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Techniques for Assessment of Stratification and Effects of Mechanical Mixing in Tropical Fish Ponds

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ABSTRACT

Density stratification isolates near-surface from bottom pond waters and prevents exchange of dissolved oxygen (DO) and nutrient elements, potentially restricting photosynthesis and production. Destratification strategies have become important for cost-effective intensification of pond aquaculture. Evaluation of methods and devices has emphasized effects on production, with little detailed description of effects on physico-chemical components of pond ecosystems.

This paper describes short-term effects of mechanical mixing on temporal and spatial distribution of temperature and DO in tropical freshwater fish ponds. Intensely stratified ponds of 1·5 m depth were monitored at eight depths for temperature and two depths for DO every 30 min with a modest-cost automated system of commercially available hardware. Results are presented as time-series plots, isotherm diagrams of temperature distribution with time and depth, and a stability index of energy required to mix a pond to uniform temperature.

Required mixing energy is minuscule compared with electrical energy consumption of the lowest-powered mixing devices discussed in literature. Strategy for application of mechanical energy to water is critical for efficiency. A relatively subtle difference between two mixing regimes

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(daytime mixing for one 2-h period or two 1-h periods) produced potentially important differences in temperature and DO distribution.

INTRODUCTION

Mechanical devices are often used to destroy density stratification and to increase or maintain dissolved oxygen (DO) concentrations in ponds. Interest in intensification of production strategies has led to demonstrations that night-long or continuous active aeration can improve production over emergency aeration practices (reviewed by Boyd & Watten, 1989). Daytime aeration is usually avoided to conserve the commonly observed surface supersaturations of DO for nighttime use (Busch *et al.*, 1978). Mixing or circulation of pond waters without increasing the surface area of air–water contact is relatively inexpensive compared with active aeration (Busch & Goodman, 1981), and has the potential, by redistributing DO in time and space, to reduce the cost of aeration needed to optimize production. Water circulation alone failed to increase shrimp production in brackishwater ponds (Fast *et al.*, 1988), but Lorio (1990) showed mixing to be advantageous in channel catfish production.

Development of cost-effective aeration and mixing strategies will require careful evaluation of effects on pond ecosystems and cultured animals. This task is complicated by such factors as limited knowledge of animals' requirements and responses to stress, their abilities to move away from stressful conditions in ponds (Boyd & Lichtkoppler, 1979), and both gross and subtle differences in microbial rate processes during diel cycles (Boyd & Watten, 1989; Chang, 1989; Costa-Pierce & Laws, 1985). In addition, there is no standardization of observational and analytical techniques for assessment of aeration and mixing effects.

This paper presents an assessment of a simple mixing strategy in intensely stratified tropical freshwater fish ponds, using an automated monitoring system simplified from a previously described design (Ebeling & Losordo, 1989).

MATERIALS AND METHODS

Observations were made in earthen ponds of 376 m^2 surface area and 1·4–1·6 m depth at the Asian Institute of Technology near Bangkok, Thailand, during the 'cooler' dry season in February, when the diel range of air temperature was about 22–32°C. The ponds had been stocked 4 months earlier with juvenile nile tilapia (*Oreochromis niloticus*) at 3 fish/

m^2, as part of an investigation of fish growth in manure-fertilized, unfed ponds (Pond Dynamics/Aquaculture CRSP, 1990). All ponds had dense phytoplankton blooms and visible clay turbidity during these observations.

Two mixing regimes are compared here by presentation of one typical diel cycle for each. Both mixed and unmixed ponds exhibited similar conditions on 6 days during a 2 week period of similar weather. During the first trial, a 220 V AC, 373 W (0·5 hp) submersible pump was operated in Pond 1 from 15.00 to 17.00 hours; for the second trial, the pump was operated from 12.00 to 13.00 hours, and from 15.00 to 16.00 hours. Water was taken in from 80 cm depth and discharged horizontally at 10 cm from a pipe of 6·4 cm diameter. The intake depth was chosen, after examination of a typical unmixed pond condition, to ensure relocation of water from below the thermocline. The adjacent pond (Pond 2) was monitored as an unmixed control in both cases.

