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ϕ^4 in three dimensions at finite temperature

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We use renormalization-group methods to estimate the critical temperature for ϕ^4 theory in three dimensions. We show that, by delineating the range of applicability of perturbation theory, the value of the critical temperature is rather narrowly bounded.

1. Introduction

The nature and consequences of phase transitions in quantum systems at finite temperature (T) has attracted interest since the observation [1] that spontaneously broken symmetries in relativistic field theories are in general restored above some critical temperature (T_c). In particular, the phase transition associated with restoration of the electroweak symmetry of the standard model (SM) has recently been the subject of much attention and some controversy [2–4]. Ever since it was understood that baryon violation can be copious at temperatures comparable to the electroweak scale [5], there have been attempts to determine whether the baryon asymmetry of the universe can be understood as a consequence of the electroweak transition rather than a transition at some much higher temperature [6,7]. A resolution of this question in the context of the SM or some extension thereof involves a number of physics issues; in particular, a rather precise analysis of the finite-T effective potential $V(\phi)$ of the Higgs field is required. This is

[1] Current address.

rendered non-trivial by the infrared divergences (due to zero modes) characteristic of finite-T field theories.

Recent discussion [8] of how fluctuations may influence phase transitions has inspired simulations of model systems in lower dimensions [9]. In d space-time dimensions for $d < 4$, the infrared problem is generally more severe than in four dimensions. On the other hand, a theory that would be renormalizable in $d = 4$ is superrenormalizable for $d < 4$, so that its ultraviolet (UV) divergences occur up to a finite number of loops only, and the associated β-functions are therefore exactly calculable. Thus, we were invited to consider the simple case of single-component ϕ^4 in $d = 3$ dimension *. While we agree with previous treatments ** that the model cannot be solved perturbatively near the critical temperature, we shall argue (contrary to ref. [10]) that renormalization group ideas enable us to reach a reasonably reliable estimate of T_c.

Let us first digress briefly to contrast this with the generalization to an O(N) theory with N ϕ-fields. If a non-zero vacuum expectation value (VEV) for ϕ breaks a continuous global symmetry then radically different physics (from the $N = 1$ case) results because of the existence of Goldstone bosons in the broken phase. The O(N) model was analyzed in the large-N limit at $T = 0$ in ref. [11], and the existence of symmetry breaking (controlled by the sign of the scalar ((mass)^2 parameter) demonstrated. Now at finite T, as discussed further below, the effective field theory of the zero modes of a d-dimensional theory is $(d-1)$-dimensional: in this case, two. The existence of the Goldstone bosons for $N > 1$ makes this effective field theory useless for describing the finite-T behavior of the $d = 3$ theory. Indeed, as shown in ref. [11], the $d = 2$ theory does not permit a VEV for ϕ for any value of the scalar (mass)^2.

We therefore confine ourselves to the $N = 1$ case. This ϕ^4 model is in the equivalence class of the (2 + 1)-dimensional, ferromagnetic Ising model ***. Accordingly, the model is expected to have a second-order phase transition at some non-zero critical temperature T_c, and various critical-point exponents are known either from Onsager's exact solution or from numerical studies ☆. While the precise relation for the critical temperature in the Ising model is well known, the specific relation of the critical temperature to the lagrangian parameters is not to our knowledge known, so we will concentrate on that. It is also interesting to see how far analytic arguments based on perturbation theory and the renormalization group can be pushed, even though perturbation theory must break down at the critical temperature. As we shall see, the infrared-induced uncertainties can be subsumed into a single parameter, κ, upon which T_c depends only logarithmically. Consequently, we estimate T_c with a fair degree of confidence.

* We wish to thank M. Gleiser for initiating our interest in this problem.

** See for example ref. [10].

*** The relation is derived, for example, in ref. [12].

☆ For a review of the Ising model, see ref. [13].

