

Theory and Methodology

Flexible automation investments: A problem formulation and solution procedure

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Abstract: In this paper, a multi-period replacement model, based on a mixed integer nonlinear programming formulation, is developed for flexible automation investments. The model takes into account the costs, benefits and effective utilization of several types of flexibility. The decision variables pertain to the selection and optimal implementation sequence for new, CNC modules, the replacement schedule for current equipment and the aggregate production plans for transition and subsequent periods in the planning horizon. The objective function maximizes the present worth of the cash flows over the planning horizon. A two-level, exact solution method is also developed, utilizing dynamic programming methodology for the higher level sub-problem and mixed integer-linear programming for the lower level sub-problem.

Keywords: Manufacturing systems; Flexible automation; Dynamic programming

1. Introduction

The advent of multimachine systems, based on computer numerical control (CNC), has created a need for equipment replacement models that address the unique characteristics of these technologies. The thrust of recent research however, has generally been on investment justification problems, which have been identified as major deterrents to implementing these technologies (Gold, 1982; Kaplan, 1986).

The myopic and tactical procedures for capital budgeting, emphasizing quick and tangible returns, along with short term-oriented managerial reward systems, and the difficulties in quantifying improvements in lead time and quality have been widely articulated. More fundamental problems, relating to ineffective manufacturing strategies, have also been highlighted for many years (e.g., Hayes and Abernathy, 1980). There has also been a dilemma as to whether one should pursue the quantification

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more rigorously, or, should one explicitly recognize 'strategic' factors without quantification. But, as Kaplan (1986) and others have stressed, capital budgeting procedures based on discounted cash flow methods are still valid, and financial analysis cannot be bypassed.

The justification problems can also be traced to the high capital costs and risk, and the high rates of obsolescence of these technologies. There has been a need for equipment replacement models that capture the unique operational characteristics (like tool magazine-oriented setups), and the economic value of various types of flexibility offered by these systems. Equipment replacement models developed to date have mainly addressed the replacement of single machines or systems. Models devoted to multimachine systems have mostly assumed serial production contexts, with limited operational flexibility. This paper presents a new multi-period replacement model and associated solution methodology for flexible multimachine systems. Several types of flexibility are considered concurrently, and an exact solution procedure is developed, by exploiting the special characteristics of the problem.

This paper is organized as follows. A review of the related literature is presented in the next section, followed by the problem definition. The means by which several types of flexibility are considered in the model, and the model formulation, are then stated. A solution procedure is presented next, followed by a numerical example and conclusions.

2. Related literature

Equipment replacement studies, commencing with the seminal work of Terborgh (1949), have developed into a comprehensive theory. A brief survey of this literature, as well as a useful reconciliation with the engineering economics approach, are provided by Fraser and Posey (1989). Van der Veen and Jordan (1989) classify the literature into models devoted to 'serial replacements' (which assume one machine or system operating in each period), and 'parallel replacements' (involving several machines in each period). As they point out, most of the models developed to date have addressed the serial replacement problem. A one-time implementation and a single-system replacement are assumed in most of these models, along with several restrictive assumptions regarding operational costs and other input data.

Studies devoted to multimachine systems have mostly assumed linear production flows. Davis and Miller (1978) and Hayes, Davis and Wysk (1981) address the 'machine requirements planning problem', in which the objective is to determine the optimum number of machines for each station and the operating speed (capacity), given production requirements for a serial flow system. Dynamic programming and mixed integer programming methods are utilized in these studies. Stinson and Khumawala (1987) utilize a heuristic procedure to derive near-optimal machine replacement sequences in a serial production system. Given a finite planning horizon, the objective in this study is to determine the time periods in which the entire system is to be shut down to make replacements, and the specific machines to be replaced in the shutdown periods. The objective function minimizes the present worth of the relevant cash outflows.

Leung and Tanchoco (1987) present a single-period model to illustrate the complexity in multimachine replacements, which involve 'challengers' capable of replacing more than one 'defender'. Given the versatility (machine flexibility) of CNC equipment, the assignment of parts and operations are redistributed among new sets of defenders and challengers, which leads to changes in the cost structure. Among studies devoted to the economic value of flexibility, expansion flexibility is first addressed in the literature by Hutchinson (1976) and Hutchinson and Holland (1982). These involve a comparison of manufacturing costs in a full-fledged transfer line and an incrementally acquired system. Burstein and Talbi (1984) propose a mixed integer programming formulation for maximizing the net present value (NPV) of investments in flexible automation, again utilizing expansion flexibility. Product flexibility has also been addressed in economic terms, and Fine and Freund (1990) provide a summary of this literature. They also develop an economic model that captures the value of product flexibility when a firm faces uncertainty in demand. A two-product case is considered, in which the firm faces capacity investment decisions before the resolution of uncertainty in demand.

Azzone and Bertele (1989) provide an integrative framework, in which many types of flexibility are considered in the justification of a specific system. Their model is based on simulation methodology. Computer simulation has been widely used in recent years for design and justification of flexible manufacturing systems (FMS). Simulation of manufacturing systems has become increasingly sophisticated, even as the base of analytical models has grown. The main utility of simulation however, is in the lower-level, more detailed stage of the analysis, for the evaluation of specific systems under detailed and realistic conditions.

The complexities of flexible automation systems generally require the design and evaluation to be carried out in two phases: a high-level approximation phase, followed by more detailed design and evaluation at the lower level. Given the numerous choices relating to part family, configuration, operating policies and parameter assumptions, the number of candidate systems to be analyzed tends to explode. Analytical approximations utilizing mathematical programming and closed queuing networks are therefore utilized at the higher level to narrow down the choices prior to detailed analysis and multiple criteria evaluation.

The model developed in this paper is aimed at providing a high-level, deterministic approximation. It is assumed that the model is to be utilized in conjunction with several other tools and techniques, particularly closed queuing networks, in a decision support system (DSS) environment, as described in Suresh (1990). The model is aimed at deriving an optimal implementation schedule for new, CNC modules, and optimal replacement of current equipment. Refinements in the treatment of system setups and process flexibility are introduced in this model. Aggregate production plans, based on optimal utilization of flexibility, are also derived for transition and subsequent periods. The objective function involves maximizing the net present value (NPV) of the after-tax cash flows. The specific problem addressed in this paper is discussed next.

3. Problem definition

It is assumed that a candidate part family, with strategic payoffs, has been tentatively identified. These part types, denoted by the index set $i \in I$, are proposed to be eventually manufactured in a multimachine system, involving a network of primarily CNC machines. The operations required for a part type $i \in I$, are denoted by the set $j \in J_i$. These operations are performed at present using current machine types, denoted by the index set $m \in \text{CMT}$. The operation capabilities are represented by an incidence matrix element, z'_{ijm} , which assumes a value of one if operation j of part type i can be processed by machine type m , and zero otherwise. The corresponding setup and operation times are denoted by ST'_{ijm} and p'_{ijm} , respectively. Similarly, the setup cost (SC'_{ijm}), unit variable cost (δ'_{ijm}), and a fixed cost element (ϵ'_{ijm}) are specified for every operation capability. The notation is summarized in Table 1.

A planning horizon, $t \in T$, is assumed. The current machines are to be replaced during the planning horizon by the new, CNC modules. The set of candidate CNC machine types considered for the part family is denoted by $n \in \text{NMT}$. These machine types, with different capabilities, capacities, and cost parameters, are to be selected, evaluated as a system and integrated. The operational capabilities of the new modules are again denoted by an incidence matrix element, z''_{ijn} . Typically, the machine flexibility of the CNC modules is higher, and therefore, the number of elements with a value of one can be expected to be greater. Also, the setup time and cost can be expected to be lower, given such features as automatic tool changers and swift downloading of part programs. The notation for the setup parameters for the new modules reflect the difference in setup activity between current and new machine types.

