

THE ELECTRIC DIPOLE MOMENT OF W-BOSON

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Theoretical predictions of the electric dipole moment D_W of the W gauge boson are estimated for various models of CP -violation. It is shown that, for the supersymmetric model and the Weinberg–Higgs model, D_W can be of the order of 10^{-20} e cm, which is close to the upper bound derived indirectly from the neutron electric dipole moment. We also obtain smaller D_W -values of about 10^{-22} e cm and less than 10^{-38} e cm, predicted by the left–right model and the Kobayashi–Maskawa (KM) model, respectively.

1. Introduction

In the past twenty six years since the discovery of CP violation [1], even with a lot of efforts, little progress has been made in determining where this violation arises in the elementary particle interactions [2]. Immediately after the discovery of CP -violation, it was suggested by Salzman and Salzman [3] and others [4] that the observed CP -violation in the neutral kaon system might result from an intrinsic electric dipole moment of W , denoted by D_W hereafter. Indeed, the order of magnitude of the CP -violation parameter ϵ in the kaon decay is very close to α/π and hence this suggested a CP -violating electromagnetic effect on the weak interaction amplitudes. It was also realized that a finite D_W could induce an electric dipole moment for the neutron. More recently, Marciano and Queijeiro [5] updated the original analysis of Salzman and Salzman and they found that measurements of the neutron electric dipole moment of the order 10^{-25} e cm could be used to place a very stringent upper bound on D_W for its absolute magnitude,

$$D_W \lesssim 10^{-20} e \text{ cm} . \quad (1)$$

They have assumed a reasonable form factor to tame the divergence. Unless this form factor suppression is much stronger, the restriction eliminates the possibility of using D_W to explain ϵ . However, the effect of D_W of the order of 10^{-20} e cm

may still be accessible [6] in certain processes, for instance in the scattering $\gamma e^\pm \rightarrow W^\pm \nu$, for future experiments. Also, careful studies of the polar and azimuthal distributions of leptons and antileptons produced in W decays [7] may further provide useful constraints on the size of D_W . With the increasing production luminosity of W pairs in laboratories, it becomes of current interest to estimate the size of D_W in various CP-violating gauge models.

2. A general expression for D_W

It is well known that if P - and T -symmetries are violated, elementary particles with spin degrees of freedom may have electric dipole moments. The most general form of the W-boson coupled to a photon has seven terms [7], among them two of which violate P - and T - and hence CP -symmetries,

$$ikW_\mu^\dagger W_\nu \tilde{F}^{\mu\nu} + i(\lambda/M_W^2)W_{\alpha\mu}^\dagger W_\nu^\mu \tilde{F}^{\nu\alpha}. \quad (2)$$

Here W_μ is the W^- gauge potential, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + \dots$, and the dual of the photon field strength is $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}(\partial_\mu A_\nu - \partial_\nu A_\mu)$. In the momentum space, these terms can be expressed as

$$f_1\epsilon^{\mu\nu\alpha\beta}(p-p')_\beta + (f_2/M_W^2)\epsilon^{\mu\nu\rho\beta}(p-p')_\beta(p+p')^\alpha(p+p')_\rho. \quad (3)$$

Here p and p' are the incoming and the outgoing momenta of the W-boson. The form factors $f_1 = \lambda - k$ and $f_2 = \frac{1}{2}\lambda$ are functions of $(p-p')^2$ (the square of the momentum transfer). The electric dipole moment D_W can be expressed [7] in terms of these form factors in the limit $(p-p')^2 \rightarrow 0$ in the unit of $e/2M_W = 1.2 \times 10^{-16}$ e cm,

$$D_W = (f_1 - 4f_2)(e/2M_W). \quad (4)$$

In gauge theories, a CP-violating but $SU(2)_L$ -invariant term $\theta W^{\mu\nu}\tilde{W}_{\mu\nu}$ can be added to the lagrangian. However, this term can be rewritten as a total divergence and thus will not contribute to D_W perturbatively. Nonperturbative effects due to such a term are suppressed at least by a factor [8] $\exp(-8\pi^2/g^2)$, where g is the weak coupling constant, and hence extremely small. In what follows we will ignore this contribution. Also, we realize that the first term in eq. (2) has a dimensionality of four. However, this term is not invariant under $SU(2)_L \times U(1)$ and, therefore, can only be generated through higher-dimensional gauge-invariant terms of the form $\phi^n W^\dagger W \tilde{A}$. Here ϕ^n represents, generically, an interaction of n neutral Higgs fields. Since the other term already has a dimensionality of six, the CP-violating electromagnetic form factor is thus induced by operators with a dimensionality greater than four. As a result, D_W is calculable even in models with "hard" CP-violation.

