Sequential Development*

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One of the more interesting issues in economics is the conversion of land to alternative uses or densities. Decisions to convert land determine the pricing and spatial structure of urban areas. An analysis of the economic effect of urban policies like zoning or growth controls must include the impact on conversion decisions. Unfortunately a fully rational analysis that incorporates the stochastic nature of urban cash flows and the possibility of multiple future redevelopments is very complex. This paper analyzes the impact of multiple conversions under certainty on the value and density of urban land relative to models with only one conversion. We find that, even with only one additional conversion, land values are 10–15% higher. The density of the initial development is much lower (about half) and the density of the second is much higher (about twice) relative to the case with only one conversion. Hurdle rents also vary widely from the single-conversion case. © 1993 Academic Press, Inc.

1. INTRODUCTION

One of the more interesting issues in economics is the conversion of land to alternative uses or densities. Decisions to convert land determine the pricing and spatial structure of urban areas. An analysis of the economic effect of urban policies like zoning or growth controls must include the impact on conversion decisions. While there is now a substantial literature on land conversion decisions, most of the models sidestep the issue of multiple conversions because of the complexity it introduces. It is usually assumed either that only one conversion will take place or that the developer is myopic. Under myopia the developer acts as if either no additional conversions will occur or no growth of rents will occur. Neither approach is entirely satisfactory.

In this paper we investigate models with multiple conversions under certainty and assess the impact on values, densities, and hurdle rents.[1] We find that, even with only one additional conversion, land values are 10–15% higher. The density of the initial development can be much lower (about half) and the density of the second much higher (about twice)

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[1] The hurdle rent is the critical rent at which conversion takes place.

relative to the case in which only one conversion is allowed. Hurdle rents also vary widely from the single conversion case.

We also analyze the interaction between zoning, rent controls, and land development. We show that either zoning or rent controls can be used to obtain a target density; however, if both are used simultaneously, the target density may not be achieved.

Much recent work on land conversion has focused on the effect of uncertainty (Clarke and Reed [12], Capozza and Helsley [8], Capozza and Schwann [9], Capozza and Sick [10], Williams [19], Capozza and Li [11]). In developing models with uncertainty, researchers usually have assumed that only one conversion takes place or that landowners are myopic with respect to future redevelopment and ignore redevelopment possibilities. Like the uncertainty models, the model presented in this paper is a highly streamlined model of land conversion. However, unlike these uncertainty models where the simplified structure facilitates the analysis of uncertainty, ours is a certainty model and we focus on the effect of multiple conversions. Like uncertainty, multiple conversions introduce considerable complexity and the simple structure allows insights to be more easily garnered. Our purpose is to explore how much violence is committed by ignoring future redevelopment.

While most models of land use since the seminal work of Mills [17] address land use over space, our model, like those of Evans [13] and Arnott and Lewis [2], is non-spatial and focuses on land use over time. Unlike much of the research on deterministic models, our research suppresses the consumption side. In the spirit of many of the uncertainty models, rents are exogenous. Both rents and the production function for space are specified in general terms, but the illustrative examples use linear rent growth and Cobb–Douglas production technology.

Development of land can be economically irreversible if the costs of demolition are high enough to prohibit any redevelopment (Anas [1]). An alternative assumption, and the one used in this paper, is that durable capital can be demolished and replaced (Brueckner [5], Wheaton [18]).

Models of residential growth vary in assuming myopia or perfect foresight. The myopic developer takes some conditions of the present to be those of the future (Anas [1], Brueckner [5], Harrison and Kain [15]). If developers have perfect foresight, development and redevelopment maximize profits over an infinite horizon (Braid [4], Brueckner [6], Capozza and Helsley [7], Fujita [14], Mills [16]). We investigate myopia with respect to future redevelopments in this paper.

Brueckner [6] was the first author to consider redevelopment with perfect foresight. Rents are constant over time in his model. Our model is in the spirit of other perfect foresight models but with multiple conversions and rising rents incorporated into the decision. The complexity arises

because redevelopment takes place at increasing densities when rents are rising over time. The problem is not stationary as in the classic tree cutting models and cannot be solved by standard dynamic programming methods.

The next section describes the problem and provides some preliminary results on multiple conversions. The third section considers zoning and rent controls in the multiple conversion context. The fourth presents the general problem and the final section summarizes and concludes.