Temperatures were monitored in each pond at 15 m distance downstream from the mixing pump (away from banks at opposite ends of the 15 m × 25 m rectangular ponds) every 30 min by Type T thermocouples attached to a plastic pipe suspended from a float, at depths of 10, 20, 30, 40, 50, 70, 100, and 130 cm. Hand-held thermometer readings confirmed that temperatures at pond margins corresponded to those at the monitored locations. DO concentrations were recorded from a polarographic probe (YSI, Yellow Springs, OH) mounted in a land-based plastic pipe receiver filled every 30 min by 12 V DC pumps placed in the ponds at 10 and 130 cm depth, about 5 m behind the mixing pump. DO concentrations at this location changed as a result of the beginning of mixing within 30 min, as did temperature, providing additional evidence of reasonable horizontal uniformity in the ponds on that time scale. Two adjacent ponds, one mixed and the other unmixed, were monitored for DO alone. All pumps in the system provided water to the receiver sequentially during the first half of each 30 min period, with the receiver draining after each sample. Data were stored in a data logger (Campbell Scientific, Inc., Logan UT) which also controlled the sampling pumps. This system incorporated parts of the system designed and constructed at the University of Hawaii Institute of Marine Biology's Mariculture Research and Training Center (Ebeling & Losordo, 1989), and utilized solely commercially available devices with the exception of a hand-wired mounting board for the relays controlling the sampling pumps.

A simple index for exposure of pond organisms to temperature and DO (in units of degree-hours and mg/litre-hours or 'ppm-hours', respectively) was calculated as the area under time-series plots for the 10 cm and 130 cm depths, and for the whole pond in the case of temperature.

Whole-pond indices could not be calculated for oxygen because only two depths were sampled and details of vertical gradients were unknown.

A water-body's inertial resistance to mixing to uniform temperature may be quantified by the stability index S, commonly used by limnologists (Schmidt, 1915 — not seen in original; Wetzel, 1975). S has the units g-cm/cm^2, and was calculated from the following discrete-interval form of the theoretical integral formula:

$$S = 1/A_0 \sum_{i=1}^{8} A_{zi}(z_i - z_g)(\rho_{zi} - \rho_m)\Delta z_i \tag{1}$$

where

z_i = monitored depth (cm)
Δz_i = depth intervals (cm) divided at mid-points between monitored depths
A_0 = area of pond at 0 depth (cm^2)
A_{zi} = area of pond at depth z_i (cm^2)
z_g = depth of center of gravity of isothermal pond (cm), equal to the sum of $(z_i A_{zi} \Delta z_i)$/pond volume
ρ_{zi} = density of water at depth z_i (g/cm^3)
ρ_m = density of isothermal pond (g/cm^3), equal to the sum of $(\rho_{z_i} A_{zi} \Delta z_i)$/pond volume

RESULTS

Temperature

The diel cycle of temperature in unmixed Pond 2 during Trial 1 is shown in the lower portion of Fig. 1. Data from 20 and 40 cm depths are left off the plot for visual clarity. Maximum temperatures at 10 cm depth and greatest itensity of stratification (distance between curves for neighboring depths) were apparent in mid-afternoon. Near-surface (10–30 cm) waters began to cool before sunset (about 18.30 hours), with mixing of the upper 50 cm, indicated by close proximity of the curves, after 19.00 hours. Cooling and convective mixing of upper waters continued through 22.00 hours, when mixing reached 70 cm. In this pond the deeper layer, from the bottom up to at least 100 cm, never mixed with the upper waters.

