symmetry model it will be important to
choose the SU(3) subgroup labels, α_ω, α_0, $\alpha_{\omega'}$, in an SU(3) $\supset$ SO(3) basis including
the k-space angular momentum quantum numbers KM_K, so that the full SU(3)
Wigner coefficient will factor into a product of an ordinary k-space angular momen-
tum Wigner coefficient and an SU(3) $\supset$ SO(3) reduced Wigner coefficient. The latter
can be obtained in an orthonormalized basis from the code of Draayer and
Akiyama [18]).

Since eq. (24) will be used repeatedly to convert a z-space matrix element into
a standard Hilbert space matrix element or vice versa, it will be termed the master
equation. Note again that round parentheses, $|\ldots)$, are used to signify z-space
integrations, whereas carets, $|\ldots\rangle$, signify standard Hilbert space matrix elements.

When $[n] = [n'] = [0]$, the master eq. (24) collapses to

$$([\sigma']; I' \| \Gamma(O)^{[\omega_0]I_0} \| [\sigma]; I) = \langle [\sigma']; I' \| O^{[\omega_0]I_0} \| [\sigma]; I \rangle , \tag{27}$$

where we have used the fact that K and K^{-1} are simple unit operators when acting
on the "intrinsic" states $[\sigma]$ or $[\sigma']$.

Eq. (24) is given for the most general case, including all SU(3) multiplicity labels.
Since the $[\sigma]$'s for low S, D-pair seniorities are such that the products $[n] \times [\sigma]$ are
generally free of multiplicities [14]), the labels ρ and ρ' are generally not needed.
Matrix elements of $b^\dagger$ and b, with $(\omega_0) = (10)$ and (01), are also free of the label
ρ_0. Since the method of calculation will be illustrated in detail through the operators
$b^\dagger$, b, all multiplicity labels ρ will be omitted in the remainder of this section and
in sect. 3 where it will be assumed that all SU(3) couplings are free of multiplicities.
In sect. 4, however, where some multiplicity labels come into play, all formulae are
displayed with their full multiplicity labeling.

Since the intrinsic operators, such as $\mathbb{b}^\dagger, \mathbb{b}, \mathbb{A}, \mathbb{P}, \ldots$ in their left action on the
intrinsic or vacuum states in general change the Sp(6) irreducible representation,
the first step in any calculation involves the evaluation of the reduced matrix elements
of such operators. Since the operator $\mathbb{b}^\dagger$ acting to the left on a state with $[\sigma']$, $u' =
\sigma'_1 + \sigma'_2 + \sigma'_3 = u - 1$, must lower the seniority to $u' - 1 = u - 2$, we have the obvious
result

$$u' = u - 1: \qquad ([\sigma']; I' \| \mathbb{b}_i^\dagger \| [\sigma]u; I) = 0 . \tag{28}$$

Note that $[\sigma]$ specifies both the U(3) representation of the intrinsic state and the Sp(6) representation via eq. (5).

The reduced matrix element of the intrinsic operator b_i, on the other hand, will be different from zero in this case. It can be related to the corresponding standard Hilbert space matrix element. With $u' = u - 1$ the left action of b_i will convert the intrinsic state $[\sigma']$ to an intrinsic state $[\sigma]$ with heritage u. Using $\Gamma(\tilde{b}_i) = \tilde{b}_i$, eq. (27) yields

$$u' = u - 1 : \quad ([\sigma']; I' \|\|\tilde{b}_i\|\| [\sigma]; I) = \langle [\sigma']; I' \|\|\tilde{b}_i\|\| [\sigma]; I \rangle . \tag{29}$$

On the other hand, with $u' = u + 1$, the left action of b_i on an intrinsic state will now have to convert this state to one with particle number $n = u + 2$ which is no longer an intrinsic state of the Sp(6) representation specified by the intrinsic $[\sigma]$ but instead corresponds to a state created from the intrinsic $[\sigma]$ by the Sp(6) excitation operator $A^{\dagger[2]}$. The normalized z-space realization of this state is given by

$$|[[2] \times [\sigma]][\bar{\sigma}] \alpha_{\bar{\sigma}}) = [Z^{[2]}(z) \times [[\sigma]])]^{[\bar{\sigma}]}_{\alpha_{\bar{\sigma}}} . \tag{30a}$$

The corresponding normalized state in standard Hilbert space is

$$|[\sigma][\bar{\sigma}] \nu = [2], \alpha_{\bar{\sigma}} \rangle = \frac{1}{K([\sigma][\bar{\sigma}])_{[2][2]}} [A^{\dagger[2]} \times [[\sigma]]]^{[\bar{\sigma}]}_{\alpha_{\bar{\sigma}}} . \tag{30b}$$

For the first symplectic excitation the K-matrix is a 1×1 matrix so that the label ν can be identified with $\nu \equiv [n] = [2]$ and K^{-1} serves as a simple normalization factor. Using the master equation (24), together with eq. (12a)

$$([\sigma']; I' \|\| K^{-1} \Gamma(\tilde{b}_i) K \|\| [[2] \times [\sigma]][\bar{\sigma}]; I)$$

$$= K([\sigma][\bar{\sigma}])_{[2][2]} ([\sigma']; I' \|\| \tilde{b}_i \|\| [[2] \times [\sigma]][\bar{\sigma}]; I)$$

$$= \langle [\sigma']; I' \|\| \tilde{b}_i \|\| [\sigma][\bar{\sigma}] \nu = [2]; I \rangle$$

$$= K^{-1}([\sigma][\bar{\sigma}])_{[2][2]} \langle [\sigma']; I' \|\| \tilde{b}_i \|\| [A^{\dagger[2]} \times [\sigma]][\bar{\sigma}]; I \rangle , \tag{31}$$

leading to

$$([\sigma']; I' \|\| \tilde{b}_i^{[11]} \|\| [[2] \times [\sigma]][\bar{\sigma}]; I)$$

$$= \frac{1}{K^2([\sigma][\bar{\sigma}])_{[2][2]}} \langle [\sigma']; I' \|\| \tilde{b}_i^{[11]} \|\| [A^{\dagger[2]} \times [\sigma]][\bar{\sigma}]; I \rangle . \tag{32}$$

The right-hand side can be simplified by using the SU(3) recoupling relation for SU(3)-tensors $T^{[\omega]}$

$$\langle [\sigma'] \| T^{[\omega]} \| [T^{[\omega']} \times [\sigma]][\bar{\sigma}] \rangle$$

$$= \sum_{[\omega_0]} U([\sigma][\omega'][\sigma'][\omega]; [\bar{\sigma}][\omega_0]) \langle [\sigma'] \| [T^{[\omega]} \times T^{[\omega']}]^{[\omega_0]} \| [\sigma] \rangle , \tag{33a}$$

with inverse

$$\langle [\sigma']\|[T^{[\omega]} \times T^{[\omega']}]^{[\omega_0]}\|[\sigma]\rangle$$

$$= \sum_{[\bar\sigma]} U([\sigma][\omega'][\sigma'][\omega]; [\bar\sigma][\omega_0])\langle [\sigma']\| T^{[\omega]}\|[T^{[\omega']} \times [\sigma]][\bar\sigma]\rangle . \qquad (33b)$$

With (33a), eq. (32) can be rewritten

$$([\sigma']; I'\|\tilde{\mathbb{b}}_i^{[11]}\|[[2] \times [\sigma]][\bar\sigma]; I) = \frac{1}{K^2([\sigma][\bar\sigma])_{[2][2]}} \sum_{[\omega_0]} U([\sigma][2][\sigma'][11]; [\bar\sigma][\omega_0])$$

$$\times \langle [\sigma']; I'\|[\tilde{b}_i^{[11]}, A^{\dagger[2]}]^{[\omega_0]}\|[\sigma]; I\rangle , \qquad (34)$$

where the U(3) coupled operator $[\tilde{b}_i^{[11]} \times A^{\dagger[2]}]^{[\omega_0]}$ has been converted to a U(3) coupled commutator by using the fact that $A^\dagger$ annihilates the intrinsic state $\langle [\sigma']\alpha'|$ in its left action on such a state. The U(3)-coupled commutator is defined by

$$[\tilde{b}_{im_i}^{[11]}, A^{\dagger[2]}]_\alpha^{[\omega_0]} = \sum_{\alpha_{11},\alpha_2} \langle [2]\alpha_2; [11]\alpha_{11}|[\omega_0]\alpha\rangle[\tilde{b}_{\alpha_{11}im_i}^{[11]}, A_{\alpha_2}^{\dagger[2]}]$$

$$= 2\delta_{[\omega_0][1]}b_{\alpha im_i}^{\dagger[1]} , \qquad (35)$$

where straightforward anticommutation relations of the fermion operators have been used to evaluate the right-hand side of eq. (35). This leads to the result

for $u' = u + 1$: $([\sigma']; I'\|\tilde{\mathbb{b}}_i\|[[2] \times [\sigma]][\bar\sigma]; I)$

$$= \frac{2}{K^2([\sigma][\bar\sigma])_{[2][2]}} U([\sigma][2][\sigma'][11]; [\bar\sigma][1])$$

$$\times \langle [\sigma']; I'\|b_i^\dagger\|[\sigma]; I\rangle . \qquad (36)$$

The required matrix element resulting from the left action of the intrinsic operator $\mathbb{b}_i$ on the intrinsic state $[\sigma']$ has thus been expressed in terms of a simple standard fermion matrix element of the operator $b_i^\dagger$, connecting the u-particle state to a $u' = (u+1)$-particle state. Examples of these simple "starting matrix elements" are given in the appendix.

In similar fashion eq. (27), together with (12b), leads to

$$\langle [\sigma']; I'\|b_i^\dagger\|[\sigma]; I\rangle = ([\sigma']; I'\|\Gamma(b_i^\dagger)\|[\sigma]; I)$$

$$= ([\sigma']; I'\|\mathbb{b}_i^\dagger\|[\sigma]; I)$$

$$-2([\sigma']; I'\|[\tilde{\mathbb{b}}_i^{[11]} \times Z^{[2]}(z)]^{[1]}\|[\sigma]; I) . \qquad (37)$$

Eq. (33b) then gives

$$([\sigma']; I' \| [\tilde{\mathbf{b}}_i^{[11]} \times Z^{[2]}]^{[1]} \| [\sigma]; I)$$
$$= \sum_{[\bar{\sigma}]} U([\sigma][2][\sigma'][11]; [\bar{\sigma}][1])([\sigma']; I' \| \tilde{\mathbf{b}}_i^{[11]} \| [[2] \times [\sigma]][\bar{\sigma}]; I) . \qquad (38)$$

The matrix element on the right-hand side is given by eq. (36) so that

$$\text{for } u' = u + 1 : \quad ([\sigma']; I' \| \mathbf{b}_i^\dagger \| [\sigma]; I) = \left\{ 1 + 4 \sum_{[\bar{\sigma}]} \frac{U^2([\sigma][2][\sigma'][11]; [\bar{\sigma}][1])}{K^2([\sigma][\bar{\sigma}])_{[2][2]}} \right\}$$
$$\times \langle [\sigma']; I' \| b_i^\dagger \| [\sigma]; I \rangle , \qquad (39)$$

with a nonzero value only for $u' = u + 1$.

Eqs. (36) and (39) can be evaluated in terms of simple U(3) or equivalent SU(3) Racah coefficients and the K^2 matrix elements for the first symplectic excitation with $[n] = [2] = [200]$. These are simple 1×1 submatrices with values which follow at once from eq. (24) of ref. [14]).

$$K^2([\sigma][\bar{\sigma}])_{[2][2]} = (\tfrac{2}{3}\Omega - 2\sigma_1) \qquad \text{for } [\bar{\sigma}] = [\sigma_1 + 2, \sigma_2, \sigma_3] ,$$
$$= (\tfrac{2}{3}\Omega - 2\sigma_2 + 2) \qquad \text{for } [\bar{\sigma}] = [\sigma_1, \sigma_2 + 2, \sigma_3] ,$$
$$= (\tfrac{2}{3}\Omega - 2\sigma_3 + 4) \qquad \text{for } [\bar{\sigma}] = [\sigma_1, \sigma_2, \sigma_3 + 2] ,$$
$$= (\tfrac{2}{3}\Omega - \sigma_1 - \sigma_2 + 2) \qquad \text{for } [\bar{\sigma}] = [\sigma_1 + 1, \sigma_2 + 1, \sigma_3] ,$$
$$= (\tfrac{2}{3}\Omega - \sigma_1 - \sigma_3 + 3) \qquad \text{for } [\bar{\sigma}] = [\sigma_1 + 1, \sigma_2, \sigma_3 + 1] ,$$
$$= (\tfrac{2}{3}\Omega - \sigma_2 - \sigma_3 + 4) \qquad \text{for } [\bar{\sigma}] = [\sigma_1, \sigma_2 + 1, \sigma_3 + 1] . \qquad (40)$$

The techniques illustrated in connection with the results given by eqs. (36) and (39) can be used to evaluate the required reduced matrix elements of intrinsic operators such as $\mathbb{A}(ii')$ or $\mathbb{P}(ii')$. These are collected in table 3. The results of table 3 are valid for the most general representations $[\sigma]$ and $[\sigma']$ and therefore include some SU(3) multiplicity labels ρ_0. In most cases of practical interest, however, these labels are not needed. For low values of u (or u'), in particular for $u(u') \leqslant 2$, the couplings $[\sigma] \times [\omega_0] \to [\sigma']$ are entirely multiplicity-free so that the labels ρ_0 and the ρ_0-sums can be omitted.

