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Fractal physiography?

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Abstract

Fractal geometric parameters were employed to assess the morphometric viability of traditionally defined physiographic provinces using relatively coarse 30-arc-second digital elevation models (DEM) within $1° \times 1°$ blocks of the conterminous United States. Four DEM samples were analyzed for each of the eight areas examined. Six of the eight physiographic provinces plot as strong clusters in fractal parameter space, indicating regional terrain homogeneity. Two displayed weak spatial clustering, suggesting that samples from these regions lacked the spatial homogeneity to be easily detectable in digital form. The terrain parameters for these problematic blocks were calculated using a multi-cell area with a pseudo-inversion solution to the resulting over-constrained matrix equations. The study indicates that use of the Hausdorff–Besicovitch Dimension as an index of surface roughness, the explained variance of the power-law fit to the variance spectrum, and the elevation range of the terrain block are powerful parameters for abstracting surface information. Use of relatively coarse DEMs in high-relief terrain produced fractal dimensions somewhat larger than those reported by other workers who employed finer arrays covering smaller areas. In regions of low relief there was little disparity, suggesting invariance across the scale of the DEMs. The 30-arc-second DEM is capable of resolving features with periods of up to 85 km, and successfully captures the directional texture of morphotectonic terrains.

1. Introduction

Physiographic provinces are, by definition, extensive areas (Fenneman, 1928). In its traditional sense, this term refers to large areas of subcontinental dimensions in which similar climatological and surficial processes, operating on relatively uniform lithology and geological structure, produce a characteristic landform assemblage (cf. American Geological Institute, 1974, p. 380). All other exogenic factors being equal, values of morphometric descriptors for similar physiographic regions should be virtually identical at similar stages of landscape evolution.

In this paper, we use the term *morphometric region* with reference to areas of various scale, identified or discriminated from others by quantitative, geometric, or topologic criteria. The morphometric regions considered here are subsets of traditional physiographic provinces. Several fractal statistics, obtained from spectral analysis of the representative DEMs, are used to abstract topography in the morphometric regions. Our goal is to make a preliminary determination of how well such an analysis replicates the results of traditional physiographic regionalization performed by more subjective means that, nonetheless, considered a wide range of relevant phenomena including relief, lithology, climate, and structure. The proposed methodology is conducive to automation and may provide a mechanism for regional terrain recognition and classification. We also explore the possibility that application of techniques at inappropriate spatial scales may be responsible for the inability of some previous researchers to

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discover fractal parameters that might replicate the products of traditional physiographic regionalization.

2. Historical background

Delineation of physiographic provinces was long a favored activity among North American geomorphologists, beginning with John Wesley Powell (1895) and continuing through the mid-twentieth century (e.g., Bowman, 1911; Fenneman, 1928; Loomis, 1937; Atwood, 1940; Bostock, 1948; Thornbury, 1965; Hunt, 1967). This activity consisted of differentiating between areas of subcontinental dimensions containing relatively homogeneous landforms, lithology, structure, and climate; each such area presumably also experienced an internally consistent geomorphic history. The heyday of such work corresponded to and was intertwined with that of regional geography, the two sharing a strong emphasis on areal differentiation (Hartshorne, 1939). The introductory chapter of Atwood (1940) is particularly useful in pointing out the relationship between these fields, while Thornbury (1965, ch. 1) provides a good history of the regional concept in geomorphology.

The term *physiography* itself underwent an interesting evolution in the early part of the twentieth century, from having a broad meaning essentially synonymous with physical geography (e.g., Salisbury, 1910; Tarr and Martin, 1915) to a much more restricted definition, attributed by Fairbridge (1968, p. 842) to Davis, Salisbury, and Fenneman, the essence of which could be portrayed as "the regional description of landforms". Suggesting that it had fallen into disfavor as part of a general reaction against Davisian geomorphology, Fairbridge (1968) characterized "physiography" as "a dead term," but did leave open the possibility of its resuscitation.

Physiographic regionalization was in essence a qualitative science not overly concerned with replicability or precise definition. As is the case with climatic geomorphology and the delineation of morphogenetic regions (see Chorley et al., 1984, p. 468), *n* physiographers working with similar tools and data sources were likely to produce *n* different physiographic regionalizations, although broad agreement is evident in the major divisions of several of the authors cited above. As dissatisfaction with the Davisian paradigm grew and emphasis shifted to mathematical characterization, statistical analysis, and geomorphic process, the scale of concern to geomorphologists necessarily changed, *dramatically*, to local areas dominated by such features as individual hillslopes and small drainage basins. Activities analogous to physiographic regionalization, initiated in an extraordinary series of papers by the Columbia school (Strahler, 1992), came to be known as landform *morphometry* or *geomorphometry*. Many subsequent papers focused on indices and ratios that treat landform geometry or topology at spatial scales inappropriate for regionalization, or that did not lend themselves well to mapping. Reviews by Clarke (1966) and Zakrzewska (1967) catalog a very large number of such methods; the lack of a spatial focus in much of this literature is striking, particularly in light of the fact that maps provided the overwhelming proportion of their data bases. A useful critique of such work was provided by Evans (1972).

Numerous attempts were made over the years to discriminate morphometrically between regions of contrasting climate, structure, or depositional history (e.g., Ruhe, 1952; Chorley, 1957; Chorley and Morgan, 1962; Gregory and Gardiner, 1975; Toy, 1977; O'Neill and Mark, 1987). As noted in the latter reference, until very recently few studies employed preexisting digital elevation matrices for such purposes as comparative investigation of slope frequency distributions. Geomorphometry has not, to the present authors' knowledge, ever produced a scheme of "morphometric regions" comparable in scope or ambition to the systems of physiographic provinces presented early in this century.

Evans (1972), building on some early digital terrain analysis by Waldo Tobler in the University of Michigan's geography department, proposed a unified system of terrain morphometry based on elementary calculus and descriptive statistics. In a less widely circulated document Evans (1979) presented a more comprehensive and integrated version of his terrain analysis system. Demonstrating remarkable prescience in his concluding chapters, Evans (1979) anticipated the potential of fractal analysis in *general geomorphometry*, "the measurement and analysis of those characteristics of landform applicable to any continuous rough surface." Many papers concerned with applications of fractal concepts have appeared in the intervening years; a synoptic view of this work is contained in the volume edited by Snow and Mayer (1992).

A recently published map by Thelin and Pike (1991) has revived interest in physiographic regions through the context of modern digital computing. This stunning map has received widespread attention and acclaim (e.g., Lewis, 1992), and is accompanied by a manuscript explaining the technical aspects of data collection, cartographic transformations, image processing, and map production. Traditional physiographic divisions of the United States (Fenneman and Johnson, 1946) were overlaid onto the relief portrayal and discussed in reference to landforms of interest. Morrison (1992) presented this digitally shaded-relief map side-by-side with the hand-drawn map of physiographic divisions by Raisz (1957) — a graphic visual comparison of the computer-derived image with the traditional portrayal.