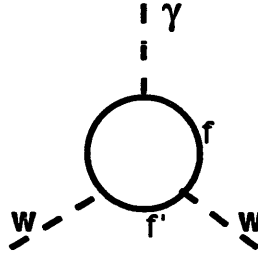


Fig. 1. One-loop Feynman graphs for calculating D_W due to the left-handed and the right-handed currents.

An obvious distinction between the CP -conserving and the CP -violating electromagnetic form factors of W is that the CP -violating terms are directly proportional to the Levi-Civita tensor. Such a tensor occurs naturally in the spinor trace of the Dirac matrices, $\epsilon^{\mu\nu\alpha\beta} = \frac{1}{4}i \text{Tr}(\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\gamma_5)$. Thus the fermion loop is required to give D_W . This fact can be understood in a different way. A perturbative renormalizable theory that contains only gauge bosons and Higgs bosons (without fermions) is always invariant under the symmetry \tilde{P} : $x^\mu \rightarrow x_\mu, W^\mu \rightarrow W_\mu, \phi \rightarrow \phi$. Consequently, the lowest-order contribution that may potentially contribute to D_W must contain fermion loops. To study the size of D_W quantitatively, we consider the following general W -fermion interaction:

$$\mathcal{L}^{cc} = -\frac{g}{\sqrt{2}}W_\mu^+ \sum_{i,j} \bar{f}_i \gamma^\mu (V_{ij}L + U_{ij}R) f_j. \tag{5}$$

Here $L, R = \frac{1}{2}(1 \mp \gamma_5)$, i and j are generation indices, f and f' represent fermion fields with charges different by one unit. The phases in the mixing matrices V and U are the sources of the CP -violation. For simplicity we will not consider the lepto-quark theories where the mixing between quark and lepton is not zero. The one-loop contributions to D_W are depicted in fig. 1. Evaluating these graphs, we find

$$D_W = \frac{g^2 C}{8\pi^2} \sum_{a,i,j} \frac{e}{2M_W} \frac{m_i m_j'}{M_W^2} \text{Im}(V_{ij}U_{ij}^*) \left(Q_i \mathcal{J} \left(\frac{m_i^2}{M_W^2}, \frac{m_j'^2}{M_W^2} \right) + Q_j' \mathcal{J} \left(\frac{m_j'^2}{M_W^2}, \frac{m_i^2}{M_W^2} \right) \right), \tag{6}$$

where

$$\mathcal{J}(x, y) = \int_0^1 d\alpha \frac{\alpha}{\alpha(\alpha - 1) + \alpha x + (1 - \alpha)y - i\epsilon}. \tag{7}$$

Here Q_i and Q'_j are the charges for fermions f and f' , respectively. The color factor C is 1 or 3 for the lepton or for the quark. The fermion masses m_i and m'_j occur explicitly in the factor $m_i m'_j / M_W^2$ due to the helicity argument. The above expression can be integrated directly. When $\Delta \equiv 1 + x^2 + y^2 - 2x - 2y - 2xy \leq 0$,

$$\mathcal{F}(x, y) = \frac{1}{2} \ln \frac{x}{y} + \frac{1-x+y}{\sqrt{-\Delta}} \left(\arctan \frac{1+x-y}{\sqrt{-\Delta}} + \arctan \frac{1+y-x}{\sqrt{-\Delta}} \right). \quad (8)$$

A typical value of this function at the electroweak scale is about unity, e.g. $\mathcal{F}(1, 1) = \sqrt{3} \pi / 9 \approx 0.6$. When $\Delta > 0$,

$$\mathcal{F}(x, y) = \Delta^{-1/2} \left[z_+ \ln |z_+^{-1} - 1| - z_- \ln |z_-^{-1} - 1| - i\pi(1-x+y) \theta(1-\sqrt{x}-\sqrt{y}) \right], \quad (9)$$

where $z_{\pm} = \frac{1}{2}(1-x+y \pm \sqrt{\Delta})$. As indicated by the step function θ , for sufficiently small values of x and y (i.e. $1 > \sqrt{x} + \sqrt{y}$), the internal fermions can be on-shell and as a result $\mathcal{F}(x, y)$ picks up an imaginary part. However, in contrast to the CP -violating phases, this “loop” phase will not change its sign under the hermitian conjugation. In the absence of CP -violation, $\text{Im}(V_{ij}U_{ij}^*) = 0$, we have $D_W = 0$ as it should.

