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ACOUSTICAL PROPERTIES OF INTERACTING AND AGGLOMERATED PARTICLES

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Acoustical cross-sections of random homogenous dense systems of interacting and agglomerated Rayleigh particles are studied analytically. Both the interaction among particles within the agglomerate and the interaction among the agglomerates are taken into account. A model for the radial distribution function is proposed. The interaction effect on the attenuation of a plane wave is studied.

1. INTRODUCTION

Acoustic agglomeration has potential application in improving the efficiency of conventional particle removal devices, particularly cyclones, where the particulate system is radiated with high intensity sound to increase the agglomeration rate of particles [1, 2]. Also, the phenomenon of agglomeration by small spherical particles into larger clusters is encountered in the solidification of liquid metals and in combustion systems. One of the important products is soot particles. These particles are nearly spherical but they grow by agglomeration and form clusters of different types. Examples are straight chains, random clusters and fractal clusters (Figure 1). To understand the foundations of these two applications and others, the acoustical properties of agglomerated particles need to be evaluated. A usual approach in analyzing the interaction of clusters with electromagnetic waves is based on the definition of an equivalent sphere [3, 4], which is characterized in terms of an effective diameter or effective properties (density and compressibility). In general, the effective diameter is, or properties are, defined as the diameter or properties of an isolated sphere that has the same specified characteristics as the agglomerated clusters. In dealing with the acoustical properties of agglomerations, two interactions need to be considered; the interaction among the particles within the cluster, and the interaction among clusters.

2. ANALYSIS

Consider a homogeneous system of randomly positioned and uniformly distributed identical clusters in a fluid of density ρ and compressibility κ. Let the dimensions of clusters be small compared to the wavelength λ (the Rayleigh limit), the mean distance between clusters be no larger than λ, and the dimensions of the fluid be considerably larger than λ. The interaction of a plane acoustic wave proportional to e^{ikz}, k $(=2\pi/\lambda)$ being the propagation constant, with a cloud of particles is described by the extinction and scattering

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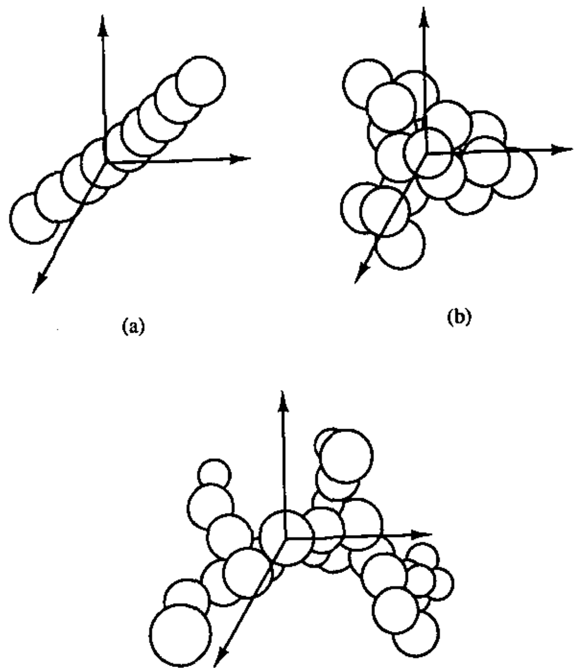

Figure 1. Types of clusters: (a) straight chains; (b) fractal clusters; (c) random clusters.

cross-sections [5]

$$C_{sN}=(1/k^2)\sum_l\sum_j\int_{4\pi}|S_j(0)|^2\, e^{[ik(\hat{e}_z-\hat{e}_r)\cdot(\hat{r}_j-\hat{r}_l)]}\, d\Omega, \tag{1}$$

$$C_{eN}=(4\pi/k)\,\mathrm{Im}\left[\sum_j S_j(0)\right], \tag{2}$$

where S is the angle distribution factor, $\hat{r}_j$ is the position vector of the jth particle, $\hat{e}$ is the unit vector and Im refers to the imaginary part; for a single particle, the angle distribution factor $S(\theta)$ is

$$S(\theta)=\tfrac{1}{3}k^2a^3[(\varepsilon_\kappa-1)+3((1-\varepsilon_\rho)/(2+\varepsilon_\rho))\cos(\theta)], \tag{3}$$

where a is the radius of particle, $\varepsilon_\kappa=\kappa_p/\kappa$, $\varepsilon_\rho=\rho/\rho_p$, and subscript p refers to the property of the particle. In terms of this factor, the independent cross sections for a particle are

$$C_{sp}=(4/9)\pi a^2(ka)^4[|\varepsilon_\kappa-1|^2+3|(1-\varepsilon_\rho)/(2+\varepsilon_\rho)|^2], \tag{4}$$

$$C_{ap}=\tfrac{4}{3}\pi a^2(ka)\,\mathrm{Im}\,[\varepsilon_\kappa-1+3((1-\varepsilon_\rho)/(2+\varepsilon_\rho))]. \tag{5}$$