Operation of the mixing pump from 15.00 to 17.00 hours (Trial 1) created a dramatically different pattern of temperature curves (Fig. 1, upper). Near-surface (10–30 cm) waters were promptly cooled, and waters of intermediate depths (50–70 cm) were warmed, by the mixing.

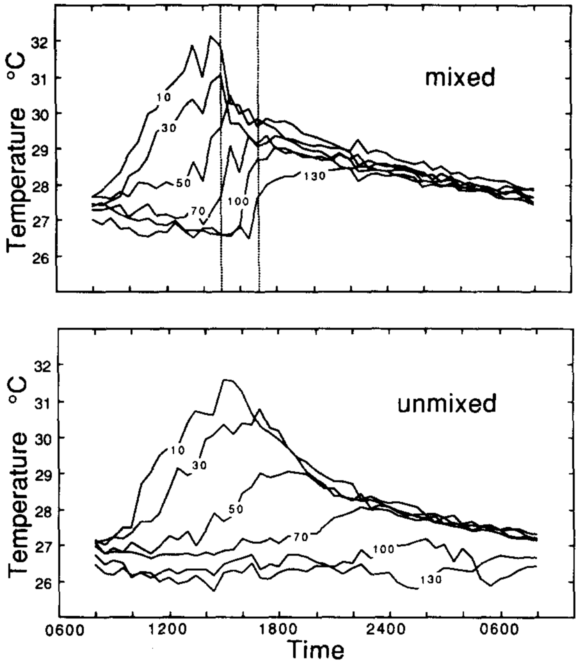

Fig. 1. Time-series plot of temperature at different depths in a pond mixed from 15.00 to 17.00 hours (Pond 1: upper) and in an unmixed pond (Pond 2: lower) during Trial 1. Dotted lines encompass the mixing period.

Involvement of deeper waters required a longer pumping period: 100-cm water was warmed by mixing after 1 h; the 130-cm water was affected after about 1·5 h. Pre-dawn temperatures (which were nearly isothermal with depth) and diel minimum bottom temperatures were approximately 1 °C greater in Pond 1.

Further insight to this comparison is provided in Fig. 2, in which the distribution of isotherms is shown through depth and time. Temporal changes or spatial gradients are indicated by closely spaced isotherms. For example, in the lower plot representing unmixed Pond 2, solar heating of the surface is indicated by the vertical isotherms near the surface between 09.00 and 15.00 hours; development of an intense thermocline is indicated by the horizontally oriented lines between top and bottom of the plot at 1200–1800. As cooling and mixing progress through late afternoon and evening, isotherms previously involved in the thermocline diverge, and surface temperatures decrease through time.

The contrasting temperature regime in the mixed pond is apparent in the upper portion of Fig. 2, as it was in Fig. 1. The two ponds exhibited similar thermal regimes until mixing began in Pond 1 at 15.00 hours. The

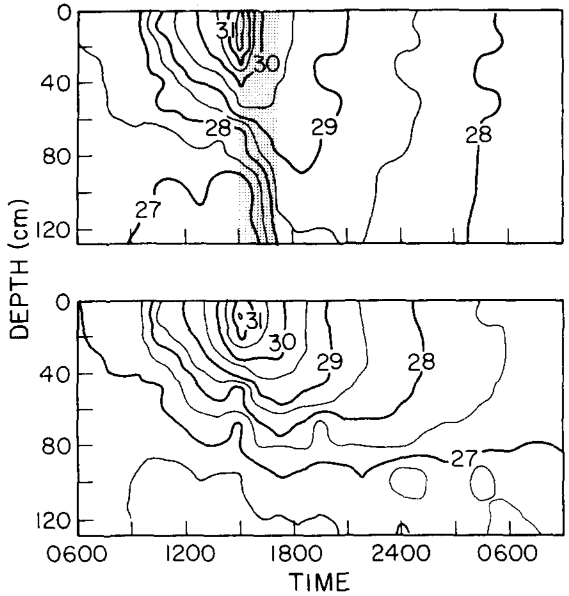

Fig. 2. Isotherm diagram of temperature through depth and time in a pond mixed from 15.00 to 17.00 hours (Pond 1: upper) and in an unmixed pond (Pond 2: lower) during Trial 1. Shaded area indicates the mixing period.

isotherms diverged rapidly after 15.00 in Pond 1, and the isothermal condition of its water column, indicating thorough mixing, persisted through the night.