Thelin and Pike (1991) used a digital data set containing elevations at regular 30-arc-second spacing. Their digital elevation model (DEM) contained approximately 12 million entries, nominally spaced at about 800 m. Originally digitized from 1:250,000 scale topographic sheets with a 100 foot contour interval, elevations were read or interpolated to the nearest one foot and later rounded to the nearest 30 m. This DEM is actually a subset of a DEM collected at a 3-arc-second (80 m) resolution, requiring about 2 billion entries to cover the mapped area. Because this higher resolution DEM was available, the decision to use the coarser data set appears to have been dictated by the cartographic purpose of the map, to show the overall pattern or trend of major landforms and physiographic units. Use of the 3-arc-second DEM would have introduced "noise" and obscured the overall pattern of the trend. This cartographic decision, therefore, properly matches the scale of the requisite data to the purpose of the message.

3. Methodology

Digital data collected on a regular lattice can easily be transformed into a gray level "elevation image", in which elevations run from dark gray in the lowest region of the image to near-white at the highest; the images contained within provide examples. DEMs are also conducive to analysis of elevation along N–S columns or E–W rows that can be presented as topographic profiles. The elevation series can be synthesized by Fourier methods to produce variance spectra (Outcalt and Melton, 1992) where spectral density is plotted against wavelength to determine if a periodic component exists in the landscape. Strong periodicity might be expected, for example, in the Valley and Ridge province of the Appalachian Highlands.

Analytical techniques based upon variance spectra have been developed (Mandelbrot, 1975; Voss, 1988) to calculate the Hausdorff–Besicovitch or "fractal dimension" (D) (Hausdorff, 1919; Besicovitch and Ursell, 1937). This statistic describes the "space-filling" nature of the abstracted array, and can be calculated by a variety of methods recently reviewed by Xu et al. (1994). In the variance spectra technique, an array is transformed by Fourier methods and variance is plotted as a function of wavelength on a log–log plot. The slope of the best-fit line to the spectral points is the power law exponent. D is calculated as:

$$D = E + \left(\frac{3-\beta}{2}\right) \qquad (1)$$

where E is the Euclidean Dimension and β is slope of the best-fit line to the power spectrum.

The method can be utilized on series ranging from 1 to n dimensions. When applied to a two-dimensional array of elevations ($E=2$), the analysis attempts to characterize the land surface "roughness," as affected by lithology, geologic structure, climate, time, and geomorphic processes — an aim entirely consistent with that of traditional physiographic regionalization. The fractal character of three-dimensional landscapes has been investigated by numerous researchers (e.g., Mandelbrot, 1983; Hallett, 1990; Snow and Mayer, 1992) who have made extensive investigations of methods for calculating D and its relationship to the topography of previously defined physiographic units (e.g., Mark and Aronson, 1984; Culling and Datco, 1987).

Permissible values of D for a two-dimensional space range from 2.0 to 3.0. The slope of β is therefore limited to the range ($1<\beta<3$) if the land surface is truely fractal, exhibiting self-similarity across the entire spectrum. The coefficient of determination (R^2) can also be calculated as the fraction of explained variance of the power-law fit used to determine β. As D increases from 2.0 toward 3.0, the terrain is more space-filling or rougher. In a similar vein, as R^2 approaches 1.0 from lower values, the terrain is considered to fit more closely the "pure fractal model," where variance increases linearly across the spectrum (Outcalt and Melton, 1992). D and R^2 are used in the same manner as the mean and standard deviation are used in parametric statistics to abstract the information in a histogram. Thus, the spectrum is analogous to the histogram and contains more information than the abstraction parameters (D and R^2). However, the fractal parameters do not carry the excess baggage of deviation magnitude probability implicit in the mean and standard deviation.

A Discrete Fourier Transform (DFT) was performed on each row and column in the elevation array. The speed of the DFT is enhanced by pre-calculating the real and imaginary coefficients and later accessing them using modular indices. Row and column variance is used to produce average north/south and east/west variance spectra for each of the 1°× 1° terrain blocks. The variance bins were zeroed at the start of row/column calculations, yielding total row/column spectra.

The series were not smoothed and linear trends were not removed prior to application of the DFT. Detrending removes some of the first harmonic variance from further analysis. In the scale of this investigation, an oriented longwave component would be expected in morphotectonic terrains. Elimination of this information would have a deleterious impact on the analysis. As done by other researchers using the variance spectra technique (see Klinkenberg and Goodchild, 1992; Xia et al., 1991b), the average of the 120 N–S and the 120 E–W spectra are presented separately as a check for directional texture. The summary fractal statistics reported within are an average of the mean orthogonal spectra and are, in effect, averages of 240 variance spectra.

3.1. Limitations of the variance spectra technique

In some cases, use of the variance spectra methodology to abstract topography yields values of D outside the legal theoretical range of 2.0–3.0. Pentland (1984) and Carr and Benzer (1991) have discussed some of the causes. Strong periodic terrain elements or analog/digital data aliasing at short wavelengths influence the slope of the best-fit line to the variance spectrum. For descriptive purposes, however, these "out-of-bounds" values are retained here without modification.

Some researchers (e.g., Mark and Aronson, 1984) calculated D using only the self-similar portion of the terrain variance spectrum or that portion of the spectrum that precedes the inception of apparent periodicity (e.g., Xia et al., 1991a). This entails visual inspection of the spectrum and application of regression techniques to calculate the best-fit line to that portion of the spectrum judged scale-invariant. Typically, the very short and very long wavelength components are deleted. Yokoya et al. (1989) developed an automated method to select that part of the spectrum that is linear, and across which self-similarity exists.

In contrast, we used a least-squares fit to the entire variance spectrum. The "true" spectrum of a terrain block extends from the maximum block length to the millimeter range. When much of the variance in the "true" spectrum occurs at wavelengths shorter than the grid interval, an elevation image of the block appears much like a pin cushion. This effect is enhanced by artifacts introduced by the data system such as block smoothing, data interval resolution, and the resolution limits of the interpolation procedure. These artifacts are concentrated at the grid wavelength and are aliased into the shorter wavelengths of the calculated spectrum, appearing as spectral peaks at and near the Nyquist frequency (twice the grid spacing). This analog/digital chuffing can be removed by spline interpolation, but this is an artificial solution as some of the Nyquist density peak is certainly "real." However, retention of high frequency spectral points has the effect of reducing the slope of the power law, increasing the fractal dimension and decreasing the coefficient of determination. The possible impact can be assessed by R^2, which is used as a measure of fractal purity. In this sense, the high frequency portion of the spectrum carries necessary information. Our goal is to explore the possibility of discriminating between terrain types using relatively coarse DEMs. The summary D and R^2 values reported within are the averages of 240 separate linear variance spectra and therefore yield high R^2 values while minimizing directional textural effects. Although our techniques may be termed "reductionist," they produce the rapid surface parameterization necessary for automated detection systems.

