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Modelling radiative heat transfer in packed beds

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Abstract—A comprehensive approach for modelling dependent radiative heat transfer in beds of large (geometric range) spherical particles is presented. Such a system of large spheres lies in the dependent range even for large porosities. We show that the dependent properties for a bed of opaque spheres can be obtained from their independent properties by scaling the optical thickness while leaving the albedo and the phase function unchanged. The scaling factor is found to depend mainly on the porosity and is almost independent of the emissivity. We show that such a simple scaling for non-opaque particles is not feasible. The transparent and semi-transparent particles are treated by allowing for the displacement across an optical thickness (because of transmission through a particle) while solving the equation of radiative transfer. When combined with the scaling approach, this results in a powerful method of solution called the dependence included discrete ordinates method (DIDOM). The results obtained from the DIDOM give good agreement with the results obtained from the Monte Carlo method.

1. INTRODUCTION

RADIATIVE heat transfer in participating media consisting of large (geometric range) spheres is considered. The solution can either be obtained from a *direct* (*Monte Carlo*) *simulation* or by following a single continuum treatment and then solving the equation of radiative transfer. The solution of the equation of radiative transfer requires knowledge of the radiative properties of the medium, i.e. the absorption and scattering coefficients $\langle \sigma_a \rangle$, $\langle \sigma_s \rangle$ and the scattering phase function $\langle \Phi \rangle$. If the theory of *independent* scattering is valid, then the radiative properties of the bed are obtained from the properties of an individual particle [1]. However, the independent theory fails when:

- the ratio of the interparticle distance to the wavelength is small [1], or
- the porosity is small [2].

The first condition will generally be satisfied for large particles. However, Singh and Kaviany [2] show that the scattering and absorption of radiation in media consisting of large spherical particles is *dependent* even for porosities as high as 0.93. They compare the results of a direct simulation (ray tracing by the Monte Carlo method) to those based on a single continuum treatment using the properties obtained from the theory of independent scattering. Two distinct *dependent scattering effects* are identified. The first is an increase in the cross-section due to the *multiple scattering* in the representative elementary volume (the local volume averaging for heat transfer in porous media is discussed by Kaviany [3]). The second effect occurs only in *transparent* and *semi-transparent* particles and is due to the '*transportation*' of a ray across a substantial optical thickness when it is transmitted through the particle.

The Monte Carlo method often requires extensive computation. Here, we model dependent scattering and absorption with the aim of arriving at a simple method of analyzing the heat transfer in packed and fluidized beds.

One approach is to *scale* the independent properties so that dependent computations can be carried out using the equation of radiative transfer with these scaled properties. However, since the *deviations* from the independent theory are a function of the *porosity* and the *complex index of refraction*, we will show that a *simple* scaling of the extent of dependence is *not feasible*. This will be done by examining the probability density functions for independent and dependent scattering from both opaque and transparent particles.

Then, a novel approach that separately accounts for multiple scattering in the representative elementary volume and the transportation of radiation through a particle (across a substantial optical thickness) is presented. Multiple scattering depends on the *porosity alone* and is accounted for by scaling the *optical thickness* using the porosity. The transmission through semi-transparent particles is modelled by allowing for the transportation effect while describing the intensity field by the method of discrete ordinates. This is done by taking into consideration the *spatial difference* between the point where a ray first interacts with a sphere and the point from which it finally leaves the sphere. This spatial difference corresponds to an *optical thickness* (for a given porosity) across which the ray is transported while undergoing scattering by a particle.

The results of the application of this '*dependence included discrete ordinates method*' are shown to be in

NOMENCLATURE

A	cross-section [m^2]
C	average interparticle clearance [m]
d	diameter [m]
I	radiation intensity [$W m^{-2}$]
L	depth of the bed [m]
n	index of refraction, or total number of grids in the DIDOM
P	integer that defines reflected or refracted rays
pdf	probability density function
S	distance travelled [m]
S_r	scaling factor
R	radius [m]
T_r	transmittance
x	coordinate axis along the bed [m]
x', y'	coordinate axes with y'-axis along the incident radiation.

Greek symbols

α_r	size parameter, $2\pi R/\lambda$
Γ	inscattering term
Δk	number of grids by which energy is transported on being scattered
η	efficiency
ε	porosity
ε_r	emissivity
θ	polar angle [rad]
θ_0	angle between incident and scattered beam [rad]
κ	index of extinction
μ	$\cos\theta$
λ	wavelength [m]
ρ	reflectivity
σ_a	absorption coefficient [m^{-1}]
σ_{ex}	extinction coefficient, $\sigma_a+\sigma_s$ [m^{-1}]
σ_s	scattering coefficient [m^{-1}]
τ	optical thickness
ϕ	azimuthal angle [rad]
Φ	particle scattering phase function
ω_a	scattering albedo, $\sigma_s/(\sigma_s+\sigma_a)$.

Superscript

$'$	directional quantity, or coordinate axes.

Subscripts

a	absorption
b	black body radiation
d	diffuse
ex	extinction
i	incident
ind	independent
max	maximum
n	normal
r	reflected or refracted or radiation
s	scattering or specular
$\perp$	axial (or longitudinal) component
$\parallel$	lateral (or transverse) component.