3. The values of D_W in some theories of CP -violation

(1) In the KM model [9], the electric dipole moment vanishes at one-loop level because there is no right-handed current ($U_{ij} = 0$). The reason is very similar to that of calculating the neutron electric dipole moment [10]: finite value of D_W in the KM model can only appear at least from two-loop diagrams. Including the GIM cancellation, we estimate that

$$D_W(\text{KM model}) \lesssim s_1 s_2 s_3 s_\delta \left(\frac{g^2}{8\pi^2} \right)^2 \left(\frac{e}{2M_W} \right) \frac{m_b^4 m_s^2 m_c^2}{M_W^8} \lesssim 10^{-38} e \text{ cm}. \quad (10)$$

Here $s_1 s_2 s_3 s_\delta$ is a combination of CP -violating factors of the order 10^{-4} . The t -quark mass m_t at the scale of M_W has been assumed. In fact, one can argue that even at the two-loop level the contribution is probably zero. The argument goes as follows. There are only four fermion lines in the two-loop diagram. In the unitary gauge, all the interactions are left handed, therefore the quark masses must appear quadratically. In the KM model, the CP -violation disappears when any two of the up- or the down-type quarks are degenerate in mass. Therefore we expect any CP -violating effect to carry a factor of $\prod_{i < j} (m_{d_i}^2 - m_{d_j}^2)(m_{u_i}^2 - m_{u_j}^2)$. As a result, there are a total of 6 powers of quadratic mass differences. The GIM effect at each

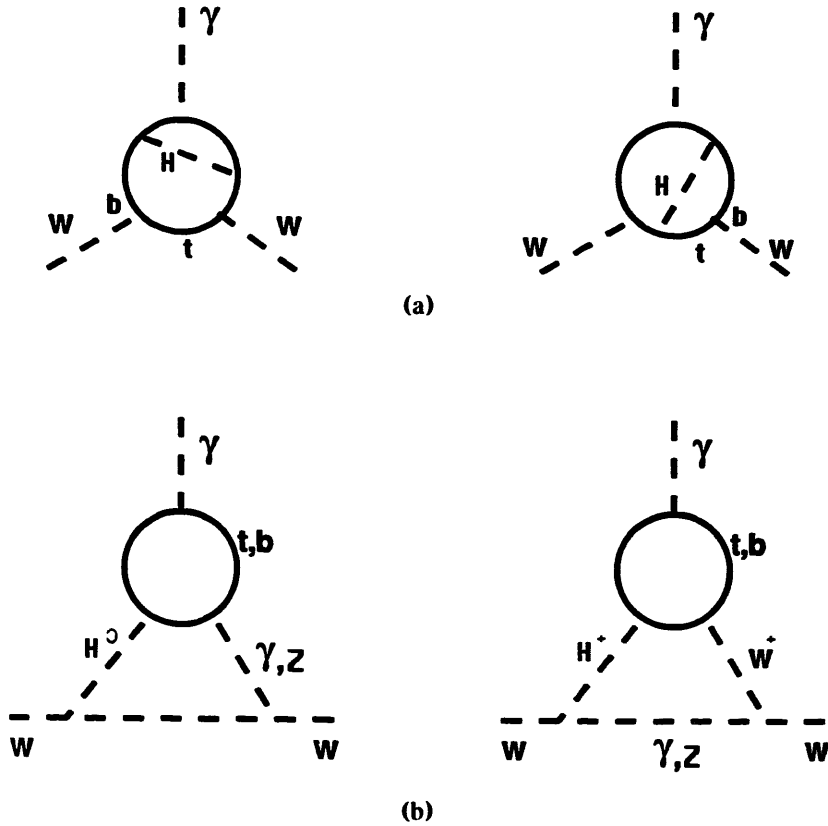


Fig. 2. A sample of two-loop Feynman graphs for calculating D_W in the Weinberg–Higgs model.

fermion line in the loop yields one factor of quadratic mass differences. Therefore, from a two-loop diagram, it can only produce four powers of quadratic mass differences which is smaller than the 6 powers as needed for CP -violation.