Under the assumption that each particle in the agglomerate scatters and absorbs sound unaffected by the presence of other particles, the extinction and scattering by the agglomerate may be expressed by a simple algebraic addition of the energy extinguished and scattered by each primary particle. The cross-section for the agglomerate is the sum of the cross-sections of each particle, and the individual particles are assumed to scatter and absorb sound independent of others. For identical particles this leads to the following expressions

$$C_{eA} = N_c C_{ep}, \qquad C_{sA} = N_c C_{sp}, \tag{6}$$

where the subscript A denotes the agglomerate of N_c particles.

The equivalent sphere of the agglomerate is defined in terms of equations (4)–(6) as

$$(D_{eff}/D_p)_e = N_c^{1/3}, \qquad (D_{eff}/D_p)_s = N_c^{1/6}, \tag{7, 8}$$

where the subscripts e and s indicate the effective diameter, based on extinction and scattering cross-sections, respectively. The close spacing of particles in an agglomerate invalidates the usual assumption of independent interaction. This is due to the two fundamental effects. The first one is the near-field effect due to the multiple scattering, which modifies the internal field of the primary particle and consequently changes both the extinction and scattering characteristics of the agglomerate. The second one is the far-field effect which results from the coherent addition of scattered waves and only changes the scattering characteristics. Equations (1) and (2) still describe the acoustical properties of interacting particles if S_j stands for the actual angle distribution factor. For small agglomerates, where the particles are very close together, the resulting phase difference of the scattered radiation from different particles is very small. Ignoring the near-field interaction, S_j can be described by the angle distribution factor of an isolated particle. Then, an asymptotic expression for the scattering cross-section in terms of equation (1) leads to

$$C_{sA} = N_c^2 C_{sp}, \tag{9}$$

which implies, in view of equation (4),

$$(D_{eff}/D_p)_s = N_c^{1/3}. \tag{10}$$

In tightly packed small agglomerates, the far field is the dominant mechanism and cannot be ignored. Thus, equation (9) is closer than equation (6) to the actual value of the effective diameter of a sphere that is equivalent to the agglomerate in terms of the scattering cross-section. In this case, equations (7) and (10) neglect the near-field interactions, and in addition, equation (10) is based on the asymptotic behavior of coherent addition. To account for these effects, the following are defined

$$(D_{eff}/D_p)_e = N_c^{1/3}\eta_e, \qquad (D_{eff}/D_p)_s = N_c^{1/3}\eta_s, \tag{11, 12}$$

where η_e is the correction factor which accounts for the modification due to the near-field interactions on the extinction cross-section and η_s is a correction factor which represents the deviation of the scattering cross-section from the asymptotic behavior of coherent addition, as well as the modification due to near-field interaction. The next section describes the method of evaluation of these factors.

3. CORRECTION FACTORS

Consider a homogeneous system of randomly positioned and uniformly distributed identical particles in a fluid of density ρ and compressibility κ. Let the dimensions of particles be small compared to the wavelength λ (the Rayleigh particles), the mean distance between particles be no larger than λ, the dimensions of the fluid be considerably larger than λ, and the particles be spheres having a (or diameter D); with compressibility κ_p, density ρ_p, and the location of the center of each sphere being represented by the vector

$\bar{r}$. The scattering of an incident wave $P_i(r)$ by a single scatterer is known to satisfy reference [5]:

$$P_w(r) = P_i(r) + \int_{V'} [k^2 \gamma_\kappa P_w(r') - \vec{\nabla}' \cdot [\gamma_\rho \vec{\nabla}' P_w(r')]] G(r, r') \, \mathrm{d}V', \tag{13}$$

where $G(r, r')$ is the Green's function $= (\mathrm{e}^{\mathrm{i}k|r-r'|})/(4\pi|r-r'|)$, $\gamma_\kappa = (\kappa_p - \kappa)/\kappa$ and $\gamma_\rho = (\rho_p - \rho)/\rho_p$. The first term on the right side of equation (13) is the imposed incident wave and the integral term represents the incidence due to scattering from the rest of the volume. Equation (13) assumes the space to be a single entity in which γ_κ and γ_ρ vary with position. For a cloud of particles, after dividing the space into a number of small spherical volumes V_j, within which $(\gamma_\kappa, \gamma_\rho) \neq 0$ and outside which $(\gamma_\kappa, \gamma_\rho) = 0$, equation (13) becomes