The number of current machines of type m and new modules of type n operating in a given period t is denoted by $M_{m,t}$ and $N_{n,t}$, respectively. $M_{m,0}$ represents the number of machines of type m (operating in various work centers) initially earmarked for replacement. As the current machines are phased out during the transition periods, the salvage value, $S_{m,t}$, contributes to the cash flow. The capital costs for new modules, which may be introduced in various periods, $C''_{n,t}$, have to be estimated based on price

Table 1
The notation

<i>Indices:</i>	
$i \in I$	Index set of part types
$j \in J_i$	Index set of operations required for part type i
$m \in \text{CMT}$	Index set of current machine types
$n \in \text{NMT}$	Index set of new machine types
$t \in T$	Index set of time periods in the planning horizon
 <i>Parameters:</i>	
z'_{ijm}, z''_{ijn}	= 1 if operation (i, j) can be performed on current and new machine types; 0 otherwise
$\delta'_{ijm}, \delta''_{ijn}$	Unit variable cost for operation (i, j) on machine types m and n
$\varepsilon'_{ijm}, \varepsilon''_{ijn}$	Fixed costs for an operation capability (i, j) on machine types m and n
p'_{ijm}, p''_{ijn}	Processing time for operation (i, j) on machine types m and n
ST'_{ijm}, SC'_{ijm}	Setup time and cost for operation (i, j) on current machine type m
ST'', SC''_n	System setup time and setup cost for a new module of type n
$D_{i,t}$	Demand forecast for part type i in period t
$P_{i,t}, MC_{i,t}$	Price and material cost forecast for part type i in period t
C'_m, C''_n	Capital costs for one unit of machine type m and n at the time of installation
$S_{m,t}$	Salvage value, after tax on capital gain/loss, of one unit of machine type m disposed of in period t
d'_m, d''_n	Straight line depreciation factors for current and new machine types
r_t	Discounting factor = $1/(1 + \text{discount rate})^t$
TR_t	Tax rate for period t
η_t	Adjusted tax factor for discount i.e., $(1 - TR_t)r_t$
$h_{i,t}$	Unit inventory carrying cost for part type i in period t
K	Number of hours of available capacity per year
γ	Slack for routing flexibility
τ_m	Last period by which a current machine of type m has to be phased out
 <i>Decision variables:</i>	
$M_{m,t}$	Number of current machines of type m in period t
$N_{n,t}$	Number of new machines of type n in period t
$X_{i,t}, Y_{i,t}$	Production quantity for part type i on current and new machine types in period t , respectively
$s_{i,t}$	Number of setups for part type i on current machines in period t
L_t	Number of operating cycles in a period
$y_{ijn,t}$	Production quantity for operation (i, j) on new machine type n in period t
$DP'_{m,t}, DP''_{n,t}$	Depreciation for current and new machine types in period t , respectively
BV_T	Book value of assets at end of planning horizon

trends for such equipment. Several other parameters shown in Table 1, including demand, price and material costs, have to be forecasted.

As the new modules are installed, and the current machines phased out, the parts may be produced using both types of machines in the transition periods. The assignment of operations and production quantities among the various machine types is based on the effective utilization of flexibility, as denoted by the decision variables $X_{i,t}$ and $s_{i,t}$ for current machines, and $Y_{i,t}$, L_t and $y_{ijn,t}$ for new machine types.

The objectives of the model are to: 1) select a subset of new machine types from the set NMT, 2) determine the optimum replacement sequence (decision variables $M_{m,t}$ and $N_{n,t}$), and, 3) derive the optimal assignment of operations and production quantities (decision variables $X_{i,t}$, $s_{i,t}$, $Y_{i,t}$, L_t and $y_{ijn,t}$). Other decision variables may also be involved, depending on the model formulation. The objective function attempts to maximize the net present value of the after-tax cash flows over the planning horizon. While it is recognized that non-financial criteria are quite important in the evaluation of advanced manufacturing systems, financial analysis cannot be bypassed altogether (Kaplan, 1986; Miltenburg and Krinsky, 1987; Hundy and Hamblin, 1988; Azzone and Bertele, 1989). As Dean (1954) articulated in an early work, "another fallacy is the notion that some projects are so pivotal for the long-run welfare of the enterprise that they possess high strategic value; (this) lifts their evaluation into a

mystic realm beyond the ken of financial analysis; the idea that there is such a thing as strategic value, not ultimately rooted in economic worth, is demonstrably wrong”.

The problem addressed corresponds to the aggregate production and resource planning phases in the hierarchy of production planning and control. Essentially, machine-level equipment replacement, typically requiring tactical-level decisions, is now elevated to considering a part family and its manufacturing processes as a whole. We next discuss the types of flexibility which are considered in the model.

4. Flexibility

The flexibility of CNC-based systems is due to reprogrammability and versatility of the equipment, i.e., the ability to perform several operations for a given job on the same setup in a machining center, as well as the capability of performing a given operation on several types of machines. When such machines are operated as a system, other types of flexibility also arise. These have been analyzed and incorporated into production planning models for flexible manufacturing systems (see Stecke and Suri, 1986, for instance). But, incorporating flexibility in economic evaluation models has continued to be a challenge.

The definitions assumed in this paper correspond mostly to Browne et al. (1984). For modeling purposes, flexibility is viewed as the *capability*, as well as the *ease* (i.e., time, cost, or any other resource) with which an internally- or externally-induced change can be accommodated by a manufacturing system. An elemental capability (like machine flexibility) may be represented by a binary-valued (0/1) parameter, and a capability considered for introduction, by a binary-valued decision variable. Providing a capability, with a certain degree of ease (i.e., speed and cost of response to changes) may involve capital costs and/or recurring operating expenses, which need to be justified in the overall system context.

Consider *machine flexibility*, for instance, which is the capability and the ease of change in processing a given set of part types. The *capabilities* are represented by the binary-valued parameters z'_{ijm} and z''_{ijn} , and the *ease*, by the relevant setup times and costs (ST'_{ijm} , ST''_n , SC'_{ijm} and SC''_n).

Expansion flexibility refers to the ability to add capacity and capability in a modular approach, by providing for that possibility in the original design. Apart from the capability of expanding/contracting a new system, an incremental implementation of a system is also viewed as the utilization of expansion flexibility.

An incremental implementation may lead to foregoing strategic opportunities, but may also result in some key advantages (Jablonowski, 1987; Talavage and Hannam, 1988):

- a) lower capital outlays in each period;
- b) a hedge against several factors of uncertainty: later investments may be made in the light of the experience with earlier investments, and a partial resolution of uncertainty in demand and technological factors; modules for which better technology is anticipated, or capital costs expected to decrease, can be stalled to later periods, etc.;
- c) a slower transition, providing for a more effective learning (especially in part programming) and absorption of CNC technologies within the firm: given the high capital costs of FMSs, there has been a tendency to forego learning and experimentation, and flexibility not utilized effectively in several US firms (Jaikumar, 1986);
- d) pilot projects: initially implementing modules leading to tangible benefits, in a bottleneck area, for instance, may serve to reduce internal resistance and justification problems, apart from ‘concept verification’ (Jablonowski, 1987).

It must be mentioned however, that a one-time installation of an integrated system may still be recommended as an optimal solution by the model. For the given problem, we assume that an incremental implementation is feasible, and its worth is reflected in the resulting NPV as a function of various implementation paths.

Volume flexibility refers to the ability to operate profitably at varying overall levels. The volume flexibility of a given system may be explored with an output measure such as the NPV, or unit manufacturing cost over the planning horizon, in response to various volume-variety scenarios.

Routing flexibility is the ability to process a given set of parts on alternate machines, in response to unanticipated events like machine breakdowns. Alternate routings are made possible by the redundancy within a machine group, as well as the versatility of other machine groups. Additional loads are imposed on alternate machines, and a factor γ may be introduced as a capacity slack to provide for this flexibility (as in Azzone and Bertele, 1989). But, it also requires duplications in tooling, adding to the costs of providing this flexibility. In the model described below, redundancy in each machine group, as well as adequate amounts of machine and process flexibility may be ensured through appropriate constraints.

Process (mix) flexibility refers to the ability to produce a given set of part types in several ways (i.e., the ability to load tooling for a variety of parts in the tool magazines), so that changeovers can be effected with little time and cost. This flexibility can be measured by the number of part types that can be simultaneously processed without using batches (Browne et al., 1984). A high level of process flexibility requires loading tools and fixtures for a broader subset of parts in a system setup, which is restricted by the capacities of the tool magazines, fixtures and pallets. Assumptions relating to system setups are discussed in a later section.