2. $(\phi^4)_3$ at zero temperature

We begin, however, by reviewing the properties of the theory at zero temperature. We write the tree potential as

$$V_0 = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4, \tag{2.1}$$

where, in three dimensions, the coupling λ has dimensions of mass. The β-function for the mass m^2 vanishes at one-loop but receives a positive contribution proportional to λ^2 at two-loops,

$$\beta_{m^2} = +\frac{1}{6}\left(\frac{\lambda}{4\pi}\right)^2, \tag{2.2}$$

so that

$$m^2(\mu) = m^2(\mu_1) + \frac{1}{6}\left(\frac{\lambda}{4\pi}\right)^2 \ln\left(\frac{\mu}{\mu_1}\right). \tag{2.3}$$

In fact, in a renormalization prescription such as minimal subtraction, these two-loop results are exact *. It follows that we can say definitely that $m^2(\mu)$ is positive at sufficiently large scales and negative at small scales. This suggests that the theory (at zero temperature) undergoes spontaneous symmetry breaking at some scale. This conclusion can be drawn within the context of perturbation theory by working at a sufficiently small scale so that $\lambda/|m|$ is small. As the argument parallels the one that we shall use below at finite temperature, we shall present it here. Even though the exact β-functions and anomalous dimensions are known for this model, it is not known how to calculate V_{eff} analytically except perturbatively. The one-loop effective potential may be written as

$$V_{\text{eff}} = V_0 - \frac{1}{12\pi}\left(m^2 + \tfrac{1}{2}\lambda\phi^2\right)^{3/2}. \tag{2.4}$$

Simple power counting arguments suggest that a minimal requirement for a reasonable perturbation expansion is

$$\frac{\lambda}{4\pi\sqrt{m^2 + \frac{1}{2}\lambda\phi^2}} \ll 1, \tag{2.5}$$

for m^2 and ϕ in the range of interest. Even though $m^2(\mu)$ is not scale dependent

* As this is a superrenormalizable theory, the anomalous dimension γ and β_λ both vanish.

at one-loop, it is at two loops, so there is a scale choice implicit in eqs. (2.4) and (2.5). Let us normalize at some small scale where m^2 is negative. In mean-field theory, one would conclude that the theory undergoes spontaneous symmetry breaking with $\langle\phi\rangle^2 = -6m^2/\lambda$. Under what conditions is this conclusion justified? Taking into account the one-loop correction, we find the shift in $\langle\phi\rangle$ from its mean field is

$$\frac{\delta\langle\phi\rangle}{\langle\phi\rangle} = \frac{\sqrt{2}\,\lambda}{16\pi|m|}. \tag{2.6}$$

Thus, it would appear that, simply by choosing the normalization scale sufficiently small, so that $-m^2$ is sufficiently large, the mean-field conclusion would be correct.

On the other hand, if we choose μ such that $m^2(\mu)$ is sufficiently large and positive, a similar argument in the neighborhood of $\phi = 0$ suggests that there would be no symmetry breaking. This is paradoxical to say the least, since the conclusion that the theory does or does not undergo symmetry breaking cannot depend on the arbitrary choice of normalization scale. The flaw in the reasoning is this: consider the situation when we have normalized at a scale where m^2 is large and negative. Then, in tree approximation, $(m^2 + \frac{1}{2}\lambda\langle\phi\rangle^2) = -2m^2$. At the two-loop level, the effective potential will involve $\ln[(m^2 + \frac{1}{2}\lambda\phi^2)/\mu^2]$. To avoid large logarithmic corrections that might lead to a breakdown in perturbation theory near the tree minimum, the normalization scale must be comparable to $(m^2 + \frac{1}{2}\lambda\langle\phi\rangle^2)$. This may or may not be compatible with choosing the scale such that eq. (2.5) is satisfied *. Similarly, simply by choosing μ sufficiently large, one may not be able to conclude that there is a local minimum at $\phi = 0$ because it may be that perturbation theory is unreliable in that region.

3. $(\phi^4)_3$ at finite temperature

Now let us turn to the behavior of the theory at finite temperature. In general, the equilibrium properties of a quantum field theory in d space-time dimensions may be written as a euclidean field theory in d euclidean dimensions in which the imaginary time coordinate plays the role of the inverse temperature and the bosonic (fermionic) fields are periodic (antiperiodic) with period $\beta \equiv 1/T$ **. The

* Our purpose here is to show that one cannot always reliably conclude that there is a local minimum away from the origin. It is also the case that, for m^2 negative, there is a range of ϕ where $m^2 + \frac{1}{2}\lambda\phi^2$ is so small so that radiative corrections become uncontrollably large. In addition, when $m^2 < 0$, the effective potential will develop an imaginary part for ϕ sufficiently small, so that one cannot trust the loop expansion in that region.

