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# Extracting energies and intensities from complex coincidence matrices

M.G. Chemaly *, H.C. Griffin

*Department of Chemistry, University of Michigan, Ann Arbor, MI 48109-1055, USA*

**Abstract**

Coincidence intensities and uncertainties are extracted at the statistical limits from γ–γ coincidence matrices. The matrices are decomposed into continuum, ridges and peaks. The continuum is successfully modeled by the product of two vectors which describe the Compton distributions in the coincident detectors or groups of detectors. The ridges are represented by the corresponding continuum vector scaled according to the intensity and energy of the associated γ-ray. The peaks are fitted as the product of two, one-dimensional Gaussians. This technique has been applied to the analysis of prompt gamma-rays from the spontaneous fission of $^{252}$Cf and the high spin states in $^{163}$Lu populated via the $^{122}$Sn($^{45}$Sc, 4n) reaction.

## 1. Introduction

Level schemes of excited states of nuclei can be deduced from the analysis of gamma–gamma coincidence spectra. Larger arrays of more efficient, escape-suppressed HPGe detectors allow the investigation of complex decays from heavy-ion reactions or prompt fission products, for example. However, when the average spacing between the gamma-rays is comparable to the energy resolution, the traditional digital gate method for extracting coincidence energies and intensities can be ineffective.

Emelianov et al. [1] developed a method for the direct decomposition of coincidence spectra that used a priori information on energies and intensities of γ-rays obtained from other experiments. This approach did involve a large number of degrees of freedom as brought up by Vanin and Aïche [2], who proposed a return to one-dimensional fits with greater emphasis on the propagation of statistical errors.

We have developed a technique for decomposing the coincidence matrix into its main features, namely a continuum (from coincidences of Compton distributions), ridges (from coincidences of full energy events with Compton distributions) and peaks (coincidences of two full energy events). We have greatly reduced the number of parameters needed by successfully modeling the continuum and the ridges. The peaks are located by a two-dimensional peak search. The most probable values and the uncertainties of energies and intensities are extracted from a two-dimensional fit. The process has been tested with coincidence data acquired in two different experiments. The heavy-ion reaction $^{122}$Sn($^{45}$Sc, 4n)$^{163}$Lu [3] data was acquired with an array of eight unsuppressed Ge detectors gated by high gamma-ray multiplicity. The data from the spontaneous fission of $^{252}$Cf [4] consisting of prompt coincidences were obtained with seven suppressed Ge detectors and a small planar detector (LEPS).

## 2. Method

Our first approach to direct decomposition of a coincidence matrix resulted in a successful simplification of the continuum function [5]. In subsequent developments we assumed the ridge associated with each peak found in the projection required its own description and that the γ–γ coincidences were defined as the intersections at one of those peaks in each axis. This brought us back to the problems faced by the traditional method, due to the poor correlation between peaks in the projection and individual gamma rays. However, we noticed that the shape of the ridges for the most intense gamma-rays had the same profile as the corresponding continuum vector. This led to a universal model for all the ridges parallel to a given axis. Once the continuum and ridges were accounted for, a two-dimensional peak search gave us the location of the coincidence peaks and we were able to take full advantage of the two-dimensional nature of the data.

* Corresponding author.


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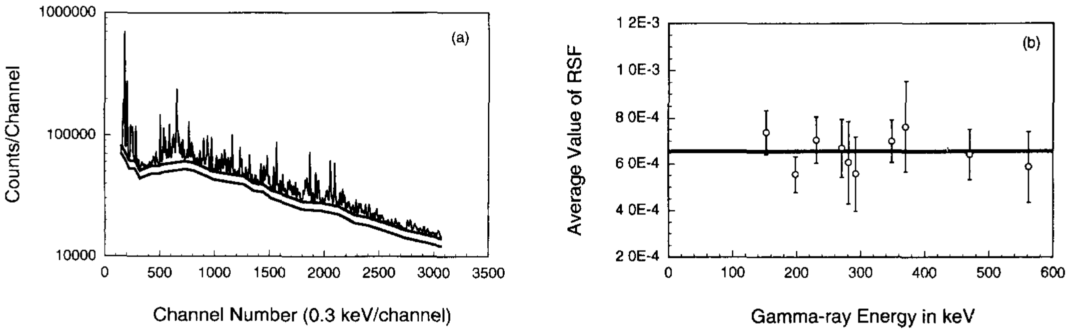


Fig. 1. Comparison of models with $^{163}$Lu data obtained with unsuppressed Ge detectors. (a) The projection of the modeled continuum (lower curve) does not reach the baseline for the total projection (upper curve) since the ridges parallel to the axis of projection also contribute. However, if multiplied by a single factor CSF (middle curve) we get a good agreement. (b) Average value of RSF determined for some strong gamma-rays which shows that a good approximation for the ridge intensity can be obtained from Eq. (3).