Product flexibility refers to the capability and ease in responding to new design requirements reactively, or in introducing new designs proactively. The ease is reflected in the engineering lead times, developmental costs of design, tooling, and part programs. All these are affected by the configuration chosen. Providing a high degree of product flexibility, in excess of current processing requirements, may result in higher capital costs and underutilization of capabilities, but it may serve as insurance for the future.

The product flexibility requirements are determined through technological forecasting of the rate of new product introduction, the life cycle functions, and the capabilities required for future part family. The evolution of the part family and scenarios of products, processes, volumes and variety are best handled in lower-level, stochastic and nonlinear models. In higher-level, deterministic approximations, the index sets of parts and processes assumed should simply include the product-process domain anticipated over the planning horizon.

For the estimation of various cost parameters, empirical data for CNC equipment has been available for several years (e.g., Steffy et al., 1973; Smith and Evans, 1977). As an example, we consider the reduction in non-perishable tooling costs (part of product flexibility). Two types of savings are known to materialize when switching to CNC equipment: first, since CNC involves computer-guided tool movements instead of manual operations, the jigs and fixture requirements are reduced; second, for operations which do require jigs and fixtures, standardized work holding devices meet the requirements in many cases. Steffy et al. (1973), for instance, provide empirical data on the relative magnitude of these two types of savings for various applications. In addition, data on part programming costs, savings in the downstream assembly operations (due to the consistency of parts manufactured upstream, with CNC equipment), and several other costs and benefits have been provided. Various methods for the comprehensive identification and estimation of cost parameters are also presented by Klahorst (1983) and others.

5. Model formulation

The proposed model is formulated as a nonlinear integer programming problem (P1), as shown in Table 2. We first discuss the objective function, followed by the various constraints.

The objective function maximizes the NPV of the after-tax cash flows arising from several sources. The NPV is now generally accepted as the proper evaluation criterion for these investments (Kaplan, 1986; Miltenburg and Krinsky, 1987). The various expressions for the after-tax cash flows to be considered in this context are listed below (adapted from Suresh, 1992).

Table 2
Problem (P1)

$$\begin{aligned}
 \text{Maximize NPV} &= + \sum_t \sum_i (P_{i,t} - MC_{i,t}) D_{i,t} (1 - TR_t) r_t & (a) \\
 &- \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C_{n,t}'' (1 - IC_{n,t}) r_t & (b) \\
 &+ \sum_t \sum_m (M_{m,t-1} - M_{m,t}) S_{m,t} r_t & (c) \\
 &+ \sum_t \left(\sum_n DP_{n,t}'' + \sum_m DP_{m,t}' \right) TR_t r_t & (d) \\
 &- \sum_t \left[\sum_i \sum_j \sum_n \delta_{ijn}'' y_{ijn,t} + \sum_i \sum_j \sum_m \delta_{ijm}' X_{i,t} \right] (1 - TR_t) r_t & (e)\&(f) \\
 &- \sum_t \left[\left(\sum_i \sum_j \sum_n \varepsilon_{ijn}'' N_{n,t} + \sum_i \sum_j \sum_m \varepsilon_{ijm}' M_{m,t} \right) \right] (1 - TR_t) r_t & (g)\&(h) \\
 &- \sum_t \left[L_t \sum_n SC_n'' N_{n,t} + \sum_i \sum_j \sum_m s_{i,t} SC_{ijm}' \right] (1 - TR_t) r_t & (i)\&(j) \\
 &- \sum_t \left[\sum_i \frac{1}{2} (D_{it} / L_t) h_{i,t} + g(L_t) \right] (1 - TR_t) r_t & (k) \\
 &+ BV_T r_T & (l) \\
 \text{s.t. } M_{m,t} &\leq M_{m,t-1} \quad \forall m, t, & (1) \\
 N_{n,t} &\geq N_{n,t-1} \quad \forall n, t, & (2) \\
 M_{m,t} &= 0 \quad \forall m, t = \tau_m, & (3) \\
 Y_{i,t} + X_{i,t} &= D_{i,t} \quad \forall i, t, & (4) \\
 y_{ijn,t} &\leq z_{ijn}'' D_{i,t} \quad \forall i, j, m, t, & (5) \\
 \sum_n y_{ijn,t} &= Y_{i,t} \quad \forall i, j, t, & (6) \\
 \sum_i \sum_j p_{ijn}'' y_{ijn,t} + L_t ST'' N_{n,t} &\leq K \gamma N_{n,t} \quad \forall n, t, & (7) \\
 \sum_i \sum_j p_{ijm}' X_{i,t} + \sum_i \sum_j ST_{ijm}' s_{i,t} &\leq K \cdot M_{m,t} \quad \forall m, t, & (8) \\
 X_{i,t} &\leq V v_t \quad \forall i, t, & (9) \\
 s_{i,t} - L_t &\leq V(1 - v_t) \quad \forall i, t, & (10) \\
 L_t - s_{i,t} &\leq V(1 - v_t) \quad \forall i, t, & (11) \\
 DP_{n,t}'' &= DP_{n,t-1}'' + (N_{n,t} - N_{n,t-1}) C_{n,t}'' d_n'' \quad \forall n, t, & (12) \\
 DP_{m,t}' &= M_{m,t} C_m' d_m' \quad \forall m, t, & (13) \\
 BV_T - \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C_{n,t}'' (1 - IC_{n,t}) &+ \sum_t \sum_n DP_{n,t}'' = 0. & (14)
 \end{aligned}$$

(a) Sales revenue less material costs:

$$+ \sum_t \sum_i (P_{i,t} - MC_{i,t}) D_{i,t} (1 - TR_t) r_t. \tag{a}$$

The above expression, denoted by ϕ , is constant as per assumption of the model. The parameters $P_{i,t}$, $MC_{i,t}$, and $D_{i,t}$ are the forecasts of price, material cost and demand, respectively, for part type i for period t . TR_t is the tax rate and r_t is the discounting factor assumed.

(b) *Capital costs of new modules:*

$$- \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C_{n,t}'' (1 - IC_{n,t}) r_t. \quad (b)$$

The term $N_{n,t} - N_{n,t-1}$ represents the number of new modules of type n installed in time t , and $C_{n,t}''$ is the capital cost applicable for that period. $IC_{n,t}$ is the investment credit factor, if applicable.

(c) *Salvage value of current machines:*

$$+ \sum_t \sum_m (M_{m,t-1} - M_{m,t}) S_{m,t} r_t. \quad (c)$$

The term $M_{m,t-1} - M_{m,t}$ denotes the number of machines of type m disposed of in time t , and $S_{m,t}$ is the salvage value, after adjusting for tax on capital gain/loss. As per the Tax Reform Act of 1986 in the US, the tax rate TR_t applies to capital gains as well (Canada and Sullivan, 1989, p. 186). All current machines of a given type are assumed to have the same salvage value, which requires that the current machines be grouped accordingly.

(d) *Depreciation charges:*

$$+ \sum_t \left(\sum_n DP_{n,t}'' + \sum_m DP_{m,t}' \right) TR_t r_t. \quad (d)$$

The depreciation charges for the current and new machine types are computed in constraint sets (12) and (13) below.

(e)–(h) *Variable and fixed costs:* The variable and fixed costs on new and current machines are given by

$$- \sum_t \sum_i \sum_j \sum_n \delta_{ijn}'' y_{ijn,t} (1 - TR_t) r_t, \quad (e)$$

$$- \sum_t \sum_i \sum_j \sum_m \delta_{ijm}' X_{i,t} (1 - TR_t) r_t, \quad (f)$$

$$- \sum_t \sum_i \sum_j \sum_n \epsilon_{ijn}'' N_{n,t} (1 - TR_t) r_t, \quad (g)$$

$$- \sum_t \sum_i \sum_j \sum_m \epsilon_{ijm}' M_{m,t} (1 - TR_t) r_t. \quad (h)$$

(i) & (j) *Setup costs:* The expressions for setup costs consider the differences in setup activity between traditional and CNC equipment. For the new modules, the setup costs depend on L_t , the number of system setups in a period, and the costs of setting up machines of various types in every system setup:

$$- \sum_t L_t \left(\sum_n SC_n'' N_{nt} \right) (1 - TR_t) r_t. \quad (i)$$

Since both L_t and $N_{n,t}$ are decision variables, this term is nonlinear. Another type of cost which is related to the number of system setups is the *process flexibility cost*, which, for convenience, is considered along with inventory costs below. The setup costs for the current machine types, based on traditional setups, are given by

$$- \sum_t \left(\sum_i \sum_j \sum_m SC_{ijm}' s_{i,t} \right) (1 - TR_t) r_t, \quad (j)$$

where $s_{i,t}$ is the number of lots or setups for part type i on current machines in period t .

