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# Geometrical optimization of longitudinally pumped solid-state lasers

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We use a simplified analysis of gain saturation in longitudinally pumped lasers or amplifiers to study the effect of the relative pump and laser beam size. We derive a simple rule of thumb allowing one to optimize the efficiency.

End pumped or longitudinally pumped solid-state lasers have proven to be very efficient sources. They differ from side pumped lasers, regardless of the nature of the pump, by the fact that the gain is concentrated in a very well defined region. Furthermore the gain exhibits a transverse profile directly related to the pump beam profile which induces new requirements on the relative sizes of the pump beam and cavity mode. For example when an amplifying medium is pumped by a gaussian beam the unsaturated gain is higher on axis than on the edges of the gain profile. The average gain seen by a beam sent through this amplifier becomes a function of the beam size: the smaller the beam, the higher the small signal gain but also, the higher the saturation. There is then a trade-off between large small signal gain (i.e. small beam) and little saturation (i.e. large beam). Modeling of longitudinally pumped lasers has been extensively worked out [1–6]. The optimization of the different parameters influencing the efficiency of those lasers has been studied. In general it leads to complicated relations between these parameters which are difficult of use. We used a simplified analysis of gain saturation including transverse profiles of both pump and laser beams in order to find a simple rule of thumb giving the optimum ratio of the pump and laser beams sizes. This rule is found to work with gaussian and super-gaussian pump beams.

We consider a laser medium of length $l$ pumped by a beam which transverse profile is $f(r)$. Only beams with cylindrical symmetry are considered in the following. The intensity of the pump beam is written as

$$I_P(r) = I_{P0} f(r) , \quad (1)$$

where $I_{P0}$ is the pump intensity on axis.

This laser medium is used to amplify a laser beam which is supposed to be gaussian with an intensity

$$I_L(r,z) = [2P_L(z)/\pi w_L^2] \exp(-2r^2/w_L^2) , \quad (2)$$

where $P_L$ is the average power of the beam, $w_L$ is the amplitude radius at 1/e and $z$ is the coordinate along the axis of propagation.

In this simplified study we assume that the size of both the pump and the laser beam are constant over the length of the amplifying medium. We also neglect the loss of the medium at the laser wavelength and assume a small gain per pass. We can define a local small signal gain coefficient as

$$G(r,z) = \alpha\,(\lambda_P I_P(r)/\lambda_L I_{sat}) \exp(-\alpha z) , \quad (3)$$

where $\alpha$ is the absorption at the pump wavelength, $\lambda_P$ and $\lambda_L$ are the pump and laser wavelengths and $I_{sat}$ is the saturation intensity of the amplifying medium ($I_{sat} = h\nu/\sigma\tau$ where $\nu$ is the laser frequency, $\sigma$ is the emission cross-section at the laser wavelength and $\tau$ is the fluorescence lifetime). The laser intensity in the amplifying medium can be described by [1–5]

$$\frac{\partial I_L(r,z)}{\partial z} = \frac{G(r,z)\, I_L(r,z)}{1 + sI_L(r,z)} , \quad (4)$$

where $s$ is the saturation parameter: $s = 1/I_{sat}$. The intensity after one pass through the amplifier is obtained by integration of (4). In the case of small gain per pass one obtains

$$I_L(r, z=l) = \int_0^l G(r,z) \frac{I_L(r,z=0)}{1+sI_L(r,z=0)} \, dz \,. \qquad (5)$$

Substituting (3) in (4) yields

$$I_L(r, z=l) = sI_P(r)\,[1-\exp(-\alpha l)]\,\frac{\lambda_P}{\lambda_L}\,\frac{I_L(r,z=0)}{1+sI_L(r,z=0)} \,. \qquad (6)$$

The average power in the laser beam is

$$P_L(z) = \int_0^\infty 2\pi r I_L(r,z)\, dr \,. \qquad (7)$$

One can then define an average gain for this particular combination of pump and laser beams by

$$G_{av} = \frac{P_L(z=l)}{P_{L0}} = \frac{2g_0}{\pi w_L^2} \int_0^\infty \frac{f(r)\exp(-2r^2/w_L^2)}{1+(2sP_{L0}/\pi w_L^2)\exp(-2r^2/w_L^2)}\, 2\pi r\, dr \,, \qquad (8)$$

where $P_{L0} = P_L(z=0)$ is the input laser beam average power and $g_0$ is the unsaturated gain on axis:

$$g_0 = sI_{P0}\frac{\lambda_P}{\lambda_L}\,[1-\exp(-\alpha l)] \,. \qquad (9)$$

All these expressions have been derived for cw beams but the results are the same for pulsed beams. If the laser and pump pulses are shorter than the fluorescence lifetime of the amplifying medium, one can obtain the average gain by replacing the laser and pump intensities ($I_L$ and $I_P$) by the corresponding fluences ($J = 2E/\pi w^2$ for a gaussian beam with $E$ the pulse energy) and the saturation intensity $I_{sat}$ by the saturation fluence $J_{sat} = h\nu/\sigma$.