2. PROBLEM FORMULATION

We begin by considering optimal land conversion for a plot of land in a market with rising urban rents. As rents rise, the optimal intensity of land use also rises. Because existing capital must be demolished before redevelopment, the sequence of redevelopments is discrete and lumpy.

The following notation will be used:

Notation

q_i = Space per unit of land; i.e., the density of development after i conversions, $i \in \{1, 2, \ldots, \infty\}$.

K_i = Quantity of capital employed in the ith conversion.

t = Time (> 0).

$R(t)$ = Rent at time t.

r = Real interest rate (assumed constant).

C = Cost of a unit of capital K (assumed constant).

t_i = Time of the ith conversion.

Assumptions

A1.   Capital is durable and does not depreciate.

A2.   The existing capital is costlessly demolished when redevelopment takes place.

A3.   The production function for space per unit of land q is Cobb–Douglas normalized,

$$q(K) = K^\gamma \qquad 0 < \gamma < 1.$$

A4.   The cost of development is linear in the quantity of capital employed; i.e., it costs $CK\$$ to construct $q = K^\gamma$ units of space.

A5. $R(t)$ is a deterministic non-decreasing, unbounded function of time such that $R(t) \leq \bar{R} + R(0)\exp(gt)$ for constants $\bar{R}$ and g such that $g < r$ and $\bar{R} \geq 0$. This condition guarantees that the present value of future rents is finite.

The objective of a landowner is to choose a sequence of redevelopments that maximizes the value of the land starting from land in a vacant state (i.e., $q_0 = 0$) at time t. This implies that he must maximize the present value of future rents less the present value of the costs incurred in future redevelopment.

The problem can be stated as

$$V[R(t)] = \max_{t_i, K_i, N} \sum_{i=1}^{N} \left\{ \int_{t_i}^{t_{i+1}} q(K_i) R(\tau) e^{-r(\tau - t)} d\tau - CK_i e^{-r(t_i - t)} \right\}, \quad (1)$$

where $N \in \{1, 2, \ldots, \infty\}$ is the number of conversions.

Hence, we have the sequence illustrated in Fig. 1. Given that the rents are deterministic, the optimal redevelopment times and densities as well as the total number of redevelopments are deterministic. Note that (1) can be readily extended to include demolition costs by adding inside the braces the term $-D(K_{i-1}) e^{-r(t_i - t)}$, where $D(K)$ is the cost of demolishing K units of capital. The subsequent analysis follows through with only minor modifications. For simplicity we ignore demolition costs.

For ease of subsequent notation, define

$$p(t, s) = \int_t^s R(\tau) e^{-r(\tau - t)} d\tau, \quad (2)$$

i.e., the present value at time t of future rents (for one unit of space) from time t to time s given that the current rent is $R(t)$. In analyzing the
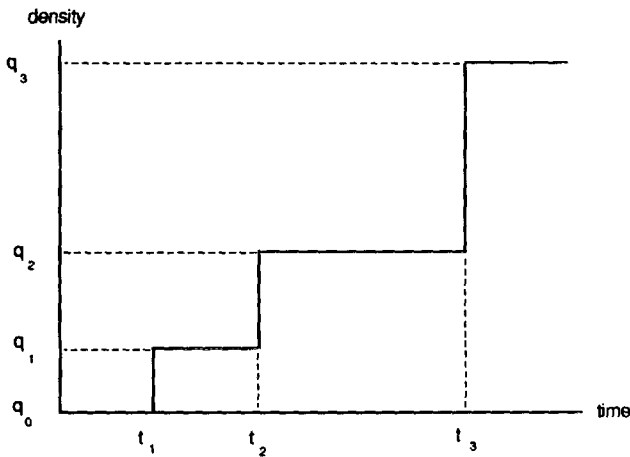

FIG. 1. Sequential development.

decision to convert from q_i to q_{i+1}, the following first-order conditions must be satisfied.

First-order condition with respect to capital K at conversion time t_i

Since capital created at time t_i is demolished at time t_{i+1}, the marginal revenue product of capital given by $q'[K_i]p(t_i, t_{i+1})$ must equal the cost of capital. The first-order condition can be written as

$$C = q'(K_i)p(t_i, t_{i+1}). \tag{3}$$

With the Cobb–Douglas production function this becomes

$$\gamma K_i^{\gamma-1} p(t_i, t_{i+1}) = C. \tag{3a}$$

First-order condition with respect to conversion times

The conversion from a density $q_{i-1} = q(K_{i-1})$ to density $q_i = q(K_i)$ will occur when the rent foregone by waiting for an additional unit of time $\Delta t, (q_i - q_{i-1})R(t)(\Delta t)$, equals the opportunity cost of the capital, $rCK_i(\Delta t)$, needed in the conversion. Therefore, the F.O.C. with respect to the ith conversion time is