In Trial 2, when the mixing periods were 12.00–13.00 and 15.00–16.00 hours, unmixed Pond 2 (Fig. 3, lower) exhibited a similar pattern to its earlier one (Fig. 1), but with diel minima and maxima 0·5–1°C higher. The first hour's mixing appeared to affect the water column to a depth of 70 cm in Pond 1 (Fig. 3, upper), but did not make the pond isothermal. Solar heating promptly began to restore stratification when the first mixing period ended at 13.00, and continued to do so until the second mixing began at 15.00. Waters down to 100 cm were mixed to within 1°C during that period, but not the 130-cm water.

The first of the two mixing periods deepened near-surface isotherms and slowed surface heating in Pond 1 (Fig. 4, upper), preventing establishment of the typical intense thermocline seen in Pond 2. The second mixing period created nearly isothermal conditions, but as noted above and as is apparent also in Fig. 4, did not mix the deepest water. Minor temperature inversions are apparent in Figs 2 and 4 after 19.00 hours; similar observations were reported by Cathcart and Wheaton (1987). We have no information to explain this phenomenon; it is

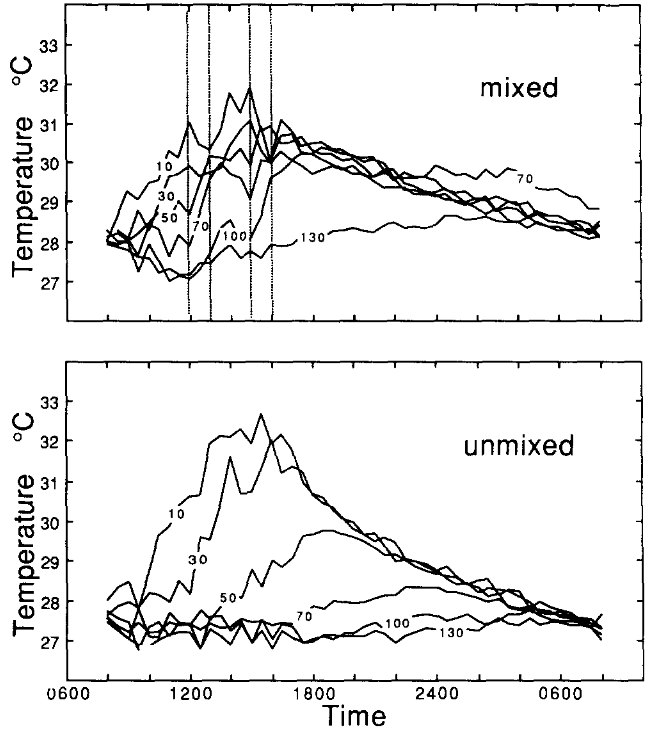

Fig. 3. Time-series plot of temperature at different depths in a pond mixed from 12.00 to 13.00 hours and from 15.00 to 16.00 (Pond 1: upper) and in an unmixed pond (Pond 2: lower) during Trial 2. Dotted lines encompass the mixing periods.

possible that mineral turbidity was not uniform with depth and influenced density of some layers to allow inversions to persist.

Dissolved oxygen

Pre-dawn dissolved oxygen concentrations were typically depleted to below 2 mg/litre in both surface and bottom waters of both ponds by overnight respiration (Fig. 5). In both trials, bottom water in unmixed Pond 2 exhibited DO concentrations less than 1 mg/litre from 10.00 to 22.00 hours, after which convective mixing brought levels to 2–3 mg/litre, followed in turn by slow decrease to the pre-dawn severely depleted condition. Pond 2 exhibited wide-ranging diel cycles on both dates, with near-surface maxima above 16 mg/litre (greater than 200% saturation) in mid-to-late afternoon.