** The Boltzmann constant k_B has been set equal to one. For a review of the formalism, see, for example, the text in ref. [14].

"energy" then becomes quantized with $\omega_n = 2n\pi T$ for bosons (and $\omega_n = (2n+1)\pi T$) for fermions.) Following Ginsparg [15] for sufficiently high temperatures (at scales below $2\pi T$) one may integrate out the heavy modes to write this as an effective field theory of the zero-energy modes of the bosonic fields in $d-1$ euclidean dimensions. The effective lagrangian takes the form

$$\mathscr{L}_{\text{eff}} = \tfrac{1}{2}(\partial_\mu \phi)^2 + \frac{m_T^2}{2}\phi^2 + \frac{\lambda_T}{4!}\phi^4. \tag{3.1}$$

In going from d to $d-1$ dimensions, we have rescaled the parameters and field, $\sqrt{T}\phi \to \phi$, $\lambda T \to \lambda_T$, so that, as usual, λ_T has dimensions $\mu^{4-(d-1)}$, where μ is some mass scale. Thus, if we start with a four-dimensional theory, then λ_T has dimensions of mass, whereas if we begin with a three-dimensional theory, λ_T has dimensions of mass^2. The parameters m_T and λ_T include the virtual effect of the heavy "energy" modes.

In the case at hand, where we have begun with $d=3$, we are led to consider eq. (3.1) in two dimensions. In this effective field theory, the only UV divergence is the one-loop contribution to m_T^2, giving

$$\beta_{m_T^2} = -\frac{\lambda_T}{4\pi}, \tag{3.2}$$

so that

$$m_T^2(\mu) = m_0^2 - \frac{\lambda_T}{4\pi}\ln(\mu/\mu_0), \tag{3.3}$$

exactly. It is extremely convenient to make μ_0 a physical parameter by choosing it so that $m_0 = 0$; thus μ_0 is defined as the scale at which the renormalized mass vanishes. The parameters m_T and λ_T are related to the original parameters of the three-dimensional theory by certain "matching conditions" *. Up to corrections of order m^2/T^2, we may take $\lambda_T = \lambda T$. To obtain the relation for m_T, one may calculate the inverse propagator at zero momentum from the original theory and compare it with the result from the effective theory. Through two loops, this leads to the relation **

$$m_T^2(\mu) = m^2(\mu) + \frac{\lambda T}{4\pi}\ln\frac{T}{\mu} + \frac{1}{6}\left(\frac{\lambda}{4\pi}\right)^2 \ln\left(\frac{T}{\mu}\right). \tag{3.4}$$

In particular,

$$m_T^2(T) = m^2(T). \tag{3.5}$$

* The method of effective field theories is elaborated in ref. [16].

** Terms finite in λ^2 can be absorbed by an appropriate renormalization prescription.

Eqs. (3.3) and (3.4) together determine the physical scale μ_0 as a function of the temperature T and the original lagrangian parameters m^2 and λ, to wit,

$$\ln\left(\frac{T}{\mu_0}\right) = -\frac{4\pi m^2(T)}{\lambda T}. \tag{3.6}$$

Note that μ_0 is a function of temperature, and for sufficiently large T, $\mu_0(T) \sim T$. Although in general μ_0 may be larger or smaller than $2\pi T$, the presumption throughout is that

$$|m_T(2\pi T)| \ll 2\pi T; \tag{3.7}$$

otherwise, the $n = 0$ mode will not be light compared to the modes that have been integrated out. Eventually, this will be seen to provide an upper limit on λ/T_c.