$$(q_i - q_{i-1})R(t) = rCK_i. \tag{4}$$

Under Assumptions A1–A5, the solution to the landowner's problem is an infinite sequence of redevelopments in which the density increases without bound. The following argument illustrates by contradiction. Assume that there are only n redevelopments ($n < \infty$); i.e., the number of redevelopments is finite. Let the nth (and final) redevelopment occur at time t_n and density q_n. Fix some $q > q_n$. The cost of construction for q units of space is $Cq^{1/\gamma}$. Conversion to density q at time $t > t_n$ is a positive NPV transaction if

$$(q - q_n)p(t, \infty) \geq Cq^{1/\gamma}. \tag{5}$$

From definition (2), $p(t, \infty)$ is unbounded since $R(t)$ is unbounded, and since rents are non-decreasing, $p(t, \infty) \geq (R(t)/r)$. Therefore (5) can always be satisfied eventually. The developer will always convert at least one more time to a higher density. This also implies that the density of development is unbounded. This follows since successive conversions must be at strictly increasing densities from Eq. (4).

PROPOSITION 1. *With multiple conversions, the first conversion date is earlier and the development density is smaller than those obtained with only one development if $p(t, \infty)/R(t)$ is non-increasing in t.*

Proof. From the F.O.C. for the development time,

$$q_1 R(t_1) = rCK_1 = rCq_1^{1/\gamma}$$

$$\therefore R(t_1) = rCq_1^{1/\gamma - 1} \tag{6}$$

Since $1/\gamma - 1 > 0$, the conversion density is an increasing function of rent. If the rent is higher at a conversion date, then the density will also be higher. This is independent of how many conversions occur subsequently or the functional form for the rent.

First consider the multiple conversions case. The first-order condition with respect to K implies that

$$\gamma K^{\gamma - 1} p(t_1, t_2) = C. \tag{7}$$

Combining (6) and (7), defining $R(t_i) = R_i$, and noting that $p(t_1, t_2) = p(t_1, \infty) - e^{-r(t_2 - t_1)} p(t_2, \infty)$, we have

$$\frac{\gamma r C}{R_1} \left[p(t_1, \infty) - e^{-r(t_2 - t_1)} p(t_2, \infty) \right] = C.$$

Therefore,

$$\frac{R_1}{r} = \gamma \left[p(t_1, \infty) - e^{-r(t_2 - t_1)} p(t_2, \infty) \right].$$

Rearranging gives

$$\frac{R_1}{r} - \gamma p(t_1, \infty) = -e^{-r(t_2 - t_1)} p(t_2, \infty). \tag{8a}$$

If we permit only one development, (8a) can be rewritten as

$$\frac{R_1}{r} - \gamma p(t_1, \infty) = 0. \tag{8b}$$

Clearly, with multiple conversions, the right-hand side of (8a) is a negative quantity. Therefore, the left-hand side with multiple conversions must be less than the left-hand side with only one conversion. If this term is an increasing function of t_1, then the value of t_1 which solves (8b) must always be greater than any corresponding solution of (8a). To complete the proof we need to show that

$$\frac{R(t)}{r} - \gamma p(t, \infty)$$

is increasing in t or that

$$\frac{R(t)}{r}\left(1 - \frac{r\gamma p(t,\infty)}{R(t)}\right)$$

is increasing in t. Since $R(t)/r$ is increasing, it is sufficient for $p(t,\infty)/R(t)$ to be non-increasing in t.   Q.E.D.

The condition that $p(t,\infty)/R(t)$ be non-increasing in t holds when $R(t)$ is no more convex than an exponential function. Exponential growth is an extreme case in models of urban land use. Corner solutions and perverse comparative statics can arise when this condition is violated. If the growth rate dR/R is increasing, it will exceed the interest rate at some finite time. As a result the present value, $p(t,\infty)$ will be infinite. Since we do not observe infinite prices (notwithstanding golf course values in Japan in the late 1980s) the condition is fairly weak.