Other

$\langle\ \rangle$	volume average.

good agreement with those obtained from the Monte Carlo method. The correct modelling of the physics results in the applicability over the full range of porosity and optical properties and obviates the need for calculating and presenting scaling factors in a three-dimensional array (table).

2. GOVERNING EQUATION

The one-dimensional, steady-state equation of radiative transfer for an absorbing, emitting and scattering continuum is [4]

$$\frac{\partial I}{\partial S} = \langle\sigma_a\rangle I_b(S) - \langle\sigma_{ex}\rangle I(S) + \frac{\langle\sigma_s\rangle}{2}\int_{-1}^{1} I(S,\mu_i)\langle\Phi\rangle(\mu_i,\mu)\,d\mu_i \quad (1)$$

where I is the intensity, S is the distance travelled, σ_{ex}, σ_a and σ_s are the absorption, extinction and scattering coefficients, I_b is the black body emission, and $\Phi(\mu_i, \mu)$ is the phase function for scattering from a direction μ_i to a direction μ ($\mu = \cos\theta$). For simplicity, the properties are assumed to be wavelength independent. In general, because of the dependence of the optical properties on the wavelength, the *spectral* variation of the properties must be considered [2, 3].

The calculation of independent radiative properties of the medium from the properties of a single particle can be done as discussed by refs. [1, 2]. For large particles ($\alpha_r > 100$) the diffraction is focused in a highly forward direction and can thus be neglected. Then, the extinction cross-section is equal to the geometrical cross-section, i.e. the *extinction efficiency* is equal to one.

At present, short of a Monte Carlo simulation, no satisfactory model of the dependent scattering for large particles is available. Kamiuto [5] has proposed a heuristic correlated scattering theory, which attempts to calculate the dependent properties of large particles from the independent properties. The extinction coefficient and the albedo are scaled as

$$\langle\sigma_{ex}\rangle = \gamma\langle\sigma_{ex}\rangle_{ind} \quad (2)$$

and

$$\langle\omega\rangle = 1-(1-\langle\omega\rangle_{ind})/\gamma \quad (3)$$

where

$$\gamma = 1+\tfrac{3}{2}(1-\varepsilon)-\tfrac{3}{4}(1-\varepsilon)^2, \quad \text{for } \varepsilon < 0.921. \quad (4)$$

The phase function is left *unchanged*.

3. SCALING

In this section we attempt to find scaling factors so that the independent radiative properties can be scaled to give the dependent properties of the particulate media. The *scaling factor* S_r is assumed to be scalar and scales the optical thickness leaving the *phase function* and the *albedo unchanged.*

3.1. S_r *for opaque spheres*

Consider a plane-parallel particulate medium subject to *diffuse* incident radiation at one boundary. The medium contains particles that are non-emitting in the wavelength range of interest. For opaque particles with non-zero emissivity, the slopes of the transmission curve on a logarithmic scale approach a constant value away from the boundary. The scaling factor S_r is calculated by finding the ratio of the slopes calculated by the Monte Carlo method [2] and by the independent theory. Calculation of slopes should ideally be carried out away from the boundary in order to obtain the *bulk* properties of the bed. The calculations under the assumption of independent scattering were done by the method of discrete-ordinates. A distance of about six optical thicknesses from the boundary was found to be enough to obtain a constant slope. For the Monte Carlo method, the case of low porosities and high emissivities presents some problems. The intensity can be attenuated by as much as an order of magnitude for every layer of particles. Thus calculation at large depths becomes difficult because of the very small overall transmission. Then we are forced to determine the transmission close to the boundary. Also, because of the low overall transmission, the transmission from individual rays becomes important. Therefore, apart from the difficulty in determining the transmission, the bed may actually give different values of the scaling factor at different depths. This difficulty may be overcome by noting that transmissions of the order of 10^{-5} are so small that any change in the scaling factor at large distances from the source due to transmission resulting from a small number of rays is of no significance. Thus, the value of S_r obtained can be used for emitting, highly absorbing beds with low porosity but should not be used for non-emitting, highly absorbing beds with low porosity.

Figure 1 shows the scaling factor for opaque spheres as a function of porosity for different emissivities. The values of S_r for $\varepsilon_r = 0.1$ can be curve fitted as

$$S_r = 1+1.84(1-\varepsilon)-3.15(1-\varepsilon)^2+7.20(1-\varepsilon)^3 \quad \text{for } \varepsilon > 0.3. \quad (5)$$

Since the effect of emissivity on S_r is small, equation (5) can be used to obtain the value of S_r for other emissivities. The value of S_r obtained from equation

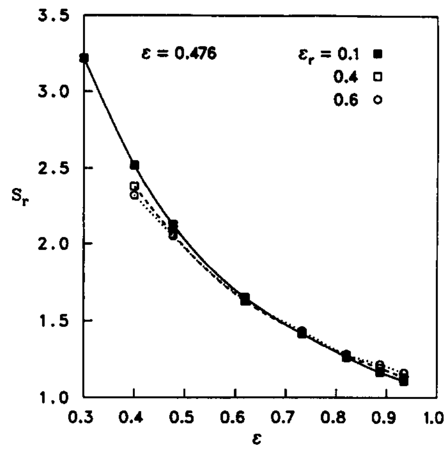

FIG. 1. Variation of scaling factors with porosity for different emissivities.