3.2. The question of scale

Standard USGS digital elevation models with 1-meter vertical resolution, derived from 1:24,000 scale topographic maps, are typically interpolated onto a regular lattice with 30-meter spacing. Constrained by computer limitations, the arrays employed for the calculation of D are often a subset of the quadrangle DEM, containing 256 rows and columns (e.g., Klinkenberg and Goodchild, 1992). In that case, the array extends less than 8 km on a side and covers only 40% of a 7.5 minute quadrangle at 40°N latitude. The size range of features detectable by spectral techniques ranges from the Nyquist wavelength to the length of the array, in this instance about 8 km. Others (e.g., Xia et al., 1991a) use square elevation arrays ranging from 290^2 to 390^2 entries, thereby extending the maximum coverage to 11.7 km. However, even if an entire 7.5 minute quadrangle of 30-m data is used, significant low-frequency physiographic information is missing. For example, the average east–west spacing of fault block mountains in the Basin and Range province is on the order of 40 km, while the east–west extent of a 7.5 minute quadrangle is about one-quarter this distance at 40°N latitude. This caveat would hold true in most continental morphotectonic regions.

It follows that calculated values of *D* for such relatively small areas will not yield spectral signatures or fractal parameters suitable for discriminating between physiographic regions at scales consistent with those at which such units are traditionally defined. For small areas, local geologic and climatologic factors tend to control topographic development and condition the fractal dimension (Clarke, 1988). The influence of microclimate on soil and vegetation, mass wasting, and the effects of lithology, jointing, and outcrop pattern will all affect the value of *D* calculated for small areas, and this value should vary with time (Outcalt and Melton, 1992). Moreover, the longer-wavelength tectonic or structural component is effectively eliminated or, at best, is part of the linear trend typically subtracted from the elevation series as a preliminary step to Fourier transformation.

Scale-dependent processes produce inhomogeneous fractal surfaces (Chase, 1992). We suggest that the fractal dimension, calculated for small areas at high sampling densities, is not representative of regional conditions and landform assemblages, and should not be used in algorithms intended for discrimination of physiographic regions. Some terrains will exhibit changes in the fractal parameters that are *dependent* on the data sampling interval, scale of roughness, areal coverage, and calculation methodology (Mark and Aronson, 1984; Roy et al., 1987; Andrle and Abrahams, 1989; Carr and Benzer, 1991). Multiple sampling within a physiographic province (e.g., Klinkenberg and Goodchild, 1992; Xia et al., 1991a) may or may not produce a converging representative value. In short, use of 30-meter data may be inappropriate for characterizing large physiographic units, as local landforms

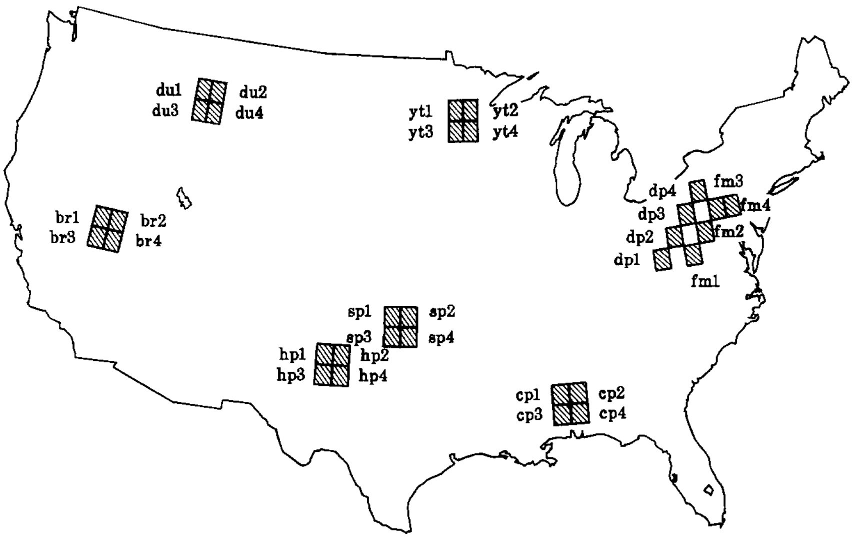


Fig. 1. Location of four 1° × 1° DEMs from each of the eight physiographic provinces. See Table 1 for code key and physiographic sections.

Table 1
D, R^2 and pZ for 1°×1° sample DEMs

Geomorphic description (Physiographic province and section[a])	Code	D	R^2	pZ[c]
High relief				
Block mtns, basins	br1	2.549	0.974	3.381
(Basin and Range, Great Basin sect, 22a)	br2	2.519	0.978	3.178
	br3	2.523	0.976	3.320
	br4	2.558	0.980	3.314
Dissected plateau[b]	dp1	3.046	0.967	2.802
(App. Plateau, Kanawha sect, 8e)	dp2	3.040	0.984	2.677
	dp3	2.833	0.988	2.838
	dp4	2.917	0.985	2.709
Dissected mtn. uplands	du1	2.594	0.989	3.265
(Northern Rocky Mtns., sect 19)	du2	2.555	0.991	3.244
	du3	2.539	0.987	3.271
	du4	2.577	0.991	3.317
Folded 2nd cycle mtns.[b]	fm1	2.651	0.980	3.071
(Valley and Ridge, Tennessee and Middle sects, 6a,b)	fm2	2.682	0.973	2.928
	fm3	2.703	0.938	2.798
	fm4	2.755	0.966	2.739
Low relief				
Fluviatile plains	hp1	2.570	0.991	2.466
(Great Plains, High Plains sect, 13d)	hp2	2.629	0.990	2.704
	hp3	2.590	0.990	2.466
	hp4	2.575	0.990	2.630
Young till plains	yt1	2.650	0.989	2.127
(Central lowlands, Western Lake sect, 12b)	yt2	2.732	0.990	2.200
	yt3	2.647	0.989	2.200
	yt4	2.679	0.989	2.200
Scarped plains	sp1	2.540	0.979	2.438
(Central lowlands, Osage Plains sect, 12f)	sp2	2.588	0.987	2.183
	sp3	2.597	0.990	2.466
	sp4	2.656	0.991	2.276
Belted coastal plain	cp1	2.711	0.987	2.276
(Coastal Plain, East Gulf sect, 3d)	cp2	2.768	0.989	2.276
	cp3	2.784	0.987	2.216
	cp4	2.728	0.992	2.262

[a]Fenneman and Johnson (1946).
[b]Non-contiguous samples, possibly non-uniform with edge contamination.
[c]$\log_{10}$ elevation range (m).

tend to mask the regional signature. Huang and Turcotte (1989) also recognized this problem in a geomorphic context. Scheidegger (1970) and Klinkenberg (1992) noted that scale invariance holds true only over a certain range, so that most surfaces are fractally inhomogeneous. A barrier island coastline, for example, has a low fractal dimension at megascopic scales. At the scale of the individual sand grains, however, the "coast" appears embayed, and self-similarity falls apart.