(k) *Inventory and process flexibility costs:* With mathematical programming models, the dynamics of multiproduct production runs cannot be fully captured. Approximations for work-in-process inventory are required, with more refined estimates being obtained through queuing network models and

simulation. An average lot size of $D_{i,t}/L_t$ is assumed, and the inventory carrying and process flexibility costs are approximated as

$$- \sum_t \left[\sum_i \frac{1}{2} (D_{i,t}/L_t) h_{i,t} + g(L_t) \right] (1 - TR_t) r_t. \quad (k)$$

The above expression is nonlinear. The second term, $g(L_t)$, is a real-valued function which represents the capital cost of providing process flexibility. With more costly tool magazines, providing higher capacity, increased number of pallets and fixtures, etc., the number of system setups (L_t) can be reduced. System setups are necessitated primarily by the capacity limitations of tool magazines, fixtures and pallets, and having to switch between subsets of the part family based on short term demand schedules, and efficient loading of the system. They typically occur in intervals of one to three weeks (Stecke, 1986), and a tight bound on the value of L_t can be arrived at. For instance, with 50 work-weeks in a year, values between 12 and 50, corresponding to monthly and weekly setups, respectively, can be reasonably assumed.

(l) *Value of the assets at the end of planning horizon:* Finally, the value of the assets at the end of the planning horizon needs to be included. The book value is assumed when the market value cannot be precisely estimated (Miltenburg and Krinsky, 1987):

$$+ BV_T r_T. \quad (1)$$

This is computed in constraint set (14) below. The model constraints are considered next, starting with the restrictions on implementation.

Implementation constraints

The following two constraint sets specify that the current machines are phased out as the new modules are installed. It is assumed that the installation of new modules and the replacement of current equipment take place at the beginning of a period. Also, the new modules are not to be replaced during the planning horizon.

$$M_{m,t} \leq M_{m,t-1} \quad \forall m, t, \quad (1)$$

$$N_{n,t} \geq N_{n,t-1} \quad \forall n, t. \quad (2)$$

Based on design and strategy considerations, relating to a specific problem situation, several other constraints may be added. Some of these, gleaned from the literature on implementation of these systems, are listed below:

- a) Technological precedence: some modules may have to be installed before certain others to ensure technical feasibility.
- b) Useful life of current equipment, denoted by the last period before which a current machine type has to be phased out:

$$M_{m,t} = 0 \quad \forall m, t = \tau_m. \quad (3)$$

- c) Ensuring adequate amount of machine, product, routing, process and other flexibilities: the selection of modules from the candidate set NMT may be constrained by the need to provide adequate amounts of each of these flexibilities. For instance, routing flexibility, 'pooling effects', etc. are enhanced by limiting the number of modules selected, so that there is redundancy in each machine group. Limiting the number of machine groups also reduces maintenance complexity and induces standardization in tooling.
- d) Financial constraints: spreading the capital costs and risk, a major consideration in practice, is ensured by specifying a capital budget for each period.

- e) Minimizing disruptions: constraints may be imposed on the number of new modules to be installed in a period.
- f) Timing of introduction for part types and modules: some technologies may be anticipated in later periods and more expensive modules may be installed only after a certain period; some part types and modules may be introduced in the beginning to maximize learning and experimentation; some part types may be introduced only in later periods; the selection of part types and modules in the initial periods may be geared to achieve immediate and tangible impact on quality and/or lead time: since, achieving a reputation for quality requires some length of time.

For clarity of presentation, only constraint b) (enforcing the useful life of current equipment) from the above set was utilized in this research. Presence of this constraint set in the model delineates the 'gradual replacement' phenomenon more effectively. Moreover, constraint set b) can result in a substantial reduction of the number of feasible states in a dynamic programming solution methodology.

Assignment of operations and production quantities

First, the annual demand for a part type i ($D_{i,t}$) is split into the quantity produced completely on current machine types ($X_{i,t}$) and on new machine types ($Y_{i,t}$). This assumption stems mainly from differences in processing requirements, and assuming a low interchangeability of work-in-process between traditional and CNC equipment. Thus the following constraint is introduced:

$$Y_{i,t} + X_{i,t} = D_{i,t} \quad \forall i, t. \quad (4)$$

After the transition periods, the $X_{i,t}$ -values are set to zero. The CNC modules are assumed to be operated as a system or, a mini-system during transition, and a system setup time, ST'' , is assumed, during which the various tool magazines are loaded.

We next consider the production assignments on new machine types. Production for an operation (i, j) in period t , can be assigned to a machine group only if it is capable of performing the operation:

$$y_{ijn,t} \leq z_{ijn}'' D_{i,t} \quad \forall i, j, n, t. \quad (5)$$

A given operation (i, j) may be assigned to different machine types, but the total quantity produced should equal $Y_{i,t}$:

$$\sum_n y_{ijn,t} = Y_{i,t} \quad \forall i, j, t. \quad (6)$$

The capacity constraint for a new module of type n is given by

$$\sum_i \sum_j p_{ijn}'' y_{ijn,t} + L_t ST'' N_{n,t} \leq K \gamma N_{n,t} \quad \forall n, t. \quad (7)$$

The first term represents the total processing time, and the second the total time lost due to system setups on a given machine type.

Similarly, the following four constraint sets hold for production assignments on current machine types:

$$\sum_i \sum_j p'_{ijm} X_{i,t} + \sum_i \sum_j ST'_{ijm} s_{i,t} \leq K \cdot M_{m,t} \quad \forall m, t, \quad (8)$$

$$X_{i,t} \leq V v_t \quad \forall i, t, \quad (9)$$

$$s_{i,t} - L_t \leq V(1 - v_t) \quad \forall i, t, \quad (10)$$

$$L_t - s_{i,t} \leq V(1 - v_t) \quad \forall i, t. \quad (11)$$

It is assumed that a given operation (i, j) can be performed on only one current machine type, which is a reasonable assumption in most cases. With this assumption of limited machine flexibility for current machines, $X_{i,t}$ and $s_{i,t}$ are the production quantity and number of setups for all the operations for a

given part type on the appropriate machines. After the transition periods, when $M_{m,t} = 0$, both the production quantities and setups are set to zero by (8). But, during transition, the number of setups for an operation (i, j) is assumed to equal L_t . Thus, the setups either equal zero or L_t , which is ensured by constraint sets (10) and (11), where V is a large positive number and ν_t is a binary variable.

Depreciation charges and final value of the assets

Assuming straight-line depreciation, the depreciation charges for new modules are computed using the following constraint:

$$DP''_{n,t} = DP''_{n,t-1} + (N_{n,t} - N_{n,t-1})C''_{n,t}d''_n \quad \forall n, t. \tag{12}$$

It may be noted that the additional depreciation due to the installation of new modules is based on the capital costs at the time of installation. For current machine types the depreciation charges are written as

$$DP'_{m,t} = M_{m,t}C'_m d'_m \quad \forall m, t. \tag{13}$$

when $M_{m,t} = 0$, $DP'_{m,t}$ becomes zero. The depreciation reduces when the number of machines is reduced, and the term $M_{m,t-1} - M_{m,t}$ in the objective function ensures that the salvage value and tax effects on capital gain/loss are taken into account in that period. Finally, the book value of the assets at the end of the planning horizon is considered if the market value cannot be estimated:

$$BV_T - \sum_t \sum_n (N_{n,t} - N_{n,t-1})C''_{n,t}(1 - IC_{n,t}) + \sum_t \sum_n DP''_{n,t} = 0. \tag{14}$$

A solution procedure for problem (P1) is developed next.