3. Matrix elements of single-nucleon operators

The single-nucleon creation (annihilation) operators can connect states of generalized S, D-pair seniority $u = \sigma_1 + \sigma_2 + \sigma_3$ to states with $u' = u \pm 1$. VCS theory leads to particularly simple formulae for the case $u' = u - 1$. These will therefore be evaluated in detail. Formulae for the case $u' = u + 1$ are then obtained most simply via hermitian conjugation.

TABLE 3

Catalogue of intrinsic operator reduced matrix elements

1. For $\mathbf{b}_i^\dagger$ see eqs. (28) and (39)
2. For $\tilde{\mathbf{b}}_i$ see eqs. (29) and (36)
3. $u' = u - 2$:

$$([\sigma']; I'\|\mathbb{A}(ii')_{I_p}^{\{\tilde{\omega}_p\}}\|[\sigma]; I) = \langle[\sigma']; I'\|A(ii')_{I_p}^{\{\tilde{\omega}_p\}}\|[\sigma]; I\rangle$$

4. $u' = u$:

$$([\sigma']; I'\|\mathbb{A}(ii')_{I_p}^{\{\tilde{\omega}_p\}}\|[[2] \times [\sigma]][\bar{\sigma}]; I)$$

$$= \frac{1}{K^2([\sigma][\bar{\sigma}])_{[2][2]}} \left\{ \sum_{[\omega_0]\rho_0} h_{(\omega_0)}^{(p)} U([\sigma][2][\sigma'][\tilde{\omega}_p]; [\bar{\sigma}]_{--}; [\omega_0]_{-}\rho_0) \right.$$

$$\times \langle[\sigma']; I'\|P(ii')_{I_p}^{(p)(\omega_0)}\|[\sigma]; I\rangle_{\rho_0}$$

$$\left. -2\sqrt{3(2i+1)} \, U([\sigma][2][\sigma][22]; [\bar{\sigma}][0])\delta_{ii'}\delta_{I_p0}\delta_{[\tilde{\omega}_p][22]}\delta_{II'}\delta_{[\sigma][\sigma']} \right\}$$

5. $u' = u - 2$:

$$([\sigma']; I'\|\mathbb{P}(ii')_{I_p}^{(p)(\omega_0)}\|[\sigma]; I)_{\rho_0} = 0$$

6. $u' = u$:

$$([\sigma']; I'\|\mathbb{P}(ii')_{I_p}^{(p)(\omega_0)}\|[\sigma]; I)_{\rho_0} = \langle[\sigma']; I'\|P(ii')_{I_p}^{(p)(\omega_0)}\|[\sigma]; I\rangle_{\rho_0}$$

$$-c_{(\omega_0)}^{(p)} \sum_{[\bar{\sigma}]} U([\sigma][2][\sigma'][\tilde{\omega}_p]; [\bar{\sigma}]_{--}; [\omega_0]_{-}\rho_0)$$

$$\times ([\sigma']; I'\|\mathbb{A}(ii')_{I_p}^{\{\tilde{\omega}_p\}}\|[[2] \times [\sigma]][\bar{\sigma}]; I)$$

where $c_{(00)}^{(s)} = -2$, $c_{(11)}^{(s)} = -\sqrt{\frac{5}{2}}$; $c_{(11)}^{(a)} = +\sqrt{\frac{3}{2}}$ [see eqs. (12a)-(12e)]

$h_{(00)}^{(s)} = 4$, $h_{(11)}^{(s)} = \sqrt{10}$; $h_{(11)}^{(a)} = -\sqrt{6}$

$(\tilde{\omega}_s) = (02)$; $(\tilde{\omega}_a) = (10)$

$U([\sigma][2][\sigma'][22]; [\bar{\sigma}][0]) = \delta_{[\sigma][\sigma']}(-1)^{\hat{\sigma}+\hat{\bar{\sigma}}}\sqrt{\dim [\bar{\sigma}]/6 \dim [\sigma]}$

$(-1)^{\hat{\sigma}} \equiv (-1)^{\lambda_\sigma+\mu_\sigma} = (-1)^{\sigma_1-\sigma_3}$

By applying the master eq. (24) to the single-nucleon creation operator and using the z-space realization $\Gamma(b^\dagger)$ of eq. (12b), we obtain

$$\langle[\sigma'][\omega']\nu'; I'\|b_i^\dagger\|[\sigma][\omega]\nu; I\rangle$$

$$= \sum_{[n][n']} K^{-1}([\sigma'][\omega'])_{\nu'[n']} K([\sigma][\omega])_{[n]\nu}$$

$$\times \{([[n'] \times [\sigma']][\omega']; I'\|b_i^{\dagger[1]}\|[[n] \times [\sigma]][\omega]; I)$$

$$-2([[n'] \times [\sigma']][\omega']; I'\|[Z^{[2]}(z) \times \tilde{\mathbb{b}}_i^{[11]}]^{[1]}\|[[n] \times [\sigma]][\omega]; I)\}. \tag{41}$$

For the case $u' = u - 1$, the matrix element of the first term on the right is zero via eq. (28). The second term involves the matrix element of the intrinsic operator $\mathbb{b}_i$ coupled to a z-space operator in a U(3)-coupled basis in which the state vectors involve the coupling of the intrinsic $[\sigma]$ to the z-space $[n]$. Note that the product

$[\tilde{\mathbb{b}}_i^{[11]} \times Z^{[2]}]^{[1]}$ is invariant under a change of coupling order and has been reordered in eq. (41) so that the coupling order of the operator is the same as that of the state vectors. For the case $u' = u - 1$ the left action of $\mathbb{b}_i$ on the intrinsic state $[\sigma']$ connects this $[\sigma']$ to a purely intrinsic state $[\sigma]$ on the right via eq. (29). Straightforward recoupling techniques therefore convert this matrix element to the pure intrinsic space reduced matrix element of b_i and the z-spaced reduced matrix element of $Z^{[2]} = z$ to yield

$$\langle [\sigma'][\omega']\nu'; I' \| b_i^\dagger \| [\sigma][\omega]\nu; I \rangle$$

$$= -2 \sum_{[n][n']} K^{-1}([\sigma'][\omega'])_{\nu'[n']} K([\sigma][\omega])_{[n]\nu} \begin{bmatrix} [\sigma] & [n] & [\omega] \\ [11] & [2] & [1] \\ [\sigma'] & [n'] & [\omega'] \end{bmatrix}$$

$$\times ([n'] \| z \| [n])\langle [\sigma']; I' \| \tilde{b}_i \| [\sigma]; I \rangle , \tag{42}$$

where the [ ] symbol is a U(3) or equivalent SU(3) 9-j type symbol in unitary form. For simplicity it is assumed that $[\sigma]$ and $[\sigma']$ belong to the simple cases of ref. [14]) for which the U(3) products $[\sigma] \times [n] \to [\omega]$ are free of multiplicity. For the reduced matrix element of z in the collective space characterized by $[n]$ and $[n']$; see e.g., p. 84 of ref. [11]). The intrinsic space reduced matrix element of $\mathbb{b}_i$ has been converted to a matrix element of b_i via eq. (29). For states of low generalized seniority, u, the reduced matrix elements of b_i are easily evaluated by standard shell-model techniques. Examples of these starting matrix elements are given in the appendix.

For the case $u' = u - 1$ the matrix element of the single nucleon annihilation operator b_i follows in similar fashion. In this case $\Gamma(b_i)$ is the pure intrinsic operator, $\mathbb{b}_i$, so that the recoupling coefficient in the final matrix element is a U-coefficient or SU(3) Racah coefficient in unitary form. The matrix element is listed as entry 2 in table 4 which gives a catalogue of all the reduced matrix elements evaluated in

TABLE 4

Catalogue for the matrix elements $([[n'] \times [\sigma']][\omega']\rho'; I' \| \Gamma(O) \| [[n] \times [\sigma]]\omega]\rho; I)_{\rho_0} \equiv M$

$$\langle [\sigma'][\omega']\nu'; I' \| O \| [\sigma][\omega]\nu; I \rangle_{\rho_0} = \sum_{[n']\rho'} \sum_{[n]\rho} K^{-1}([\sigma'][\omega'])_{\nu',[n']\rho'} K([\sigma][\omega])_{[n]\rho,\nu}$$

$$\times ([[n'] \times [\sigma']][\omega']\rho'; I' \| \Gamma(O) \| [[n] \times [\sigma]][\omega]\rho; I)_{\rho_0}$$

1. $O = b_i^\dagger$; $u' = u - 1$

$$M = -2 \begin{bmatrix} [\sigma] & [n] & [\omega] & \rho \\ [11] & [2] & [1] & - \\ [\sigma'] & [n'] & [\omega'] & \rho' \\ - & - & - & \end{bmatrix} ([n'] \| z \| [n])\langle [\sigma']; I' \| \tilde{b}_i \| [\sigma]; I \rangle$$

2. $O = \tilde{b}_i$; $u' = u - 1$

$$M = (-1)^{\hat{\sigma} - \hat{\sigma}' + \hat{\omega} - \hat{\omega}'} \delta_{[n][n']} U([11][\sigma][\omega'][n]; [\sigma']_-\rho'; [\omega]\rho_-)\langle [\sigma']; I' \| \tilde{b}_i \| [\sigma]; I \rangle$$

TABLE 4—continued

3. $O = A(ii')^{(\tilde{\omega}_p)}_{I_p}$, $(\tilde{\omega}_p) = (02), (10)$; $\quad u' = u - 2$

$$M = (-1)^{\hat{\sigma} - \hat{\sigma}' + \hat{\omega} - \hat{\omega}'} \delta_{[n][n']} U([\tilde{\omega}_p][\sigma][\omega'][n]; [\sigma']_-\rho'; [\omega]\rho_-)\langle[\sigma']; I' \| A(ii')^{(\tilde{\omega}_p)}_{I_p} \| [\sigma]; I\rangle$$

4. $O = P(ii')^{(p)(\omega_0)}_{I_p}$; $u' = u - 2$; $(\omega_0) = (00), (11)$ for $(p) = (s)$; $(\omega_0) = (11)$ for $(p) = (a)$;

$$M = (-1)^{\hat{\omega}_p} c^{(p)}_{(\omega_0)} \begin{bmatrix} [\sigma] & [n] & [\omega] & \rho \\ [\tilde{\omega}_p] & [\omega_p] & [\omega_0] & - \\ [\sigma'] & [n'] & [\omega'] & \rho' \\ - & - & \rho_0 & \end{bmatrix} ([n']\|z\|[n])\langle[\sigma']; I' \| A(ii')^{(\tilde{\omega}_p)}_{I_p} \| [\sigma]; I\rangle$$

5. $O = A^\dagger(ii')^{(\omega_p)}_{I_p}$, $(\omega_p) = (20), (01)$; $\quad u' = u - 2$

$$M = \sum_{[n_0]} g^{(p)}_{(n_0)} \begin{bmatrix} [\sigma] & [n] & [\omega] & \rho \\ [\tilde{\omega}_p] & [n_0] & [\omega_p] & - \\ [\sigma'] & [n'] & [\omega'] & \rho' \\ - & - & - & \end{bmatrix} ([n']\|Z^{[n_0]}\|[n])\langle[\sigma']; I' \| A(ii')^{(\tilde{\omega}_p)} \| [\sigma]; I\rangle$$

with

$$([n']\|Z^{[n_0]}\|[n]) = \frac{1}{\sqrt{2}} \sum_{[n'']} ([n']\|z\|[n''])([n'']\|z\|[n]) U([n][2][n'][2]; [n''][n_0])$$

$g^{(s)}_{(40)} = 2\sqrt{5}, g^{(s)}_{(02)} = -\sqrt{2}$; $\qquad g^{(a)}_{(02)} = 2\sqrt{3}$

6. $O = A(ii')^{(\tilde{\omega}_p)}_{I_p}$; $\quad u' = u$

$$M = \sum_{[\bar{\sigma}]\bar{\rho}} \sqrt{\frac{\dim[\omega]}{\dim[\omega']} \frac{\dim[\sigma']}{\dim[\bar{\sigma}]}} U([\omega_p][\sigma'][\omega][n']; [\bar{\sigma}]_-\bar{\rho}; [\omega']\rho'_-)$$

$$\times U([\sigma][2][\omega][n']; [\bar{\sigma}]_-\bar{\rho}; [n]_-\rho)([n]\|z\|[n'])\langle[\sigma']; I' \| \mathbb{A}(ii')^{(\tilde{\omega}_p)}_{I_p} \| [[2] \times [\sigma]][\bar{\sigma}]; I\rangle$$