(5) is accurate to within 2% for porosities greater than simple cubic ($\varepsilon > 0.476$), but because of the difficulty of ascertaining S_r at low porosities, up to 5% uncertainty is possible for lower porosities.

3.2. *The basis of scaling*

Figure 2 shows the probability density function (pdf) for a bed of specularly reflecting, opaque spheres of porosity 0.476. The area under this curve represents the probability that a radiation bundle (starting from a sphere surface) undergoes an interaction before travelling the distance given by the independent axis. The pdf was obtained by a direct Monte Carlo simulation of a packed bed of spheres as discussed in ref. [2]. A ray emitted in the middle of a bed of spheres was followed through successive reflections and the distances between each reflection were recorded. This information was used to calculate the pdf. Also plotted are the pdf calculated from the theory of independent scattering and the pdf for the scaled properties. The effect of increasing the emiss-

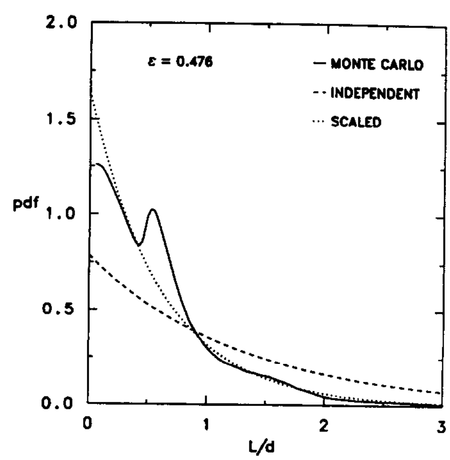

FIG. 2. Probability density function for a bed of opaque particles ($\varepsilon = 0.476$).

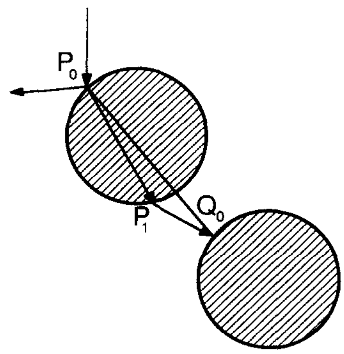

FIG. 3. Schematic of ray transmission through a particle.

ivity is to increase the relative importance of the right-hand side of the curve. This is because multiple reflections attenuate the energy of a ray undergoing a number of interactions as a result of short path lengths. Thus the net contribution to transmittance will come from the rays that include a greater number of longer paths and are thus transmitted with a lesser number of interactions. If the scaled pdf and the Monte Carlo pdf have different shapes, the scaling factor will change greatly with the particle emissivity. However, since the scaled pdf is found to conform closely to the pdf from the Monte Carlo simulation, the effect of the emissivity on S_r is small, as seen in Fig. 1. Therefore, the scaling can be carried out treating the scaling factor as a function of porosity alone.

Figure 3 shows a schematic of the interaction of a ray with a transparent sphere. The ray is intercepted by the first sphere at point P_0. Part of the energy is transmitted through the sphere and interacts with a second sphere at point Q_0. The distance P_0Q_0 is the distance that this energy travels after interaction at point P_0 and before its interaction with the next sphere. Other parts of the incident energy at P_0 travel different paths, as explained in detail in the next section. Figure 4 shows the pdf for a bed of transparent particles ($n = 1.5$). The pdf for semi-transparent particles is similar except for the fact that the fraction of rays passing through the sphere has to be modified to account for the energy attenuated on passing through the sphere. It is clear that the pdf for non-opaque particles and that obtained from the independent theory are basically *dissimilar*. Even though scaling factors can still be found for a prescribed set of n, κ and ε, a change in any one of the three parameters will change the pdf and thus affect S_r. Therefore, a scaling approach necessitates calculation and presentation of scaling factors in a three-dimensional array and is not found to be suitable.

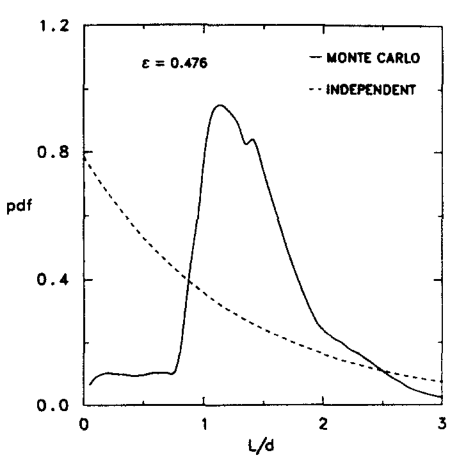

FIG. 4. Probability density function for a bed of transparent particles ($\varepsilon = 0.476$, $n = 1.5$).

4. DEPENDENCE INCLUDED DISCRETE ORDINATES METHOD (DIDOM)

DIDOM models radiation heat transfer in a packed bed of semi-transparent spheres. As mentioned in the Introduction, the deviation from the independent theory takes place because of the following two distinct effects noted in a previous communication [2]:

- multiple scattering within a small elemental volume;
- transportation across a substantial optical thickness.