$$P_w(r) = P_i(r) + k^2 \sum_{j=1}^{N_c} \int_{V_j} P_w(r_j)(\varepsilon_\kappa - 1) G(r, r_j) \, \mathrm{d}V_j + \sum_{j=1}^{N_c} \int_{V_j} [\vec{\nabla}_j \cdot [\vec{\nabla}_j P_w(r_j)(\varepsilon_\rho - 1)]] G(r, r_j) \, \mathrm{d}V_j, \tag{14}$$

where N_c is the total number of particles in the cluster, and $\bar{r}_j$ is the location of the jth particle. In terms of $k_e^2 = [(\varepsilon_\kappa - 1)/(\varepsilon_\rho - 1)]k^2$, equation (14) may be written as

$$P_w(r) = P_i(r) + k_e^2 \sum_{j=1}^{N_c} \int_{V_j} P_w(r_j)(\varepsilon_\rho - 1) G(r, r_j) \, \mathrm{d}V_j + \sum_{j=1}^{N_c} \int_{V_j} [\vec{\nabla}_j \cdot [\vec{\nabla}_j P_w(r_j)(\varepsilon_\rho - 1)]] G(r, r_j) \, \mathrm{d}V_j. \tag{15}$$

If equation (15) is applied to describe the internal field of a particle l, then the two summations on the right side for $j \neq l$ represent the contribution to the scattered field from the surrounding particles, and the term for $j = l$ contributes to the field as a result of scattering from the rest of the particle itself. For a large number of particles, the contribution of the surrounding particles scattering to the net field is significant, and is taken into account by the introduction of a near-field correction factor. Note that if $P(r)$ is assumed to be the one component of the vector $\vec{P}(r) = P(r)\vec{e}_r$, and that $\vec{\nabla} \cdot (\vec{\nabla} P(r))\vec{e}_r = \vec{\nabla}(\vec{\nabla} \cdot \vec{P}(r))$, then equation (15) is similar to that given by Saxon [6], which describes the internal field of each particle resulting from a plane electromagnetic wave incident on a cloud of particles. Assuming a uniform pressure field within each particle j, and following Jones [3], equation (15) may be rearranged as

$$\left[P_w(r_l) - \frac{\mathrm{i}}{3} \left(\frac{\varepsilon_\rho - 1}{\varepsilon_\rho + 2} \right) \left(\frac{\varepsilon_\kappa - 1}{\varepsilon_\rho - 1} \right)^{3/2} \alpha_p^3 \sum_{j=1, j \neq l}^{N_c} T_{lj} P_w(r_j) \right] = \frac{3}{\varepsilon_\rho + 2} P_i(r_l), \tag{16}$$

where $T_{jl} = a = -h_0^{(1)}(k_e r_{jl}) - h_2^{(1)}(k_e r_{jl})[P_2(\cos\theta_{jl}) - 0{\cdot}5 \cos(2\phi_{jl}) P_2^{(2)}(\cos\theta_{jl})]$, $h_m^{(n)}$ and $P_m^{(n)}$ are the spherical Bessel functions and the associated Legendre functions, respectively, and θ and ϕ are the polar and azimuthal angles. The subscript jl indicates the relative angle

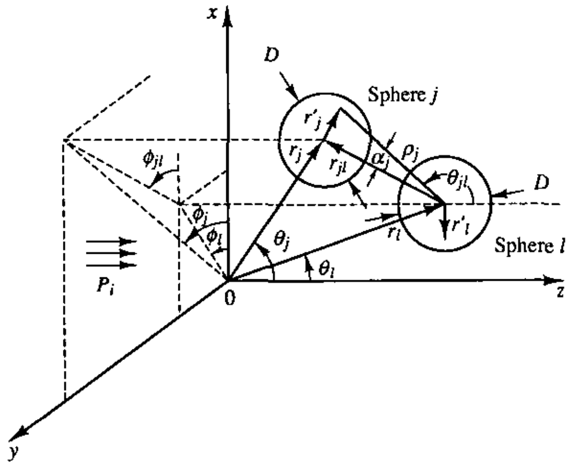

Figure 2. Co-ordinate geometry.