6. The algorithm

Problem (P1) has several special characteristics which may be utilized to simplify the formulation. They may also be used for developing an efficient solution algorithm. While computational efforts for this problem may be easily justified, given the high capital costs and risk of these systems, one is still interested in efficient solution procedures to facilitate sensitivity analysis, and interactive decision making.

To begin with, (P1) can be simplified by: 1) writing the after-tax, discounted cash flow from sales less material costs as ϕ ; 2) substituting the depreciation and book values from constraints (12) to (14) into the objective function; 3) combining the objective function terms involving decision variable $N_{n,t}$; 4) combining the objective function terms involving decision variable $M_{m,t}$; and 5) eliminating constraint set (5). The reduced problem, (P2), is summarized in Table 3.

The objective function terms involving $N_{n,t}$, in (P1), include capital costs (b), depreciation (d), fixed costs (g), and book value (l). The depreciation and book value expressions, given by constraints (12) and (14), are substituted into the above terms. They can be reduced, as shown in the Appendix, to the following form:

$$+ \sum_t \sum_n \alpha_{n,t} N_{n,t}.$$

Similarly, the terms involving $M_{m,t}$ include salvage value (c), depreciation (d), and fixed operating costs (h) in the objective function, along with constraint set (13). These are reduced to the following terms, as shown in the Appendix:

$$+ \beta_{m0} M_{m0} + \sum_t \sum_m \beta_{m,t} M_{m,t}.$$

The values for the parameters, $\alpha_{n,t}$ and $\beta_{m,t}$, are also provided in the Appendix.

Table 3
Problem (P2)

$\max \text{NPV} = \phi - \sum_t \sum_n \alpha_{nt} N_{nt} + \sum_t \sum_m \beta_{mt} M_{mt}$	(a)&(b)
$- \sum_t \sum_i \sum_j \sum_n \delta''_{ijn} y_{ijn,t} \eta_t$	(c)
$- \sum_t \sum_i \sum_j \sum_m \delta'_{ijm} X_{i,t} \eta_t$	(d)
$- \sum_t \left[L_t \sum_n \text{SC}''_n N_{nt} + \sum_i \sum_m s_{i,t} \text{SC}'_{ijm} \right] \eta_t$	(e)&(f)
$- \sum_t \left[\sum_i \frac{1}{2} (D_{it} / L_t) h_{i,t} + g(L_t) \right] \eta_t$	(g)
<p>s.t. $M_{m,t} \leq M_{m,t-1} \quad \forall m, t,$</p>	(1)
<p>$N_{n,t} \geq N_{n,t-1} \quad \forall n, t,$</p>	(2)
<p>$M_{m,t} = 0 \quad \forall m, t = \tau_m,$</p>	(3)
<p>$Y_{i,t} + X_{i,t} = D_{i,t} \quad \forall i, t,$</p>	(4)
<p>$\sum_n y_{ijn,t} = Y_{i,t} \quad \forall i, j, t,$</p>	(6)
<p>$\sum_i \sum_j p''_{ijn} y_{ijn,t} + L_t \text{ST}'' N_{n,t} \leq K \cdot N_{n,t} \quad \forall n, t,$</p>	(7)
<p>$\sum_i \sum_j p'_{ijm} X_{i,t} + \sum_i \sum_j \text{ST}'_{ijm} s_{i,t} \leq K \gamma M_{m,t} \quad \forall m, t,$</p>	(8)
<p>$X_{i,t} \leq V \nu_t \quad \forall i, t,$</p>	(9)
<p>$s_{i,t} - L_t \leq V(1 - \nu_t) \quad \forall i, j, t,$</p>	(10)
<p>$L_t - s_{i,t} \leq V(1 - \nu_t) \quad \forall i, j, t,$</p>	(11)

Constraint set (5) of (P1) ensures the assignment of production quantities consistent with the operation capabilities of various machine types, as indicated by the parameters z''_{ijn} . These machine flexibility parameters assume a value of one for a capability and a value of zero for an incapability. These constraints can be eliminated by assigning large values for the variable cost parameters (δ''_{ijn} in the objective function ('the big M method') corresponding to incapacibilities. At the expense of intuitive appeal, an incapability is thus treated as a capability with a low degree of ease. Alternatively, a matrix generator may be utilized to generate the problem. In this case, the $y_{ijn,t}$ -variables corresponding to zero-valued z''_{ijn} may be eliminated altogether while generating the problem.

In (P2), the three nonlinear terms remain, in expressions (e) and (g) of the objective function, and in constraint set (7). One approach to remove the nonlinearity is to linearize these terms through suitable approximations. One can then solve the linearized problem through a branch and bound procedure. But, in the proposed algorithm, the nonlinearity is retained, and the problem is solved as a partitioned, two-level problem. The higher-level problem, referred to as the configuration-level sub-problem (CLSP), consists of a recursive dynamic programming (DP) equation. The lower-level problem, referred to as the operational-level sub-problem (OLSP), is a mixed integer linear program, as shown in Table 4. Values for the configuration-related variables ($M_{m,t}$ and $N_{n,t}$) are generated by (CLSP), and values for the operational variables ($X_{i,t}$, $s_{i,t}$, $Y_{i,t}$, $y_{ijn,t}$ and L_t) determined by (OLSP).

The configuration-level sub-problem (CLSP) is formulated as a forward-inductive dynamic programming problem. The *stages* of the DP denote the time periods in the planning horizon, while the *states* represent the number of machines of each type, operating in a period, as shown in Figure 1. This multi-dimensional state vector, consisting of $\{M_{1,t}, M_{2,t}, \dots, M_{|CMT|,t}\}$ and $\{N_{1,t}, N_{2,t}, \dots, N_{|NMT|,t}\}$, is

Table 4
Operational level sub-problem (OLSP)

$$\begin{aligned}
 & Z_t^*([M_t, N_t], [M_{t-1}, N_{t-1}]) \\
 & = \max - \sum_i \sum_j \sum_n \delta''_{ijn} y_{ijn,t} \eta_t - \sum_i \sum_j \sum_m \delta'_{ijm} X_{i,t} \eta_t - \left[L_t \sum_n SC''_n N_{n,t} + \sum_i \sum_m s_{i,t} SC'_{ijm} \right] \eta_t \\
 & \quad - \left[\sum_i \frac{1}{2} (D_{it} / L_t) h_{i,t} + g(L_t) \right] \eta_t - \sum_n \alpha_{n,t} N_{n,t} + \sum_m \beta_{m,t} M_{m,t} \\
 \text{s.t. } & Y_{i,t} + X_{i,t} = D_{i,t} \quad \forall i, & (4) \\
 & \sum_n y_{ijn,t} = Y_{i,t} \quad \forall i, j, & (6) \\
 & \sum_i \sum_j p''_{ijn} y_{ijn,t} + L_t ST'' N_{n,t} \leq K \gamma N_{n,t} \quad \forall n, & (7) \\
 & \sum_i \sum_j p'_{ijm} X_{i,t} + \sum_i \sum_j ST'_{ijm} s_{i,t} \leq K \cdot M_{m,t} \quad \forall m, & (8) \\
 & X_{i,t} \leq V \nu \quad \forall i, & (9) \\
 & s_{i,t} - L_t \leq V(1 - \nu) \quad \forall i, j, & (10) \\
 & L_t - s_{i,t} \leq V(1 - \nu) \quad \forall i, j, & (11) \\
 & \text{and, any additional side-constraints involving } [M_t, N_t] \text{ and } [M_{t-1}, N_{t-1}].
 \end{aligned}$$

denoted by $[M_t, N_t]$. Let ψ_t be the set of all possible combinations of $[M_t, N_t]$. ψ_t is the resulting state space in stage t , and its size can be reduced substantially by considering several design and implementation requirements.