Multiple scattering is a function of porosity alone and is accounted for by scaling, as shown in the previous section. The transportation effect is modelled by allowing for transmission through a sphere while solving the equation of radiative transfer. For this, the method of discrete ordinates has been found to be most suitable. The key to understanding and modelling the transportation effect is that a ray may be scattered by a particle from a point that is different from the point at which the ray first interacts with the particle. This is because of transmission through a particle. In highly porous media ($\varepsilon \to 1$), this effect is of no consequence because the particle size is small as compared to the inter-particle distance. However, in packed beds the ray may be transported through a distance that corresponds to a substantial optical thickness. Thus not only is it important to know the direction in which a particle scatters, it is also essential to know the displacement undergone by the ray as it passes through the particle. In this section, we first examine the properties of a single particle. Then, the properties of beds are discussed. Finally, the DIDOM is presented.

4.1. *Properties of a single particle*

The theory of geometric scattering is used to calculate the properties of a single sphere. The details are discussed in van de Hulst [6]. Here a brief treatment is presented and the procedure for obtaining the direction as well as the location of the scattered rays is

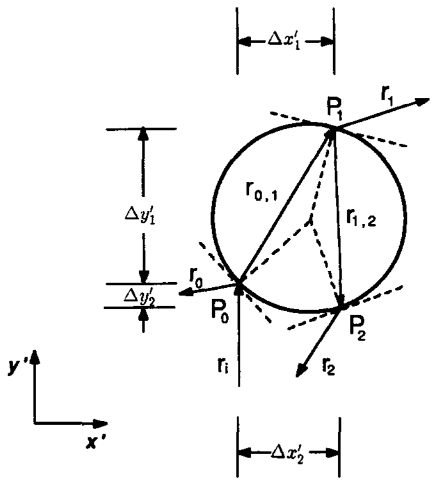

FIG. 5. Ray tracing through a single particle.

outlined. Figure 5 shows an incident ray r_i that strikes a sphere at an angle θ_i with the tangent at point P_0. A fraction of the energy is reflected as the ray r_0 while the rest is refracted as the ray $r_{0,1}$ which undergoes multiple internal reflections while some energy leaves the sphere as rays r_1, r_2, ... at points P_1, P_2,... The ray $r_{0,1}$ makes an angle θ_r with the tangent to the surface at the point P_0.

Then, the energy carried by various rays is

$$\beta_{\parallel} = \rho'_{\parallel} \quad \text{for } P = 0 \tag{6}$$

and

$$\beta_{\parallel} = (1-\rho'_{\parallel})^2(\rho'_{\parallel})^{P-1}\exp(-4\kappa P\alpha_r \sin\theta_r) \quad \text{for } P = 1,2,3,\ldots \tag{7}$$

where P represents the number of internal reflections. For the other *polarization*, we replace $\parallel$ with $\perp$. The total deviation from the original direction is

$$\theta' = 2\theta_i - 2P\theta_r. \tag{8}$$

The *scattering angle* in the interval $(0, \pi)$ is given by

$$\theta_0 = k2\pi + q\theta' \tag{9}$$

where k is an *integer* and $q = +1$ or -1. Differentiating and using the Snell law, we have

$$\frac{d\theta'}{d\theta_i} = 2 - 2P\frac{\tan\theta_i}{\tan\theta_r} \tag{10}$$

$$d\theta_0 = \left|\frac{d\theta'}{d\theta_i}\right| d\theta_i. \tag{11}$$

Then following the steps given by ref. [6], the *gain* G of a ray relative to *isotropic scattering* can be written as

$$G_{\parallel}(\theta) = 4\beta_{\parallel} D \tag{12}$$

where

$$D = \frac{\sin\theta_i\cos\theta_i}{\sin\theta_0\left|\frac{d\theta'}{d\theta_i}\right|}. \tag{13}$$

The gain for *non-polarized* incident radiation is

$$G = \tfrac{1}{2}(G_{\parallel} + G_{\perp}). \tag{14}$$

The fraction of energy scattered, or the *scattering efficiency*, is calculated using

$$\eta_s = \frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{1}\sum_{P=0}^{P_{max}} G(P,\theta_0)\,d\cos\theta_0\,d\phi = \frac{1}{2}\int_{-1}^{1}\sum_{P=0}^{n} G(P,\theta_0)\,d\cos\theta_0. \tag{15}$$

The angle θ_i varies from 0 to $\pi/2$, and the calculation of the gain is carried out at 720 points at equal intervals. The integral over θ_0 is replaced by a summation for carrying out the calculation. The *absorption efficiency* is given by

$$\eta_a = 1 - \eta_s. \tag{16}$$

For independent scattering, the sum of the gains in a particular direction resulting from various values of P gives the phase function Φ for a non-absorbing sphere. For an absorbing sphere, the resulting values of the phase function must be divided by the scattering efficiency.