7. $O = P(ii')^{(p)(\omega_0)}_{I_p}$; $(\omega_0) = (00), (11)$ for $(p) = (s)$; $\quad (11)$ for $(p) = (a)$, $u' = u$

$$M = \sum_{\rho_\sigma} \begin{bmatrix} [\sigma] & [n] & [\omega] & \rho \\ [\omega_0] & [0] & [\omega_0] & - \\ [\sigma'] & [n] & [\omega'] & \rho' \\ \rho_\sigma & - & \rho_0 & \end{bmatrix} ([\sigma']; I' \| \mathbb{P}(ii')^{(p)(\omega_0)}_{I_p} \| [\sigma]; I)_{\rho_\sigma}$$

$$+ c^{(p)}_{(\omega_0)} \sum_{[\omega'']} \sum_{[n'']\rho''} U([\omega][2][\omega'][\tilde{\omega}_p]; [\omega'']_--; [\omega_0]_-\rho_0)$$

$$\times U([\sigma][n][\omega''][2]; [\omega]\rho_-; [n'']_-\rho'')([n'']\|z\|[n])$$

$$\times \sum_{[\bar{\sigma}]\bar{\rho}} \sqrt{\frac{\dim[\omega'']}{\dim[\omega']} \frac{\dim[\sigma']}{\dim[\bar{\sigma}]}} U([\omega_p][\sigma'][\omega''][n']; [\bar{\sigma}]_-\bar{\rho}; [\omega']\rho'_-)$$

$$\times U([\sigma][2][\omega''][n']; [\bar{\sigma}]_-\bar{\rho}; [n'']_-\rho'')([n'']\|z\|[n'])\langle[\sigma']; I' \| \mathbb{A}(ii')^{(\tilde{\omega}_p)}_{I_p} \| [[2] \times [\sigma]][\bar{\sigma}]; I\rangle$$

where $\quad c^{(s)}_{(00)} = -2, c^{(s)}_{(11)} = -\sqrt{\frac{5}{2}}$; $c^{(a)}_{(11)} = +\sqrt{\frac{3}{2}}$

this investigation. For easy reference, eq. (42) is also included as entry 1 of this table, but now for the most general case for which the product $[\sigma] \times [n] \to [\omega]$ might involve the multiplicity label, ρ. The k-space SU(3), i-space SU(2) reduced matrix elements of $b^\dagger$, b for the case $u' = u - 1$ can thus be evaluated in terms of SU(3) recoupling coefficients which are readily available through the code of ref. [18]) and the K-matrix elements of VCS theory. The needed K-matrix elements are evaluated by the techniques of ref. [14]). For states with $u = 0, 1, 2$ very specific analytic formulae are given in ref. [14]). Note also that for most of the states with $u = 0, 1, 2$ the $[n]$ is uniquely specified by $[\sigma]$ and $[\omega]$, the K-matrices are one-dimensional, and no $[n]$-sums are needed in eq. (42).

Matrix elements of $b^\dagger$ and b for the case $u' = u + 1$ could be obtained directly with the use of eqs. (36) and (39). However, the simplest expressions are obtained through the hermitian conjugation of the matrix elements for the case $u' = u - 1$.

$$\langle [\sigma'][\omega']\nu'; I'\|b_i^\dagger\|[\sigma][\omega]\nu; I\rangle = \sqrt{\frac{(2I+1)}{(2I'+1)}} (-1)^{I+i-I'} \sqrt{\frac{\dim[\omega]}{\dim[\omega']}} (-1)^{\hat{\omega}+1-\hat{\omega}'}$$

$$\times \langle [\sigma][\omega]\nu; I\|\tilde{b}_i\|[\sigma'][\omega']\nu'; I'\rangle, \tag{43}$$

where the U(3) or equivalent SU(3) dimension factor is given by $\dim[\omega] = \frac{1}{2}(\lambda_\omega + 1)(\mu_\omega + 1)(\lambda_\omega + \mu_\omega + 2)$. The phase factors follow from the $1 \to 3$ interchange symmetry property of the k-space SU(3) Wigner coefficients and the i-space angular momentum Wigner coefficients. In the case of SU(3) these have the simple value shown for the multiplicity-free coupling $(\omega) \times (10) \to (\omega')$. [For the definition of the phase $\hat{\omega}$, see eq. (10).]

Since the operators $b_i^\dagger$, b_i span the six-dimensional Sp(6) representation (100), the reduced matrix elements of these single-nucleon operators can be expressed in their most convenient form through a set of Sp(6) $\supset$ U(3) reduced Wigner coefficients for the Sp(6) coupling $(\Omega_1\Omega_2\Omega_3) \times (100) \to (\Omega_1'\Omega_2'\Omega_3')$ [see eq. (5)]. For the case $u' = u - 1$ the reduced matrix elements of $b_i^\dagger$ and b_i (entries 1 and 2 of table 4) can be expressed as a product of a Sp(6) $\supset$ U(3) reduced Wigner coefficient and a Sp(6) reduced matrix element. There is, however, no need to introduce a special new notation (such as a quadruple-barred matrix element!) for this Sp(6) fully reduced matrix element. For the case $u' = u - 1$ it is simply the starting matrix element, $\langle [\sigma']; I'\|\tilde{b}_i\|[\sigma]; I\rangle$; i.e.

$$\langle [\sigma'[\omega']\nu'; I'\|\tilde{b}_i\|[\sigma][\omega]\nu; I\rangle$$

$$= \langle (\Omega_1\Omega_2\Omega_3)[\sigma][\omega]\nu; (100)[0\ 0\ -1]\|(\Omega_1'\Omega_2'\Omega_3')[\sigma'][\omega']\nu'\rangle$$

$$\times \langle [\sigma']; I'\|\tilde{b}_i\|[\sigma]; I\rangle. \tag{44a}$$

Here, the double-barred coefficient is the Sp(6) $\supset$ U(3) reduced Wigner coefficient. This follows from the fact that this coefficient describes a unique 1×1 unitary transformation for the special case $[\omega'] = [\sigma'](\nu' = [0])$, $[\omega] = [\sigma](\nu = [0])$, $u' =$

$u - 1$; and is chosen to have the value $+1$. Note also that *both* entries 1 and 2 of table 4 are proportional to the Sp(6) reduced matrix element $\langle[\sigma']; I'\|\tilde{b}_i\|[\sigma]; I\rangle$. It is therefore easy to read off the orthonormal Sp(6) $\supset$ U(3) reduced Wigner coefficients for the case $u' = u - 1$; or, vice versa, from the reduced Wigner coefficients to obtain the matrix elements of $\tilde{b}_i$ and $b_i^\dagger$ by using eq. (44a) or its companion

$$u' = u - 1 : \quad \langle[\sigma'][\omega']\nu'; I'\|b_i^\dagger\|[\sigma][\omega]\nu; I\rangle$$

$$= \langle(\Omega_1\Omega_2\Omega_3)[\sigma][\omega]\nu; (100)[100]\|(\Omega_1'\Omega_2'\Omega_3')[\sigma'][\omega']\nu'\rangle$$

$$\times\langle[\sigma']; I'\|\tilde{b}_i\|[\sigma]; I\rangle . \tag{44b}$$

The conjugation relation of eq. (43) can be translated into the symmetry relation for the Sp(6) $\supset$ U(3) reduced Wigner coefficient

$$\langle(\Omega_1\Omega_2\Omega_3)[\sigma][\omega]\nu; (100)[\sigma'']\|(\Omega_1'\Omega_2'\Omega_3')[\sigma'][\omega']\nu'\rangle$$

$$= \pm(-1)^{\hat{\sigma}-\hat{\sigma}'}\sqrt{\frac{\dim(\Omega_1'\Omega_2'\Omega_3')\dim[\omega]}{\dim(\Omega_1\Omega_2\Omega_3)\dim[\omega']}}$$

$$\times\langle(\Omega_1'\Omega_2'\Omega_3')[\sigma'][\omega']\nu'; (100)[\tilde{\sigma}'']\|(\Omega_1\Omega_2\Omega_3)[\sigma][\omega]\nu\rangle , \tag{45}$$

with upper (lower) sign for $[\sigma''] = [100]([0\ 0 - 1])$.

The symmetry relation (45) is somewhat dependent on phase choices for the Sp(6) states. Eq. (43), on the other hand, is completely independent of such choices and depends solely on the standard angular momentum phase conventions and the SU(3) phase conventions of the computer code of ref. [18]. For the case $u' = u + 1$ the Sp(6) fully reduced matrix element has the more complicated value

$$\sqrt{\frac{\dim(\Omega_1\Omega_2\Omega_3)\dim[\sigma']}{\dim(\Omega_1'\Omega_2'\Omega_3')\dim[\sigma]}}(-1)^{\hat{\sigma}-\hat{\sigma}'}\langle[\sigma']; I'\|b_i^\dagger\|[\sigma]; I\rangle .$$

The Sp(6) $\supset$ U(3) reduced Wigner coefficients for the couplings $(\Omega_1\Omega_2\Omega_3) \times (100) \to (\Omega_1'\Omega_2'\Omega_3')$ for states with heritage u and $u' = 0, 1, 2$ can be given in very specific analytic form. These are collected in tables 5–11. In these tables, as well as in eqs. (44), (45) the U(3) representations for the single-nucleon creation and annihilation operators are given in their full $[\sigma_1''\sigma_2''\sigma_3'']$ form; [100] for $b_i^\dagger$, [00 − 1] for b_i. This has the advantage that $\sum_i \omega_i' = \sum_i (\omega_i + \sigma_i'')$ in tables 5–11.

4. Matrix elements of one-body and pair operators

One-body operators, $P(ii')$, and nucleon pair creation and annihilation operators, $A^\dagger(ii')$, $A(ii')$, can connect states of heritage $u = \sigma_1 + \sigma_2 + \sigma_3$ to states with $u' = u \pm 2, u$. VCS theory again leads to the simplest formulae for states with $u' = u - 2$. Detailed formulae are therefore given only for states with $u' = u - 2$ and $u' = u$. The remaining cases can again be obtained from hermitian conjugation. Although SU(3) outer multiplicities will not be encountered in most formulae involving heritage

TABLE 5

$$\left\langle \left(\frac{\Omega}{3}\frac{\Omega}{3}\frac{\Omega}{3}-1\right)[1][\omega]; (100)[\sigma''] \middle\| \left(\frac{\Omega}{3}\frac{\Omega}{3}\frac{\Omega}{3}\right)[0][n]\right\rangle$$

$[\omega]$	$[\sigma'']$	$\langle \ \| \ \rangle$
$[n_1-1\ n_2\ n_3]$	$[1\,0\,0]$	$-\sqrt{\dfrac{(n_1+2)(n_1-n_2)(n_1-n_3+1)}{2\Omega(n_1-n_2+1)(n_1-n_3+2)}}$
$[n_1\ n_2-1\ n_3]$	$[1\,0\,0]$	$\sqrt{\dfrac{(n_2+1)(n_1-n_2+2)(n_2-n_3)}{2\Omega(n_1-n_2+1)(n_2-n_3+1)}}$
$[n_1\ n_2\ n_3-1]$	$[1\,0\,0]$	$-\sqrt{\dfrac{n_3(n_1-n_3+3)(n_2-n_3+2)}{2\Omega(n_1-n_3+2)(n_2-n_3+1)}}$
$[n_1+1\ n_2\ n_3]$	$[0\,0-1]$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_1)(n_1-n_2+2)(n_1-n_3+3)}{2\Omega(n_1-n_2+1)(n_1-n_3+2)}}$
$[n_2\ n_2+1\ n_3]$	$[0\,0-1]$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_2+1)(n_1-n_2)(n_2-n_3+2)}{2\Omega(n_1-n_2+1)(n_2-n_3+1)}}$
$[n_1\ n_2\ n_3+1]$	$[0\,0-1]$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_3+2)(n_1-n_3+1)(n_2-n_3)}{2\Omega(n_1-n_3+2)(n_2-n_3+1)}}$

TABLE 6

$$\left\langle \left(\frac{\Omega}{3}\frac{\Omega}{3}\frac{\Omega}{3}\right)[0][\omega]; (1\,0\,0)[\sigma''] \middle\| \left(\frac{\Omega}{3}\frac{\Omega}{3}\frac{\Omega}{3}-1\right)[1][\omega']\nu'\equiv[n']\right\rangle$$

(1) $[\omega']=[n_1'+1\ n_2'\ n_3']$

$\quad [\omega]=[n]=[n_1'\ n_2'\ n_3']; [\sigma'']=[1\,0\,0]$ $\qquad +\sqrt{\dfrac{(2\Omega-3n_1')}{2(\Omega+6)}}$