### 2.1. The continuum

The continuum function is obtained by projecting the coincidence matrix onto the axes and using all local minima that are at least 2–3 channels wide and that appear to be peak-free as a trial vector from which to generate the continuum. The average counts per channel at the intersection of all those selected regions can be modeled with Eq. (1):

$$\overline{M}(i \pm \Delta i,\ j \pm \Delta j) = \mathrm{VI}(i) \times \mathrm{VJ}(j), \qquad (1)$$

where $\Delta i$ and $\Delta j$ are the widths of the regions centered at $i$ and $j$. The value of the continuum vectors VI and VJ for points in-between the peak-free regions are obtained by linear interpolation. The continuum has been covered in more details in Ref. [5].

### 2.2. The ridges

We assume that the ridges arise from coincidences of peaks and Compton distributions. To the extent that the Compton distributions of the γ-rays in coincidence with a given γ-ray are typical of the decays being studied, the ridge should have the same profile as the continuum vector. (In effect, we have tested the adequacy of this assumption.) The intensity of a ridge should be given by the product of the intensity of the associated γ-ray and the parallel continuum vector. However, we do not know the intensity of the γ-rays. A reasonable approximation can be obtained from the projection, but we go back to the original problem of unresolved multiplets. Our approach is to use the information from the projection without having to decompose the peaks into their components. We first find the continuum scale factor, CSF, for each axis such that the projected modeled continuum times CSF equals the matrix projection at the peak-free regions:

$$\mathrm{CSF}_i \mathrm{VI}(i \pm \Delta i) \sum_{j=1}^{N} \mathrm{VJ}(j) = \sum_{j=1}^{N} M(i \pm \Delta i, j), \qquad (2)$$

where $N$ is the number of channels and $\Delta i$ is the width of the peak-free regions in the projection onto the i-axis, in this case. Figs. 1a, 2a and 3a show that Eq. (2) holds well for different cases. For all the other channels in the

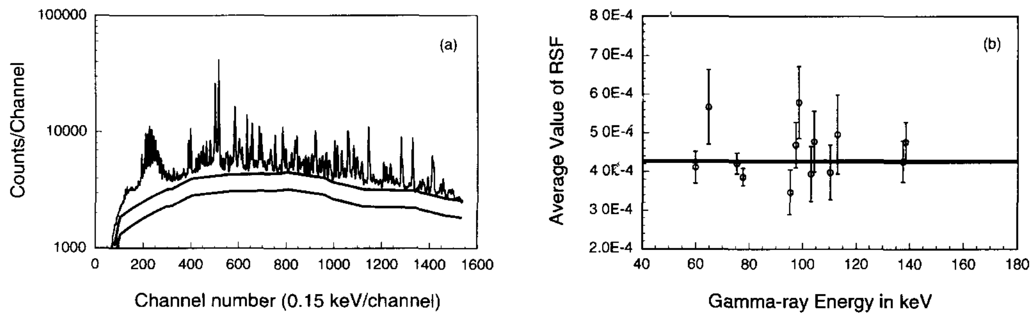


Fig. 2. Comparison of models with $^{252}$Cf data involving the LEPS detector and Ge suppressed detectors. (a) There is good agreement between model and data except at low energies (around X-rays) where it is difficult to find suitable peak-free regions for the continuum function determination. (b) Average value of RSF for some strong gamma-rays and X-rays. The model function does not seem to be affected by the different detectors involved.