Illustrative Example: Linear Growth in Rents and Two Conversions

The following example illustrates the previous proposition. Let the rent process be additive, which satisfies A5:

$$R(t) = R(0) + Gt. \tag{9}$$

Case A.   For the single-conversion case the first-order conditions are

$$q'(K^*)p(t^*,\infty) = C \tag{10a}$$

and

$$q^*R^* = rCK^* = rCq^{*1/\gamma}, \tag{10b}$$

where asterisks indicate the optimal values in the single-conversion case. Equation (10a) can be rewritten as

$$\gamma q^{*(\gamma - 1/\gamma)}\left[\frac{R^*}{r} + \frac{G}{r^2}\right] = C$$

since $p(t^*,\infty) = R^*/r + G/r^2$.

Solving these equations simultaneously yields the hurdle rent for conversion,

$$R^* = \frac{\gamma}{1 - \gamma}\left[\frac{G}{r}\right]. \tag{11}$$

Note that the hurdle rent is increasing in G and γ, but decreasing in r and is invariant to construction cost. As construction costs increase, the marginal product of capital increases as K declines. With the unit elasticity of substitution of the Cobb–Douglas production function, the decline in K offsets the increase in C so that the first-order conditions are still satisfied.

Case B.   With two conversions, the first-order condition with respect to K implies that

$$q'(K_1)p(t_1,t_2) = C,$$

$$\gamma q_1^{(\gamma-1)/\gamma}\left[\frac{G}{r^2} + \frac{R_1}{r} - e^{-r(t_2-t_1)}p(t_2,\infty)\right] = C. \tag{12}$$

Substituting in the first-order condition with respect to time,

$$q_1 R_1 = rCq_1^{1/\gamma}, \tag{13}$$

and solving yield

$$R(t_1) = \frac{\gamma}{1-\gamma}\left(\frac{G}{r} - rx\right), \tag{14}$$

where

$$x = e^{-r(t_2-t_1)}p(t_2,\infty) > 0.$$

Therefore, the hurdle rent with multiple conversions is lower than the hurdle rent with a single conversion. This also implies that the density of development in the first case is also lower than that in the second case.

Some authors (for example, Brueckner [5]) have solved the problem by applying a myopic optimality condition in which the developer assumes that the building will last forever even though at some future date he will redevelop. The developer anticipates future rents but not future redevelopments.

PROPOSITION 2.   *Myopia with respect to future redevelopment (but not future rents) will cause higher densities to be attained earlier. Correspondingly, the number of conversions in any given period also will be smaller under myopia.*

Proof.   Under myopia, the first-order condition with respect to K is

$$C = q'[K(R_i)] \cdot p(t_i,\infty)$$

rather than

$$C = q'(K(R_i)) \cdot p(t_i, t_{i+1}).$$

This condition can be rewritten as

$$q_{i+1} = \left[\frac{\gamma p(t_{i+1}, \infty)}{C} \right]^{\gamma/(1-\gamma)}.$$

Since $p(t_{i+1}, \infty) > p(t_{i+1}, t_{i+2})$, myopia will induce higher densities at each conversion date.   Q.E.D.

Under myopic optimality, the general solution is simple. One can start from Date 0 and solve the first-order conditions using a forward recursion. Under full optimality, the solution must follow backward recursion. Therefore, to obtain an explicit solution one must fix the number of conversions. Before addressing the backward recursion problem, we establish results for zoning and rent control.

3. THE EFFECT OF ZONING AND RENT CONTROL

A common feature in most urban areas is the existence of zoning laws which bound the density of development. In this section, we study the effect of such laws.

PROPOSITION 3.   *If zoning laws limit the density of development to q_{max}, then the maximum density will always be attained.*

Proof. Suppose that the maximum density is not attained and development stops at $\bar{q} < q_{max}$. But, conversion from $\bar{q}$ to q_{max} is a positive NPV transaction at time t if $(q_{max} - \bar{q})p(t, \infty) \geq Cq_{max}^{1/\gamma}$. Since rents are unbounded and $p(t, \infty) \geq R(t)/r$, this inequality will always be satisfied eventually. Therefore, the maximum density will always be attained.
                                                    Q.E.D.

Note that in the proof of Proposition 3, the production function that we assumed played a role only to the extent that the marginal cost of production was increasing and that for $q < q_{max}$, the marginal cost of an additional unit of space was finite. Hence, the result holds for any production function satisfying the above properties.