However, for cases where the transportation effect is important, the addition of gains for different values of P is not permissible. This is because rays scattered in the same direction from different points (P_0 for $P = 0$, P_1 for $P = 1$) will not have the same effect on transmission in a packed bed. Therefore, along with the gain, information regarding the point from which the ray leaves the sphere must be mentioned. Thus, the phase function will be reported as a three column array, i.e. $[\Phi(\theta_0, P), \Delta x', \Delta y']$. Here $\Phi(\theta_0, P) = G(\theta_0, P)/\eta_s$, $\Delta x' = x'_P - x'_0$ and $\Delta y' = y'_P - y'_0$ represent the displacement undergone by the ray in a direction perpendicular and parallel to the incident ray respectively. Thus, $A_a = \eta_a\pi R^2$, $A_s = \eta_s\pi R^2$ and $[\Phi(\theta_0, P), \Delta x', \Delta y']$ are determined. $\Delta x'$ and $\Delta y'$ are given by

$$\Delta x' = 0, \quad \Delta y' = 0 \quad \text{for } P = 0 \tag{17}$$

and

$$\Delta x' = d\sin\theta_r \sum_{P'=1}^{P} \sin[(2P'-1)\theta_r - \theta_i]$$
$$\Delta y' = d\sin\theta_r \sum_{P'=1}^{P} \cos[(2P'-1)\theta_r - \theta_i], \quad \text{for } P = 1,2,\ldots. \tag{18}$$

4.2. *Properties of beds*

In this section, we will relate the radiative properties of a single particle determined in the previous section to the radiative properties of the particulate medium. We assume a one-dimensional plane-parallel slab geometry. The required properties are $\langle\sigma_a\rangle$, $\langle\sigma_s\rangle$ and $[\langle\Phi\rangle(\mu_j \to \mu_i), \Delta k]$. The last one represents the phase function from a direction μ_j to a direction μ_i, and Δk represents the number of grids through which it is transported in the direction perpendicular to the slab boundaries.

For *mono-sized* scatterers of porosity ε, we have

$$\langle\sigma_s\rangle = N_s A_{\lambda s} S_r. \tag{19}$$

Similarly, $\langle\sigma_a\rangle = N_s A_a S_r$.

The procedure for computing $[\langle\Phi\rangle(\mu_j \to \mu_i), \Delta k]$ is outlined below.

(i) To find the phase function for scattering into a direction μ_i from a direction μ_j, we must integrate over the azimuthal angle ϕ. For this purpose we employ a Gaussian quadrature and find discrete values of $\phi_i - \phi_j$ between 0 and π at 24 points.

(ii) At every point, we find $\theta_0 = \cos^{-1}[\mu_i\mu_j + \sqrt{(1-\mu_i^2)}\sqrt{(1-\mu_j^2)}\cos(\phi_i - \phi_j)]$.

(iii) Up to this point, the treatment is similar to that used when employing a standard DOM with the phase function available at discrete values of θ_0, except that each value of P has its own phase function for every θ_0. However, here the numerical integration over ϕ to evaluate

$$\langle\Phi\rangle(\mu_j \to \mu_i) = \frac{1}{\pi}\int_0^{\pi} \sum_P \langle\Phi\rangle \times (\theta_0(\mu_j, \phi_j \to \mu_i, \phi_i), P)\,\mathrm{d}(\phi_j - \phi_i) \tag{20}$$

is not performed. This is because we cannot add $\langle\Phi\rangle[\theta_0(\mu_j, \phi_j \to \mu_i, \phi_i), P]$ terms unless they have the same Δk.

(iv) For every $\langle\Phi\rangle[\theta_0(\mu_j, \phi_j \to \mu_i, \phi_i), P]$, we find $\Delta k = \text{Integer}\ \{\Delta k_{x'} + \Delta k_{y'}\}$ where $\Delta k_{x'}$ and $\Delta k_{y'}$ are the contributions of $\Delta x'$ and $\Delta y'$ to Δk. Δk is rounded-off to the nearest integer. Figure 6 shows how $\Delta x'$ and $\Delta y'$ contribute to Δk. Equations for $\Delta k_{x'}$ and $\Delta k_{y'}$ are written as

$$\Delta k_{x'} = \Delta x' \zeta_k \sqrt{(1-\mu_j^2)}\frac{\mu_j}{|\mu_j|}, \quad \Delta k_{y'} = \Delta y' \zeta_k \mu_j \tag{21}$$

where

$$\zeta_k = \frac{\text{radius of sphere}}{\text{distance between two grids}} = \frac{0.75(1-\varepsilon)\cdot S_r}{\Delta\tau}. \tag{22}$$

(v) We calculate $\langle\Phi\rangle(\mu_j \to \mu_i, \Delta k)$, i.e. the phase function from μ_j to μ_i that is transported by Δk number of grid points. Note that

$$\langle\Phi\rangle(\mu_j \to \mu_i) = \sum_{\Delta k} \langle\Phi\rangle(\mu_j \to \mu_i, \Delta k). \tag{23}$$

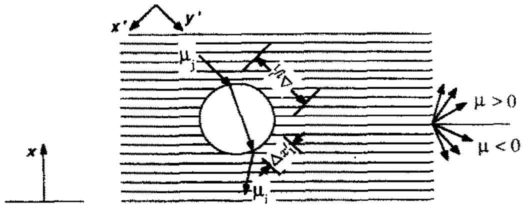

FIG. 6. Sketch of transportation effect in a particulate medium.