(2) In the Weinberg–Higgs model [11], $\text{Im}(V_{ij}) = 0$ and $U_{ij} = 0$. Consequently, $D_W = 0$ at one-loop level. However, a nonzero D_W can arise through two-loop graphs. The dominant graphs are shown in fig. 2 where the exchanged Higgs boson can be charged or neutral. Some of the neutral Higgs contributions (fig. 2a) have recently been studied by He and McKellar [12]. The complete two-loop amplitude still requires more detailed calculations. Here we only estimate the size of D_W to be of the order

$$D_W \sim \sin \delta_H \left(\frac{g^2}{8\pi^2} \right)^2 \left(\frac{m_f}{M_{H^0}} \right)^2 \left(\frac{e}{2M_W} \right). \quad (11)$$

The CP -violating phase δ_H characterizes the complex mixing in the Higgs sector. It could be of the order of unity. The contribution is large if the mass m_{H^0} of the

neutral Higgs is small. With three generations of fermions, it gives

$$D_W(\text{Higgs model}) \lesssim 10^{-20} \left(\frac{m_t}{100 \text{ GeV}} \right)^2 \left(\frac{10 \text{ GeV}}{m_H} \right)^2 e \text{ cm}. \quad (12)$$

For $m_t > 100 \text{ GeV}$ and $m_{H^0} \sim 10 \text{ GeV}$, this yields $D_W \lesssim 10^{-20} e \text{ cm}$.

(3) In left–right models [13], the leading contribution should be due to the CP -violating phase associated with the left–right mixing. The electric dipole moment D_W arises even for the case of only one generation. We can consider the dominant contribution from the top and the bottom quark generation (assuming $g_L = g_R$).

$$\text{Im}(V_{tb}U_{tb}^*) \simeq \xi \sin \delta_{LR}, \quad (13)$$

where ξ is the left–right mixing which is bound [14] by $\xi \leq 5 \times 10^{-3}$. We find the dominant contribution is of the order

$$D_W(\text{LR model}) = \xi \sin \delta_{LR} \frac{g^2}{8\pi^2} \frac{m_b m_t}{M_W^2} \left(\frac{e}{2M_W} \right) \times \left[2\mathcal{F}\left(\frac{m_t^2}{M_W^2}, \frac{m_b^2}{M_W^2}\right) - \mathcal{F}\left(\frac{m_b^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right) \right] \lesssim 10^{-22} e \text{ cm}. \quad (14)$$

The numerical analysis for the t – b quark contribution is shown in fig. 3. One should be aware of potential off-diagonal contribution. Again, a much larger