Proposition 3 yields an interesting result. Suppose that there exists an urban property with $q < q_{max}$. This proposition guarantees that it will be redeveloped to q_{max}. But will the property be developed to its maximum density in only one redevelopment or will it undergo one or more intermediate redevelopments? If we consider the possibility of redeveloping today,

then it is easy to see that:

COROLLARY 3.   *If*

$$q_{\max} \leq \left[\frac{\gamma p(t, \infty)}{C} \right]^{\gamma/(1-\gamma)},$$

then full development will take place in only one conversion.

Proof.   The result follows from the first-order condition for the density applied to the current date. Conversions at future dates will always occur at (weakly) higher densities.   Q.E.D.

PROPOSITION 4.   *If rents are controlled, i.e., the rent is bounded, then the maximum density of development is also bounded. Both zoning and rent control result in finite densities. Indeed, given an initial density, q_j, and any density $q_{max} > q_j$, a rent level R_{max} can be chosen so that there is no redevelopment to a density higher than q_{max}.*

Proof.   Consider a situation in which the property owner is trying to redevelop. Let t_i be the time at which the next redevelopment occurs and t_{i+1} be the time at which the subsequent redevelopment occurs. As before, for redevelopment to be worthwhile, the value of the marginal product of capital must equal or exceed the cost of another unit of capital. Here this implies that

$$\gamma q_i^{(\gamma-1)/\gamma} \int_{t_i}^{t_{i+1}} R(\tau) \exp[-r(\tau - t)] \, d\tau \geq C$$

or

$$q_i \leq \left\{ \frac{\gamma}{C} \int_{t_i}^{t_{i+1}} R(\tau) \exp[-r(\tau - t)] \, d\tau \right\}^{\gamma/(1-\gamma)}.$$

Note that $\gamma/(1 - \gamma) > 0$ because $0 < \gamma < 1$. Let the bound on rent be $R_{\max}$; then the R.H.S. is bounded by $[(\gamma/C)(R_{\max}/r)]^{\gamma/(1-\gamma)}$. Hence, the density of redevelopment is bounded by $[(\gamma/C)(R_{\max}/r)]^{\gamma/(1-\gamma)}$.   Q.E.D.

Consider now what happens when both zoning and rent control are in force. Then, the result in Proposition 3 is no longer valid; i.e., it is possible that the maximum density of development may not actually equal the density imposed by the zoning law.

COROLLARY 4A.   *With both zoning and rent controls in force, the maximum density may not be attained.*

A trivial condition can be obtained from Proposition 3. Suppose that $q_{max} > [(\gamma/C)(R_{max}/r)]^{\gamma/(1-\gamma)}$; then q_{max} cannot be attained. An alternative condition that may be more stringent can be developed as follows.

Suppose that (currently or at any subsequent time) the density is $q_{max} - \varepsilon = q_t$ for $\varepsilon > 0$. No redevelopment will occur if

$$(q_{i+1} - q_i)R(\tau) < rCK_{i+1} = rCq_{i+1}^\gamma \le rCq_{max}^\gamma.$$

Since $q_{i+1} - q_i \le q_{max} - q_i = \varepsilon$, no redevelopment will occur if $\varepsilon R(\tau) < rCq_{max}^\gamma$. With an upper bound on rents equal to R_{max}, this will be satisfied when $\varepsilon < rCq_{max}^\gamma/R_{max}$. Hence, no subsequent redevelopment will occur if the current density is less than $q_{max} - rCq_{max}^\gamma/R_{max}$.

COROLLARY 4B. *If rents are bounded (no matter how large the bound relative to the zoning restriction), then full development to the maximum density allowed by zoning cannot be guaranteed.*

Proof. Follows easily from the proof of Proposition 4.

In this section we have highlighted the role of zoning and rent control in the conversion process. We turn now to the general problem.

4. THE GENERAL PROBLEM

We have already proven some general results in our framework, but we are far from solving the complete problem. A direct solution to the general problem (even with specific functional forms for the rent) is not available. Since there are an infinite number of sequential redevelopments where the solution at each stage depends on the solution in the next stage, and the density increases without bound, the problem lacks the necessary stationarity for the application of standard dynamic programming techniques.

Three approaches can be taken to solve the problem. One method is to fix the number of conversions to a finite number. The problem can then be solved working backward from the last conversion. The second method is to relax the optimality condition and replace it with a myopic optimality condition. Individuals assume that rents are constant forever (even though in actuality they are not) when they decide to redevelop. The problem can be solved even if we allow the individual to incorporate future growth, but do not allow the current decision to depend on future possible conversions. This is slightly more general, but the solution is more complicated. A third approach is to fix the sequence of densities at conversion and solve for the time of conversion or alternatively fix the sequence of development

times and solve for densities (Evans [13], Briad [3]). Under some conditions "quasi-stationarity" arises and the solution can be obtained.