4.3. *DIDOM*

The one-dimensional radiative transfer equation at x and in direction μ_i can be written as

$$\mu_i \frac{\mathrm{d}I_i(x)}{\mathrm{d}x} = -\langle\sigma_{ex}\rangle I_i(x) + \langle\sigma_a\rangle I_b(x) + \frac{\langle\sigma_s\rangle}{2}\Gamma_i(x)$$

$$\text{for } i = -M, -M+1, \ldots, M, i \neq 0 \tag{24}$$

where the Γ_i term represents the in-scattering term and accounts for in-scattering into the direction μ_i at location x from all directions at x as well as from all directions at other x locations. After discretization, by evaluating equation (24) at the mid-point between two nodes (k and $k+1$) as in Fiveland [7], the in-scattering term can be written as

$$\Gamma_{i,k+1/2} = \sum_{k'=1}^{n} \sum_{j=-M, j\neq 0}^{M} \Delta\mu_j I_{j,k'+1/2} \langle\Phi\rangle(\mu_j \to \mu_i, \Delta k)$$

$$\text{for } \Delta k = k - k' \tag{25}$$

where $\Delta\mu_j$ are the quadrature weights corresponding to the direction μ_j and

$$\sum_{j=-M, j\neq 0}^{M} \Delta\mu_j = 2. \tag{26}$$

The boundary conditions are

$$I_i = \varepsilon_r I_b + \rho_s I_{-i} + 2\rho_d \sum_{j=-1}^{M} \Delta\mu_j I_j \mu_j$$

$$i = 1, \ldots, M \quad \text{at } x = 0 \tag{27}$$

$$I_i = \varepsilon_r I_b + \rho_s I_{-i} + 2\rho_d \sum_{j=1}^{M} \Delta\mu_j I_j \mu_j$$

$$i = -1, \ldots, -M \quad \text{at } x = L. \tag{28}$$

In the case of incident radiation on a transparent boundary, the above equation is used with $\varepsilon_r = 1$, $\rho_s = 0$ and $\rho_d = 0$. The intensity at the boundary, in a direction μ_i, is equal to the intensity of the incident radiation in that direction.

4.4. *Solution procedure*

As a first step, the values of $\langle\Phi\rangle[\theta_0(\mu_j, \phi_j \to \mu_i, \phi_i), P]$ are calculated at discrete values of $(\phi_j - \phi_i)$ for all combinations of μ_i and μ_j, and the corresponding Δk values are calculated to obtain $\langle\Phi\rangle(\mu_j \to \mu_i, \Delta k)$. For $\mu_i > 0$, the intensities at $x = 0$ ($k = 1$) are known from the boundary conditions. $I_i(\mu_i > 0)$ is evaluated at $k = 1, \ldots, n$. Similarly, $I_i(\mu_i < 0)$ is evaluated at $k = n, \ldots, 1$.

The in-scattering term $\Gamma_{i,k+1/2}$ is stored in a two-dimensional array, which is updated at every point, e.g. while calculating the scattering phase function from direction j to direction i at point $k+1/2$ for $\mu_i > 0$

$$\Gamma_{i,k+1/2+\Delta k} = \Gamma_{i,k+1/2+\Delta k} + \Delta\mu_j I_{j,k+1/2}\langle\Phi\rangle(\mu_j \to \mu_i, \Delta k). \quad (29)$$

This calculation is carried out for scattering into other directions, i.e. for different values of i. It is then repeated for all positive values of μ_j. Then, the $I_{j,k+1}$ ($\mu_j > 0$) are calculated and updated and $\Gamma_{j,k+1/2}$ ($\mu_j > 0$) [used to calculate $I_{j,k+1}$ ($\mu_j > 0$)] is set to zero. This procedure is carried out for $k = 1, \ldots, n$.

After the sweep for $\mu_j > 0$ is complete, the calculation for I_i ($\mu_j < 0$) is carried out at $k = n, \ldots, 1$ in a similar manner.

5. RESULTS AND DISCUSSION

Figures 7(a) and (b) show the transmittance through a bed of specularly reflecting opaque spheres as a function of the bed thickness. The particles are assumed to have a constant reflectivity. Figure 7(a) is plotted for a porosity of 0.476 and Fig. 7(b) for a porosity of 0.732. As expected, the scaled results show the same bulk behavior as the Monte Carlo results. However, the results are offset by a difference that occurs at the boundaries, where the bulk properties are no longer valid. The difference is more pronounced for $\varepsilon_r = 0.1$ than for $\varepsilon_r = 0.4$. This is because for $\varepsilon_r = 0.1$ a large amount of energy is reflected at the surface of the bed (before the continuum treatment becomes applicable). The results obtained from the Kamiuto correlated theory are found to overpredict the transmission.