4. Results and discussion

For reasons cited above, we employed the NOAA 30-arc-second data set (Row and Kozleski, 1992). This is the same data set used and described by Thelin and Pike (1991), except that the vertical resolution of the NOAA data set is 20 feet. One-degree blocks yield an array of 120 entries on a side. The observations have a lattice spacing at latitude 40°N of 925 (N–S)×710 (E–W) meters. The 30-arc-second data cannot resolve

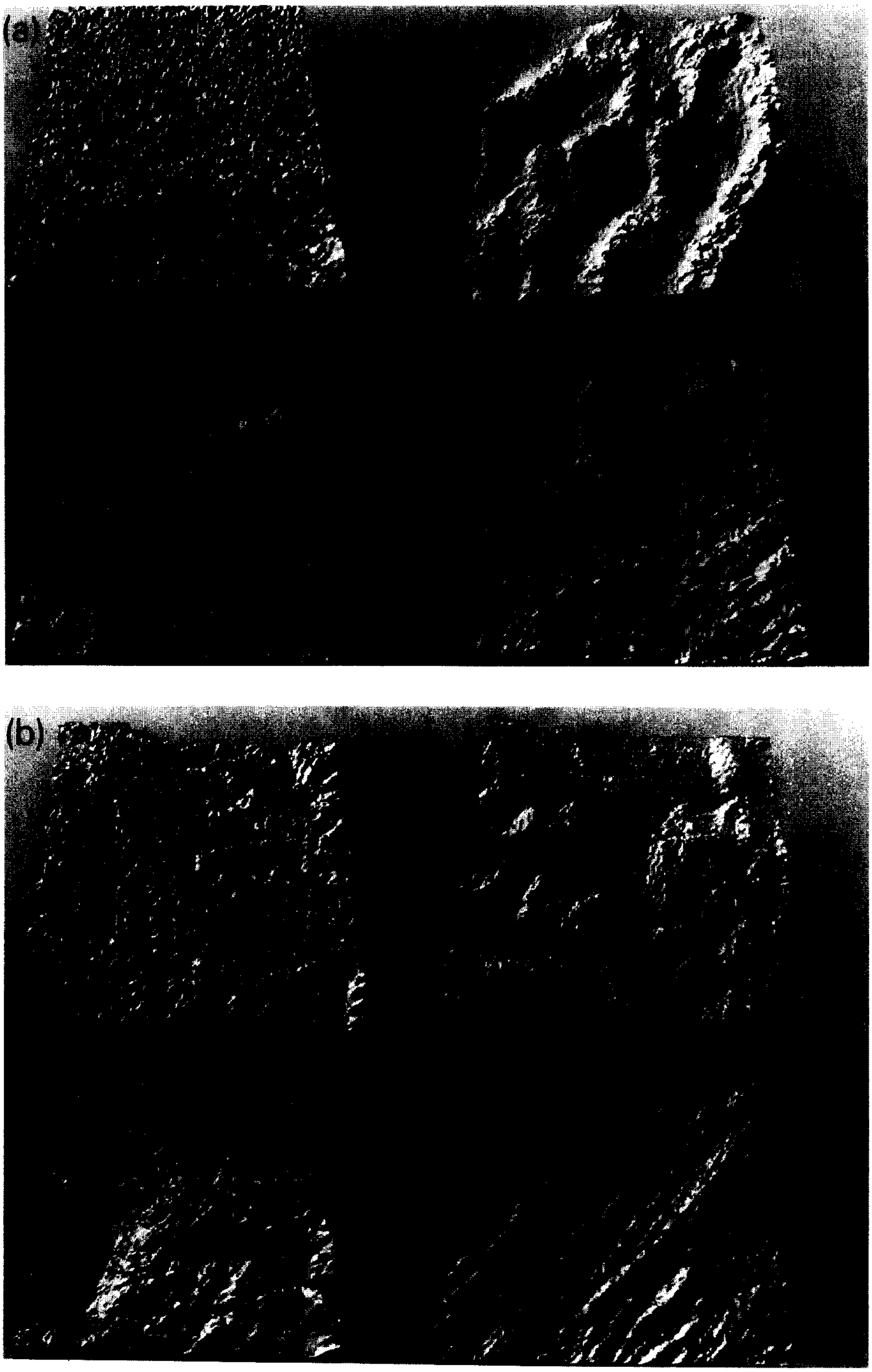


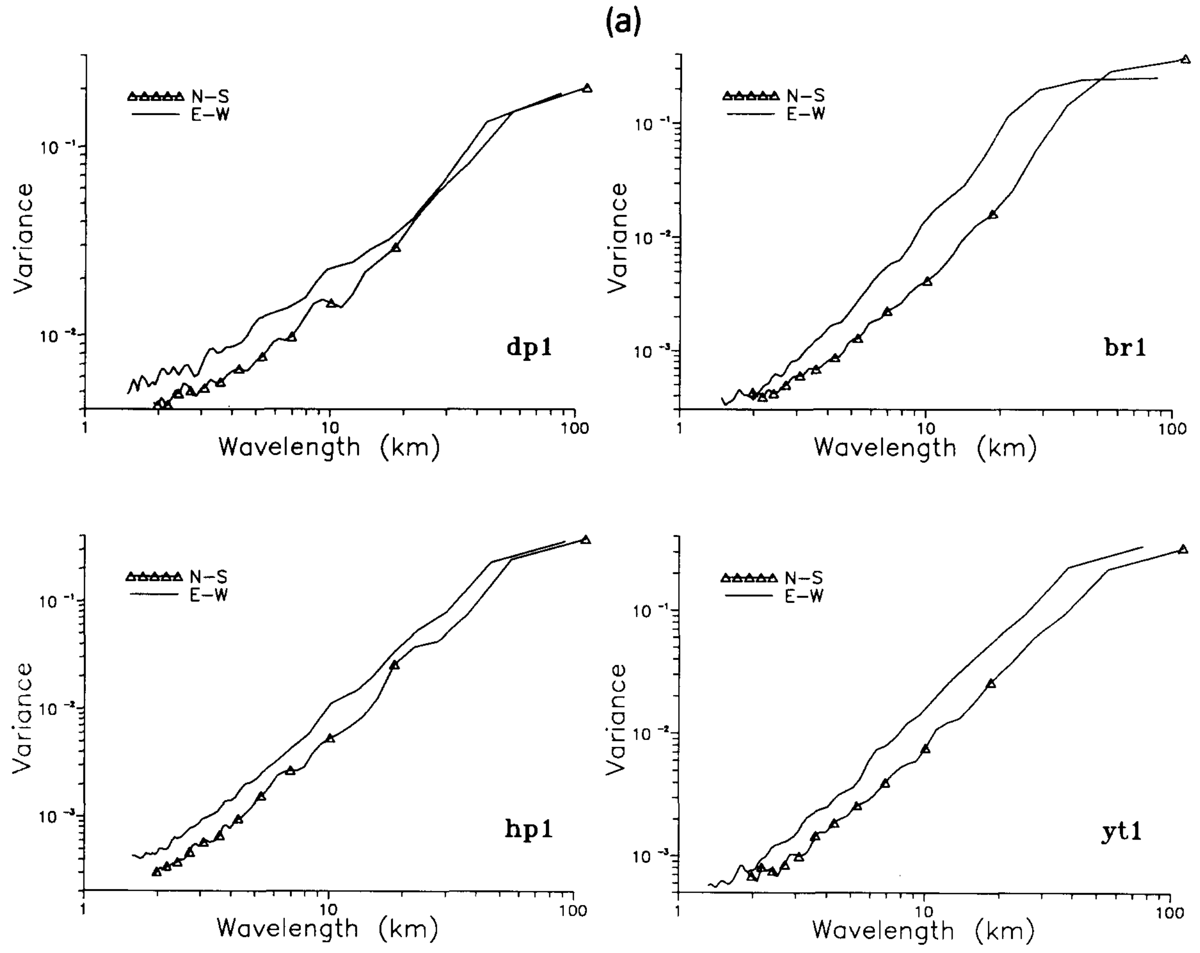
Fig. 2. Sample block image from each of the eight test regions, identified by the coordinates of the NW corner (upper left).

variance at wavelengths below 1 minute, but can detect periodicities up to at least 85 km. One-degree blocks represent an area 64 times larger than a single 7.5 minute topographic map. In this study, four samples from each of eight physiographic provinces were used, so the total area abstracted for each province is 256 times larger than a 7.5-minute quadrangle.