None of these approaches is completely satisfactory since all involve some form of sub-optimal behavior. We pursue the first approach here since we wish to explore the relative effect of multiple conversions. It also has the advantage of approaching full optimality as the fixed number of conversions becomes large.

Finite Number of Conversions

Since the problem can be solved for any finite number of conversions, for exposition, we consider two. Let the current time be zero and the two conversion dates be t_1 and t_2 corresponding to development to densities q_1 and q_2.

In Proposition 1, we showed that with multiple conversions, the first conversion date and development density are smaller than those obtained with only one development. Consider now what happens to the final density when only two developments are permitted.

PROPOSITION 5. *With two conversions the final development density is higher than that with only one development.*

Proof. With a single development, from (8b), $R_1/r = \gamma p(R_1, t_1\infty)$. Development occurs when rents have increased sufficiently to satisfy this condition.

With two developments, the first-order conditions are

$$\gamma q_2^{(\gamma-1)/\gamma} p(R_2, \infty) = C$$

$$(q_2 - q_1) R_2 = rCq_2^{1/\gamma}.$$

Combining these two equations yields

$$\frac{R_2}{r} = \gamma p(R_2, \infty)\left(\frac{q_2}{q_2 - q_1}\right).$$

The second development occurs when rents satisfy the above equation. Since $q_2/(q_2 - q_1) > 1$, the right-hand side is higher than if $q_1 = 0$. This implies that the value of the rent required to initiate the second development is higher than the value of the rent required to initiate the development with only one conversion. As the conversion density is a monotonic function of rents irrespective of the number of conversions (from the first-order condition for density of development), this implies the required result.   Q.E.D.

Illustrative Example (Continued): Linear Rent Growth

Returning to our previous example with linear rent growth, we have the first-order conditions with respect to the second conversion

$$q_2 = \left\{ \frac{C}{\gamma\left[\dfrac{R_2}{r} + \dfrac{G}{r^2}\right]} \right\}^{-\alpha}$$

and

$$(q_2 - q_1)R_2 = rCq_2^{1/\gamma},$$

where $\alpha = \gamma/(1 - \gamma)$.

The first-order conditions with respect to the first development are as before. These four equations must be solved simultaneously to obtain the values of the unknown variables t_1, t_2, q_1, and q_2. Unfortunately, these equations cannot be solved in closed form and we rely on numerical results.

Comparative Statics Results

How do the parameter values r, G, C, and γ influence the development process? Further, how do the dynamics for the rent process influence development? We now analyze these issues.

First, consider the effect of these parameters on the development process if there is a single development. With only one development, the hurdle rent is given in (11). The density at conversion then is (from (10b) and (11))

$$q^* = \left[\frac{R^*}{rC}\right]^{\alpha} = \left[\frac{\alpha G}{r^2 C}\right]^{\alpha},$$

where $\alpha = \gamma/(1 - \gamma)$.

Recalling that $q_0 = 0$, property value is

$$V(R(t)) = \int_{t^*}^{\infty} e^{-r(\tau - t)} q^* R(\tau)\, d\tau - CK^* e^{-r(t^* - t)}.$$

The effects of r, G, C, and γ are readily apparent. An increase in the growth rate makes the developer wait for a higher value of R and develop

to a higher density. The effects of increased interest rates are exactly the opposite. An increase in construction costs, as expected, reduces the intensity of development, but does not affect the hurdle rent. As γ increases, the value of α also increases. This implies that the developer waits longer and converts at higher rents and to higher densities as γ increases.