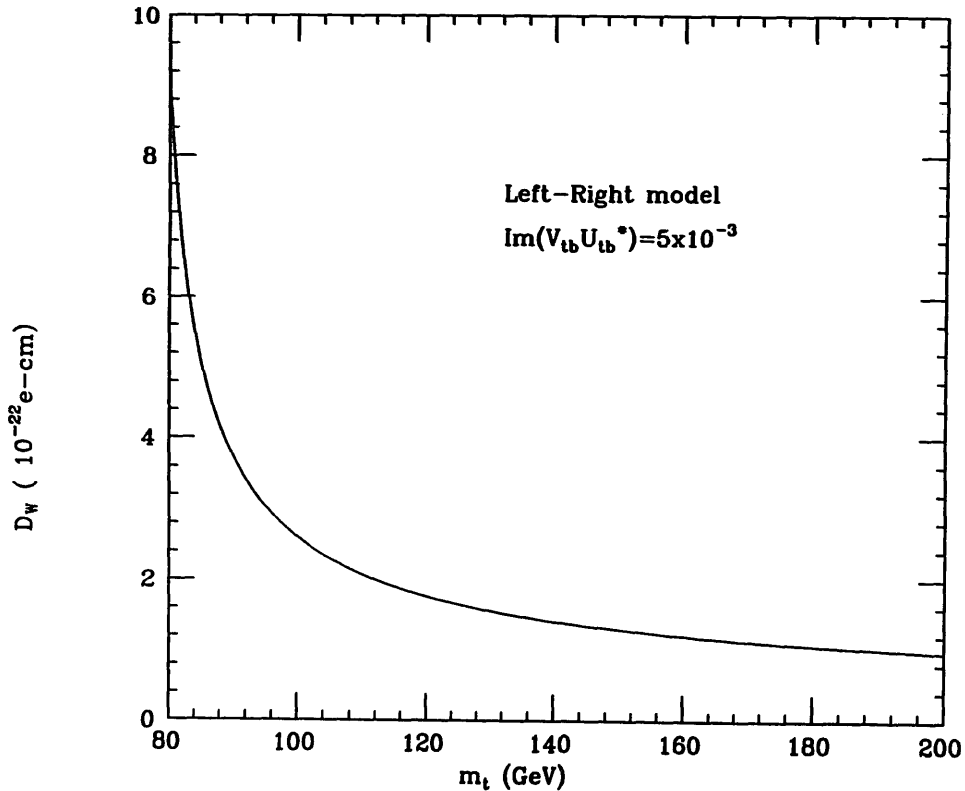


Fig. 3. D_W in the left–right model due to the t – b generation, assuming $\xi \sin \delta_{LR} = 5 \times 10^{-3}$.

contribution could arise (like in the Weinberg–Higgs model discussed above) through Higgs boson exchanges if the specific model has a complicated Higgs structure.

(4) In supersymmetric (SUSY) models [15], the internal fermions in fig. 1 can be supersymmetric particles – the charginos and the neutralinos. To show the basic mechanism of the CP -violation, we consider only the case when the neutralino is the photino $\tilde{\gamma}$ and the chargino is the wino ω . In general, the photino will mix with the neutral higgsino, the zino and the neutrinos; and the wino will mix with the charged higgsino. We avoid this extra complication in the simplified scenario in order to illustrate the physics involved. One can easily extend our approach to the general case. In terms of the independent Weyl’s fields ω_L^+ , ω_L^- and $\tilde{\gamma}_L$, the relevant lagrangian is

$$\mathcal{L} = -eW_\mu^+ (\overline{\omega_L^+} \gamma^\mu \tilde{\gamma}_L + \overline{\tilde{\gamma}_L} \gamma^\mu \omega_L^-) - m_{\tilde{\gamma}} \overline{\tilde{\gamma}_L} \tilde{\gamma}_L - m_\omega e^{i\delta_s} \overline{\omega_L^+} \omega_L^{-c} + \dots + \text{h.c.} \quad (15)$$

The mass terms of the wino and the photino break supersymmetry softly. A phase δ_s in the mass term is usually allowed and it cannot be totally absorbed by redefining the fields. Consequently, this causes CP non-conservation. To recover the usual form of the mass expression, we define the Dirac field $\omega^+ = \omega_L^+ + e^{i\delta_s} (\omega_L^-)^c$ and the Majorana field $\tilde{\gamma} = \tilde{\gamma}_L + \tilde{\gamma}_L^c$. The above lagrangian becomes

$$\mathcal{L} = -eW_\mu^+ \overline{\omega^+} + \gamma^\mu (L - e^{i\delta_s} R) \tilde{\gamma} - \frac{1}{2} m_{\tilde{\gamma}} \overline{\tilde{\gamma}} \tilde{\gamma} - m_\omega \overline{\omega^+} \omega^+ + \dots \quad (16)$$

Both the left-handed and the right-handed currents appear with a relative phase δ_s . We can obtain D_W from eqs. (5)–(7),

$$D_W = \frac{e^2}{4\pi^2} \left(\frac{e}{2M_W} \right) \frac{m_{\tilde{\gamma}} m_\omega}{M_W^2} \sin \delta_s \mathcal{I} \left(\frac{m_\omega^2}{M_W^2}, \frac{m_{\tilde{\gamma}}^2}{M_W^2} \right). \quad (17)$$