It is particularly useful to apply queuing network approximations at this stage, to narrow the design requirements, and screen out states corresponding to poor configurations. Multimachine systems like the FMS typically involve only three or four machine groups, with one or more machines in each group. To ensure adequate amounts of routing flexibility, the number of machine types selected may be restricted to a certain limit, so that there is redundancy of machines of each type. The state space is also reduced due to implementation requirements. After the transition period, for instance, the state vector corre-

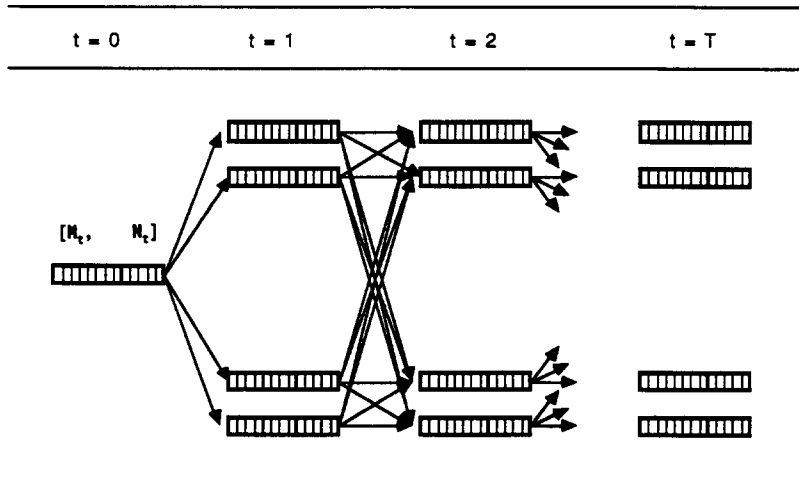


Fig. 1. Dynamic programming formulation

sponds only to the new machines of various types. A tight upper bound on the number of machines required in every period can be established by assessing the capacity required. At every stage, several states are infeasible, and so are several transitions from the feasible states of one stage to the next.

The solution to the operational-level sub-problem (OLSP) yields the optimal (negative) cost of utilizing M_t current machines and N_t new modules in period t , given M_{t-1} current machines and N_{t-1} new modules in operation in period $t-1$. Let $Z_t^*([M_t, N_t], [M_{t-1}, N_{t-1}])$ be this optimal (negative) cost. Further, let $f_t([M_t, N_t])$ be the optimal cost of utilizing M_t current machines and N_t new modules. In dynamic programming, this is the optimal-value function. For the first period, only the transition from the current machines to $[M_1, N_1]$ is involved. Hence,

$$f_1([M_1, N_1]) = Z_1^*([M_1, N_1][M_0, N_0]), \quad [M_1, N_1] \in \psi_1. \quad (15)$$

For subsequent stages (periods), the following recursion can be applied to calculate the cost:

(CLSP)

$$f_t([M_t, N_t]) = \max\{Z_t^*([M_t, N_t], [M_{t-1}, N_{t-1}]) + f_{t-1}([M_{t-1}, N_{t-1}])\} \quad (16)$$

$$M_t \leq M_{t-1}, \quad (17)$$

$$N_t \geq N_{t-1}, \quad (18)$$

$$M_t = 0: t \geq \tau_m. \quad (19)$$

The value of $[M_t, N_t]$ that maximizes (16), for a given stage t , is denoted by $[M_t, N_t]^*$ and the optimal decision function is denoted by $d_t([M_t, N_t]^*)$. The optimal decision function stores the previous optimal state vector.

The value of $f_T[M_T, N_T]$, found by applying the recursion, gives the maximum NPV for utilizing $[M_T, N_T]$ in the last period T . The value of $[M_T, N_T]$ that maximizes $f_T(\cdot)$, denoted by $[M_T, N_T]^*$, yields the optimal solution to (CLSP). The decisions corresponding to this solution are found, beginning with $d_T([M_T, N_T]^*)$ and back-tracking through the optimal decision function. This yields the optimal number of current and new machines in each period. The optimal decisions may then be substituted into (OLSP) and the optimal solution of the resulting (OLSP) gives the optimal values of the operational decision variables.

The operational-level sub-problem (OLSP) is a mixed integer linear programming problem (MILP), as mentioned earlier, and it can be solved by a standard MILP package. The lower-level problem is executed for several feasible states in the higher-level routine. Since $M_{m,t}$ and $N_{n,t}$ are passed on as parameters, two of the three nonlinear terms are linearized: the term (e) in the objective function of (P2) and constraint set (7). In addition, since (OLSP) is executed for various states (periods), the decision variables all correspond to a single time period. This eliminates the time subscript, and significantly reduces the number of variables and constraints. It is also worth mentioning that (OLSP) has only one binary integer variable (ν). Consequently, the problem can be reduced to two standard linear programs by letting $\nu = 0$ once and then $\nu = 1$. The better of the two solutions is then passed back to (CLSP) as the optimal solution. The only nonlinear term remaining corresponds to inventory and process flexibility-related costs:

$$-\left[\sum_i \frac{1}{2}(D_{it}/L_t)h_{i,t} + g(L_t) \right] (1 - TR_t)r_t.$$

This is replaced by a piecewise linear approximation, involving feasible ranges of values for L_t . Values between 12 and 50 may be reasonably assumed, for instance, as mentioned earlier. It is also difficult to envision the costs associated with process flexibility as a continuous function. We next consider a numerical example to illustrate the basic capabilities of the model.

Table 5
Machine-related data

<i>Current machines:</i>							
Type	$M_{m,0}$	C'_m (000s)	τ_m	Salvage values $S_{m,t}$ (000s) in year			
				1	2	3	4
1	2	500	4	125	100	60	30
2	1	400	4	100	80	60	25
3	3	250	3	80	55	20	0
4	2	250	3	75	65	15	0
5	2	175	2	90	45	0	0
6	2	200	3	75	55	25	0

<i>New modules:</i>								
Type	Capital costs ($C'_{n,t}$) (000s) in year							
	1	2	3	4	5	6	7	8
1	1050	1050	1000	1000	950	950	950	950
2	1100	1050	1050	1000	1000	1000	1000	1000
3	950	950	950	900	900	900	900	900
4	1000	1000	975	975	975	975	950	950
Budget	3300	3300	1100	0	0	0	0	0

7. A numerical example

The inputs for the numerical example are presented in Tables 5 to 7. There are six types of current machines, with one to three machines of each type to be replaced. The salvage values, initial capital costs, the last period before which each current machine type must be replaced are shown in Table 5. Four new machine types, or modules, are considered for this limited example. The capital cost for these machine types, and the capital budget are also shown. A planning horizon of 8 years, a tax rate of 34%, and a discounting factor of 0.12 are assumed.

The part family consists of 3 part types, having the demand forecasts, prices, and material costs shown in Table 6. This scenario is based on assumptions of increasing demand and material costs, but declining prices. The three part types require 5, 5, and 4 operations, respectively. The operation capabilities of current and new machines are shown in Table 7. For brevity, data on setup and operation times and costs have been omitted.

Table 6
Part-related data

Part i	Year							
	1	2	3	4	5	6	7	8
<i>Demand forecasts ($D_{i,t}$):</i>								
1	8000	8250	8500	8750	9000	9250	9500	9500
2	7000	7500	8000	8500	9000	10000	10000	10500
3	6000	6000	6500	6500	6500	7000	7000	7000
<i>Price forecast ($PR_{i,t}$):</i>								
1	165.00	160.00	155.00	150.00	145.00	145.00	140.00	140.00
2	175.00	165.00	157.00	153.00	150.00	147.00	145.00	142.00
3	175.00	165.00	160.00	160.00	160.00	160.00	160.00	150.00
<i>Material costs ($MC_{i,t}$):</i>								
1	95.00	97.85	100.79	103.81	106.92	110.13	113.43	116.84
2	75.00	78.00	81.12	84.36	87.74	91.25	94.90	98.69
3	80.00	81.60	83.23	84.90	86.59	88.33	90.09	91.89

Table 7
Machine capabilities

Capabilities of current machines (z'_{ijm})								Capabilities of new modules (z''_{ijn})			
Operation (i, j)		Machine						Module			
i	j	1	2	3	4	5	6	1	2	3	4
1	1	1	0	0	0	0	0	1	0	0	0
	2	0	1	0	0	0	0	1	1	0	0
	3	0	0	0	1	0	0	0	1	0	0
	4	0	0	0	0	1	0	0	1	1	1
	5	0	0	0	0	0	1	0	0	1	1
2	1	1	0	0	0	0	0	1	0	0	0
	2	0	1	0	0	0	0	1	1	0	0
	3	0	0	1	0	0	0	0	1	0	0
	4	0	0	0	1	0	0	0	1	1	1
	5	0	0	0	0	1	1	0	1	1	1
3	1	1	0	0	0	0	0	1	1	0	0
	2	0	0	1	0	0	0	1	1	0	0
	3	0	0	0	1	0	0	0	1	1	1
	4	0	0	0	0	0	1	0	1	1	1

The optimal solution for the numerical example is presented in Tables 8 to 10. The current machines are seen to be utilized in period 1 and salvaged at the end of this period. However, due to the overall cost-effectiveness, one new module each of types 1, 2 and 4 are obtained at an early stage (period 1).