Naively, the critical temperature T_c would be determined by requiring that $m_T = 0$ and neglecting fluctuations (the so-called mean-field value). It is not at all clear what interpretation could be given to that, since, according to eq. (3.3), m_T is explicitly dependent on the choice of scale. One may try to determine under what conditions the physical value of the scalar mass vanishes, but one encounters infrared problems that undermine such an approach [10]. By carefully exploring the limits of perturbation theory, we will be led to make an estimate of the critical temperature that, while not calculable, in fact has relatively small uncertainty.

To that end, consider the one-loop effective potential

$$\begin{aligned} V_{\text{eff}} = {} & \frac{m_T^2}{2}\left(\phi^2 - v_0^2\right) + \frac{\lambda_T}{4!}\left(\phi^4 - v_0^4\right) \\ & - \frac{1}{8\pi}\left[\left(m_T^2 + \tfrac{1}{2}\lambda_T\phi^2\right)\ln\left(\frac{m_T^2 + \frac{1}{2}T\phi^2}{\mu^2}\right) - \left(m_T^2 + \tfrac{1}{2}\lambda_T v_0^2\right)\ln\left(\frac{m_T^2 + \frac{1}{2}Tv_0^2}{\mu^2}\right)\right], \end{aligned} \tag{3.8}$$

where v_0 is $\langle\phi\rangle$ in tree approximation, viz., $v_0 = 0$ on scales where $m_T^2(\mu) \geqslant 0$, and $v_0^2 = -6m_T^2(\mu)/\lambda_T$ on scales where $m_T^2(\mu) < 0$. The terms in V_{eff} involving v_0 are field independent but necessary if V_{eff} is to satisfy the renormalization group equation through one-loop order *. As in the zero temperature case, power counting arguments suggest that the perturbative expansion parameter is of the order

$$\frac{\lambda_T}{4\pi\left(m_T^2 + \frac{1}{2}T\phi^2\right)} \ll 1, \tag{3.9}$$

* This prescription differs from the one in ref. [17], where we chose $v_0 = 0$ on all scales. The present prescription avoids spurious imaginary contributions to V_{eff}.

for scales μ and field values ϕ in the range of interest. This "weak coupling" condition may be seen to be equivalent to $1 \ll |\ln(\mu_0/\mu) + 2\pi\phi^2|$. Typically, perturbation theory requires that we work at large fields and/or at scales μ either much larger or much smaller than μ_0.

It is clear from eqs. (3.5) and (2.3) that, at sufficiently high temperature, $m_T^2(T) > 0$. At smaller scales, $m_T^2(\mu)$ is larger and positive, so there would appear to be no symmetry breaking for sufficiently high temperature. This inference is reliable only if perturbation theory is valid. Expanding the one-loop effective potential about $\phi = 0$, we find the leading contribution to be

$$\left[m_T^2(\mu) + \frac{\lambda_T}{8\pi}\left\{\ln\left(\frac{m_T^2}{\mu^2}\right) + 1\right\}\right]\frac{\phi^2}{2}. \tag{3.10}$$

For the perturbative expansion to be valid, the second term in the square brackets must be small compared to the first. This constraint may be seen to be equivalent to the dual requirements of eq. (3.9) and

$$\ln\left(\mu_0^2/m_T^2(\mu)\right) \gg 1. \tag{3.11}$$

Since m_T^2 grows indefinitely as the scale μ is diminished, it is not certain that a region of scales can be found satisfying both constraints. The consistency condition can easily be shown to be

$$\frac{\lambda_T}{4\pi\mu_0^2(T)} \ll 1, \tag{3.12}$$

in order that one can have confidence in the perturbative conclusion that the symmetry is unbroken. This determines the high-temperature range within which we can be confident that the symmetry is unbroken.