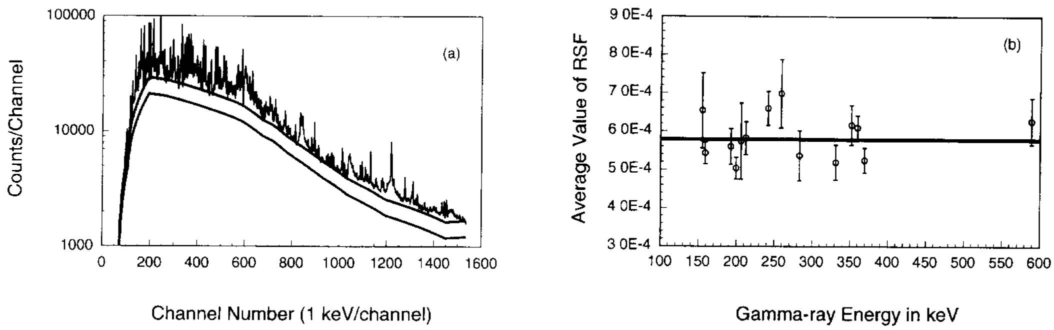


Fig. 3. Comparison of models with $^{252}$Cf data involving only the suppressed Ge detectors. (a) There is good agreement between model and data. (b) The ridge model (Eq. (3)) still holds and does not seem to be influenced by the suppressed detectors.

projection, the difference, DP($i$), between the two terms on opposite sides of the equality in Eq. (2) is proportional to the intensity of the gamma-ray and therefore the associated ridge. The intensity of a ridge is then obtained from

$$\mathrm{RHI}(i, j) = \mathrm{DP}(i)\mathrm{VJ}(j)\mathrm{RSF}_j, \tag{3}$$

where RSF is a scale factor that determines how the intensity in the projection is distributed to the channels in the two-dimensional matrix. Figs. 1b, 2b and 3b show that RSF is basically independent of γ-ray energy and intensity.

### 2.3. The peaks

The peaks are located by a peak search algorithm that calculates the statistical significance, SS($i$, $j$), for each matrix element using Eq. (4) adapted from the one-dimensional method described in SAMPO80 [6]:

$$\mathrm{SS}_{(i,j)} = \mathrm{dd}_{(i,j)}/\mathrm{sd}_{(i,j)}, \tag{4}$$

where dd($i$, $j$) and sd($i$, $j$) are given by

$$\mathrm{dd}_{(i,j)} = \sum_{k=-m}^{m} C_k M(i+k, j) + \sum_{l=-n}^{n} C_l M(i, j+l), \tag{5}$$

$$\mathrm{sd}_{(i,j)} = \left[ \sum_{k=-m}^{m} C_k^2 M(i+k, j) + \sum_{l=-n}^{n} C_l^2 M(i, j+l) \right]^{1/2}. \tag{6}$$

If SS($i$, $j$) is greater than about 2.5 and is a local maximum, then ($i$, $j$) is near the center of a peak.

The peaks are fitted as two-dimensional Gaussians superimposed on the continuum and the intersection of the two ridges associated with the gamma-rays:

$$P(i, j) = H \exp\left( \frac{-(i-i_0)^2}{2\sigma_i^2} \right) \exp\left( \frac{-(j-j_0)^2}{2\sigma_j^2} \right) + (\mathrm{VI}(i)\mathrm{VJ}(j)) + \mathrm{RHI}(i, j) + \mathrm{RHJ}(i, j). \tag{7}$$

The height of a peak, $H$, is first estimated from a linear fit using a Singular Value Decomposition algorithm [7] where the position from the peak search is used. The actual height and position with the associated uncertainties are obtained from a non-linear fit using a Levenberg–Marquardt fitting routine [7].

## 3. Conclusions

We have developed a method for decomposing complex coincidence matrices that does not require an unusually large number of degrees of freedom. We are able to model the continuum and the ridges with a relatively small number of parameters, about 20 for the continuum and 2 for the ridges (per axis).

This two-dimensional peak search and fitting approach allows us many potential advantages. From each coincidence peak we can get independent energies and intensities and their statistical uncertainties. We can find weak coincidences that are not part of a long cascade and would not stand-out in the projection. Additionally, because of the greater number of channels available in the fitting region when comparing with one-dimensional fits, we get a narrower $\chi^2$ distribution that gives us better sensitivity in our goodness-of-fit parameters.

## References

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