Mechanical mixing between 15.00 and 17.00 hours in Trial 1 (Fig. 5, upper plot, P1) increased bottom DO to more than 4 mg/litre, while

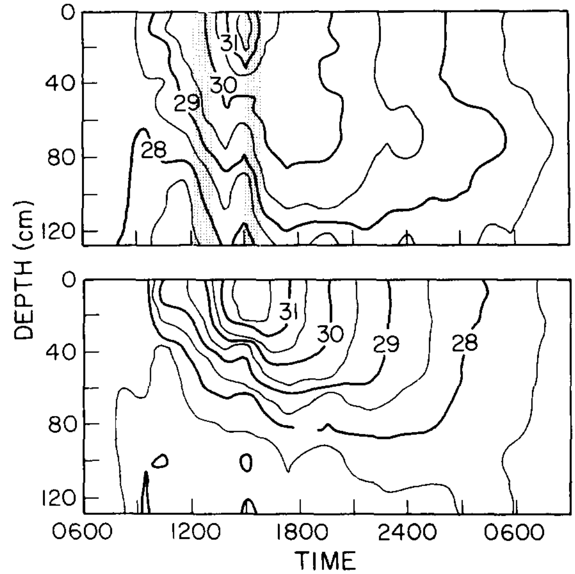

Fig. 4. Isotherm diagram of temperature through depth and time in a pond mixed from 12.00 to 13.00 hours and from 15.00 to 16.00 (Pond 1: upper) and in an unmixed pond (Pond 2: lower) during Trial 2. Shaded areas indicate the mixing periods.

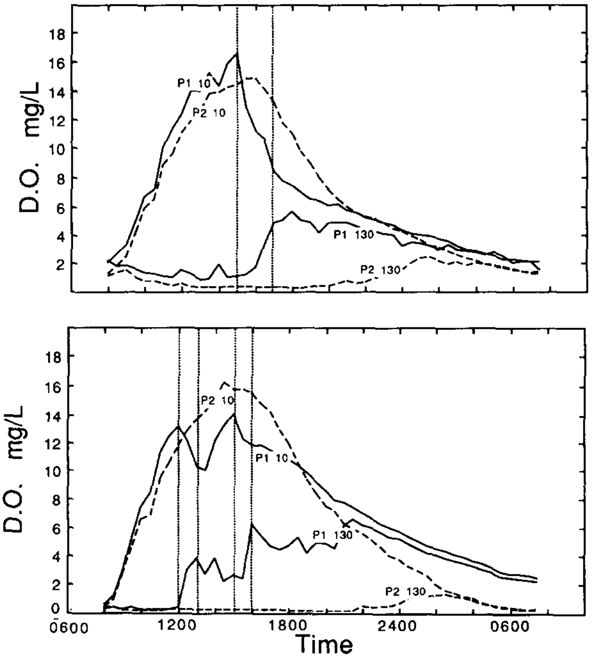

Fig. 5. Time-series plots of dissolved oxygen concentration at different depths in a mixed (Pond 1, solid line) and an unmixed pond (Pond 2, dashed line) during Trials 1 and 2. Dotted lines encompass the mixing periods.

leaving concentrations near 8 mg/litre in upper (10 cm) water. The first of two mixing periods in Trial 2 (lower plot, P1) resulted in bottom DO concentrations greater than 3 mg/litre; the second period brought DO above 5 mg/litre; a further increase was associated with early nighttime convective mixing (20.00–22.00 hours). The second pair of ponds, monitored for DO but not temperature on these dates, exhibited similar patterns (Fig. 6).