between particles j and l (Figure 2). Equation (16) represents a system of N_c coupled linear algebraic equations for the N_c unknown pressure at the center of each particle. For a large number of particles, equation (16) is difficult to solve. Also, the evaluation of the spherical Bessel and associated Legendre functions in T_{jl} requires the position of each particle be specified. To overcome these difficulties, two approximations are introduced. First, the pressure field at the secondary particle j is related to that of the primary particle l by the leading order approximation

$$P_w(r_j) = \mathrm{e}^{ik(z_j - z_l)} P_w(r_l). \tag{17}$$

This approximation implies that the magnitude of the pressure field of the secondary particles j is taken to be the same as that of the primary particle l, and that each pressure field has a phase difference equal to the instantaneous phase difference of the wave incident to the respective particle. Note that in a dense randomly packed isotropic medium, no particle is uniquely defined in terms of its acoustical interactions with the rest of the particles. Each particle has a similar interaction with the surrounding medium. Consequently, the problem is reduced to N_c uncoupled linear algebraic equations by inserting equation (17) into equation (16):

$$\left[1 - \frac{\mathrm{i}}{3}\frac{(\varepsilon_\kappa - 1)^{3/2}}{(\varepsilon_\rho + 2)(\varepsilon_\rho - 1)^{1/2}}\,\alpha_p^3 \sum_{j=1, j\neq l}^{N_c} T_{lj}\,\mathrm{e}^{ik(z_j - z_l)}\right] P_w(r_l) = \frac{3}{\varepsilon_\rho + 2} P_i(r_l). \tag{18}$$

Second, the scattered sound from the surrounding particles is averaged over the solid angle and the summation is replaced by an integration under the assumption of a continuous distribution of particles. Accordingly, equation (18) becomes

$$\left[1 - \frac{\mathrm{i}}{3}\frac{(\varepsilon_\kappa - 1)^{3/2}}{(\varepsilon_\rho + 2)(\varepsilon_\rho - 1)^{1/2}}\,\alpha_p^3(24 f_{vc}) \int_1^{R_A/2} \bar{a}(k_e D R) G_c(R) R^2\,\mathrm{d}R\right] P_w(R) = \frac{3}{\varepsilon_\rho + 2} P_i(R), \tag{19}$$

where $R = r/D$, $\bar{a} = (1/4\pi)\int_0^{2\pi}\int_0^{\pi} \mathrm{e}^{ikr\cos(\theta)} a(k_e r) \sin(\theta)\, d\theta\, \mathrm{d}\phi$, f_{vc} is the volume fraction within the cluster $= N_c(\alpha_p/\alpha_A)^3$, $G_c(R)$ is the radial distribution function within the cluster, $R_A = D_A/D_p$, D_A is the diameter of the smallest sphere that completely encloses the cluster, and the factor R^2 is introduced so that for $G_c(R) = \bar{a} = 1$; then, $24 f_{vc}\int_1^{R_A/2} R^2\,\mathrm{d}R = N_c$. The statistical models available lead to a non-linear integral equation for $G_c(R)$, which needs to be solved numerically. Recognizing that the incident acoustic wave satisfies a relation identical to that satisfied by a one-component electromagnetic wave propagating in the z-direction, the averaging of T_{jl} over the solid angle leads to a diagonal matrix. Since $\bar{P}(r) = P(r)\bar{e}_r$ has only one component, $\overline{T_{jl}} = \overline{a_{jl}}$. Now, in terms of the spherical Bessel functions and the definition

$$\xi = \left[1 - \frac{\mathrm{i}}{3}\frac{(\varepsilon_\kappa - 1)}{(\varepsilon_p + 2)}\varepsilon_r \alpha_p^3\right]^{-1}, \tag{20}$$

equation (19) is reduced to

$$P_w(r) = \xi(3/(\varepsilon_p + 2))P_i(r), \tag{21}$$

where

$$A = 24 f_{vc} \int_1^{R_A/2} R^2[A_1(\alpha_p R) + A_2(\alpha_p R)A_3(\alpha_p R)]G_c(R)\, \mathrm{e}^{2\mathrm{i}\alpha_p R}\, \mathrm{d}R, \tag{22}$$

and

$$\varepsilon_r = \left(\frac{\varepsilon_\kappa - 1}{\varepsilon_p - 1}\right)^{1/2}, \qquad A_1(\alpha_p R) = \frac{\mathrm{i}\sin(2\alpha_p R}{(2\alpha_p R)^2}, \qquad A_2(\alpha_p R) = \frac{\mathrm{i}}{2\alpha_p R}\left(\frac{3}{(2\alpha_p R)^2} - 1\right) + \frac{3}{(2\alpha_p R)^2},$$

$$A_3(\alpha_p R) = \frac{\sin(2\alpha_p R)}{2\alpha_p R} + 3\frac{\cos(2\alpha_p R)}{(2\alpha_p R)^2} - 3\frac{\sin(2\alpha_p R)}{(2\alpha_p R)^3}.$$