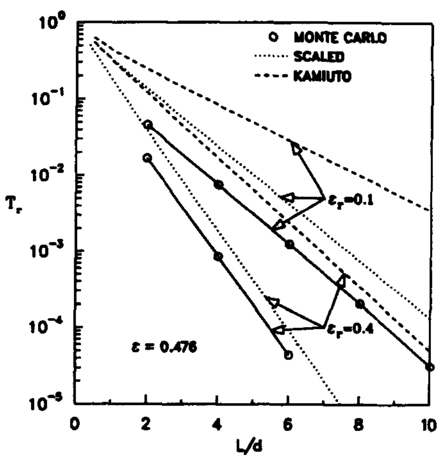

FIG. 7(a). Transmittance through a medium of specularly reflecting, opaque particles ($\varepsilon = 0.476$).

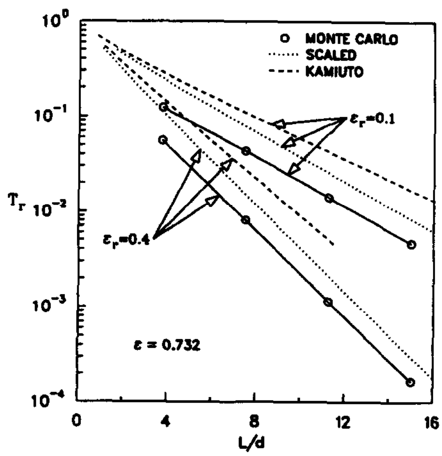

FIG. 7(b). Transmittance through a medium of specularly reflecting, opaque particles ($\varepsilon = 0.732$).

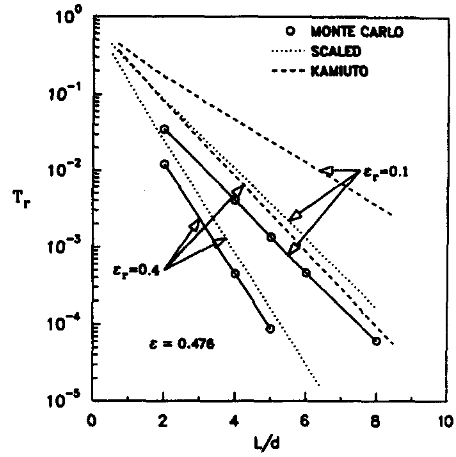

FIG. 8. Transmittance through a medium of opaque particles with a diffuse phase function ($\varepsilon = 0.476$).

Figure 8 is plotted for diffusely reflecting particles ($\varepsilon = 0.476$). The results are similar to Fig. 7(a) except that the transmittances are lower because the diffuse phase function is more backscattering. The scaling factor was calculated from equation (5). Again, the results obtained from the Kamiuto correlated theory are found to overpredict the transmission while the scaled solution shows good agreement with the Monte Carlo solution. Thus, the scaling factors obtained from equation (5) can be used for different phase functions as long as the particles are opaque.

Figures 9(a) and (b) illustrate the change in the transmittance as a function of bed thickness for *transparent* and *semi-transparent* particles. Figure 9(a) is plotted for a porosity of 0.476 and Fig. 9(b) for a porosity of 0.732. The spheres have a refractive index, $n = 1.5$. Three different absorptivities are considered, i.e. $\kappa\alpha_r = 0$, $\kappa\alpha_r = 0.05$ and $\kappa\alpha_r = 0.2$, giving $\eta_a = 0$, $\eta_a = 0.158$ and $\eta_a = 0.479$, respectively. The results of the DIDOM are in good agreement with those of the Monte Carlo method for all these cases and for both of the porosities considered. The results from the Kamiuto correlated theory underpredict the transmission for transparent particles. As the absorption of the particle is increased, the results become closer to the Monte Carlo results. In the limiting case of opaque particles, the correlated theory overpredicts the transmittance.

6. CONCLUSIONS

A method for modelling dependent radiative heat transfer through beds of large spherical particles is

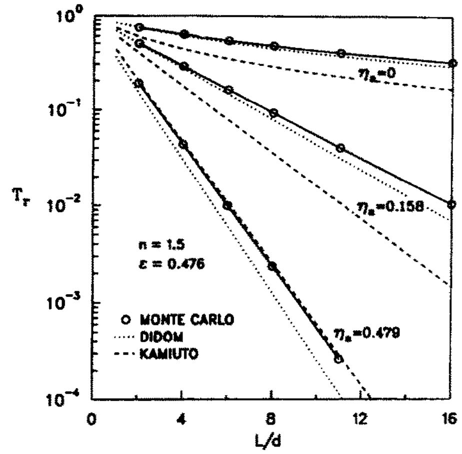

FIG. 9(a). Transmittance through a medium of semi-transparent particles ($\varepsilon = 0.476$, $n = 1.5$).

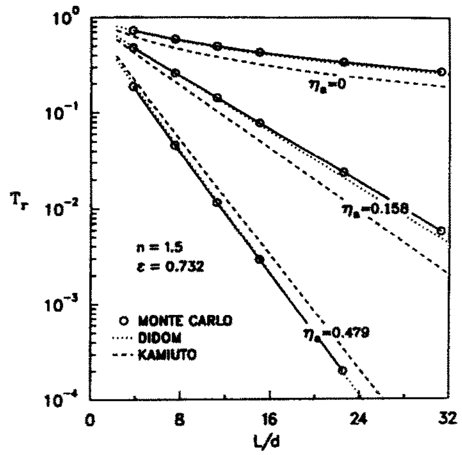

FIG. 9(b). Transmittance through a medium of semi-transparent particles ($\varepsilon = 0.732$, $n = 1.5$).

developed. The dependent properties for opaque particles are obtained by scaling the optical thickness obtained from the independent theory. Radiative transfer through semi-transparent particles is modelled by allowing for the transmission through the particle while solving the equation of radiative transfer, thus resulting in the dependence included discrete ordinates method (DIDOM). The main conclusions obtained from this study are given below.