4.1. Terrain samples and variance spectra

Sample locations within physiographic regions were determined on the basis of an examination of a well-known physiographic map of the United States (Fenneman and Johnson, 1946) and Raisz's (1957) map "Landforms of the United States". The 1°×1° elevation matrices were, where possible, conterminous and thus covered 2°×2° blocks. The sampling design for this reconnaissance study was dictated by size and shape limitations of the smaller physiographic regions; formal sampling strategies with greater areal coverage are possible.

Four regions of relatively low local relief were selected, representing young glacial till plains (yt), scarped plains (sp), coastal plains (cp) and high plains of the continental interior (hp). These are contrasted with four regions of relatively high local relief, including glaciated dissected uplands (du), basin and range (br), folded mountains (fm) and dissected plateau (dp). Owing to the shape of the Appalachian Plateau (dp) and Valley and Ridge (fm) provinces in the Appalachian Highlands Division, samples in these regions do not form conterminous 2°×2° terrain



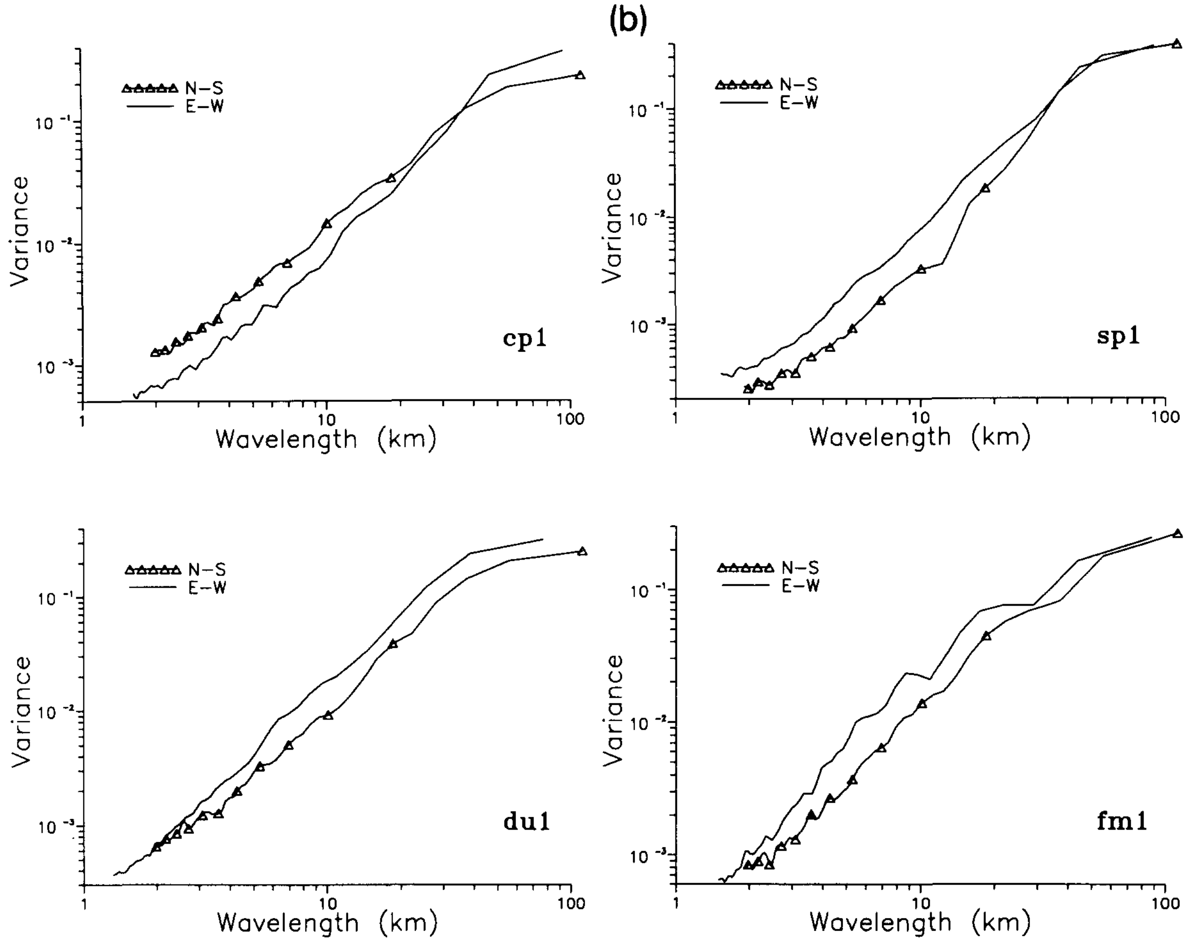


Fig. 3. Representative orthogonal variance spectra for block images shown in Fig. 2. Note differences in scale of *y*-axes.

blocks. Locations of the sampled areas are shown in Fig. 1 and the traditional physiographic regions of which they are part are listed in Table 1.

A sample block of terrain from each physiographic province is presented in Fig. 2 and the associated N–S and E–W orthogonal variance spectra in Fig. 3. The fractal parameters D and R^2, and the $\log_{10}$ of the elevation range in meters (pZ), are listed for each sample terrain block in Table 1. The parameter pZ is similar to log R used by Melton (1958) as a vector in his morphometric analysis of small drainage basins. Using pZ as an index of the magnitude of the maximum spectral amplitude can provide an additional level of description. This may be necessary, since it is possible for regions of different relief to have similar or identical values of D and R^2.

Pronounced separation of the N–S and E–W spectra exists in the sp, yt and, especially, in the br samples. Divergence of the spectra at some wavelength indicates a directional texture at that wavelength. At this scale, it must reflect the underlying anisotropy of the geologic structure. Spectral divergence is maximized around 30 km in the Great Basin example (br) and relates to the direction of late Cenozoic crustal extension and subsequent fault plane orientation. In the case of the young till plains (yt), it probably reflects the orientation of stream valleys and late Wisconsin glacial moraines.

The terrain block models in Fig. 2 were constructed using three-dimensional transforms of the 1°×1° elevation matrices using the program FRACTINT (Wegner and Peterson, 1991, pp. 111–126). Some portions of the sample block models have a "pin cushion" appearance which, as mentioned earlier, occurs when there is considerable variance at wavelengths shorter than the sampling wavelength of 8.33×10^{-3} degrees. This effect demonstrates that the results of all attempts to parameterize surface characteristics are dependent upon sampling grid density and total sample area. Comparisons between shapes or forms must be made using the same grid mesh size and sample area, even though the parameters are based on a dimensionless "fractal metric."

4.2. Terrain samples in (D, R^2) and (D, pZ) space

The fundamental purpose of any classification procedure is to minimize intraclass variation while maximizing interclass separation. To implement a physiographic regionalization, we wish to discriminate between morphometric regions based on fractal parameters from representative terrain samples.