In deriving equation (22), it is assumed that the agglomerate contains a large number of particles, and each particle in the agglomerate has identical characteristics. However, for an agglomerate of a small number of particles, the system of equations represented by equation (16) has to be solved for $P_w(r_l)$ of each particle, and then ξ needs to be defined as

$$\xi_l = \frac{P_w(r_l)}{3P_i(r_l)/(\varepsilon_p + 2)}. \tag{23}$$

In this case, each particle has its own correction factor. In terms of ξ the pressure field and the angle distribution factor S become

$$P_{wD}(r) = \xi P_{wI}(r), \qquad S_D = \xi S_I, \tag{24}$$

where the subscripts D and I refer to dependent and independent particles. Inserting equation (24) into equations (1) and (2) yields

$$C_{sD} = (1/k^2)\sum_l \sum_j \int_{4\pi} |\xi_j|^2 |S_j(0)|^2\, \mathrm{e}^{[\mathrm{i}k(e_i - e_s)(r_j - r_l)]}\, \mathrm{d}\Omega, \tag{25}$$

$$C_{eD} = (4\pi/k)\,\mathrm{Im}\left[\sum_j \xi_j S_j(0)\right]. \tag{26}$$

The double summation in equation (25) gives coherent scattering for $j \neq l$ and incoherent scattering for $j = l$. Accordingly, the coherent intensity is proportional to N_c^2 and the incoherent intensity is proportional to N_c. Assuming identical particles, equations (25) and

(26) may then be written as

$$C_{sD} = N_c|\xi|^2 F(\theta) C_{sI}, \qquad C_{eD} = (4\pi N_c/k)\,\mathrm{Im}\,[\xi S(0)], \tag{27, 28}$$

where

$$C_{sI} = \int_{4\pi} \frac{|S(0)|^2}{k^2}\,\mathrm{d}\Omega, \tag{29}$$

and $F(\theta)$ is the form factor

$$F(\theta) = (1/N)\sum_l \sum_j \mathrm{e}^{[ik(e_s - e_r)(r_j - r_l)]}, \tag{30}$$

which describes the coherent addition of the scattered waves from different particles [7]. Equation (30), averaged over the solid angle, gives

$$\gamma = (1/4\pi)\int_0^{2\pi}\int_0^{\pi} F(\theta)\Phi(\theta)\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi, \tag{31}$$

where $\Phi(\theta)$ is the phase function

$$\Phi(\theta) = \pi^2 D^2 I_{sI}(\theta)/(C_{sI} I_i), \tag{32}$$

I_i is the intensity of the incident acoustic wave, and I_{sI} is the independent scattering intensity. For particles with $a_p \ll 1$,

$$I_{sI} = (16\pi^4 v^4 a^6 I_i/(9c^4 r^2))(1 - \tfrac{3}{2}\cos(\theta))^2, \tag{33}$$

where v is the frequency, a is the radius of the sphere and c is the speed of sound.

In terms of equations (4) and (33), the phase function reduced to

$$\Phi(\theta) = \frac{\pi(1 - 1{\cdot}5\cos(\theta))^2}{[|\varepsilon_\kappa - 1|^2 + 3|(1-\varepsilon_\rho)/(2+\varepsilon_\rho)|^2]}. \tag{34}$$

The extinction correction factor η_e can be evaluated from equations (11) and (28) as

$$\eta_e = \left(\left\{\frac{1}{N_c}\sum_{l=1}^{N_c} \mathrm{Im}\,[\xi_l S(0)]\right\} \mathrm{Im}\,[S(0)]\right)^{1/3} \tag{35}$$

and the scattering correction factor η_s can be evaluated from equations (12) and (27) as

$$\eta_s = \left[\sum_{l=1}^{N_c} \gamma|\xi_l|^2/N_c\right]^{1/6}. \tag{36}$$

Now, for a homogeneous, continuous, isotropic and infinite distribution of particles, the form factor is, after averaging over all orientations of particles and replacing the double sum in equation (30) by an integral [8, 9],

$$F(\theta) = 1 + 24 f_{vc}\int_1^{R_A/2} R^2[G_c(R) - 1]\frac{\sin(\beta R)}{\beta R}\,\mathrm{d}R, \tag{37}$$

and $\beta = 4\alpha_p \sin(\theta/2)$. Now, within the Rayleigh limit, and noting that $G_c(R) - 1 \approx 0$ for $R > 2$, and $0 < \theta < \pi$, $\sin(\beta R)/(\beta R) \approx 1$ and equation (37) is reduced to

$$F(\theta) = 1 + 24 f_{vc}\int_1^{R_A/2} R^2[G_c(R) - 1]\,\mathrm{d}R. \tag{38}$$

Thus, equations (27) and (28) become, in terms of equations (20), (31) and (38),

$$C_{sD}=N_c|\xi|^2\gamma C_{sI}, \tag{39}$$

$$C_{eD}=(4\pi/k)\xi_R N_c \operatorname{Im}[S(0)]+(4\pi/k)\xi_I N_c \operatorname{Re}[S(0)], \tag{40}$$

where the subscripts Im and Re refers to the imaginary and real parts, respectively.