Diel means ($n = 48$ sampling periods per day), standard deviations, ranges of temperature and DO, and exposure indices for Ponds 1 and 2 are presented in Table 1. Only small differences between mixed and unmixed ponds, and between the two mixing regimes, are apparent for most properties, except for DO at 130 cm. Diel mean DO at 130 cm depth was significantly greater in the mixed pond than in the control under both regimes ($P < 0{\cdot}01$, *t*-tests; Sokal & Rohlf, 1981). Diel minimum bottom DO was greater in the mixed pond than in the control during Trial 1; bottom exposure indices were greater for the mixed pond in both trials, with a greater mixing advantage seen for Trial 2. Minima and exposure indices could not be compared between ponds statistically because only single values of these quantities pertain to a given day.

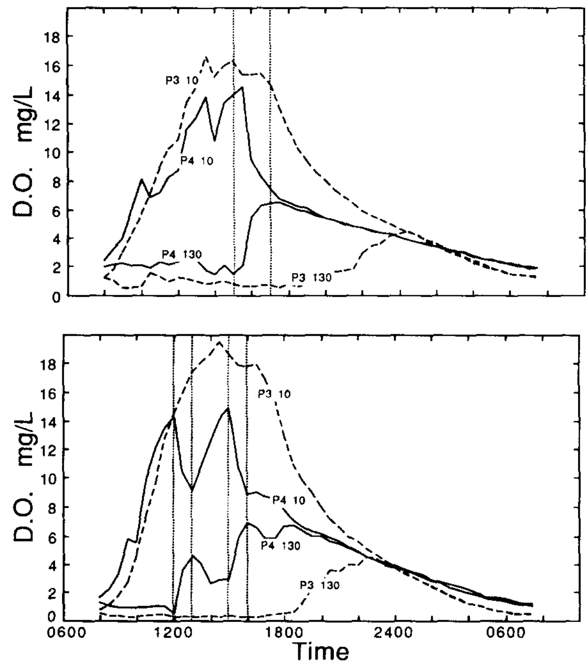

Fig. 6. Time-series plots of dissolved oxygen concentration at different depths in a mixed (Pond 4, solid line) and an unmixed pond (Pond 3, dashed line) during Trials 1 and 2. Dotted lines encompass the mixing periods.

TABLE 1

Summary of Dissolved Oxygen (DO) and Temperature Data for Mixed and Unmixed Ponds

	Diel mean[a] (SD)		*Diel range*		*Exposure index[b]*	
	Mixed pond	*Unmixed control*	*Mixed pond*	*Unmixed control*	*Mixed pond*	*Unmixed control*
Dissolved oxygen (ppm)						
One 2-hour mixing period						
Top (10 cm)	6·6 (4·2)	6·7 (4·6)	2·0–16·6	1·3–14·8	160	162
Bottom (130 cm)	2·8 (1·4)	1·1 (0·7)	0·9–5·6	0·3–2·5	68	25
Whole pond summary			0·9–16·6	0·3–14·8		
Two 1-hour mixing periods						
Top (10 cm)	7·3 (3·8)	6·8 (5·4)	0·6–14·0	0·3–16·3	176	162
Bottom (130 cm)	3·4 (1·9)	0·4 (0·3)	0·2–6·6	0·1–1·3	82	10
Whole pond summary			0·2–14·0	0·1–16·2		
Temperature (°C)						
One 2-hour mixing period						
Top (10 cm)	28·5 (1·2)	28·2 (1·3)	27·7–32·1	27·1–31·6	685	676
Bottom (130 cm)	27·0 (0·7)	25·9 (0·4)	26·5–28·5	25·7–27·2	648	621
Whole pond summary	27·7	26·9	26·5–32·1	25·7–31·6	666	646
Two 1-hour mixing periods						
Top (10 cm)	29·1 (1·0)	28·9 (1·6)	28·2–31·9	27·4–32·7	698	695
Bottom (130 cm)	27·5 (0·4)	26·7 (0·2)	27·2–28·6	26·8–27·7	660	641
Whole pond summary	28·5	27·6	27·0–31·9	26·8–32·7	684	661

[a] $n = 48$ samplings per day.