$\quad [\omega]=[n]=[n_1'+2\ n_2'\ n_3']; [\sigma'']=[0\,0-1]$ $\qquad +\sqrt{\dfrac{3(n_1'+4)}{2(\Omega+6)}}$

(2) $[\omega']=[n_1'\ n_2'+1\ n_3']$

$\quad [\omega]=[n]=[n_1'\ n_2'\ n_3']; [\sigma'']=[1\,0\,0]$ $\qquad -\sqrt{\dfrac{(2\Omega-3n_2'+3)}{2(\Omega+6)}}$

$\quad [\omega]=[n]=[n_1'\ n_2'+2\ n_3']; [\sigma'']=[0\,0-1]$ $\qquad +\sqrt{\dfrac{3(n_2'+3)}{2(\Omega+6)}}$

(3) $[\omega']=[n_1'\ n_2'\ n_3'+1]$

$\quad [\omega]=[n]=[n_1'\ n_2'\ n_3']; [\sigma'']=[1\,0\,0]$ $\qquad +\sqrt{\dfrac{(2\Omega-3n_3'+6)}{2(\Omega+6)}}$

$\quad [\omega]=[n]=[n_1'\ n_2'\ n_3'+2]; [\sigma'']=[0\,0-1]$ $\qquad +\sqrt{\dfrac{3(n_3'+2)}{2(\Omega+6)}}$

TABLE 7

$$\left\langle \left(\frac{\Omega}{3}\,\frac{\Omega}{3}\,\frac{\Omega}{3}-1\right)[1][\omega]\nu \equiv [n];\,(1\,0\,0)[\sigma'']\right\| \left(\frac{\Omega}{3}\,\frac{\Omega}{3}-1\,\frac{\Omega}{3}-1\right)[11][\omega']\nu' \equiv [n']\right\rangle$$

$[\omega]$	$[n]$	$[\sigma'']$	$\langle \ \| \ \rangle$
(1) $[\omega']=[n_1'+1\ n_2'+1\ n_3']$			
$[n_1'+1\ n_2'+2\ n_3']$	$[n_1'\ n_2'+2\ n_3']$	$[0\,0-1]$	$\sqrt{\dfrac{(n_2'+3)(n_1'-n_2')}{2(\frac{2}{3}\Omega+5)(n_1'-n_2'+1)}}$
$[n_1'+2\ n_2'+1\ n_3']$	$[n_1'+2\ n_2'\ n_3']$	$[0\,0-1]$	$-\sqrt{\dfrac{(n_1'+4)(n_1'-n_2'+2)}{2(\frac{2}{3}\Omega+5)(n_1'-n_2'+1)}}$
$[n_1'+1\ n_2'\ n_3']$	$[n_1'\ n_2'\ n_3']$	$[1\,0\,0]$	$-\sqrt{\dfrac{(\frac{2}{3}\Omega-n_2'+1)(n_1'-n_2'+2)}{2(\frac{2}{3}\Omega+5)(n_1'-n_2'+1)}}$
$[n_1'\ n_2'+1\ n_3']$	$[n_1'\ n_2'\ n_3']$	$[1\,0\,0]$	$-\sqrt{\dfrac{(\frac{2}{3}\Omega-n_1')(n_1'-n_2')}{2(\frac{2}{3}\Omega+5)(n_1'-n_2'+1)}}$
(2) $[\omega']=[n_1'+1\ n_2'\ n_3'+1]$			
$[n_1'+2\ n_2'\ n_3'+1]$	$[n_1'+2\ n_2'\ n_3']$	$[0\,0-1]$	$-\sqrt{\dfrac{(n_1'+4)(n_1'-n_3'+3)}{2(\frac{2}{3}\Omega+5)(n_1'-n_3'+2)}}$
$[n_1'+1\ n_2'\ n_3'+2]$	$[n_1'\ n_2'\ n_3'+2]$	$[0\,0-1]$	$\sqrt{\dfrac{(n_3'+2)(n_1'-n_3'+1)}{2(\frac{2}{3}\Omega+5)(n_1'-n_3'+2)}}$
$[n_1'+1\ n_2'\ n_3']$	$[n_1'\ n_2'\ n_3']$	$[1\,0\,0]$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_3'+2)(n_1'-n_3'+3)}{2(\frac{2}{3}\Omega+5)(n_1'-n_3'+2)}}$
$[n_1'\ n_2'\ n_3'+1]$	$[n_1'\ n_2'\ n_3']$	$[1\,0\,0]$	$-\sqrt{\dfrac{(\frac{2}{3}\Omega-n_1')(n_1'-n_3'+1)}{2(\frac{2}{3}\Omega+5)(n_1'-n_3'+2)}}$
(3) $[\omega']=[n_1'\ n_2'+1\ n_3'+1]$			
$[n_1'\ n_2'+2\ n_3'+1]$	$[n_1'\ n_2'+2\ n_3']$	$[0\,0-1]$	$-\sqrt{\dfrac{(n_2'+3)(n_2'-n_3'+2)}{2(\frac{2}{3}\Omega+5)(n_2'-n_3'+1)}}$
$[n_1'\ n_2'+1\ n_3'+2]$	$[n_1'\ n_2'\ n_3'+2]$	$[0\,0-1]$	$\sqrt{\dfrac{(n_3'+2)(n_2'-n_3')}{2(\frac{2}{3}\Omega+5)(n_2'-n_3'+1)}}$
$[n_1'\ n_2'+1\ n_3']$	$[n_1'\ n_2'\ n_3']$	$[1\,0\,0]$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_3'+2)(n_2'-n_3'+2)}{2(\frac{2}{3}\Omega+5)(n_2'-n_3'+1)}}$
$[n_1'\ n_2'\ n_3'+1]$	$[n_1'\ n_2'\ n_3']$	$[1\,0\,0]$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_2'+1)(n_2'-n_3')}{2(\frac{2}{3}\Omega+5)(n_2'-n_3'+1)}}$

$u \leqslant 2$, one-body operators $[b^\dagger \times b]$ coupled to the 8-dimensional SU(3) representation $(\omega_0)=(11)$ will lead to multiplicity in their coupling to arbitrary (ω). The reduced matrix elements of such operators will therefore require the SU(3) multiplicity label ρ. All formulae will therefore be given for the most general intrinsic state $[\sigma]$ for which the coupling $[\sigma]\times[n]\to[\omega]$ will in general also require the

TABLE 8

$$\left\langle \left(\frac{\Omega}{3}\,\frac{\Omega}{3}-1\,\frac{\Omega}{3}-1\right)[11][\omega]\nu \equiv [n]; (1\,0\,0)[\sigma''] \middle\| \left(\frac{\Omega}{3}\,\frac{\Omega}{3}\,\frac{\Omega}{3}-1\right)[1][\omega']\nu' \equiv [n']\right\rangle$$

$[\omega]$	$[\sigma'']$	$[n]$	$\langle\ \|\ \rangle$
(1) $[\omega']=[n_1'+1\ n_2'\ n_3']$			
$[n_1'+1\ n_2'-1\ n_3']$	$[1\,0\,0]$	$[n_1'\ n_2'-2\ n_3']$	$\sqrt{\dfrac{(n_2'+1)(n_2'-n_3')}{2(\frac{2}{3}\Omega+1)(n_2'-n_3'+1)}}$
$[n_1'+1\ n_2'\ n_3'-1]$	$[1\,0\,0]$	$[n_1'\ n_2'\ n_3'-2]$	$-\sqrt{\dfrac{n_3'(n_2'-n_3'+2)}{2(\frac{2}{3}\Omega+1)(n_2'-n_3'+1)}}$
$[n_1'+1\ n_2'+1\ n_3']$	$[0\,0\,-1]$	$[n_1'\ n_2'\ n_3']$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_2'+1)(n_2'-n_3'+2)}{2(\frac{2}{3}\Omega+1)(n_2'-n_3'+1)}}$
$[n_1'+1\ n_2'\ n_3'+1]$	$[0\,0\,-1]$	$[n_1'\ n_2'\ n_3']$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_3'+2)(n_2'-n_3')}{2(\frac{2}{3}\Omega+1)(n_2'-n_3'+1)}}$
(2) $[\omega']=[n_1'\ n_2'+1\ n_3']$			
$[n_1'-1\ n_2'+1\ n_3']$	$[1\,0\,0]$	$[n_1'-2\ n_2'\ n_3']$	$\sqrt{\dfrac{(n_1'+2)(n_1'-n_3'+1)}{2(\frac{2}{3}\Omega+1)(n_1'-n_3'+2)}}$
$[n_1'\ n_2'+1\ n_3'-1]$	$[1\,0\,0]$	$[n_1'\ n_2'\ n_3'-2]$	$-\sqrt{\dfrac{n_3'(n_1'-n_3'+3)}{2(\frac{2}{3}\Omega+1)(n_1'-n_3'+2)}}$
$[n_1'+1\ n_2'+1\ n_3']$	$[0\,0\,-1]$	$[n_1'\ n_2'\ n_3']$	$-\sqrt{\dfrac{(\frac{2}{3}\Omega-n_1')(n_1'-n_3'+3)}{2(\frac{2}{3}\Omega+1)(n_1'-n_3'+2)}}$
$[n_1'\ n_2'+1\ n_3'+1]$	$[0\,0\,-1]$	$[n_1'\ n_2'\ n_3']$	$\sqrt{\dfrac{(\frac{2}{3}\Omega-n_3'+2)(n_1'-n_3'+1)}{2(\frac{2}{3}\Omega+1)(n_1'-n_3'+2)}}$
(3) $[\omega']=[n_1'\ n_2'\ n_3'+1]$			
$[n_1'-1\ n_2'\ n_3'+1]$	$[1\,0\,0]$	$[n_1'-2\ n_2'\ n_3']$	$\sqrt{\dfrac{(n_1'+2)(n_1'-n_2')}{2(\frac{2}{3}\Omega+1)(n_1'-n_2'+1)}}$
$[n_1'\ n_2'-1\ n_3'+1]$	$[1\,0\,0]$	$[n_1'\ n_2'-2\ n_3']$	$-\sqrt{\dfrac{(n_2'+1)(n_1'-n_2'+2)}{2(\frac{2}{3}\Omega+1)(n_1'-n_2'+1)}}$
$[n_1'+1\ n_2'\ n_3'+1]$	$[0\,0\,-1]$	$[n_1'\ n_2'\ n_3']$	$-\sqrt{\dfrac{(\frac{2}{3}\Omega-n_1')(n_1'-n_2'+2)}{2(\frac{2}{3}\Omega+1)(n_1'-n_2'+1)}}$
$[n_1'\ n_2'+1\ n_3'+1]$	$[0\,0\,-1]$	$[n_1'\ n_2'\ n_3']$	$-\sqrt{\dfrac{(\frac{2}{3}\Omega-n_2'+1)(n_1'-n_2')}{2(\frac{2}{3}\Omega+1)(n_1'-n_2'+1)}}$

SU(3) multiplicity label ρ. Wherever multiplicity labels are not needed in a recoupling coefficient they are to be replaced by a dash. Note also that row and column indices of the $(KK^\dagger)$ matrix for a specific $[\sigma], [\omega]$ are now labeled by both $[n]$ and ρ, although ρ will come into play only for certain $[\sigma], [n]$ combinations.