Now, the effective diameter of a cluster needs to be evaluated. In terms of this diameter, the acoustical properties of the cluster are developed where the interaction among the clusters themselves is neglected. The interaction among clusters may be modeled in a manner similar to modeling the interaction among single particles. For each cluster with an effective diameter D_{eff},

$$\xi_e=\left[1-\frac{\mathrm{i}(\varepsilon_p-1)}{3(\varepsilon_p+2)}a_e^3A\right]^{-1}, \tag{41}$$

$$A_e=24f_{ve}\int_1^\infty R^2\bar{a}(2\alpha_e R)G_e(R)\,\mathrm{d}R, \tag{42}$$

$$\gamma_e=(1/4\pi)\int_0^{2\pi}\int_0^{\pi}F(\theta)\Phi_M(\theta)\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi, \tag{43}$$

$$\Phi_m=\tfrac{3}{4}(1+\cos^2\theta), \tag{44}$$

$$F_e(\theta)=1+24f_{ve}\int_1^\infty R^2[G_e(R)-1]\frac{\sin(\beta R)}{\beta R}\,\mathrm{d}R, \tag{45}$$

and $\beta=4\alpha_e\sin(\theta/2)$, $\alpha_e=\pi D_{eff}/\lambda$, $f_{ve}=\bar{N}(\pi/6)D_A^3=(1/N_c)(\alpha_p/\alpha_A)^3 f_v$, $\bar{N}$ is the number of clusters per unit volume $=N/N_c$, N is the number of particles per unit volume and f_v is the volume fraction of the system. Now $G_e(R)$ is the radial distribution function, which describes the mechanical interaction among the clusters. Note that the diameter used in the definition of f_{ve} is D_A, and not D_{eff}. This is due to the fact that the mechanical interaction recognizes the geometrical dimension of the cluster which is better described by D_A. On the other hand, D_{eff} describes an effective diameter of the cluster as it appears to the waves. $G_e(R)$ may be described by one of the models depending on the volume fraction. Now, the acoustical properties of the system are

$$Q_{aD}=4\alpha_e\operatorname{Im}[\xi_e S(0)],\qquad Q_{sD}=Q_{sI}|\xi_e|^2\gamma_e. \tag{46, 47}$$

For a system containing non-agglomerated particles, equations (46) and (47) still apply, provided that $D_{eff}=D_p$. A cloud of particles and different types of clusters give different acoustical characteristics, related to the relative positions of particles in the cloud or within the cluster. The next section evaluates the acoustical properties of interacting particles and different types of clusters.

4. CLOUD OF INTERACTING PARTICLES AND CLUSTERS

4.1. INTERACTING PARTICLES

For a system of particles, where the particles exhibit short range potentials, it appears that the Percus–Yevick [10] model (hereafter called PY model) is the most realistic model among others. To provide a close form expressions for the sound cross-sections, the model needs to have a simple structure without sacrificing the main features of a statistical model. Two solutions to the PY integral equation are available in the literature [11, 12]. Either these solutions do not give explicit expressions for the entire domain or they are not easy to handle. Using the radial distribution function given by Thiele [11] for $0 \leqslant R \leqslant 1$, where $G(R)$ is continuous up to the third derivative at $R=1$, the following linear model is proposed:

$$G(R)=\begin{cases}0, & \text{if } 0<R<1,\\ F+ER, & \text{if } 1<R<\chi_1,\\ 1, & \text{if } R>\chi_1.\end{cases}$$

Here

$$F=(-4f_v^2+3f_v+1)/(f_v-1)^4, \qquad E=-4\cdot 5(f_v-f_v^3)/(f_v-1)^4, \qquad \chi_1=(1-F)/E,$$

and χ_l is approximated by the intersection of the linear profile with $G=1$. The comparison between the linear model and the PY model *vs.* the volume fraction is shown in Figure 3. Note that the linear model captures the essential behavior of the PY model, especially its extremum at $R = 1$. In terms of the linear model, the factors ξ and γ become

$$\xi=\left[1+8f_v\frac{\varepsilon_\kappa-1}{\varepsilon_p+2}(\mathrm{i}\overline{\beta_1}/(\alpha\varepsilon_r^3)-\overline{\beta_2}/\varepsilon_r^2+\mathrm{i}\,\overline{\beta_3}\alpha/\varepsilon_r-\overline{\beta_4}\alpha^2)+\cdots)\right], \tag{48}$$

where

$$\overline{\beta_1}=3E(Ln(\chi_1))/32, \qquad \overline{\beta_2}=(3E(\chi_1-1)-1)/8,$$

$$\overline{\beta_3}=3E(1-\chi_1^2)/8, \qquad \overline{\beta_4}=[F(\chi_1^2-1)/5+E(1-\chi_1^3)/5],$$

and

$$\gamma=\frac{22}{4}\left[\frac{1-8f_v+8f_v(F-1)(\chi_1^3-1)+6f_vE(\chi_1^4-1)}{[|\varepsilon_\kappa-1|^2+3|(1-\varepsilon_p)/(2+\varepsilon_p)|^2]}\right]. \tag{49}$$