[b] Calculated as the area under the time-series plot describing the diel cycle at the given depth. The index has units of ppm-hours for DO, and degree-hours for temperature. The whole-pond temperature values were derived by integration over both time and depth.

Stability

The diel pattern of the stability index *S* (Fig. 7) followed those of the temperature and DO time-series plots, as would be expected. The plot shows that the first mixing period in Trial 2 left considerable resistance to mixing in the pond (P1, lower plot in the figure). The second mixing period brought the index near to the values exhibited in the mixed pond in Trial 1. The unmixed pond retained more resistance to mixing overnight during Trial 1 than during Trial 2. This could be the result of differences in wind (which was not measured) during the nights following these nearly calm days. The visible clay turbidity, if uniformly distributed with depth, would have no effect on estimates of *S*. If turbidity were not uniform, it would be greater near the bottom due to settling, and the stability of the water column would be greater than calculated here. Thus these are minimum estimates of *S*.

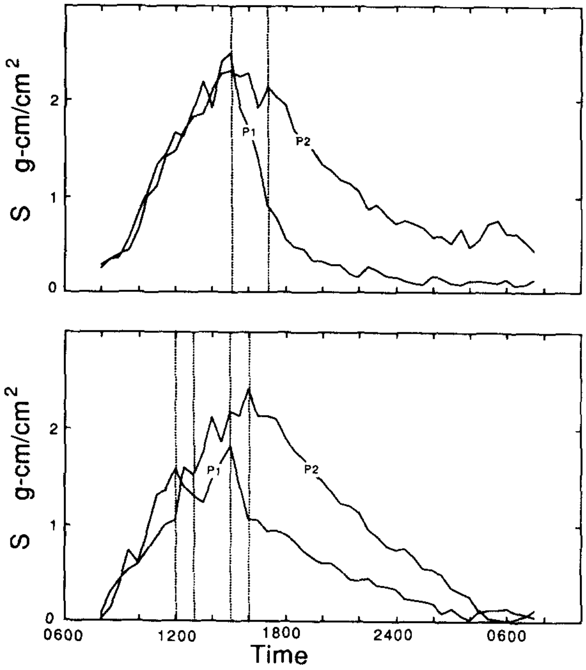

Fig. 7. Time-series plots of the value of the stability index *S* in a mixed (Pond 1) and an unmixed pond (Pond 2) during Trials 1 and 2. Dotted lines encompass the mixing periods.

DISCUSSION

Density stratification in ponds can expose cultured animals to stressful or lethal low concentrations of dissolved oxygen (DO), and may limit photosynthetic production of organic matter important to some culture strategies, such as the one under study in the longer-term experiments in these ponds. Bottom waters are depleted of DO by respiration, and isolated from replenishment by atmospheric transfer and photosynthetic production in surface waters. Upper-layer production of both DO and organic matter may be limited by depletion of nutrient elements when surface waters are isolated from deep-water or benthic organic remineralization. Although fishes and shrimps can relocate from DO-depleted layers, convective overturn after nighttime cooling can create inescapable whole-pond low levels if (1) the hypolimnion is large compared with the epilimnion, or (2) respiration in sediments and bottom water has been rate-limited by low DO before exposure to the DO newly mixed in from the surface (Boyd, 1979; Chang, 1989).

These conditions are typical of ponds involved in manure-fertilization experiments, which harbor dense phytoplankton blooms and show high rates of respiration. Also typically, the fish survive in high percentage, but it is presently unknown whether their growth is affected by these conditions.