Observing the aggregate production schedule (bottom of Table 8) during period 1, it is seen that the demands for part types 1 and 2 have been split among current machines and new modules. Table 9

Table 8
Optimal replacement and production plan

m	Year									
	0	1	2	3	4	5	6	7	8	
<i>Current machines ($M_{m,t}$):</i>										
1	2	1	-	-	-	-	-	-	-	-
2	1	1	-	-	-	-	-	-	-	-
3	3	1	-	-	-	-	-	-	-	-
4	2	1	-	-	-	-	-	-	-	-
5	2	1	-	-	-	-	-	-	-	-
6	2	1	-	-	-	-	-	-	-	-
<i>New modules ($N_{n,t}$):</i>										
1		1	2	2	2	2	2	2	2	2
2		1	1	1	1	2	2	2	2	2
4		1	2	2	2	2	2	2	2	2
<i>Part i</i>										
<i>Production on current machines ($X_{i,t}$):</i>										
1	5576	-	-	-	-	-	-	-	-	-
2	790	-	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	-	-
<i>Production on new machines ($Y_{i,t}$):</i>										
1	2424	8250	8500	8750	9000	9250	9500	9500	9500	
2	6210	7500	8000	8500	9000	9500	10000	10000	10500	
3	6000	6000	6500	6500	6500	7000	7000	7000	7000	

Table 9
Operation assignments

Assignments on current machines			Assignments on new modules $y_{ijn,t}$									
m	$i:j$	Year 1	n	$i:j$	Year							
					1	2	3	4	5	6	7	8
1	1:1	5576	1	1:1	2424	8250	8500	8750	9000	9250	9500	9500
	2:1	790		1:2	2424	8250	8500	8750	9000	9250	9500	9500
				2:1	6210	7500	8000	8500	9000	9500	10000	10500
2	1:2	5576		2:2	528	7500	8000	8433	7400	5367	4333	3800
	2:2	790		3:1	6000	6000	6500	6500	6500	7000	7000	7000
				3:2	6000	6000	6500	6500	6500	7000	7000	7000
3	2:3	790	2	1:3	2424	8250	8500	8750	9000	9250	9500	9500
4	1:3	5576		2:2	5682	-	-	67	1600	4133	5667	6700
	2:4	790		2:3	6210	7500	8000	8500	9000	9500	10000	10500
5	1:4	5576	2:5	6210	4500	3789	3026	9000	9500	10000	10500	
	2:5	790	1:4	-	-	-	-	4553	-	5421	3605	
6	1:5	5576	4	3:4	-	-	-	-	6500	-	-	-
				1:4	2424	8250	8500	8750	4447	1408	4079	5895
				1:5	2424	8250	8500	8750	9000	9250	9500	9500
				2:4	6210	7500	8000	8500	9000	9500	10000	10500
				2:5	-	3000	4211	5474	-	-	-	-
3:3	6000	6000	6500	6500	6500	7000	7000	7000	7000			
3:4	6000	6000	6500	6500	-	7000	7000	7000	7000			

presents the operation assignments on current and new machines. Consider part type 1, for instance, which requires five operations. The production has been assigned to current machines 1, 2, 4, 5 and 6 in period 1, and new modules 1, 2 and 4. Table 10 shows the cash flows corresponding to the above replacement and production plans. It is seen that the NPV amounts to \$4.463 million.

Table 10
Cash flows for optimal solution

Cash flows (000s) in year								Book value
1	2	3	4	5	6	7	8	
<i>Sales less material cost:</i>								
1830.00	1695.64	1601.06	1475.74	1347.67	1325.76	1172.73	1081.48	
<i>Capital costs:</i>								
-3150.00	-2050.00	0.00	0.00	-1000.00	0.00	0.00	0.00	
<i>Depreciation:</i>								
133.88	221.00	221.00	221.00	263.50	263.50	263.50	263.50	756.25
<i>Salvage value of current machines:</i>								
525.00	400.00	0.00	0.00	0.00	0.00	0.00	0.00	
<i>Depreciation:</i>								
75.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
<i>Operating costs:</i>								
-219.34	-87.58	-122.93	-155.98	-79.92	-124.23	-139.13	-183.60	
<i>Total cash flow:</i>								
-805.03	179.05	1699.12	1540.77	531.25	1465.04	1297.09	1161.38	
<i>Present worth:</i>								
-805.03	159.87	1354.53	1096.69	337.62	831.30	657.15	525.35	305.4
Net present value (@ 12%) = \$4.4629 million.								

The above problem was solved using a computer implementation of the algorithm. The program was developed in FORTRAN IV on an IBM 3084 under CMS environment. The dynamic programming part of the algorithm was coded using a forward-inductive approach, emphasizing computational efficiency (instead of memory efficiency). That is, the transitions between all current feasible states and potential next states were first evaluated. A linear list was utilized to store the end-nodes (corresponding to the current and next states) and the associated costs of all feasible arcs. Next, all of the generated arcs were scanned to identify optimal current states for feasible next states. The optimal current states were then stored in the decision vector. This approach requires additional computer storage (memory) but results in a computationally more efficient code. The reason is that once a state becomes infeasible, all arcs (next states) emanating from it can be skipped. Whereas, if the next states were evaluated, for optimal current states, one at a time, an infeasible current state would be tested many times.

Initially, there were 648 states, resulting in 17 036 feasible arcs (OLSPs). Each OLSP included 40 variables and 45 constraints. The (OLSP)s associated with feasible arcs were solved using LINDO (Schrage, 1984). LINDO was utilized because of its relative ease of use, availability on a mainframe with virtual memory technology, and having the capability of being interfaced with the DP code. Overall, the case problem had 3314 consistent next states and 34 297 feasible arcs. The solution process required about 4 hours of execution time. Clearly, the required computational effort is well justified for the magnitude of savings achieved.

8. Conclusions

In this paper, a multi-period replacement model was developed for flexible automation systems. Several types of flexibility were considered concurrently in the economic evaluation. The model was aimed at deriving an optimal schedule for the replacement of current machines, implementation of new modules, and production plans capitalizing on flexibility during transition and subsequent periods. The model was intended to serve as an analytical approximation, along with closed queueing networks, prior to detailed analysis of specific system configurations. The formulation was simplified, and a solution procedure developed by partitioning into a two-level problem. Dynamic programming methodology was utilized at the higher, configuration level, while a mixed integer program was employed at the lower, operational level. The objective function involved maximizing the net present value of the cash flows.

Several extensions of this work are warranted. Further refinements in modeling process flexibility, as well as other types of flexibility, are likely to be of significant value. The impact of other production planning assumptions, particularly those involving system setups and lot sizes, needs to be investigated. On the financial dimension, the influence of accelerated depreciation methods (ACRS) on the investment decisions is one area for further investigation. The formulation also needs to be extended into a multiple-criteria decision problem, in which the goal of maximizing the net present value has to be tempered by the pursuit of other, conflicting strategic goals. The model developed in this paper represents an initial step in structuring this critical investment problem.