Similar results are available when there is more than one conversion by solving numerically. The numerical results are displayed in Figs. 2 to 4. In Fig. 2 values in the one- and two-development cases are plotted. Values
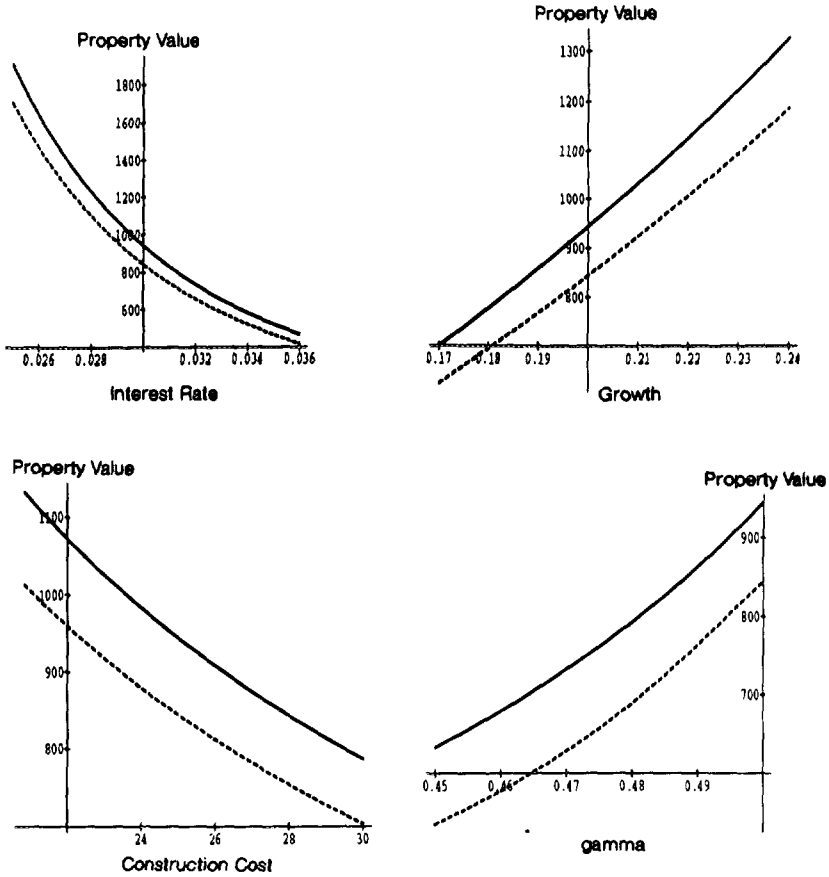

FIG. 2. Comparative statics for property value as a function of the interest rate, growth rate (G), cost of construction (C), and productivity parameter (γ). Default parameter values: interest rate = 0.03, G = 0.2, C = 25, γ = 0.5. Full line, property value with two conversions; dotted line, property value with only one conversion.
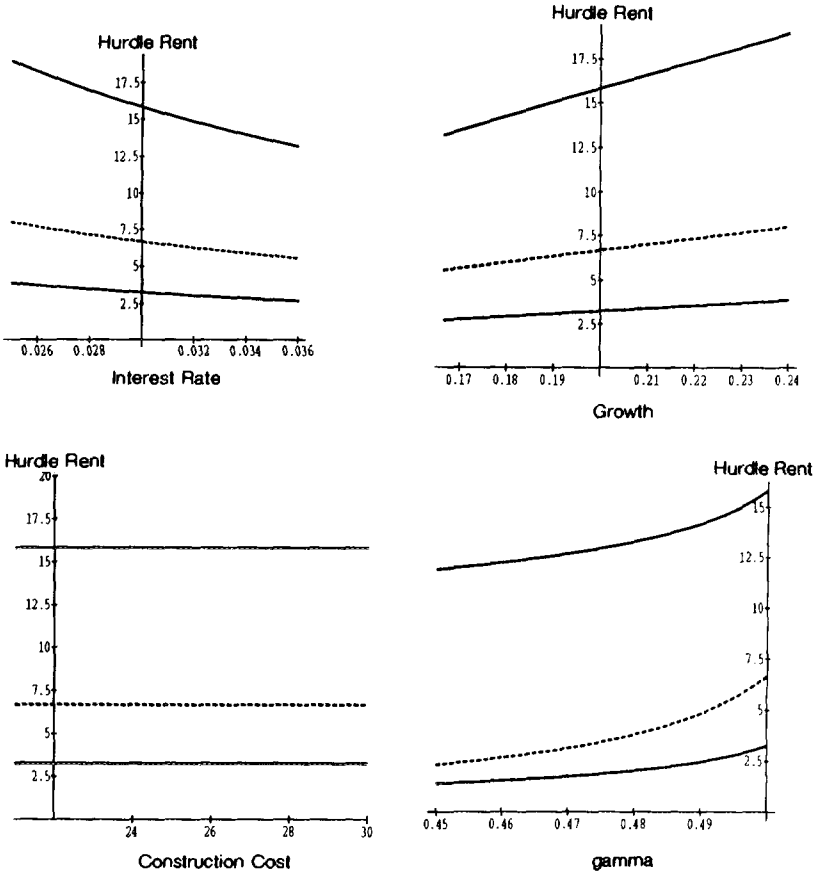
FIG. 3. Comparative statics for hurdle rents as a function of the interest rate, growth rate (G), cost of construction (C), and productivity parameter (γ). Default parameter values: interest rate = 0.03, G = 0.2, C = 25, γ = 0.5. Upper full line; second conversion with two conversions; lower full line, first conversion with two conversions; dotted line, conversion with only one conversion.

are 10–15% higher with only one additional conversion. The comparative statics are similar in the two cases.