Fig. 4 shows the positions of the 32 test 1° × 1° samples in (D, R^2) and $(D, \mathrm{p}Z)$ space. The four sample members from each physiographic province are enclosed by "convex hulls," a boundary that minimizes the area enclosing the samples (Sedgewick, 1983). Two attributes of interest are contained in these diagrams: the degree of regional sample clustering and the degree of regional cluster separation.

Tight clustering indicates strong correlation between the qualitative criteria originally used to define a physiographic province and the fractal geometry of the regional terrain. Weak clustering may suggest contamination by other regions along sample edges and/or significant geographic variation within the physiographic section. The latter condition may indicate that the qualitative criteria used originally to establish the region do not translate well into the digital realm. Note, for example, that two convex hulls (fm and dp) cover large areas of (D, R^2) space, indicating poor discrimination. The $(D, \mathrm{p}Z)$ space, however, appears to minimize the area contained within the hulls.

Spatial separation between the convex hulls of the regions is desirable, but appears to be inadequate in (D, R^2) space. This is illustrated by the overlapping hulls in the upper left of Fig. 4a. By contrast, $(D, \mathrm{p}Z)$ space is characterized by strong clustering of samples, yielding small-area hulls with spatial separation between the regional hulls — a necessary feature of an automatic recognition system.

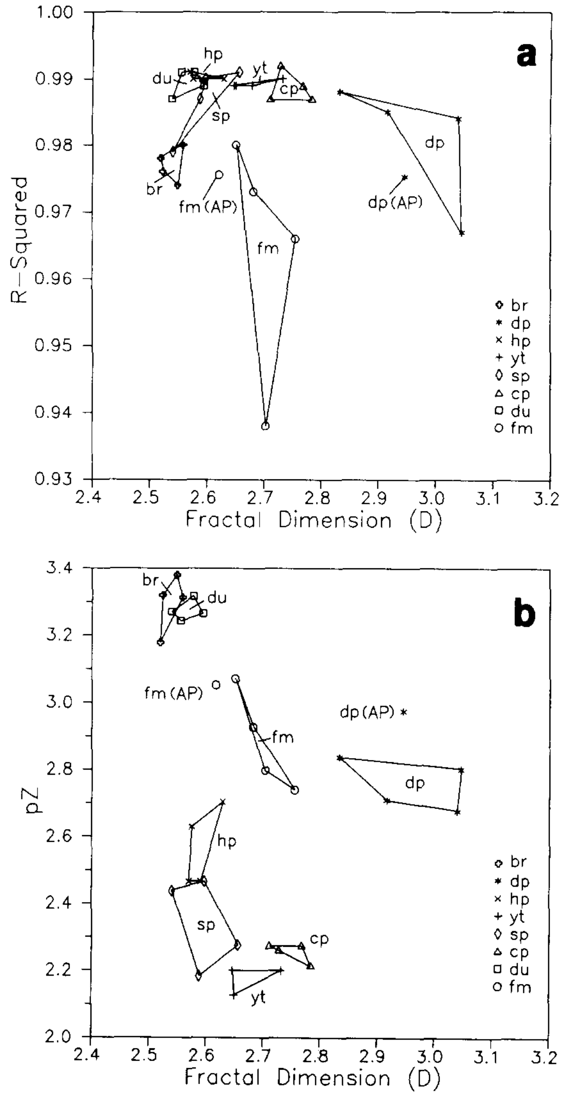


Fig. 4. Plots of samples in (D, R^2) space (a) and $(D, \mathrm{p}Z)$ space (b). The points *dp(AP)* and *fm(AP)* are regional values calculated using the pseudo-inverse method for the area shown in Fig. 6.

As noted, all morphometric test regions except the folded mountains (fm) and dissected plateaus (dp) of the Appalachian Highlands show rather tight clustering in (D, R^2) space. Elevation images of all the samples from these problematic regions are presented in Fig. 5. Large spatial variations in the intensity, orientation, and wavelength occur within the thin-skinned fold belt (fm) samples, and significant drainage density variations exist in the dissected plateau (dp) samples. Con-

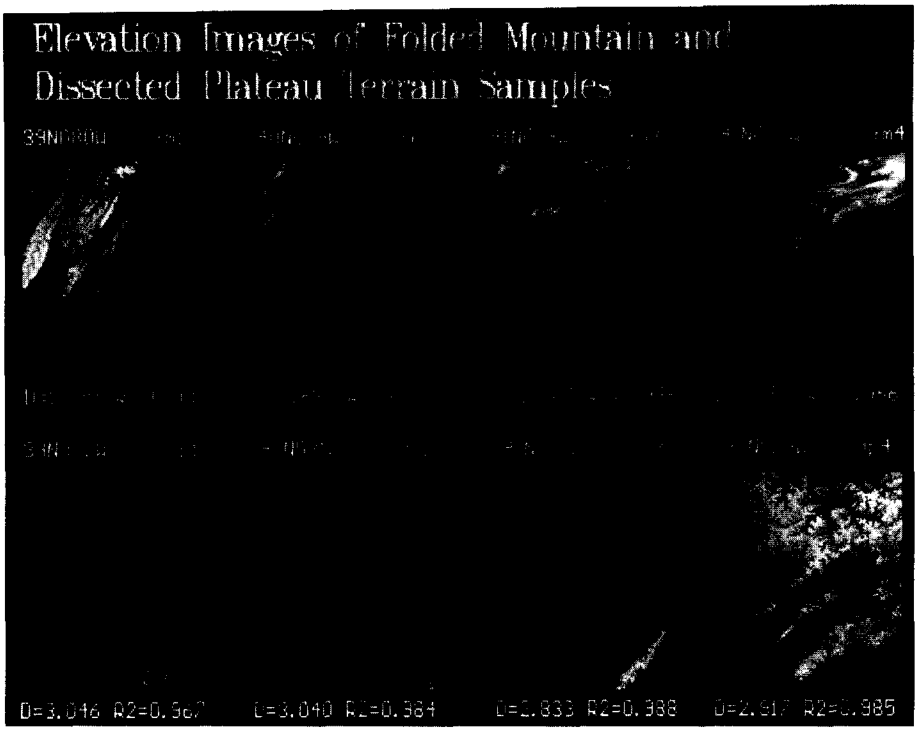


Fig. 5. Elevation image mosaic of the eight samples from the folded mountain (*fm*, upper row) and dissected plateau (*dp*, lower row) regions, using the same notation as in Fig. 2.

tamination from adjacent but geomorphically dissimilar regions is evident near the edges of the elevation images. The relatively low R^2 value of the folded mountain (fm) region is an artifact of the concentration of variance at the characteristic fold wavelength (uncorrected for fold crest orientation).

Table 2 lists the results of the fractal calculations for each of the morphometric regions, reported as an average of the four samples with the range given in parentheses below the mean value of D. In addition, the average D reported by Xia et al. (1991a) and Klinkenberg and Goodchild (1992) are presented. These were derived from 30-meter USGS digital elevation models of various sizes for the same physiographic provinces. The value in parentheses below their mean D is the difference between their D and ours; a negative number indicates that their estimate of D is lower. Strictly speaking, comparison between fractal statistics calculated using different algorithms for different sized DEMs is not legitimate, for the reasons cited earlier. It does, however, provide some insights.