Appendix

Objective function coefficients of $N_{n,t}$ ($\alpha_{n,t}$)

The following terms in the objective function of Problem (P1) are combined first:

$$- \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C_{n,t}'' (1 - IC_{n,t}) r_t \quad (b)$$

$$+ \sum_t \sum_n DP_{n,t}'' TR_t r_t \text{ from} \quad (d)$$

$$- \sum_t \sum_i \sum_j \sum_n \varepsilon''_{ijn} N_{n,t} (1 - TR_t) r_t \tag{g}$$

$$+ BV_T r_T. \tag{1}$$

From constraint (14),

$$BV_T r_T = \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C''_{n,t} (1 - IC_{n,t}) r_T - \sum_t \sum_n DP''_{n,t} r_T.$$

This is substituted in the above expressions to get

$$- \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C''_{n,t} (1 - IC_{n,t}) r_t + \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C''_{n,t} (1 - IC_{n,t}) r_T$$

$$+ \sum_t \sum_n DP''_{n,t} TR_t r_t - \sum_t \sum_n DP''_{n,t} r_T - \sum_t \sum_i \sum_j \sum_n \varepsilon''_{ijn} N_{n,t} (1 - TR_t) r_t.$$

This summarizes to

$$- \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C''_{n,t} (1 - IC_{n,t}) (r_t - r_T) + \sum_t \sum_n DP''_{n,t} (TR_t r_t - r_T)$$

$$- \sum_t \sum_i \sum_j \sum_n \varepsilon''_{ijn} N_{n,t} (1 - TR_t) r_t.$$

Let $(1 - IC_{n,t}) (r_t - r_T) = a_t$, $TR_t r_t - r_T = b_t$ and $(1 - TR_t) r_t = \eta_t$. The above terms then reduce to

$$- \sum_t \sum_n (N_{n,t} - N_{n,t-1}) C''_{n,t} a_t + \sum_t \sum_n DP''_{n,t} b_t - \sum_t \sum_i \sum_j \sum_n \varepsilon''_{ijn} N_{n,t} \eta_t.$$

The first and third terms can be expanded as follows, for $t = 1, \dots, T$, for a given n (with $N_{n,0} = 0$):

$$+ \left(-C''_{n,1} a_1 + C''_{n,2} a_2 + \sum_i \sum_j \varepsilon''_{ijn} \eta_1 \right) N_{n,1} + \left(-C''_{n,2} a_2 + C''_{n,3} a_3 + \sum_i \sum_j \varepsilon''_{ijn} \eta_2 \right) N_{n,2} + \dots$$

$$+ \left(-C''_{n,T-1} a_{T-1} + C''_{n,T} a_T + \sum_i \sum_j \varepsilon''_{ijn} \eta_{T-1} \right) N_{n,T-1} + \left(-C''_{n,T} a_T + \sum_i \sum_j \varepsilon''_{ijn} \eta_T \right) N_{n,T}. \tag{A}$$

We next expand the second (depreciation) terms, $\sum_t \sum_n DP''_{n,t} b_t$, using constraint set (12):

$$DP''_{n,1} b_1 = (N_{n,1} - N_{n,0}) d''_n C''_{n,1} b_1,$$

$$DP''_{n,2} b_2 = (N_{n,1} - N_{n,0}) d''_n C''_{n,1} b_2 + (N_{n,2} - N_{n,1}) d''_n C''_{n,2} b_2,$$

$$\vdots$$

$$DP''_{n,T-1} b_{T-1} = (N_{n,1} - N_{n,0}) d''_n C''_{n,1} b_{T-1} + \dots + (N_{n,T-1} - N_{n,T-2}) d''_n C''_{n,T-1} b_{T-1},$$

$$DP''_{n,T} b_T = (N_{n,1} - N_{n,0}) d''_n C''_{n,1} b_T + \dots + (N_{n,T} - N_{n,T-1}) d''_n C''_{n,T} b_T.$$

Summing the above, we get

$$+ \sum_t \sum_n DP''_{n,t} b_t = + C''_{n,1} d''_n (b_1 + b_2 + \dots + b_T) - C''_{n,2} d''_n (b_2 + b_3 + \dots + b_T) N_{n,1}$$

$$+ C''_{n,2} d''_n (b_2 + b_3 + \dots + b_T) - C''_{n,3} d''_n (b_3 + b_4 + \dots + b_T) N_{n,2}$$

$$+ C''_{n,T-1} d''_n (b_{T-1} + b_T) - C''_{n,T} d''_n (b_T) N_{n,T-1} + C''_{n,T} d''_n b_T N_{n,T}. \tag{B}$$

Combining (A) and (B), the coefficients for $N_{n,t}$ are as follows:

$$N_{n,1}: \quad \alpha_{n,1} = -C''_{n,1}a_1 + C''_{n,2}a_2 + \sum_i \sum_j \varepsilon''_{ijn}\eta_1 + C''_{n,1}d''_n(b_1 + b_2 + \dots + b_T) \\ - C''_{n,2}d''_n(b_2 + b_3 + \dots + b_T),$$

$$N_{n,2}: \quad \alpha_{n,2} = -C''_{n,2}a_2 + C''_{n,3}a_3 + \sum_i \sum_j \varepsilon''_{ijn}\eta_2 + C''_{n,2}d''_n(b_2 + b_3 + \dots + b_T) \\ - C''_{n,3}d''_n(b_3 + b_4 + \dots + b_T),$$

⋮

$$N_{n,T-1}: \quad \alpha_{n,T-1} = -C''_{n,T-1}a_{T-1} + C''_{n,T}a_T + \sum_i \sum_j \varepsilon''_{ijn}\eta_{T-1} + C''_{n,T-1}d''_n(b_{T-1} + b_T) - C''_{n,T}d''_n(b_T),$$

$$N_{n,T}: \quad \alpha_{n,T} = -C''_{n,T}a_T + \sum_i \sum_j \varepsilon''_{ijn} + C''_{n,T}d''_nb_T\eta_T.$$

Objective function coefficients of $M_{m,t}$ ($\beta_{m,t}$)

The following terms in the objective function are combined:

$$+ \sum_t \sum_m (M_{m,t-1} - M_{m,t})S_{m,t}r_t \tag{c}$$

$$+ \sum_t \sum_m DP'_{m,t}TR_t r_t \tag{d}$$

$$- \sum_t \sum_i \sum_j \sum_m \varepsilon'_{ijm}M_{m,t}(1 - TR_t)r_t. \tag{h}$$

The depreciation term can be substituted from constraint set (13):

$$DP'_{m,t} = M_{m,t}C'_m d'_m \quad \forall m, t. \tag{13}$$

Letting $C'_m d'_m TR_t r_t = c_{m,t}$, the above terms reduce to:

$$+ \sum_t \sum_m (M_{m,t-1} - M_{m,t})S_{m,t}r_t + \sum_t \sum_m M_{m,t}C'_m d'_m TR_t r_t - \sum_t \sum_i \sum_j \sum_m \varepsilon'_{ijm}M_{m,t}\eta_t.$$

Expanding the above terms for every period, we get (with $M_{m,0} \geq 0$):

$$t = 1: \quad S_{m,1}r_1M_{m,0} - \left(S_{m,1}r_1 - c_{m,1} + \sum_i \sum_j \varepsilon'_{ijm}\eta_1 \right) M_{m,1},$$

$$t = 2: \quad S_{m,2}r_2M_{m,1} - \left(S_{m,2}r_2 - c_{m,2} + \sum_i \sum_j \varepsilon'_{ijm}\eta_1 \right) M_{m,2},$$

⋮

$$t = T: \quad S_{m,T}r_T M_{m,T-1} - \left(S_{m,T}r_T - c_{m,T} + \sum_i \sum_j \varepsilon'_{ijm}\eta_1 \right) M_{m,T}.$$

Summarizing the above, the coefficients for $M_{m,t}$ -variables are given by

$$M_{m,0}: \quad \beta_{m,0} = S_{m,1}r_1,$$

$$M_{m,1}: \quad \beta_{m,1} = -S_{m,1}r_1 + S_{m,2}r_2 + c_{m,1} - \sum_i \sum_j \varepsilon'_{ijm}\eta_{1j},$$

⋮

$$M_{m,T-1}: \quad \beta_{m,T-1} = -S_{m,T-1}r_{T-1} + S_{m,T}r_T + c_{m,T-1} - \sum_i \sum_j \varepsilon'_{ijm}\eta_{T-1j},$$

$$M_{m,T}: \quad \beta_{m,t} = -S_{m,T}r_T + c_{m,T} - \sum_i \sum_j \varepsilon'_{ijm}\eta_{Tj}.$$

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