Usually there could be additional factors due to the mixings among charginos and neutralinos. At present, no direct phenomenological constraint on these mixings is available. Also, the CP -violating phase δ_s , allowed by the soft supersymmetry breaking lagrangian, could be naturally of the order of unity. The only natural suppression on D_W in this class of models is thus the loop factor $e^2/4\pi^2$. As a result, we expect that in supersymmetric models D_W could be as large as the present limit given by eq. (1),

$$D_W(\text{SUSY model}) \lesssim 10^{-20} e \text{ cm}. \quad (18)$$

(5) In mirror models (for a recent review see ref. [16]), the presence of mirror quarks and mirror leptons introduces right-handed currents with W . The mixing between a quark and its mirror image, ξ_q , is strongly constrained by the absence of

flavor changing neutral current, where one finds [17]

$$\xi_q \lesssim 10^{-3} - 10^{-4}. \tag{19}$$

The relevant lagrangian can be written as

$$\begin{aligned} \mathcal{L} = & -\frac{g}{\sqrt{2}} W_\mu (\bar{u}_L \gamma^\mu d_L + \bar{U}_R \gamma^\mu D_R) \\ & + M_u \bar{u}_R U_L + M_d \bar{d}_R D_L + M_2 (\bar{u}_L U_R + \bar{d}_L D_R) \\ & + m_u \bar{u}_L u_R + m_d \bar{d}_L d_R + m_U \bar{U}_L U_R + m_D \bar{D}_L D_R + \dots \end{aligned} \tag{20}$$

Note that M_i are $SU(2)_L$ -invariant masses and m_i are $SU(2)_L$ broken masses. It is reasonable to assume that m_u, m_d are the smallest massive parameters. Also, the condition $m_U, m_D \gg M_{u,d,2}$ is imposed for the constraint of mirror mixing (eq. (19)). It is possible to define the fields so that only m_u, m_d are complex, i.e. CP -violating parameters. Therefore the CP -violating effect should be proportional to either m_u or m_d . Now, it is easy to draw diagrams that will contribute to D_W . Some typical ones are shown in fig. 4. Each M_i insertion corresponds a factor of ξ_q mixing. It then follows from eqs. (5)–(7) that contributions to D_W from virtual quark–mirror-quark exchange is typically of the order

$$D_W(\text{mirror model}) \sim \frac{g^2}{8\pi^2} \xi_q^2 \sin \delta_M \left(\frac{e}{2M_W} \right) \frac{m_q}{m_Q} \lesssim 10^{-25} e \text{ cm}. \tag{21}$$

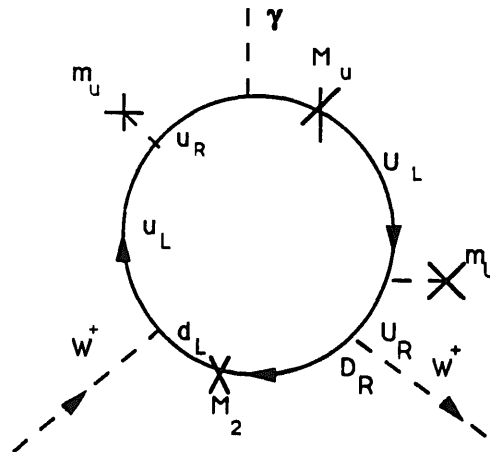


Fig. 4. Typical graph for D_W in the mirror models with the relevant mass insertions illustrated. Here CP -violation requires all parameters in the u - U (or d - D) mass matrix appear.

TABLE 1

Upper limit of D_W in various CP -violation models. Here it is assumed that (1) there are only three generation of fermions and (2) in left–right and mirror models Higgs contributions to D_W are negligible

CP -violation models	Upper limit on D_W (e cm)
KM	10^{-38}
Weinberg–Higgs	10^{-20}
left–right	10^{-22}
SUSY	10^{-20}
mirror	10^{-25}

The phase δ_M characterizes the complex mixing among the quark and its mirror. A similar size of contribution can also be generated from lepton–mirror-lepton mixings. It should also be pointed out that the constraint on ξ_q can be evaded if (1) there is a fourth generation and (2) its mixing with the rest of the generations are negligible. In that case, we find D_W can be as large as of the order $10^{-20} e$ cm.

4. Conclusion

Our results for D_W in different CP -violation models are summarized in table 1. Here it is assumed that there are only three generations of fermions and the Higgs contribution to D_W is negligible except in models where CP -violation is due to the Higgs boson exchange. In SUSY and Higgs models, we find that D_W could be as large as the present upper limit while in other models it appears to be small.

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