The beginning of the 15.00–17.00 mixing period was chosen to coincide approximately with the time of maximal surface DO concentration, in an attempt to conserve the supersaturation excess from escape to the atmosphere. The two 2-hour periods constituted a successful attempt to relieve bottom depletion for a longer period (Fig. 5; exposure index, Table 1), while maintaining the same mixing time and energy expenditure for comparison. This benefit might be enhanced by beginning mixing earlier, for example when the DO first approaches saturation (7·7 mg/litre at 29°C), near 10.00 hours on these days. There is great potential to tune mixing and aeration effects at minimal energy cost by having the data logger control devices under specified criteria.

DO increased at the 130 cm depth more quickly when the first mixing of the day began at 12.00 than when it began at 15.00 (Figs 5 and 6). This may be explained by the lesser thermocline development at the earlier hour (Figs 2 and 4), and the associated lesser stability (Fig. 7). It appears that complete mixing to isothermal conditions is not required for transfer of some oxygen to deep layers.

The stability index S provides the potential for comparison of the energy required to mix a pond with that applied to water by mixing devices. S estimates the extent to which the pond's mass is distributed non-uniformly in the direction of positive vertical stability, i.e. the center of gravity of the pond is at a lower point than that of an isothermal pond of equal size and average temperature. The mass unit (g) in the dimension of S (g-cm/cm^2) may be converted to the cgs unit representing the force of weight (dynes) by multiplication by the acceleration of gravity, 980 cm/s^2, thus converting S to units of work/energy per unit area (1 erg $= 10^{-7}$ J). This factor was taken into consideration in the conversion table presented by Fast (1968). Further multiplication by the pond area ($3{\cdot}76 \times 10^6$ cm^2) leads to an estimate of the work required to mix the pond. The maximum observed S was 2·51 in Pond 1 at 15.00 during Trial 1:

$$2{\cdot}51 \text{ g-cm/cm}^2 \times 980 \text{ cm/s}^2 \times 3{\cdot}76 \times 10^6 \text{ cm}^2 = 9{\cdot}08 \times 10^9 \text{ ergs} = 908 \text{ J}$$

The 373 W pump would be expected to use this amount of electrical energy in approximately 2·4 s. Because pumps are not perfectly efficient, more time would be required for the pump to provide equivalent mechanical energy to the water.

By any consideration of load or efficiency, however, the energy required to mix a pond is minuscule compared with the rate of energy application to water by even the lowest-powered devices (0·19 kW = 190 J/s) reviewed by Boyd and Watten (1989). It is clear that the mode of energy application is critical to efficient mixing. The stability energy of a stratified pond is extremely diffuse in space, as its acquisition by solar heating is relatively slow in time. It is possible that diffusely or slowly applied mixing strategies will be more efficient than those applying energy rapidly from a point-source. Our strategy applied energy in a concentrated manner, but should have gained some efficiency by the distribution of denser 80-cm water near the surface over a considerable area. It would be useful to attempt to maximize such an effect, perhaps using dye-tracer studies. Of the mixing and circulation strategies reviewed by Boyd and Watten (1989), continuously operated air-lift pumps would seem to provide the slowest and most diffuse application of energy, but power requirements were not detailed there, and the authors' reasonable cautions about inconvenience of installations should be noted in practical application.

The two modes of graphical presentation used here (time-series plots and isotherm diagrams) are partially redundant but take different perspectives on the information. For example, details of timing for selection of mixing periods are best seen in the time-series plots, while thermocline properties and responses are most readily apparent in the isotherm diagrams. The stability index and its energy-unit derivative discussed here might be useful for classifying water bodies or destratification tasks. The small energetic efficiency of mixing strategies (far less than 1% in our experiments) may be taken as an engineering challenge for further development.

Data like these are readily obtained in ponds, and could be valuable at developmental or verification stages of temperature-distribution models such as that of Cathcart and Wheaton (1987), who in fact made similar temperature measurements. These observations were made with an automated system of modest cost (less than $5000), which has the potential for inexpensive expansion to automatic control output and multiplexed observations.

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