Figures 3 and 4 illustrate the bracketing of the one-conversion case by the two-conversion case. In Fig. 3 note that construction costs have no effect on the hurdle rent. This follows from Eq. (11). Intuitively, as construction costs increase, the density of development declines and offsets the need for a higher rent.

Figures 3 and 4 illustrate the dramatic effect of multiple conversions on the hurdle rent and density. The possibility of a second conversion permits
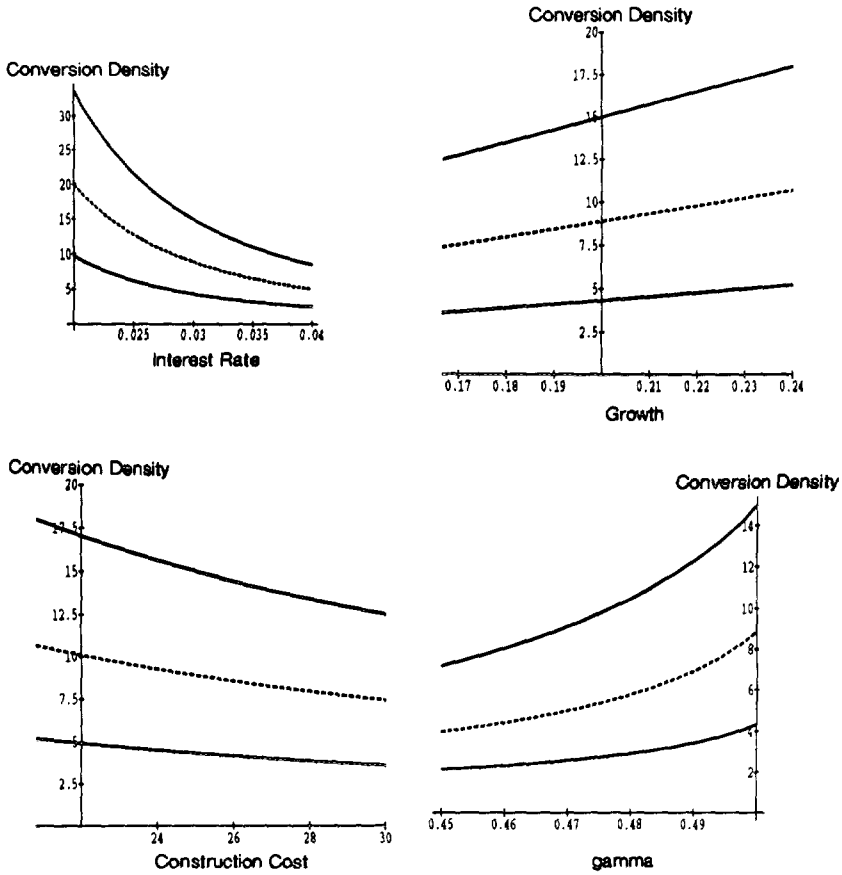
FIG. 4. Comparative statics for conversion density as a function of the interest rate, growth rate (G), cost of construction (C), and productivity parameter (γ). Default parameter values: interest rate = 0.03, G = 0.2, C = 25, γ = 0.5. Upper full line, second conversion with two conversions; lower full line, first conversion with two conversions; dotted line, conversion with only one conversion.

the first conversion to occur at about half the density of the one-conversion case. This implies that results from models with only one conversion or models with myopic optimality may be quantitatively misleading.

5. CONCLUSION

In this paper we have modeled the land conversion decision with multiple conversions, full rationality, and growing rents. This takes us a step closer to understanding the full complexity of the land conversion decision.

We find that the qualitative results under multiple conversion are similar to those of the single-conversion case. However, quantitative results can be dramatically different. Therefore models that assume a single conversion can enhance our intuition of urban spatial development. They will not, however, be useful guides to quantitative values.

We have also shown that either zoning or rent controls can be used to obtain a target density. However, if both are used simultaneously, the target density may not be obtained.

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