These expressions can be used to optimize longitudinally pumped amplifiers or lasers. First, one can use eq. (4) to show how the gain profile in an amplifier is reshaped by saturation. Fig. 1 shows the evolution of the gain profile as the average power of the beam to be amplified increases. The amplifier is pumped by a gaussian beam and the size of the laser beam is half that of the pump beam. One can see that saturation first flattens the gain profile and ends up digging a hole on the axis. Note that most of the amplifiers work in the saturation regime which corresponds to the last part of fig. 1 where the gain is minimum on axis.

Second, using eq. (8) one can try to determine what is the optimum size of the laser beam to be used for a given pump beam profile and radius. In order to be able to predict what happens with gaussian pump beam as well as with uniform (or "top hat") pump beam we used the following super-gaussian profiles:

$$f(r) = \exp[-2(r/w_P)^n] \,, \qquad (10)$$

where $w_P$ is the amplitude radius of the pump beam and $n$ the order of the super-gaussian. We compared three different beam profiles: a gaussian beam ($n=2$) and two super-gaussian beams ($n=4$ and $n=8$). For $n=8$ the beam is almost uniform over the range $\{-w_P, +w_P\}$. The input laser beam is gaussian. As a practical example we consider a longitudinally pumped Ti:$Al_2O_3$ rod. The pumped beam radius $w_P$ is 50 μm and the incident laser power is in the range 0–10 W (typical intracavity power in a cw laser). The on axis small signal gain is constant ($g_0 = 0.25$) which means that the average small signal gain increases with the order $n$ of the super-gaussian profile. Fig. 2 shows the average single pass gain versus the size $w_L$ of the laser beam for three laser average powers and three pump shapes. One can see that there is an optimum value of $w_L$ which gives the highest gain. This beam size depends on the shape of the pump beam and on the laser power. It does not depend on the small signal gain as long as the approximations used in the calculations stay valid. The presence of a maximum in the curves of fig. 2 means that, in order to obtain the best gain from a highly saturated amplifier, one has to calculate the optimum size of the beam. In any case, the best solution is almost never to use an equal beam size for the pump and the laser.

One can also use these results to optimize the average power obtained from a longitudinally pumped laser. In a cw pumped laser the steady state round trip gain in laser power equals the loss due to mirror transmission $T$ plus unwanted losses $L$. Neglecting

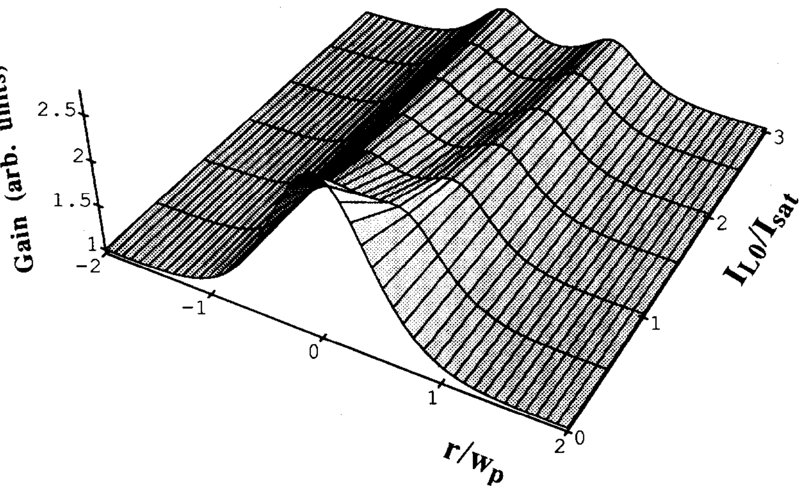


Fig. 1. Transverse gain profile as a function of the average power $P_L$ of the laser beam. The pump beam and the laser beam are gaussian. The laser beam radius is half that of the pump beam. The graph is normalized in units of $I_{L0}/I_{sat}$ where $I_{L0}$ is the laser beam intensity on axis ($I_{L0}=2P_L/\pi w_L^2$).