- The scaling factor for *opaque* spheres is primarily a function of the porosity and is almost independent of the emissivity and the phase function. Thus the optical thickness obtained from the theory of independent scattering can be scaled by the scaling factor (S_r) leaving the albedo and the phase function unchanged.
- The transportation of rays through an optical thickness while passing through a *semi-transparent* particle can be modelled by allowing for this phenomenon while solving the equation of radiative transfer.
- The DIDOM allows for the *transportation* effect and is found to be in good agreement with the results obtained from the Monte Carlo method.
- The results obtained from the Kamiuto correlated scattering theory do not in general show good agreement with the results obtained from the Monte Carlo method.

It must be noted that radiative heat transfer in packed beds is influenced by the solid conductivity because radiation absorbed at one face of the particle may be emitted from the other face. Therefore the DIDOM as described here should not be used to predict radiative heat transfer in packed beds when the solid conductivity is large. The effect of solid conductivity on radiative heat transfer is currently being modelled and will be the subject of a forthcoming communication.

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MODELISATION DU TRANSFERT THERMIQUE RADIATIF DANS DES LITS FIXES

Résumé—On présente une approche par modélisation du transfert thermique radiatif dans des lits fixes à grosses particules sphériques (domaine géométrique). Pour un tel système de grosses sphères opaques, les propriétés dépendantes peuvent être obtenues à partir des grandeurs indépendantes par une analyse d'échelle sur l'épaisseur optique en laissant inchangés l'albedo et la fonction de phase. Le facteur d'échelle dépend principalement de la porosité et il est plutôt indépendant de l'émissivité. On montre qu'une telle approche pour les particules semi-transparentes n'est pas possible. Celles-ci sont traitées en considérant un déplacement à travers une épaisseur optique (à cause de la transmission à travers une particule) pendant la résolution de l'équation du transfert radiatif. Ceci conduit à une méthode puissante de résolution appelée méthode de dépendance incluse dans les ordonnées discrètes (DIDOM). Les résultats obtenus par DIDOM sont en bon accord avec ceux donnés par la méthode Monte Carlo.

MODELLIERUNG DES STRAHLUNGSWÄRMEAUSTAUSCHS IN SCHÜTTUNGEN

Zusammenfassung—Es wird eine weithin gültige Annäherung für den modellabhängigen Strahlungswärmeaustausch in Schüttungen mit geometrisch großen kugelförmigen Partikeln vorgelegt. Solch ein System großer Kugeln liegt selbst für große Porositäten im abhängigen Bereich. Es wird gezeigt, daß die abhängigen Stoffeigenschaften einer Schüttung opaker Kugeln aus ihren unabhängigen Stoffeigenschaften gewonnen werden können, indem die optische Dicke skaliert wird. Dabei bleiben das Albedo und die Phasenfunktion unverändert. Der Skalierungsfaktor erwiest sich als primär von der Porosität abhängig und ist fast unabhängig von der Emissivität. In der Arbeit wird gezeigt, daß solch ein einfaches Skalierungsverfahren für nicht-opake Partikel nicht durchführbar ist. Transparente und halbtransparente Partikel werden behandelt, indem die Verschiebung parallel zu einer optischen Dicke (wegen der Transmission durch ein Partikel) bei der Lösung der Gleichung für den Strahlungswärmeaustausch berücksichtigt wird. In Kombination mit der Skalierungsannäherung ergibt dies eine überzeugende Lösungsmethode, deren Ergebnisse gut mit Resultaten aufgrund des Monte-Carlo-Verfahrens übereinstimmen.

МОДЕЛИРОВАНИЕ РАДИАЦИОННОГО ТЕПЛОПЕРЕНОСА В УПАКОВАННЫХ СЛОЯХ

Аннотация—Описывается способ моделирования радиационного теплопереноса с учетом коллективного рассеяния в слоях крупных сферических частиц (приближение геометрической оптики). Такая система крупных сфер должна рассматриваться с учетом коллективного рассеяния даже при больших порозностях. Показано, что при этом свойства слоя непрозрачных сфер можно определить по их характеристикам независимого рассеяния посредством пересчета оптической толщины без изменения альбедо и фазовой функции. Найдено, что коэффициент пересчета зависит преимущественно от порозности и почти не зависит от степени черноты. Показано, что такой простой пересчет невозможен для прозрачных частиц. Прозрачные и полупрозрачные частицы исследуются с учетом смещения по оптической толщине (обусловленного пропусканием через частицу) при решении уравнения радиационного переноса. Такой подход при его комбинации с методом пересчета дает удобную модификацию метода дискретных ординат (DIDOM). Результаты, полученные методом DIDOM, хорошо согласуются с результатами, найденными методом Монте Карло.