The grand average of D for the 32 samples we examined is 2.671, which is slightly greater ("pinker") than the value of 2.50 expected for a Brownian surface. The dissected plateau region (dp) has the largest values of D and is thus the most "space filling" of the regions examined. This can hardly be surprising to anyone who has traveled secondary roads in the Appalachian Plateau. By contrast, high-relief mountains of the west (du, br) have values of D near the lower end of the scale and are much less "space filling." The range of values we calculated tends to be maximized in non-uniform regions with edge contamination.

In the four regions of high relief (large pZ), including br, dp, du, and fm, our D is substantially larger than those calculated by others, reflecting the influence of sampling interval, data resolution, methodology, *and* an extreme difference in the extent of areal coverage. There is much less disparity in values of D calculated for the morphometric regions with low relief (hp, yt, sp, and cp). Since local reduction of relief typically occurs in tectonically inactive regions, high-density

Table 2
Mean D for eight morphomorphic test regions

Code	D (range)	[1][b] (Δ)[d]	[2][c] (Δ)
High relief			
br	2.538 (0.039)	2.364 (−0.174)	2.29 (−0.248)
dp[a]	2.959 (0.213)	2.567 (−0.392)	2.30 (−0.659)
du	2.566 (0.055)	2.434 (−0.132)	– –
fm[a]	2.698 (0.104)	2.525 (−0.173)	2.36 (−0.338)
Low relief			
hp	2.591 (0.059)	2.496 (−0.095)	– –
yt	2.677 (0.085)	2.597 (−0.080)	2.41 (−0.267)
sp	2.595 (0.116)	2.597 (0.002)	2.41 (−0.185)
cp	2.748 (0.073)	2.706 (−0.042)	2.65 (−0.098)

[a]Non-contiguous samples, possibly non-uniform with edge contamination.
[b][1] Xia et al. (1991a); 30-m DEM, 290^2 to 390^2 arrays.
[c][2] Klinkenberg and Goodchild (1992); 30-m DEM, 256^2 arrays.
[d]Difference in D (30-m DEM minus 30-arc-second DEM).

coverage of small areas of low relief would necessarily entail extensive sampling of depositional landforms. Thus, the consistency of D for low-relief regions may reflect fractal self-similarity across the scale of the data sets. In a sense, the 30-meter DEM preferentially samples higher-order planar landforms within the larger, lower-order plains (hp, yt, sp, and cp). In such cases, true scale invariance is encountered.

4.3. Parameter calculation in large heterogeneous areas

Regions whose most prominent morphometric characteristics cannot be captured in 1°×1° block units are not uncommon. A 2° latitude × 4° longitude test area of the Appalachian Highland Division, containing Plateau (dp), Valley and Ridge (fm), and Piedmont (pm) physiographic provinces, is shown in Fig. 6 with the terrain parameter values for each 1×1 block. The image was computer enhanced to show terrain structure. In each block, the fraction (F) of the physiographic type (F_{dp}, F_{fm}, F_{pm}) ranges from zero to unity, and can easily be measured with a dot planimeter or more sophisticated devices. Using D as the parameter in this example, the equation for a single block is of the form:

$$D_{block} = F_{dp}D_{dp} + F_{fm}D_{fm} + F_{pm}D_{pm} \quad (2)$$

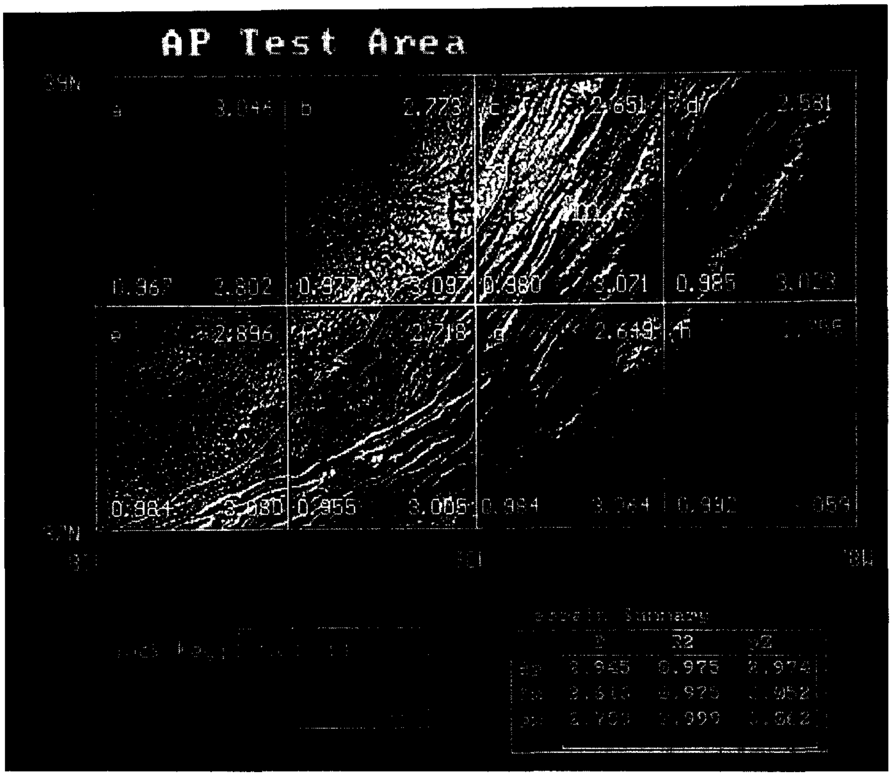
where D_{block} is the block value of D and D_{dp}, D_{fm}, and D_{pm} are terrain parameters for the 2°×4° test area.