The matrix elements of operators $A(ii')$, $A^\dagger(ii')$, and $P(ii')$ for states with $u'=u-2$ all three lead to intrinsic reduced matrix elements of the intrinsic operator of type

TABLE 9a

$$\left\langle \left(\frac{\Omega}{3}\frac{\Omega}{3}\frac{\Omega}{3}-1\right)[1][\omega]\nu \equiv [n]; (1\,0\,0)[\sigma''] \middle\| \left(\frac{\Omega}{3}\frac{\Omega}{3}\frac{\Omega}{3}-2\right)[2][\omega']\nu' \right\rangle$$

$[\sigma'']$	$[\omega]$	$[n]$	$\langle\ \|\ \rangle$
(1) $[\omega'] = [n_1'+1\ n_2'+1\ n_3']$ $(\nu' \equiv [n_1'\ n_2'\ n_3'])$			
$[1\,0\,0]$	$[n_1'+1\ n_2'\ n_3']$	$[n_1'\ n_2'\ n_3']$	$-\sqrt{\dfrac{(\frac{2}{3}\Omega - n_2'+1)(n_1'-n_2')}{2(\frac{2}{3}\Omega+3)(n_1'-n_2'+1)}}$
$[1\,0\,0]$	$[n_1'\ n_2'+1\ n_3']$	$[n_1'\ n_2'\ n_3']$	$\sqrt{\dfrac{(\frac{2}{3}\Omega - n_1')(n_1'-n_2'+2)}{2(\frac{2}{3}\Omega+3)(n_1'-n_2'+1)}}$
$[0\,0-1]$	$[n_1'+2\ n_2'+1\ n_3']$	$[n_1'+2\ n_2'\ n_3']$	$\sqrt{\dfrac{(n_1'+4)(n_1'-n_2')}{2(\frac{2}{3}\Omega+3)(n_1'-n_2'+1)}}$
$[0\,0-1]$	$[n_1'+1\ n_2'+2\ n_3']$	$[n_1'\ n_2'+2\ n_3']$	$\sqrt{\dfrac{(n_2'+3)(n_1'-n_2'+2)}{2(\frac{2}{3}\Omega+3)(n_1'-n_2'+1)}}$
(2) $[\omega'] = [n_1'+1\ n_2'\ n_3'+1]$ $(\nu' \equiv [n_1'\ n_2'\ n_3'])$			
$[1\,0\,0]$	$[n_1'+1\ n_2'\ n_3']$	$[n_1'\ n_2'\ n_3']$	$\sqrt{\dfrac{(\frac{2}{3}\Omega - n_3'+2)(n_1'-n_3'+1)}{2(\frac{2}{3}\Omega+3)(n_1'-n_3'+2)}}$
$[1\,0\,0]$	$[n_1'\ n_2'\ n_3'+1]$	$[n_1'\ n_2'\ n_3']$	$\sqrt{\dfrac{(\frac{2}{3}\Omega - n_1')(n_1'-n_3'+3)}{2(\frac{2}{3}\Omega+3)(n_1'-n_3'+2)}}$
$[0\,0-1]$	$[n_1'+2\ n_2'\ n_3'+1]$	$[n_1'+2\ n_2'\ n_3']$	$\sqrt{\dfrac{(n_1'+4)(n_1'-n_3'+1)}{2(\frac{2}{3}\Omega+3)(n_1'-n_3'+2)}}$
$[0\,0-1]$	$[n_1'+1\ n_2'\ n_3'+2]$	$[n_1'\ n_2'\ n_3'+2]$	$\sqrt{\dfrac{(n_3'+2)(n_1'-n_3'+3)}{2(\frac{2}{3}\Omega+3)(n_1'-n_3'+2)}}$
(3) $[\omega'] = [n_1'\ n_2'+1\ n_3'+1]$ $(\nu' \equiv [n_1'\ n_2'\ n_3'])$			
$[1\,0\,0]$	$[n_1'\ n_2'+1\ n_3']$	$[n_1'\ n_2'\ n_3']$	$\sqrt{\dfrac{(\frac{2}{3}\Omega - n_3'+2)(n_2'-n_3')}{2(\frac{2}{3}\Omega+3)(n_2'-n_3'+1)}}$
$[1\,0\,0]$	$[n_1'\ n_2'\ n_3'+1]$	$[n_1'\ n_2'\ n_3']$	$-\sqrt{\dfrac{(\frac{2}{3}\Omega - n_2'+1)(n_2'-n_3'+2)}{2(\frac{2}{3}\Omega+3)(n_2'-n_3'+1)}}$
$[0\,0-1]$	$[n_1'\ n_2'+2\ n_3'+1]$	$[n_1'\ n_2'+2\ n_3']$	$\sqrt{\dfrac{(n_2'+3)(n_2'-n_3')}{2(\frac{2}{3}\Omega+3)(n_2'-n_3'+1)}}$
$[0\,0-1]$	$[n_1'\ n_2'+1\ n_3'+2]$	$[n_1'\ n_2'\ n_3'+2]$	$\sqrt{\dfrac{(n_3'+2)(n_2'-n_3'+2)}{2(\frac{2}{3}\Omega+3)(n_2'-n_3'+1)}}$

$\mathbb{A}(ii')$. Since the left action of $\mathbb{A}(ii')$ on an intrinsic state $[\sigma']$ with $u' = u - 2$ connects this $[\sigma']$ to a purely intrinsic state $[\sigma]$, the full reduced matrix element can be reduced to matrix elements of purely intrinsic and pure z-space type via straightforward recoupling techniques, as in eq. (42). The resulting expressions are given as entries 3–5 in table 4.

TABLE 9b

$$\left\langle \left(\frac{\Omega}{3}\,\frac{\Omega}{3}\,\frac{\Omega}{3}-1 \right)[1][\omega]\nu \equiv [n]; (100)[\sigma''] \middle\| \left(\frac{\Omega}{3}\,\frac{\Omega}{3}\,\frac{\Omega}{3}-2 \right)[2][\omega']\nu' \right\rangle [\omega_1'\,\omega_2'\,\omega_3'] = [\text{even even even}]$$

$[\sigma'']$	$[\omega]$	$[n]$	$\langle \ \| \ \rangle$
[1 0 0]	$[\omega_1'-1\;\omega_2'\;\omega_3']$	$[\omega_1'-2\;\omega_2'\;\omega_3']$	$\dfrac{(\kappa([2][\omega_1'\;\omega_2'\;\omega_3']))_{[\omega_1'-2\;\omega_2'\;\omega_3']\nu'}}{\sqrt{2}\,\Omega(\tfrac{2}{3}\Omega+3)}$
[1 0 0]	$[\omega_1'\;\omega_2'-1\;\omega_3']$	$[\omega_1'\;\omega_2'-2\;\omega_3']$	$-\dfrac{(\kappa([2][\omega_1'\;\omega_2'\;\omega_3']))_{[\omega_1'\;\omega_2'-2\;\omega_3']\nu'}}{\sqrt{\tfrac{2}{3}}\,\Omega(\tfrac{2}{3}\Omega+3)}$
[1 0 0]	$[\omega_1'\;\omega_2'\;\omega_3'-2]$	$[\omega_1'\;\omega_2'\;\omega_3'-1]$	$\dfrac{(\kappa([2][\omega_1'\;\omega_2'\;\omega_3']))_{[\omega_1'\;\omega_2'\;\omega_3'-2]\nu'}}{\sqrt{\tfrac{2}{3}}\,\Omega(\tfrac{2}{3}\Omega+3)}$
[0 0 -1]	$[\omega_1'+1\;\omega_2'\;\omega_3']$	$[\omega_1'\;\omega_2'\;\omega_3']$	$\dfrac{1}{\sqrt{\tfrac{2}{3}}\,\Omega(\tfrac{2}{3}\Omega+3)(\tfrac{2}{3}\Omega-\omega_1')}$

$$\times \left\{ (\kappa([2][\omega_1'\;\omega_2'\;\omega_3']))_{[\omega_1'-2\;\omega_2'\;\omega_3']\nu'}\, \frac{\sqrt{(\omega_1'+2)(\omega_1'-\omega_2')(\omega_1'-\omega_2'+2)(\omega_1'-\omega_3'+1)(\omega_1'-\omega_3'+3)}}{(\omega_1'-\omega_2'+1)(\omega_1'-\omega_3'+2)} \right.$$

$$+ (\kappa([2][\omega_1'\;\omega_2'\;\omega_3']))_{[\omega_1'\;\omega_2'-2\;\omega_3']\nu'}\, \frac{\sqrt{(\omega_2'+1)(\omega_2'-\omega_3')(\omega_2'-\omega_3'+3)}}{(\omega_1'-\omega_2'+1)\sqrt{(\omega_2'-\omega_3'+1)}\,(\omega_1'-\omega_3'+2)}$$

$$\left. + (\kappa([2][\omega_1'\;\omega_2'\;\omega_3']))_{[\omega_1'\;\omega_2'\;\omega_3'-2]\nu'}\, \frac{\sqrt{\omega_3'(\omega_2'-\omega_3'+2)}\,\sqrt{(\omega_1'-\omega_2'+2)}}{(\omega_1'-\omega_3'+2)\sqrt{(\omega_1'-\omega_2'+1)(\omega_2'-\omega_3'+1)}} \right\}$$

$[0\,0\,-1]$ $[\omega'_1\,\omega'_2+1\,\omega'_3]$ $[\omega'_1\,\omega'_2\,\omega'_3]$

$$\frac{-1}{\sqrt{\tfrac{2}{3}\Omega(\tfrac{1}{3}\Omega+3)(\tfrac{2}{3}\Omega-\omega'_2+1)}}$$

$$\times\left\{(\kappa([2][\omega'_1\,\omega'_2\,\omega'_3]))_{[\omega'_1-2\,\omega'_2\,\omega'_3]\nu'}\,\frac{\sqrt{(\omega'_1+2)(\omega'_1-\omega'_3+1)(\omega'_2-\omega'_3+2)}}{(\omega'_1-\omega'_2+1)\sqrt{(\omega'_1-\omega'_3+2)(\omega'_2-\omega'_3+1)}}\right.$$

$$-(\kappa([2][\omega'_1\,\omega'_2\,\omega'_3]))_{[\omega'_1\,\omega'_2-2\,\omega'_3]\nu'}\,\frac{\sqrt{(\omega'_2+1)(\omega'_1-\omega'_2)(\omega'_1-\omega'_2+2)(\omega'_2-\omega'_3)(\omega'_2-\omega'_3+2)}}{(\omega'_1-\omega'_2+1)(\omega'_2-\omega'_3+1)}$$

$$\left.-(\kappa([2][\omega'_1\,\omega'_2\,\omega'_3]))_{[\omega'_1\,\omega'_2\,\omega'_3-2]\nu'}\,\frac{\sqrt{\omega'_3(\omega'_1-\omega'_2)(\omega'_1-\omega'_3+3)}}{(\omega'_2-\omega'_3+1)\sqrt{(\omega'_1-\omega'_2+1)(\omega'_1-\omega'_3+2)}}\right\}$$

$[0\,0\,-1]$ $[\omega'_1\,\omega'_2\,\omega'_3+1]$ $[\omega'_1\,\omega'_2\,\omega'_3]$

$$\frac{-1}{\sqrt{\tfrac{2}{3}\Omega(\tfrac{1}{3}\Omega+3)(\tfrac{2}{3}\Omega-\omega'_3+2)}}$$

$$\times\left\{(\kappa([2][\omega'_1\,\omega'_2\,\omega'_3]))_{[\omega'_1-2\,\omega'_2\,\omega'_3]\nu'}\,\frac{\sqrt{(\omega'_1+2)(\omega'_1-\omega'_2)(\omega'_2-\omega'_3)}}{(\omega'_1-\omega'_3+2)\sqrt{(\omega'_1-\omega'_2+1)(\omega'_2-\omega'_3+1)}}\right.$$

$$+(\kappa([2][\omega'_1\,\omega'_2\,\omega'_3]))_{[\omega'_1\,\omega'_2-2\,\omega'_3]\nu'}\,\frac{\sqrt{(\omega'_2+1)(\omega'_1-\omega'_2+2)(\omega'_1-\omega'_3+1)}}{(\omega'_2-\omega'_3+1)\sqrt{(\omega'_1-\omega'_2+1)(\omega'_1-\omega'_3+2)}}$$

$$\left.-(\kappa([2][\omega'_1\,\omega'_2\,\omega'_3]))_{[\omega'_1\,\omega'_2\,\omega'_3-2]\nu'}\,\frac{\sqrt{\omega'_3(\omega'_1-\omega'_3+1)(\omega'_1-\omega'_3+3)(\omega'_2-\omega'_3)(\omega'_2-\omega'_3+2)}}{(\omega'_1-\omega'_3+2)(\omega'_2-\omega'_3+1)}\right\}$$

$$(\kappa([2][\omega']))_{[n']\nu'}=U^{\dagger}_{[n']\nu'}\sqrt{\frac{\lambda_{\nu'}}{\text{C.F.}}},\ \text{see ref.}\,[14]$$

TABLE 10

$$\left\langle\left(\frac{\Omega\;\Omega\;\Omega}{3\;\;3\;\;3}-2\right)[2][\omega]\nu;(100)[\sigma'']\left\|\left(\frac{\Omega\;\Omega\;\Omega}{3\;\;3\;\;3}-1\right)[1][\omega']\nu'\equiv[n']\right.\right\rangle$$