As the temperature decreases, $m_T^2(T) = m^2(T)$ eventually becomes negative. In that range, mean-field theory would lead us to expect spontaneous symmetry breakdown. Let us explore under what conditions we can trust such a conclusion. Let $\langle\phi\rangle = v$. It is convenient to choose the normalization scale $\mu \equiv \mu_v$ in the effective potential eq. (3.8) by the implicit definition * $\mu_v^2 \equiv m_T^2(\mu_v) + \frac{1}{2}\lambda v^2$. Then we find the simple formula

$$v^2 = -\frac{6m_T^2}{\lambda_T}\left(1 - \frac{\lambda_T}{8\pi\bar{m}_T^2}\right). \tag{3.13}$$

The one-loop correction to $\langle\phi\rangle$ is small only for $\lambda_T/8\pi m_T^2 \ll 1$, which may be seen to be equivalent the requirement $\ln(\mu_v/\mu_0) \gg 1$, so that the scale μ_v must be

* This choice is inessential; any other scale of the same order would be equally good.

very large compared to μ_0. Notice that these inequalities imply that $v^2 \gg 3/4\pi$, so that perturbation theory is valid only if $\langle\phi\rangle$ is not too small. At the minimum, we have

$$\mu_v^2 \approx -2m_T^2, \tag{3.14}$$

which may in turn be shown to require for consistency *

$$\frac{\lambda_T}{4\pi\mu_0^2(T)} \gg 1. \tag{3.15}$$

This is the requirement that the symmetry be broken and that perturbation theory be valid.

In summary, we have concluded that we may reliably infer in perturbation theory that there is no symmetry breaking under condition eq. (3.12), whereas there is spontaneous breaking under condition eq. (3.15). Thus, the state of the system and the validity of perturbation theory is controlled by the magnitude of

$$\frac{\lambda_T}{4\pi\mu_0^2(T)} = \frac{\lambda}{4\pi T}\exp\left[\frac{-8\pi m^2(T)}{\lambda T}\right], \tag{3.16}$$

where on the right-hand side, we used $\lambda_T = \lambda T$ and the definition of μ_0 (eq. (3.6)) to express this in terms of the parameters of the original theory. Because eq. (3.16) varies rapidly with T, perturbation theory is valid except in a rather narrow range of temperature. The critical temperature is in the regime where perturbation theory fails, viz.,

$$\frac{\lambda_{T_c}}{4\pi\mu_0^2(T_c)} \equiv \kappa, \tag{3.17}$$

where κ is some number of order one. Rewriting eq. (3.17) as an implicit equation for T_c, we arrive at an estimate of the transition temperature

$$T_c \ln\left(\frac{4\pi\kappa T_c}{\lambda}\right) = \frac{8\pi m^2(T_c)}{\lambda}. \tag{3.18}$$

Although the value of $m^2(\mu)$ depends implicitly on the scale μ in any given theory, its value varies at fixed μ from one theory to the next. So one should regard the right-hand side as an independent quantity. For the case $\kappa = 1$, we display in fig. 1 a plot of $4\pi T_c/\lambda$ as a function of $(4\pi/\lambda)^2 m^2$. The fortuitous occurrence of κ inside the logarithm makes our estimate rather insensitive to its particular value,

* It is unnecessary to choose the normalization scale to be μ_v, any scale of that order will avoid large logarithms and lead to this same consistency condition.

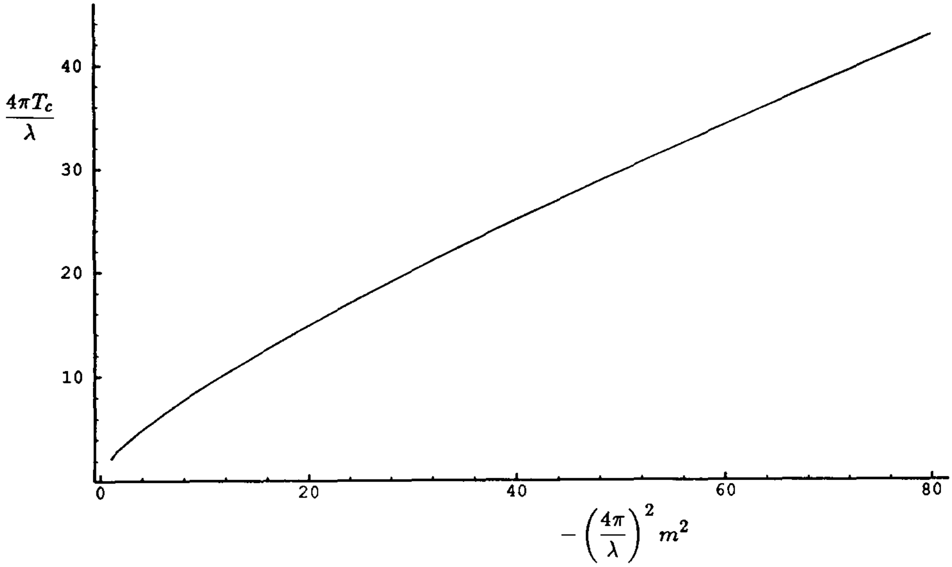

Fig. 1. The critical temperature, eq. (3.18), for $\kappa = 1$.