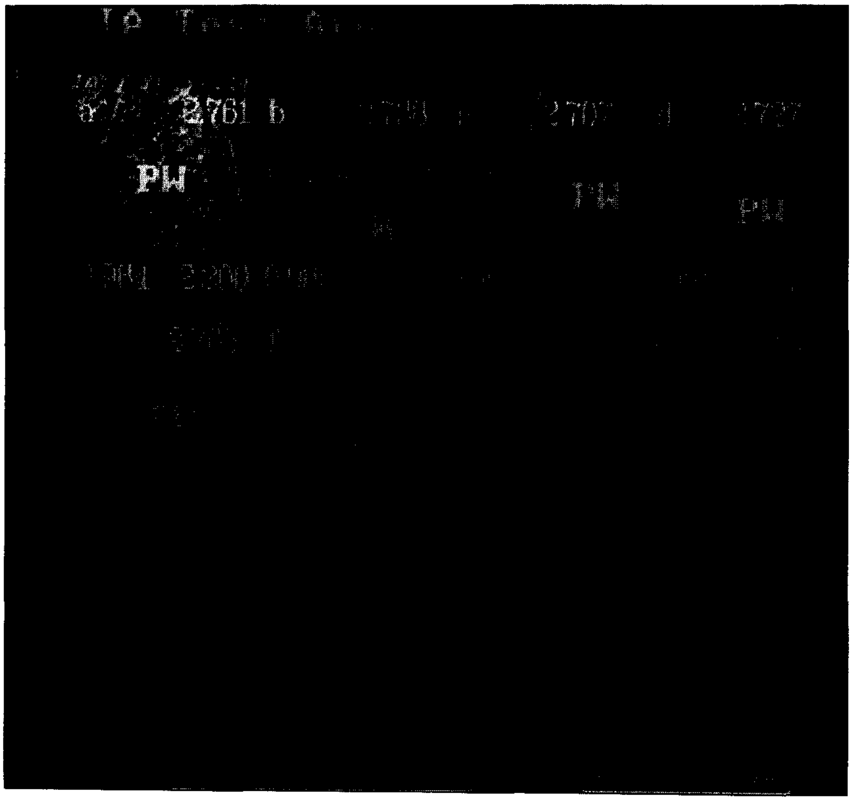
This example yields 8 linear equations (number of blocks) with three unknowns (the regional parameter values). This system of linear equations is termed an "over-constrained set," as the number of unknowns is less than the number of equations producing a rectangular coefficient matrix. Non-singular equation systems of this type are easily solved using the "pseudo-inverse method" (Bernardin, 1991, pp. 103–104), in which both sides of the matrix equation are pre-multiplied by the transpose of the coefficient matrix, followed by conventional matrix inversion. The values obtained from this test area (AP) are summarized in the table in the lower right of Fig. 6; the points are also plotted in Fig. 4 as dp(AP) and fm(AP). Although not enclosed by the convex hull for that province, the proximity of these points indicates a high degree of spatial homogeneity.

Outcalt and Melton (1992) simulated the changing "fractal dimension" of sloping surfaces undergoing several cycles of erosion. This work predicted an initial general increase in the value of D corresponding to the length of time relatively young surfaces were exposed to erosion; till sheets and lava flows were mentioned as potential candidates. In Fig. 7, an 8-cell test region in northwestern Iowa is shown as an enhanced image. The pseudo-inverse technique was applied to this 2°×4° test area. The terrain summary table shows that pre-

Fig 6. The 2°×4° Appalachian (*AP*) test region containing dissected plateau (*dp*), folded mountains (*fm*) and piedmont (*pm*). Interior Valleys and the Blue Ridge sections are included in the *fm* unit.

Fig. 7. A fractal comparison of Wisconsinan (*W*) and loess-covered pre-Wisconsinan (*PW*) 2°×4° terrain blocks in northwestern Iowa. Notation is the same as in Fig. 6.


AP Test Area
b 2.773
c 2.651
d 2.581
0.977 3.097
0.980 3.071
0.985 3.023
e 2.896
f 2.718
g 2.649
h
0.984 3.080
0.955 3.005


2.761 b
PW

Wisconsinan till beyond the prominent terminal moraine of the Des Moines lobe has a larger D than the younger Wisconsinan (yt) sample area to the north (cf. Ruhe, 1952). Although the prediction of Outcalt and Melton (1992) appears to be validated, more detailed work is necessary to refine this technique.

5. Conclusions

Comparison between variance spectra created with 3- and 30-arc-second data have shown that the spectra are strongly dependent on the data interval, calculation vector length, and spectral calculation algorithm. Such comparisons between terrain samples are only possible if these parameters remain constant. Our analytical method differs in that no prefiltering or trend removal is undertaken, and we retain the entire spectrum for calculation of the slope of the best-fit line, and thus D. Our goal is an automated textural descriptive system; therefore, the selection of only a portion of the variance spectrum that fits the pure fractal model degrades the information content of the analysis.

Plotting fractal parameters for sample DEMs derived from traditional physiographic provinces appears to be useful for testing internal homogeneity and cluster separation using relatively coarse 30-arc-second data. The test of (D, pZ) space illustrates the value of adding the Hausdorff–Besicovitch Dimension to more traditional geomorphic parameters for terrain discrimination. The large-area convex hulls in (D, R^2) space associated with the dp and fm regions are apparently produced by edge contamination and/or geographic variation, and demonstrate the internal consistency of the methodology.

Digital elevation models with 30-meter spacing may be inappropriate for calculating the fractal dimensions of large regions where there is a significant longwave terrain component. This proviso applies to regions of crustal compression and extension. Typical variance spectra methodology, employing high-density sampling of small areas, prohibits delineation of features larger than about 8 km. The resulting fractal dimension is sample-biased; local factors and higher-order landforms overwhelm the regional signal, particularly in areas with strong tectonic components. Multiple sampling will not alleviate this effect because it cannot address the fundamental mismatch between the scale of the diagnostic lower-order landscape units and the size of the DEM.

Calculation of D using 30-arc-second (800 m) DEMs can resolve features with a period of at least 85 km, which is more appropriate for regional analysis. A DEM built from a coarser lattice characterizes a much larger area, and reduces the effect of local spatial variation on the value of D. The disparity between the magnitude of D calculated for similar physiographic provinces using DEMs of different resolution and areal extent is quite pronounced. Values of D derived from 30-arc-second data are substantially larger in the four test regions of relatively high relief. In regions of low local relief, however, particularly in plains controlled by geologic structure, our values are very similar to those calculated using 30-m DEMs. We postulate that scale invariance is operating across the power spectra generated by both DEM modes. The coarse, large-area DEM captures the essence of the low-order plains. Conversely, the finer, small-area DEM preferentially reflects the influence of higher-order planar features. These landscapes contain plains within plains, and are truly self-similar over the scale of the DEMs.

Identification of inhomogeneous fractal landscapes might be possible by plotting D as a function of the DEM array spacing (ΔL) or array area (L^2). This would best be accomplished by holding the array size (rows by columns) constant. As ΔL and L^2 increase, larger wavelengths are included. D would be stationary over the wavelengths of self-similar scales, but would step to a new value at the wavelength where self-similarity breaks down and a lower-order fractal surface is encountered. This "step wavelength" can be regarded as diagnostic of important changes in physiographic properties and employed as an additional parameter in a regional discrimination procedure.

DEMs covering too large an area can be contaminated by adjacent physiographic regions near the edge of the DEM, or may not allow for regional variation in lithology, geologic structure, climate, and palimpsests, with the result that the fractal dimension could be misleading or erroneous. The pseudo-inverse technique appears effective for eliminating edge contamination, intraregional variation, and other deleterious factors on calculated values of D.

Further research would entail examination of large regions using DEMs of various horizontal spacing and vertical resolution to determine appropriate or optimal array configurations. Utilization of (D, pZ) space to test for cluster homogeneity and separation provides one method of evaluating the optimality of array characteristics.

The results of fractal analysis were successful enough to suggest that an objective physiographic regionalization based on quantitative analysis of existing digital elevation models is possible. Complemented by other parameters that describe topographic form and constitute the ''geometric signature'' (Pike, 1988), physiographic work may play a major role in the revitalization of regional geography.

The implications of this analysis extend well beyond geomorphology. These techniques may be used to build automated surface classification and recognition algorithms. The potential range of application includes all fields where surface classification and recognition are of interest — from metallurgy to medical research (Mandelbrot et al., 1984). The pseudo-inverse method of parameter estimation should prove useful in many research applications where the degree of mixing of surface types is greater than the geomorphic examples treated in this study.

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