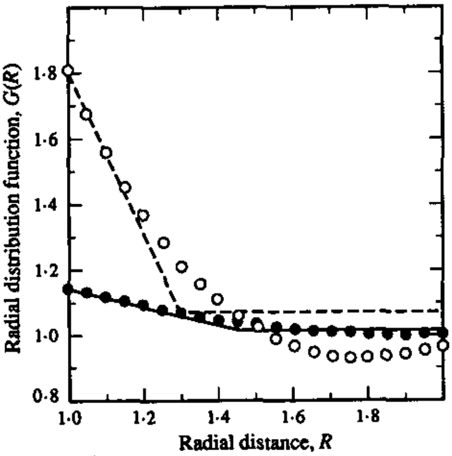

Figure 3. Radial distribution function *vs.* the radial distance. For $f_v=0{\cdot}052$: ——, linear model, ● ● ●, PY model. For $f_v=0{\cdot}209$: - - -, linear model; ○ ○ ○, PY model.

4.2. RANDOM CLUSTERS

Random clusters can be treated as small clouds of diameter D_A containing N_c particles. Due to the randomness of the position of a particle relative to others, the particle number density is uniform within the cluster and, as a result, $G_c(R)=1$. Then, equation (22) yields

$$A=\frac{3\mathrm{i}f_{vc}}{20\varepsilon_r\alpha_p^3}[\alpha_A^2-4\alpha_p^2], \qquad \alpha_p<\alpha_A\ll 1. \tag{50}$$

For a first order approximation, the far-field correction factor yields

$$\gamma=\frac{22}{4}\left[\frac{1}{[|\varepsilon_\kappa-1|^2+3|(1-\varepsilon_p)/(2+\varepsilon_p)|^2]}\right], \tag{51}$$

where $F(\theta)=1$. The correction factors related to the interaction among clusters can be evaluated under the assumption of uniform cluster density from equations (42) and (43) as

$$A_e=\frac{-3\mathrm{i}f_{ve}}{\alpha_e^3\varepsilon_r^3}[1+1{\cdot}6\alpha_e^2\varepsilon_r^2], \tag{52}$$

and

$$\gamma_e=\frac{22}{4}\left[\frac{1}{[|\varepsilon_\kappa-1|^2+3|(1-\varepsilon_p)/(2+\varepsilon_p)|^2]}\right]. \tag{53}$$

4.3. FRACTAL CLUSTERS

By definition, a fractal cluster exhibits self-similarity in shape. The radial distribution function for a fractal cluster is [13]

$$G_c(R)=\frac{2^fK}{4\pi D_p^3R^{3-f}}\mathrm{e}^{-(2pR/N^{1/f})}, \tag{54}$$

where K and p are related to the fractal dimension f by

$$K=\frac{1}{\Gamma(f)}\left(\frac{f+1}{f}\right)^{f/2}, \qquad p=\left(\frac{f+1}{f}\right)^{1/2}. \tag{55}$$

In terms of f, A and γ become

$$A=\frac{24\mathrm{i}K(2^f-1)}{(2\alpha_p)^3}\left(\frac{2p}{N^{1/f}}\right)^{3-f}\Gamma\left(f-3,\frac{2p}{N^{1/f}}\right), \tag{56}$$

$$F(\theta)=1+\frac{24f_{vc}K2^f}{(4\pi D_p^3)}\left(\frac{3p}{N^{1/f}}\right)^{f-2}\Gamma\left(2-f,\frac{2p}{N^{1/f}}\right). \tag{57}$$

The correction factors which account for the interaction among fractal clusters are the same as those given in equations (52) and (53).