$[\sigma'']=[n_1'+1\;n_2'\;n_3']$			
$[\sigma'']$	$[\omega]$	ν (or $\nu\equiv[n]$)	$\langle\;\|\;\rangle$
(1) $[\omega']=[n_1'+1\;n_2'\;n_3']$			
$[00-1]$	$[n_1'+1\;n_2'+1\;n_3']$		$\dfrac{1}{2}\sqrt{\dfrac{(\frac{2}{3}\Omega)(\frac{2}{3}\Omega-n_2'+1)(n_1'-n_2')(n_2'-n_3'+2)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_2'+2)(n_2'-n_3'+1)}}$
$[00-1]$	$[n_1'+1\;n_2'\;n_3'+1]$		$\dfrac{1}{2}\sqrt{\dfrac{(\frac{2}{3}\Omega)(\frac{2}{3}\Omega-n_3'+2)(n_1'-n_3'+1)(n_2'-n_3')}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_3'+3)(n_2'-n_3'+1)}}$
$[00-1]$	$[n_1'+2\;n_2'\;n_3']$	ν	$(\kappa([2][n_1'+2\;n_2'\;n_3']))_{[n_1'\,n_2'\,n_3']\nu}\sqrt{\dfrac{(n_1'-n_2'+3)(n_1'-n_3'+4)}{2(n_1'-n_2'+2)(n_1'-n_3'+3)(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)}}$
$[100]$	$[n_1'+1\;n_2'-1\;n_3']$		$\dfrac{1}{2}\sqrt{\dfrac{(\frac{2}{3}\Omega)(n_1'-n_2'+4)(n_2'+1)(n_2'-n_3')}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_2'+2)(n_2'-n_3'+1)}}$
$[100]$	$[n_1'+1\;n_2'\;n_3'-1]$		$-\dfrac{1}{2}\sqrt{\dfrac{(\frac{2}{3}\Omega)(n_1'-n_3'+5)n_3'(n_2'-n_3'+2)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_3'+3)(n_2'-n_3'+1)}}$
$[100]$	$[n_1'\;n_2'\;n_3']$	ν	$\dfrac{-1}{\sqrt{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(\frac{2}{3}\Omega-n_1')}}$ $\times\left\{(\kappa([2][n_1'\,n_2'\,n_3']))_{[n_1'-2\,n_2'\,n_3']\nu}\sqrt{\dfrac{(n_1'+2)(n_1'-n_2')(n_1'-n_3'+1)}{2(n_1'-n_2'+1)(n_1'-n_3'+2)}}\right.$ $+(\kappa([2][n_1'\,n_2'\,n_3']))_{[n_1'\,n_2'-2\,n_3']\nu}\sqrt{\dfrac{(n_2'+1)(n_2'-n_3')}{2(n_1'-n_2'+1)(n_1'-n_2'-n_3'+2)}}$ $\left.+(\kappa([2][n_1'\,n_2'\,n_3']))_{[n_1'\,n_2'\,n_3'-2]\nu}\sqrt{\dfrac{n_3'(n_2'-n_3'+2)}{2(n_1'-n_3'+2)(n_1'-n_3'+3)(n_2'-n_3'+1)}}\right\}$

$$(\kappa([2][\omega]))_{[n]\nu}=U^\dagger_{[n]\nu}\sqrt{\frac{\lambda\nu}{[\text{C.F.}]}},\text{ see ref. }[14]$$

(2) $[\omega] = [n_1'\, n_2'+1\, n_3']$

$[0\,0\,-1]$	$[n_1'+1\,n_2'+1\,n_3']$	$[n_1'\,n_2'\,n_3']$	$\dfrac{1}{2}\sqrt{\dfrac{\frac{2}{3}\Omega(\frac{2}{3}\Omega-n_1')(n_1'-n_2'+2)(n_1'-n_3'+3)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_2')(n_1'-n_3'+2)}}$
$[0\,0\,-1]$	$[n_1'\,n_2'+1\,n_3'+1]$	$[n_1'\,n_2'\,n_3']$	$\dfrac{1}{2}\sqrt{\dfrac{\frac{2}{3}\Omega(\frac{2}{3}\Omega-n_3'+2)(n_2'-n_3')(n_1'-n_3'+1)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_2'-n_3'+2)(n_1'-n_3'+2)}}$
$[0\,0\,-1]$	$[n_1'\,n_2'+2\,n_3']$	ν	$(\kappa([2][n_1'\,n_2'+2\,n_3']))_{[n_1'\,n_2'\,n_3']\nu}\sqrt{\dfrac{(n_1'-n_2'-1)(n_2'-n_3'+3)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)2(n_1'-n_2')(n_2'-n_3'+2)}}$
$[1\,0\,0]$	$[n_1'-1\,n_2'+1\,n_3']$	$[n_1'\,n_2'\,n_3']$	$-\dfrac{1}{2}\sqrt{\dfrac{\frac{2}{3}\Omega(n_1'+2)(n_1'-n_2'-2)(n_1'-n_3'+1)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_3'+2)(n_1'-n_2')}}$
$[1\,0\,0]$	$[n_1'\,n_2'+1\,n_3'-1]$	$[n_1'\,n_2'\,n_3']$	$-\dfrac{1}{2}\sqrt{\dfrac{\frac{2}{3}\Omega(n_3')(n_1'-n_3'+3)(n_2'-n_3'+4)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_3'+2)(n_2'-n_3'+2)}}$
$[1\,0\,0]$	$[n_1'\,n_2'\,n_3']$	ν	$\dfrac{1}{\sqrt{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(\frac{2}{3}\Omega-n_2'+1)}}$

$$\times\left\{-(\kappa([2][n_1'\,n_2'\,n_3']))_{[n_1'-2\,n_2'\,n_3']\nu}\sqrt{\frac{(n_1'+2)(n_1'-n_3'+1)}{2(n_1'-n_2')(n_1'-n_2'+1)(n_1'-n_3'+2)}}\right.$$

$$+(\kappa([2][n_1'\,n_2'\,n_3']))_{[n_1'\,n_2'-2\,n_3']\nu}\sqrt{\frac{(n_2'+1)(n_1'-n_2'+2)(n_2'-n_3')}{2(n_1'-n_2'+1)(n_2'-n_3'+1)}}$$

$$\left.+(\kappa([2][n_1'\,n_2'\,n_3']))_{[n_1'\,n_2'\,n_3'-2]\nu}\sqrt{\frac{n_3'(n_1'-n_3'+3)}{2(n_1'-n_3'+2)(n_2'-n_3'+1)(n_2'-n_3'+2)}}\right\}$$

$$(\kappa([2][\omega]))_{[n]\nu} = U^\dagger_{[n]\nu}\sqrt{\frac{\lambda\nu}{[C.F.]}},\ \text{see ref.}^{[14]}$$

(3) $[\omega'] = [n_1'\,n_2'\,n_3'+1]$

$[0\,0\,-1]$	$[n_1'+1\,n_2'\,n_3'+1]$	$[n_1'\,n_2'\,n_3']$	$\dfrac{1}{2}\sqrt{\dfrac{(\frac{2}{3}\Omega)(\frac{2}{3}\Omega-n_1')(n_1'-n_2'+2)(n_1'-n_3'+3)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_2'+1)(n_1'-n_3'+1)}}$
$[0\,0\,-1]$	$[n_1'\,n_2'+1\,n_3'+1]$	$[n_1'\,n_2'\,n_3']$	$\dfrac{1}{2}\sqrt{\dfrac{\frac{2}{3}\Omega(\frac{2}{3}\Omega-n_2'+1)(n_1'-n_2')(n_2'-n_3'+2)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n_1'-n_2'+1)(n_2'-n_3')}}$

TABLE 10—continued

$[\sigma'']$	$[\omega]$	ν (or $\nu \equiv [n]$)	$\langle \ \| \ \rangle$
$[0\,0-1]$	$[n'_1\,n'_2\,n'_3+2]$	ν	$(\kappa([2][n'_1\,n'_2\,n'_3+2])_{[n'_1\,n'_2\,n'_3]\nu}\sqrt{\dfrac{(n'_2-n'_3-1)(n'_1-n'_3)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)2(n'_1-n'_3+1)(n'_2-n'_3)}}$
$[100]$	$[n'_1-1\,n'_2\,n'_3+1]$	$[n'_1-2\,n'_2\,n'_3]$	$-\dfrac{1}{2}\sqrt{\dfrac{\frac{2}{3}\Omega(n'_1+2)(n'_1-n'_2)(n'_1-n'_3-1)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n'_1-n'_2+1)(n'_1-n'_3+1)}}$
$[100]$	$[n'_1\,n'_2-1\,n'_3+1]$	$[n'_1\,n'_2-2\,n'_3]$	$\dfrac{1}{2}\sqrt{\dfrac{\frac{2}{3}\Omega(n'_2+1)(n'_1-n'_2+2)(n'_2-n'_3-2)}{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(n'_1-n'_2+1)(n'_2-n'_3)}}$
$[100]$	$[n'_1\,n'_2\,n'_3]$	ν	$\dfrac{1}{\sqrt{(\frac{2}{3}\Omega+1)(\frac{2}{3}\Omega-2)(\frac{2}{3}\Omega-n'_3+2)}}$

$$\times\Big\{\kappa([2][n'_1\,n'_2\,n'_3])_{[n'_1-2\,n'_2\,n'_3]\nu}\sqrt{\frac{(n'_1+2)(n'_1-n'_2)}{2(n'_1-n'_2+1)(n'_1-n'_3+1)(n'_1-n'_3+2)}}$$

$$+(\kappa([2][n'_1\,n'_2\,n'_3]))_{[n'_1\,n'_2-2\,n'_3]\nu}\sqrt{\frac{(n'_2+1)(n'_1-n'_2+2)}{2(n'_1-n'_2+1)(n'_2-n'_3)(n'_2-n'_3+1)}}$$

$$-(\kappa([2][n'_1\,n'_2\,n'_3]))_{[n'_1\,n'_2\,n'_3-2]\nu}\sqrt{\frac{n'_3(n'_1-n'_3+3)(n'_2-n'_3+2)}{2(n'_1-n'_3+2)(n'_2-n'_3+1)}}\Big\}$$

$$(\kappa([2][\omega]))_{[n]\nu}=U^\dagger_{[n]\nu}\sqrt{\frac{\lambda\nu}{[\mathrm{C.F.}]}},\ \text{see ref.}^{[14]}$$

TABLE 11

$$\left\langle \left(\frac{\Omega}{3}-1\,\frac{\Omega}{3}-1\,\frac{\Omega}{3}-1\right)[111][\omega];\,(1\,0\,0)[\sigma'']\right\|\left(\frac{\Omega}{3}\,\frac{\Omega}{3}-1\,\frac{\Omega}{3}-1\right)[11][\omega']\nu'\equiv[n']\right\rangle$$

(1) $[\omega']=[n'_1+1\ n'_2+1\ n'_3]$

$[\omega]=[n'_1+1\ n'_2+1\ n'_3+1];\ [n]=[n'_1\ n'_2\ n'_3]$ $[\sigma'']=[0\ 0\ -1]$ $\sqrt{\dfrac{(\frac{2}{3}\Omega-n'_3+2)}{(\frac{2}{3}\Omega+2)}}$

$[\omega]=[n'_1+1\ n'_2+1\ n'_3-1];\ [n]=[n'_1\ n'_2\ n'_3-2]$ $[\sigma'']=[1\ 0\ 0]$ $-\sqrt{\dfrac{n'_3}{(\frac{2}{3}\Omega+2)}}$

(2) $[\omega']=[n'_1+1\ n'_2\ n'_3+1]$

$[\omega]=[n'_1+1\ n'_2+1\ n'_3+1];\ [n]=[n'_1\ n'_2\ n'_3]$ $[\sigma'']=[0\ 0\ -1]$ $-\sqrt{\dfrac{(\frac{2}{3}\Omega-n'_2+1)}{(\frac{2}{3}\Omega+2)}}$

$[\omega]=[n'_1+1\ n'_2-1\ n'_3+1];\ [n]=[n'_1\ n'_2-2\ n'_3]$ $[\sigma'']=[1\ 0\ 0]$ $-\sqrt{\dfrac{(n'_2+1)}{(\frac{2}{3}\Omega+2)}}$

(3) $[\omega']=[n'_1\ n'_2+1\ n'_3+1]$

$[\omega]=[n'_1+1\ n'_2+1\ n'_3+1];\ [n]=[n'_1\ n'_2\ n'_3]$ $[\sigma'']=[0\ 0\ -1]$ $\sqrt{\dfrac{(\frac{2}{3}\Omega-n'_1)}{(\frac{2}{3}\Omega+2)}}$

$[\omega]=[n'_1-1\ n'_2+1\ n'_3+1];\ [n]=[n'_1-2\ n'_2\ n'_3]$ $[\sigma'']=[1\ 0\ 0]$ $-\sqrt{\dfrac{(n'_1+2)}{(\frac{2}{3}\Omega+2)}}$

Matrix elements for the case $u'=u$ are somewhat more complicated. Their derivation will be illustrated in detail by the matrix element of the operator $A(ii')$. A new feature arises, since the left action of the intrinsic operator $\mathbb{A}(ii')$ on the intrinsic state $[\sigma']$ will now connect this to a right state which includes a z-space excitation (see entry 4 of table 3). Additional recoupling is therefore required to separate the full matrix element into intrinsic and z-space parts.