so one can hope that this will give a fairly reliable estimate of the critical temperature T_c. However, since the transition region is precisely where perturbation theory breaks down, we cannot say precisely how good until the results of numerical simulations are known.

We can now return to our consistency condition, eq. (3.7) to determine whether that is satisfied near T_c. We find that is a good approximation provided

$$\frac{1}{4\pi}\left(\frac{\lambda}{4\pi\,|\,m(T)\,|}\right) \ll 1. \tag{3.19}$$

If this is not the case, one cannot consistently address these issues from the effective field theory in one less dimension.

There is interest in lattice simulations of this model, in part, as a test of numerical algorithms on a simple system [18,19]. We are not aware of a previous analytic estimate of the critical temperature, so it would be very interesting to compare the results; unfortunately, the results of ref. [18] have not been presented in a manner that facilitates comparison with ours.

How sensitive are these results to the precise form of the original lagrangian? Note that the addition of a term of the form $\eta\phi^6$ to eq. (2.1) results in a term $\eta T^2\phi^6$ in the dimension $d-1$ effective theory, eq. (3.1). Thus, for $d=4$, where η has dimensions of μ^{-2}, this now becomes a relevant operator, though its coefficient is small for $T \ll 1/\sqrt{\eta}$. For $d=3$, the effective two-dimensional field theory has a dimensionless field so that all terms that were originally of higher-dimension become relevant. Thus, the high-temperature expansion is only as good an approximation as the original lagrangian, which is simply to say that T must be small compared to the cutoff scale at which new physics enters.

4. Discussion and conclusions

The properties of ϕ^4 in $d=2$ at zero temperature have been extensively studied in the context of constructive quantum-field theory [20]. That body of work is primarily concerned with the existence and mathematical consistency of the theory and is focused on the dependence on bare parameters. There has also been considerable effort devoted to establishing the existence of a phase transition, and whether it is first or second order [21]. Our orientation assumes that perturbative renormalizability is adequate to define the theory and is concentrated on establishing the existence of the phase transition and the dependence of the critical temperature on the renormalized coupling constants. The work closest to ours in spirit is that of Chang [22]; his estimate for the position of the transition corresponds in our notation to the relation $m_T^2 = -\lambda_T/8\pi$ (cf. eq. (3.18)).

We note that the finite-temperature behavior of the two-dimensional theory would be quite different. In that case, the effective field theory is in one dimension, and so is like ordinary quantum mechanics in which spontaneous symmetry breaking cannot occur. This behavior is inherently non-perturbative. In two space-time dimensions, soliton collective excitations [9] are essential for the dynamics or in the one-dimensional effective theory, tunneling or instanton solutions provide a way of describing the superposition of classically degenerate energy states *.

For field theories in $d=4$, (and in particular for gauge theories) it has been generally believed that direct perturbative examination of the effective potential is not useful because of infrared divergences. The epsilon expansion is a useful tool to determine the *order* of the transition, but in the case of a first-order transition provides little further information. Recent work on the electroweak transition has, however, demonstrated that in a gauge theory important properties of the potential near the critical temperature can, with care, be inferred. In this paper we have studied the (deceptively) simple case of ϕ^4 in $d=3$, where the infrared problems are even more pronounced. We have shown that a reasonable estimate of the critical temperature is nevertheless possible. The main ingredients of our approach are (a) application of renormalization group ideas to the effective theory of the zero modes, and (b) careful consideration of matching conditions. It is our view that this sort of approach is more elegant and more reliable than that of summing subsets of higher-order graphs in the full theory.

* See for example ref. [23].

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