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# BOOK REVIEW

**An Introduction to Splines for Use in Computer Graphics and Geometric Modeling.** R. H. BARTELS, J. C. BEATTY, AND B. A. BARSKY. Morgan Kaufmann, Los Altos, CA, 1987. 469 pp. $42.95.

Piecewise polynomials and/or splines figure prominently as all-purpose workhorses in computer-aided geometric modeling. The properties of these beasts are a part of the growing folklore of geometric modeling, but until now have not appeared as the central topic of any modeling book. Books such as Faux and Pratt's *Computational Geometry for Design and Manufacture* and Rogers and Adams' *Mathematical Elements for Computer Graphics* serve as broad introductions to the mathematical side of curve and surface modeling. DeBoor's *Practical Guide to Splines* and Schumaker's *Spline Functions: Basic Theory* offer mathematically rigorous treatments of (real-valued) splines, but are not specifically tied to the modeling arena. The principal service performed by this book is that it presents splines (more specifically, $B$-splines) and their properties in an accessible, intuitive manner, and does so within the context of geometric modeling.

The book begins immediately with vector-valued spline functions, presupposing the context of 2D and 3D design. The componentwise analysis of these splines begins with uniform linear and cubic splines and develops the corresponding $B$-spline basis for each in terms of piecewise polynomial representations. With these representations in hand, a number of standard properties of $B$-splines are noted, and in some instances proved. This development is typical of much of the presentation: a great deal of attention is paid to the computation of examples, and based upon these examples the more general theory follows.

In order to generalize uniform low degree splines to higher degree nonuniform splines, some time is spent investigating the piecewise continuity of cubic polynomials (anticipating one-sided power functions) and determining the effect that coalesced knots have on the transition continuity of quadratic polynomials. Propelled by these examples, the general one-sided power function is introduced. Without yet appealing to the formalism of divided differences, the cancellation of powers that results from successive differences of one-sided uniform cubic power functions is detailed.

Inspired by the outcome of the cubic case, the general situation is treated using divided difference formulas. Once developed, these formulas are tested on first and second order splines in their entirety and are used as computational formulas for pointwise evaluation of fourth order splines. The Cox–DeBoor recurrence formula is derived from the Leibniz formula for the divided difference of a product. With this tool a number of the previously noted properties of $B$-splines are established.

Bezier–Bernstein representations of polynomial curves are introduced for the purpose of motivating subdivision and refinement. In the case of $B$-splines the corresponding process involves knot insertion and is accomplished by using discrete $B$-splines. This insertion is illustrated by some linear and quadratic examples.

There is a short chapter which draws attention to the difference between parametric and geometric continuity. The continuity remarks seem to pave the way for the

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introduction and development of the Beta spline, to which nearly a fifth of the book is devoted.

The authors have presented the material in an informal fashion, emphasizing intuition more than rigor. The result is a gentle introduction into what can be a formidable subject. The book reads like an undergraduate text, but lacks any formal exercises identifying it as one. The notation for $B$-splines, knots, etc., is sensible and standardizing. However, the notation used to distinguish between local and global parametrizations is mathematically suspect.

For the practicing engineer the development is complete with respect to the fundamental mathematics of $B$-spline curves. The treatment of surfaces is light, and the subject of rational splines is omitted completely. Some practical remarks are offered for rapid and repeated evaluation of $B$-spline formulas and their derivatives, with some brief comments on applications such as ray-tracing, animation, and so on.

The book does not seem to offer any real defense for using $B$-splines in computer-aided modeling. There is no indication why they are more useful or better than other tools, beyond automatic continuity control and compactness of representation. The drawbacks in using $B$-splines from a variety of practical perspectives are not noted, though they certainly do exist. But for the absence of such a discussion, the book is a valuable introduction to $B$-splines since they are rapidly becoming the *de facto* standard building blocks for curve and surface modeling. This author knows of no better $B$-spline primer.

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