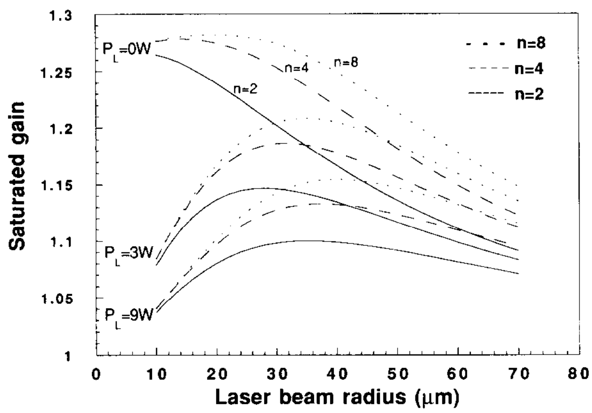


Fig. 2. Saturated gain of a single pass amplifier versus laser beam radius for three pump beam profile and three laser average power. The pump beam amplitude radius at 1/e is 50 μm. The amplifying medium is $Ti:Al_2O_3$ ($I_{sat}=300$ kW/cm$^2$). The three sets of curve correspond to a laser average power of 0 W, 3 W and 9 W. For each average power we considered three super-gaussian pump beam profiles with respectively $n=2$, $n=4$ and $n=8$.

spatial hole burning and assuming a small value of $L+T$, the intracavity power $P_L$ under equilibrium conditions can be calculated using the following equation:

$$L+T=G_{av}$$

$$=\frac{2g_0}{\pi w_L^2}\int_L^{\infty}\frac{f(r)\exp(-2r^2/w_L^2)}{1+(2sP_l/\pi w_L^2)\exp(-2r^2/w_L^2)}2\pi r\,dr\,. \tag{11}$$

Eq. (11) is an implicit relation between the intracavity power $P_L$ and the size of the beam $w_L$ in the amplifying medium. Note that in the case of a standing wave laser the constant $s$ has to be replaced by $2s$ to account for the presence of two counterpropagating beams in the cavity, both of which saturate the gain. For given values of the gain, the loss, the pump beam size and for a given pump beam shape, one can use this equation to find what is the laser beam size which leads to the highest average power. In fig. 3, we have plotted the ratio between the laser beam radius $w_L$ giving the highest average power and the pump beam radius $w_P$, as a function of the ratio between the losses $L+T$ and the gain $g_0$. We have found that there is a linear relation between these two quantities. We have tried different conditions (gain, pump beam size, losses) and have found that, as long as the ratio of gain to loss is less than 10, this relation is very well verified. The small discrepancy between the points presented in fig. 3 and a straight line comes from the limited precision of the numerical procedure used to solve eq. (11). The slope of the linear

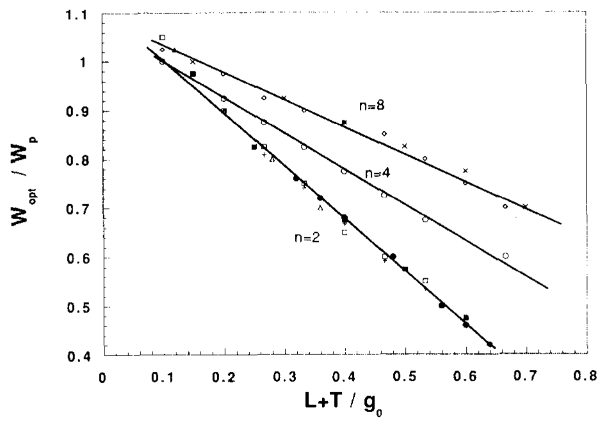


Fig. 3. Ratio of the laser beam size giving the highest average power to the pump beam size versus the ratio of the unsaturated gain to the total loss of the cavity. Three super-gaussian pump beam profiles with respectively $n=2$, $n=4$ and $n=8$ have been considered. The points correspond to the optimum laser beam size determined from eq. (11). One set of points (same symbol) is obtain by varying the cavity losses. The different sets of points correspond to different small signal gains, pump beam sizes or pump beam shapes. Straight lines are least squares fit to these data.

relation depends on the exact shape of the pump beam. We found that in the case of gaussian or super-gaussian beams the linear relations can be approximated by

$$\frac{w_L}{w_P} = 1.1\left(1 - \sqrt{\frac{2}{n}\frac{L+T}{g_0}}\right), \qquad (12)$$

where $n$ is the order of the super-gaussian ($n=2$ for a gaussian). These results show that a laser has to be optimized for a given pump power and pump beam profile. One cannot use the same cavity adjustment for a lower power laser and a high power laser regardless of the problem of thermal lensing.

In summary we developed a simple analysis of longitudinally pumped amplifiers and lasers. This analysis shows that for a given pump beam there is a laser beam size which optimized the gain in the case of an amplifier or the average power in the case of a laser. A very simple rule of thumb giving the optimum beam size has been found. Such a simplified model cannot perfectly account for the complicated mechanisms occuring in the saturated amplification of light, but we think that, at least, it gives to experimenters a rough idea of the best parameters to use in longitudinally pumped amplifiers or lasers.

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