The master equation (24) converts the matrix element of $A(ii')$ into

$$([[n']\times[\sigma']][\omega']\rho';\ I'\|\mathbb{A}(ii')^{(\tilde{\omega}_p)}_{I_p}\|[[n]\times[\sigma]][\omega]\rho;\ I).\tag{46}$$

For $u'=u$ the intrinsic operator $\mathbb{A}(ii')$ must now be worked through to the left to act on the state $[\sigma']$. For this purpose it will be useful to express the reduced matrix element of a tensor operator $T^{[\omega_0]I_0}$ not through its right-coupled form as in eq. (25) but through a left-coupled form. For multiplicity-free cases $[\omega]\times[\omega_0]\to[\omega']$

$$\langle[\omega'];\ I'\|T^{[\omega_0]I_0}\|[\omega];\ I\rangle$$

$$=\sqrt{\frac{\dim[\omega]}{\dim[\omega']}\frac{(2I+1)}{(2I'+1)}}\langle[\overbrace{T^{[\omega_0]I_0}\times[\omega']}^{};\ I']^{[\omega]I}_{\alpha_\omega M_I}|[\omega]\alpha_\omega;\ IM_I\rangle,\tag{47}$$

where we have used the symmetry property

$$\langle [\omega]\alpha_\omega ; [\omega_0]\alpha_0 | [\omega']\alpha'_\omega \rangle$$

$$= \sqrt{\frac{\dim[\omega']}{\dim[\omega]}} \langle [\omega']\alpha_{\omega'} ; [\tilde{\omega}_0]\tilde{\alpha}_0 | [\omega]\alpha_\omega \rangle (-1)^{\hat{\omega}+\hat{\omega}_0-\hat{\omega}'-\chi(\omega_0,\alpha_0)}, \quad (48)$$

together with the analogue for the *i*-space angular momentum Wigner coefficient, the hermitian conjugation of the operator $T^{[\omega_0]I_0}_{\alpha_0 M I_0}$ via eq. (8), and finally a change in the coupling order of the multiplicity-free product $[\omega'] \times [\tilde{\omega}_0] \to [\omega]$ which eliminates the phase factor $(-1)^{\hat{\omega}+\hat{\omega}_0-\hat{\omega}'}$. Note that the coupling order in eq. (47) is a left to right one which is indicated specifically by the arrow, since it is contrary to the standard right to left order which is to apply when not otherwise indicated by an arrow. Since $(\tilde{\omega}_p) = (02)$ or (10) leads to multiplicity-free couplings, eq. (47) can be applied to convert (46) to

$$([[n'] \times [\sigma']][\omega']\rho'; I' \| \mathbb{A}(ii')^{(\tilde{\omega}_p)}_{I_p} \| [[n] \times [\sigma]][\omega]\rho; I)$$

$$= \sqrt{\frac{\dim[\omega]}{\dim[\omega']} \frac{2I+1)}{(2I'+1)}} ([\overset{\frown}{\mathbb{A}^\dagger(ii')^{(\omega_p)}_{I_p} \times [[\sigma'] \times [n']]}[\omega']\rho'; I']^{[\omega]I}_{\alpha_\omega M_I} | [[n]$$

$$\times [\sigma]][\omega]\rho\alpha_\omega ; IM_I)$$

$$= \sqrt{\frac{\dim[\omega]}{\dim[\omega']} \frac{(2I+1)}{(2I'+1)}} \sum_{[\bar{\sigma}]\bar{\rho}} U([\omega_p][\sigma'][\omega][n']; [\bar{\sigma}]_-\bar{\rho}; [\omega']\rho'_-)$$

$$\times ([[\overset{\frown}{\mathbb{A}^\dagger(ii')^{(\omega_p)}_{I_p} \times [\bar{\sigma}']}; I'][\bar{\sigma}] \times [n']]^{[\omega]\bar{\rho};I}_{\alpha_\omega ; M_I} | [[n] \times [\sigma]][\omega]\rho\alpha_\omega ; IM_I). \quad (49)$$

Now we can use the result of entry 4 of table 3 to note that

$$([\overset{\frown}{\mathbb{A}^\dagger(ii')^{(\omega_p)}_{I_p} \times [\sigma']}; I']^{[\bar{\sigma}];I}_{\bar{\alpha};M_I} |$$

$$= ([\overset{\frown}{\mathbb{A}^\dagger(ii')^{(\omega_p)}_{I_p} \times [\sigma']}; I']^{[\bar{\sigma}];I}_{\bar{\alpha};M_I} | [[2] \times [\sigma]][\bar{\sigma}]\bar{\alpha}; IM_I) \times ([[2] \times [\sigma]][\bar{\sigma}]\bar{\alpha}; IM_I |$$

$$= \sqrt{\frac{\dim[\sigma']}{\dim[\bar{\sigma}]} \frac{(2I'+1)}{(2I+1)}} ([\sigma']; I' \| \mathbb{A}(ii')^{(\tilde{\omega}_p)}_{I_p} \| [[2] \times [\sigma]][\bar{\sigma}]; I)$$

$$\times ([[2] \times [\sigma]][\bar{\sigma}]\bar{\alpha}; IM_I |. \quad (50)$$

Eqs. (49) and (50) lead to

$$([[n'] \times [\sigma']][\omega']\rho'; I' \| \mathbb{A}(ii')^{(\tilde{\omega}_p)}_{I_p} \| [[n] \times [\sigma]][\omega]\rho; I)$$

$$= \sum_{[\bar{\sigma}]\bar{\rho}} \sqrt{\frac{\dim[\omega]}{\dim[\omega']} \frac{\dim[\sigma']}{\sin[\tilde{\sigma}]}} U([\omega_p][\sigma'][\omega][n']; [\bar{\sigma}]_-\bar{\rho}; [\omega']\rho'_-)$$

$$\times ([[n'] \times [[2] \times [\sigma]][\bar{\sigma}]][\omega]\bar{\rho}\alpha_\omega ; IM_I | [[n] \times [\sigma]][\omega]\rho\alpha_\omega ; IM_I)$$

$$\times ([\sigma']; I' \| \mathbb{A}(ii')^{(\tilde{\omega}_p)}_{I_p} \| [[2] \times [\sigma]][\bar{\sigma}]; I)$$

$$= \sum_{[\bar{\sigma}]\bar{\rho}} \sqrt{\frac{\dim [\omega]}{\dim [\omega']} \frac{\dim [\sigma']}{\dim [\bar{\sigma}]}} \, U([\omega_p][\sigma'][\omega][n']; [\bar{\sigma}]_{-}\bar{\rho}; [\omega']\rho'_{-})$$

$$\times U([\sigma][2][\omega][n']; [\bar{\sigma}]_{-}\bar{\rho}; [n]_{-}\rho)$$

$$\times ([n]\|z\|[n'])([\sigma']; I'\|\mathbb{A}(ii')_{I_p}^{\langle\tilde{\omega}_p\rangle}\|[[2]\times[\sigma]][\bar{\sigma}]; I) \,. \tag{51}$$

This result is tabulated as entry 6 in table 4.

Finally, the matrix element of $P(ii')_{I_p}^{(p)(\omega_0)}$ follows from

$$([[n']\times[\sigma']][\omega']\rho'; I'\|\Gamma(P(ii')_{I_p}^{(p)(\omega_0)})\|[[n]\times[\sigma]][\omega]\rho; I)_{\rho_0}$$

$$= ([[n']\times[\sigma']][\omega']\rho'; I'\|\mathbb{P}(ii')_{I_p}^{(p)(\omega_0)}\|[[n]\times[\sigma]][\omega]\rho; I)_{\rho_0}$$

$$+ c_{\omega_0}^{(p)}([[n']\times[\sigma']][\omega']\rho'; I'\|[\mathbb{A}(ii')_{I_p}^{[\tilde{\omega}_p]}\times Z^{[2]}(z)]^{[\omega_0]}\|[[n]\times[\sigma]][\omega]\rho; I)_{\rho_0} \,. \tag{52}$$

Using (33b), the second term can be reexpressed as

$$([[n']\times[\sigma']][\omega']\rho'; I'\|([\mathbb{A}(ii')_{I_p}^{[\tilde{\omega}_p]}\times Z^{[2]}]^{[\omega_0]}\|[[n]\times[\sigma]][\omega]\rho; I)_{\rho_0}$$

$$= \sum_{[\omega'']} \sum_{[n'']\rho''} U([\omega][2][\omega'][\tilde{\omega}_p]; [\omega'']_{--}; [\omega_0]_{-}\rho_0)$$

$$\times U([\sigma][n][\omega''][2]; [\omega]\rho_{-}; [n'']_{-}\rho'')([n'']\|z\|[n])$$

$$\times([[n']\times[\sigma']][\omega']\rho'; I'\|\mathbb{A}(ii')_{I_p}^{[\tilde{\omega}_p]}\|[[n'']\times[\sigma]][\omega'']\rho''; I)_{\rho_0} \,. \tag{53}$$

The matrix element of $\mathbb{A}(ii')$ follows from eq. (51). The full result for the matrix element of $P(ii')_{I_p}^{(p)(\omega_0)}$ is given as entry 7 in table 4. Note that the first term of eq. (52) leads to 9-j type SU(3) recoupling coefficient with one [0]. In the cases where multiplicity labels ρ_σ and ρ_0 are not needed this could be converted to a U-coefficient as in entry 3. In the most general case, however, it is best left in the 9-j type form since some needed $1 \leftrightarrow 2$ SU(3) reordering transformations are then no longer simple.

Entry 7 completes the list of needed reduced matrix elements for the one-body operators and pair creation and annihilation operators. Matrix elements for states with $u' = u + 2$ can be obtained from entries 3–5 via hermitian conjugation. Similarly, matrix elements of pair creation operators connecting states with $u' = u$ can be obtained from entry 6 of table 4 via

$$\langle[\sigma'][\omega']\nu'; I'\|A^\dagger(ii')_{I_p}^{(\omega_p)}\|[\sigma][\omega]\nu; I\rangle$$

$$= (-1)^{1+I+I_p-I'+\hat{\omega}+\hat{\omega}_0-\hat{\omega}'} \sqrt{\frac{(2I+1)\dim[\omega]}{(2I'+1)\dim[\omega']}}$$

$$\times\langle[\sigma][\omega]\nu; I\|A(ii')_{I_p}^{(\tilde{\omega}_p)}\|[\sigma'][\omega']\nu'; I'\rangle \,. \tag{54}$$

The final results as catalogued in table 4 give the needed matrix elements in terms of, (1), readily available [18]) SU(3) recoupling coefficients, (2), the K-matrix elements of VCS theory which can be read from ref. [14]) or calculated by the techniques of

ref. [14]), and, finally, (3), a few starting matrix elements of simple shell-model type. Examples of the latter are given for states of low heritage, u, in the appendix. In cases of greater u they can be calculated by standard cfp techniques. Table 4 includes all possible SU(3) multiplicity labels. For states with $u \leq 2$ most of these never come into play. The label, ρ_0, which is part of the k-space SU(3), i-space SU(2) reduced matrix element, is needed only for entries 4 and 7 and the special case $(\omega_0) = (11)$, $(\omega') = (\omega)$. Note that the ρ_0 dependence arises entirely from the SU(3)-recoupling coefficients in the expressions for the reduced matrix elements. In the codes of ref. [18]) these are matched by the ρ_0 dependence in the SU(3) $\supset$ SO(3) reduced Wigner coefficients which are needed to construct the full matrix elements via eq. (26).

Although some of the entries of table 4 involve a number of SU(3) Racah coefficients and summations over U(3) quantum numbers, these coefficients can be given in analytic form in certain very simple cases and the sums can be performed to lead to fairly general analytic formulae. E.g., for $[\omega_0] = [0]$ and $u = u' = 1$ so that $[\sigma] = [1]$, entry 7 of table 4, together with entries 4 and 6 of table 3, lead to the simple results, (valid for $I_p \neq 0$),

$$\langle [[1] \times [n]][\omega]; i \| P(ii')_{I_p}^{(s)[0]} \| [[1] \times [n]][\omega]; i' \rangle$$

(1) for $[\omega] = [n_1 + 1 n_2 n_3]$:

$$= \frac{(1 + \delta_{ii'})}{2} \sqrt{\frac{(2I_p + 1)}{3(2i + 1)}} \left\{ \frac{(\frac{2}{3}\Omega - 2)(\frac{2}{3}\Omega + 1) - 2n_1(\frac{2}{3}\Omega + 1) - 2(n_2 + n_3)}{(\frac{2}{3}\Omega - 2)(\frac{2}{3}\Omega + 1)} \right\}, \qquad (55a)$$

(2) for $[\omega] = [n_1 n_2 + 1 n_3]$:

$$= \frac{(1 + \delta_{ii'})}{2} \sqrt{\frac{(2I_p + 1)}{3(2i + 1)}} \left\{ \frac{(\frac{2}{3}\Omega)^2 + (\frac{2}{3}\Omega) - 2 - 2n_2(\frac{2}{3}\Omega + 1) - 2(n_1 + n_3)}{(\frac{2}{3}\Omega - 2)(\frac{2}{3}\Omega + 1)} \right\}, \qquad (55b)$$

(3) for $[\omega] = [n_1 n_2 n_3 + 1]$:

$$= \frac{(1 + \delta_{ii'})}{2} \sqrt{\frac{(2I_p + 1)}{3(2i + 1)}} \left\{ \frac{(\frac{2}{3}\Omega)^2 + 3(\frac{2}{3}\Omega) - 2 - 2n_3(\frac{2}{3}\Omega + 1) - 2(n_1 + n_2)}{(\frac{2}{3}\Omega - 2)(\frac{2}{3}\Omega + 1)} \right\}, \qquad (55c)$$

where we have used the simple starting matrix elements of $P(ii')_{I_p}^{(s)[\omega_0]}$ from the appendix.