5. ATTENUATION OF PLANE WAVE

Following Morse and Ingard [5], let the acoustics of a particulate system be described by an effective density and compressibility as

$$\rho_{eff}=\left[\frac{(1-f_v)}{\rho}+\frac{f_v}{\rho_p}\right]^{-1}, \qquad \kappa_{eff}=(1-f_v)\kappa+f_v\kappa_p, \tag{58, 59}$$

where the subscript p refers to the properties of particles. In terms of these, the independent pressure P_I is obtained from the wave equation, excluding the scattering and adsorption effects of particles. The pressure is related to the intensity as $P\sim\sqrt{\rho_{eff}c_{eff}I}$. The scatterers attenuate the intensity of a plane wave according to

$$I=I_0\,e^{-NC_{eD}X}, \tag{60}$$

where X is the distance of penetration, I_0 is the initial intensity of the wave and N is the number of scatterers (particles or clusters) per unit volume. Then, the dependent pressure distribution within the particular medium becomes

$$P_D=P_I\,e^{-NC_{eD}(X/2)}, \tag{61}$$

Now, consider the propagation of small disturbances (generated by the sinusoidal oscillations of a wall) in a semi-infinite compressible gas at rest. Assume that all sources of dissipation (viscous friction, conduction and radiation) are neglected except for the dissipation of sound due to scattering or absorption. Assume that the particles have negligible flow resistance and, as a result, $C_a=0$ and $C_e=C_s$. Then, the attenuation of pressure is described by

$$|P_D/P_I|=\exp[-(f_v/3)\alpha^4(|\varepsilon_\kappa-1|^2+3|(1-\varepsilon_\rho)/(2+\varepsilon_\rho)|^2)|\xi|^2\gamma(X/D)]. \tag{62}$$

This relation shows that the interaction effect, represented by $|\xi|^2\gamma$, may enhance or reduce the attenuation effect, depending on the properties of the particulate system and volume fraction. The effect of interaction particles on the attenuation of a sinusoidal plane as it penetrates into the particulate domain is shown in Figure 4.

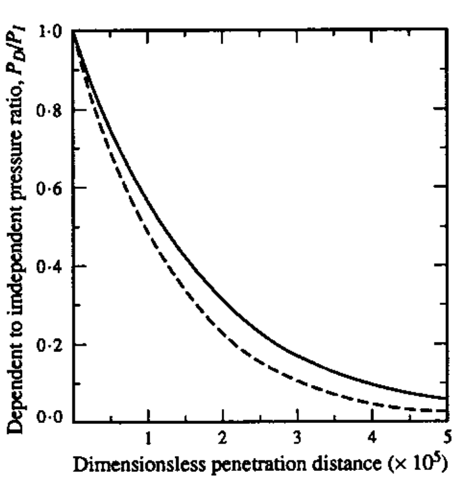

Figure 4. Dependent to independent pressure ratio *vs.* dimensionless penetration distance: - - -, interacting particles; ——, non-interacting particles.

6. CONCLUDING REMARKS

A model is proposed for the acoustic characteristics of small particles and clusters in a dense particulate system. Analytical expressions for the acoustical properties are derived based on a proposed linear model which describes the essential behavior of the pair distribution function. It is found that interaction among particles may increase among particules may increase or decrease attenuation depending on the volume fraction and properties of the particulate system.

REFERENCES

1. R. Tiwary 1985 *Ph.D. Dissertation, The Pennsylvania State University.* Acoustic agglomeration of micron and submicron fly-ash aerosols.
2. L. Song 1990 *Ph.D. Dissertation, The Pennsylvania State University.* Modeling of acoustic agglomeration of fine aerosol particles.
3. A. R. Jones 1979 *Proceedings of the Royal Society London* **A366**, 111–127. Electromagnetic wave scattering by assemblies of particles in the Rayleigh approximation.
4. A. R. Jones 1979 *Journal of Physics* **D12**, 1661–1672. Scattering efficiency factors for agglomerates of small spheres.
5. P. M. Morse and K. U. Ingard 1968 *Theoretical Acoustics.* New York: McGraw-Hill.
6. D. S. Saxon 1973 *UCLA International Conference on Radiation and Remote Probing of Atmosphere, Los Angeles, California* (J. G. Kuriyan, editor), 227–308. Lectures on the scattering of light.
7. P. Debye 1925 *Journal of Mathematical Physics* **4**, 133–147. Note on the scattering of X-rays.
8. A. Guinier 1963 *X-ray Diffraction in Crystals, Imperfect Crystals, and Amorphous Bodies.* San Francisco: W. H. Freeman. X-ray diffraction in crystals, imperfect crystals, and amorphous bodies.
9. H. C. van de Hulst 1981 *Light Scattering by Small Particles.* New York: Dover.
10. J. K. Percus and G. J. Yevick 1958 *Physics Review* **110**, 1–13. Analysis of classical statistical mechanics by means of collective coordinates.
11. E. Thiele 1963 *Journal of Chemical Physics* **39**, 474–479. Equation of state for hard spheres.
12. M. S. Wertheim, 1963 *Physics Review Letters* **10**, 321–323. Exact solution of the Percus–Yevick integral equation for hard spheres.
13. M. V. Berry and I. C. Percival 1986 *Optica Acta* **33**, 577–591. Optics of fractal clusters such as smoke.