With $I_p = 0$, the operator

$$\sum_i \sqrt{3(2i + 1)} \, P(ii)_{I_p = 0}^{(s)(00)} = N_{op}. \qquad (56)$$

is the simple number operator. In this case, entry 7 of table 4 together with entries 4 and 6 of table 3 verify that the diagonal matrix element of this operator is simply the total number of particles, $N = \sigma_1 + \sigma_2 + \sigma_3 + n_1 + n_2 + n_3$. Similarly, the operators

$$\sum_i \sqrt{(2i + 1)} \, P(ii)_{KM_K, I_p = 0}^{(s)(11)} = C_{KM_K}^{(11)} \qquad (57)$$

are the SU(3) generators. Although their matrix elements follow at once from the general theory of generator matrix elements of ref. [14]), entry 7 of table 4 verifies

that the reduced matrix elements of this operator are given in terms of the SU(3) Casimir invariant by

$$\langle[\sigma][\omega]\nu; I\|\sum_i \sqrt{(2i+1)}\ P(ii)^{(s)(11)}_{I_p=0}\|[\sigma][\omega]\nu; I\rangle_{\rho_0}$$

$$= \delta_{\rho_0 1}\sqrt{\tfrac{2}{3}}[\lambda^2_\omega + \mu^2_\omega + \lambda_\omega\mu_\omega + 3(\lambda_\omega + \mu_\omega)]\ . \tag{58}$$

Note that only the $\rho_0 = 1$ matrix element survives for this SU(3) generator.

It should be noted, in particular, that the z-space realization of the SU(3) generator, $\sum_i \sqrt{(2i+1)}\ \Gamma(P(ii)^{(s)(\omega_0)}_{\alpha, I_p=0})$, as given by eq. (12d) is quite different from the "standard" z-space realization as given by eq. (12) of ref. [14]). The latter involves z-derivative operators. It is well known that coherent state realizations of operators are not unique due to the overcompleteness of coherent states. It is, however, gratifying to note that two quite different realizations, $\Gamma(O)$, for the same operator lead to the same final matrix elements. In similar fashion, the operator

$$\sum_i \sqrt{\tfrac{3}{2}(2i+1)}\ A(ii)^{(02)}_{JM, I=0} \tag{59}$$

is the pair annihilation generator of Sp(6) whose matrix elements follow most simply from eqs. (31) and (29) of ref. [14]) through its simple z-derivative realization, eq. (12) of ref. [14]). Entry 3 of table 4 again verifies this simple result, demonstrating again that two quite different realizations, $\Gamma(O)$, of the same operator lead to the same matrix elements.

Another interesting special case involves the states with $u = 2, [\sigma] = [2]$ and $[\omega] = [n00]$ where n is the total particle number, (an even number), i.e., states of highest possible SU(3) symmetry. In this case entry 7 of table 4 leads to

$$\langle[2][\omega] = [n00]; [i \times i]I'\|P(ii)^{(s)[\omega_0]}_{I_p}\|[2][\omega] = [n00]; [i \times i]I\rangle$$

$$= \left[\frac{(\tfrac{2}{3}\Omega - 2n)}{(\tfrac{3}{2}\Omega - 4)}\right]2\sqrt{\frac{(2I+1)}{3(2i+1)}}\ U(IiI'i; iI_p)F_{[\omega_0]}\ , \tag{60a}$$

$$\text{with } F_{(00)} = 1\ , \qquad F_{(11)} = \sqrt{2(n+3)/n}\ . \tag{60b}$$

where we have used the starting reduced matrix element of $P(ii)^{(s)[\omega_0]}_{I_p}$ between states of type [2]; $[i \times i]I$ from the appendix. Other cases with different combinations of i, i' with $i \neq i'$ follow from the analogous reduced matrix elements of the appendix.

Another important matrix element for these simple $u = 2$ states is the u-breaking matrix element with $u' = 0$. Entry 4 of table 4 now leads to the simple value

$$\langle[\sigma'] = [0][n00]; I' = 0\|P(ii')^{(s)[\omega_0]}_I\|[\sigma] = [2][\omega] = [n00]; [i \times i']I\rangle$$

$$= \sqrt{2(1 + \delta_{ii'})}\sqrt{(2I+1)}\ \sqrt{\frac{(\tfrac{2}{3}\Omega - n)}{(\tfrac{2}{3}\Omega)(\tfrac{2}{3}\Omega - 2)}}\ f_{[\omega_0]}\ , \tag{61a}$$

$$\text{with } f_{(00)} = \sqrt{\tfrac{1}{6}n}\ , \qquad f_{(11)} = \sqrt{\tfrac{8}{15}(n+3)}\ . \tag{61b}$$

Eqs. (60) and (61) have been derived by Ginocchio [19]).

5. Summary

Very general expressions have been given for the matrix elements in the $Sp(6) \supset U(3)$ basis of the fermion dynamic symmetry model of single-nucleon creation and annihilation operators, of the most general one-body operators, and of the pair creation and annihilation operators. These expressions reduce the general matrix elements for arbitrary nucleon number, n, and arbitrary $U(3)$ representation, to those for nucleon numbers u and u' where these are the S, D-pair seniority numbers which come into play. Effectively, an n-particle calculation has therefore been reduced to one involving u (or u') particles. The matrix elements depend only on readily available $SU(3)$ recoupling coefficients [18] and on the K-matrix elements of VCS theory. For states with $u \leqslant 2$ the latter have been given in general analytic form [14]. If the low-lying states in real nuclei are dominated by states of low heritage it becomes feasible to evaluate nuclear matrix elements of the most general operators, with i-space spins different from zero. An earlier study by Halse [20] in the sd shell has shown that the $u = 0$ states have very little overlap with the eigenfunctions of a realistic shell-model hamiltonian, so that the validity of the low heritage model is open to serious question, at least for very light nuclei. With the techniques of this investigation it should now be feasible to make an honest test of the validity of the fermion dynamic symmetry model in rotational nuclei in the actinide region, where the $Sp(6) \supset U(3)$ branch of the model may have some applicability.

Appendix

STARTING MATRIX ELEMENTS

The simple starting matrix elements connecting states of heritage, u, to states of heritage $u' = u$, or $u \pm 1$, $u \pm 2$, via the operators b_i, $b_i^\dagger$, $A(ii')$, or $P(ii')$ can be calculated by standard shell-model techniques. For $u(u') \leqslant 2$ the results are very simple. For $u(u') \geqslant 3$, standard cfp techniques can be used. For $u(u') \leqslant 2$ the single-nucleon creation (annihilation) operator matrix elements are

$$\langle [1]; i \| b_i^\dagger \| [0]; 0 \rangle = 1 , \tag{A.1}$$

$$\langle [0]; 0 \| \tilde{b}_i \| [1]; i \rangle = \sqrt{3(2i+1)} , \tag{A.2}$$

$$\langle [\sigma']; [i \times i'] I' \| b_i^\dagger \| [1]; i \rangle = -\sqrt{(1+\delta_{ii'})} , \tag{A.3}$$

$$\langle [\sigma']; [i \times i'] I' \| b_i^\dagger \| [1]; i' \rangle = \sqrt{(1+\delta_{ii'})} \, (-1)^{i+i'-I'+\hat{\sigma}'} , \tag{A.4}$$

$$\langle [1]; i' \| \tilde{b}_i \| [\sigma]; [i \times i'] I \rangle = \sqrt{(1+\delta_{ii'})} \, \sqrt{\frac{\dim [\sigma]}{3} \frac{(2I+1)}{(2i'+1)}} , \tag{A.5}$$

$$\langle [1]; i \| \tilde{b}_i \| [\sigma]; [i \times i'] I \rangle = \sqrt{(1+\delta_{ii'})} \, \sqrt{\frac{\dim [\sigma]}{3} \frac{(2I+1)}{(2i+1)}} \, (-1)^{i+i'-I+1+\hat{\sigma}} , \tag{A.6}$$

with $[\sigma]$ or $[\sigma'] = [2]$ or $[11]$.

Matrix elements of $A(ii')$ and $P(ii')$ can be obtained from these via intermediate state sums which lead to the relations (valid for general $[\sigma]$ and $[\sigma']$).

$$\langle [\sigma']; I' \| A(ii')_{I_p}^{[\tilde{\omega}_p]} \| [\sigma]; I \rangle$$

$$= \sum_{[\sigma'']} \sum_{I''} U([\sigma][11][\sigma'][11]; [\sigma''][\tilde{\omega}_p]) U(Ii'I'i; I''I_p)$$

$$\times (-1)^{i+i'-I_p+\hat{\omega}_p} \langle [\sigma']; I' \| \tilde{b}_i^{[11]} \| [\sigma'']; I'' \rangle \langle [\sigma'']; I'' \| \tilde{b}_{i'}^{[11]} \| [\sigma]; I \rangle \qquad (A.7)$$

and

$$\langle [\sigma']; I' \| [b_i^\dagger \times \tilde{b}_{i'}]_{I_0}^{[\omega_0]} \| [\sigma]; I \rangle_{\rho_0}$$

$$= \sum_{[\sigma'']} \sum_{I''} U([\sigma][11][\sigma'][1]; [\sigma'']_{--}; [\omega_0]_{-\rho_0}) U(Ii'I'i; I''I_0),$$

$$\times (-1)^{i+i'-I_0+\hat{\omega}_0} \langle [\sigma']; I' \| b_i^{\dagger[1]} \| [\sigma'']; I'' \rangle \langle [\sigma'']; I'' \| \tilde{b}_{i'}^{[11]} \| [\sigma]; I \rangle . \qquad (A.8)$$

Interesting special cases include

$$\langle [\sigma']=[0]; 0 \| A(ii')_{I_p}^{[\tilde{\omega}_p]} \| [\sigma]; [i \times i']I \rangle = \sqrt{\dim [\sigma](2I+1)(1+\delta_{ii'})} \, (-1)^{1+\hat{\sigma}}, \qquad (A.9)$$

with $[\sigma]=[2], [11]$;

$$\langle [\sigma']=[1]; i \| P(ii')_{I_p}^{(p)(\omega_0)} \| [\sigma]=[1]; i' \rangle$$

$$= \frac{1}{2} \sqrt{\frac{(2I_p+1) \dim (\omega_0)}{3(2i+1)}} (1+\delta_{ii'}), \qquad (A.10)$$

$$\langle [\sigma']=[1]; i' \| P(ii')_{I_p}^{(p)(\omega_0)} \| [\sigma]=[1]; i \rangle$$

$$= \mp \frac{1}{2} \sqrt{\frac{(2I_p+1) \dim (\omega_0)}{3(2i'+1)}} (-1)^{i+i'-I_p}(1+\delta_{ii'}), \qquad (A.11)$$

with upper [or lower] sign for $(p)=(s)$ [or $(p)=(a)$].

$$\langle [2]; [i \times i]I' \| P(ii)_{I_p}^{(s)(\omega_0)} \| [2][i \times i]I \rangle$$

$$= 2 \sqrt{\frac{(2I+1)f(\omega_0)}{3(2i+1)}} U(IiI'i; iI_p), \qquad (A.12)$$

$$\langle [2]; [i \times i]I' \| P(ii')_{I_p}^{(s)(\omega_0)} \| [2][i \times i']I \rangle$$

$$= \sqrt{\frac{(2I+1)f(\omega_0)}{6(2i+1)}} (-1)^{I_p-I} U(Ii'I'i; iI_p), \qquad (A.13)$$

$$\langle [2]; [i' \times i']I' \| P(ii')_{I_p}^{(s)(\omega_0)} \| [2]; [i \times i']I \rangle$$

$$= \sqrt{\frac{(2I+1)f(\omega_0)}{6(2i'+1)}} U(IiI'i'; i'I_p), \qquad (A.14)$$

$$\langle [2]; [i \times i']'I' \| P(ii)_{I_p}^{(s)(\omega_0)} \| [2]; [i \times i']I \rangle$$

$$= (-1)^{i-i'-I'} \sqrt{\frac{(2I+1)f(\omega_0)}{3(2i'+1)}} U(IiI'i; i'I_p), \qquad (A.15)$$

$$\langle [2]; [i \times i']I'\| P(ii')^{(s)(\omega_0)}_{I_p} \| [2][i \times i]I \rangle$$

$$= \sqrt{\frac{(2I+1)f(\omega_0)}{6(2i+1)}}\; U(IiI'i'; iI_p)\,, \qquad\qquad (A.16)$$

with $f((00)) = 1$; and $f((11)) = 5$.

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