Implications of $\Gamma(Z \to b\bar{b})$ for supersymmetry searches and model-building

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Abstract

Assuming that the actual values of $M_t$ at FNAL and of $\Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ at LEP are within their current 1$\sigma$ reported ranges, we present a No-Lose Theorem for superpartner searches at LEP II and an upgraded Tevatron. We impose only two theoretical assumptions: the Lagrangian is that of the Minimal Supersymmetric Standard Model (MSSM) with arbitrary soft-breaking terms, and all couplings remain perturbative up to scales $\sim 10^{16}$ GeV; there are no assumptions about the soft supersymmetry breaking parameters, proton decay, cosmology, etc. In particular, if the LEP and FNAL values hold up and supersymmetry is responsible for the discrepancy with the Standard Model prediction of $\Gamma(Z \to b\bar{b})$, then we must have charginos and/or top squarks observable at the upgraded machines (for LEP the superpartner threshold is below $\sqrt{s} = 140$ GeV). Furthermore, little deviation from the Standard Model is predicted within “super-unified” supersymmetry, so these models predict that the discrepancy between experiment and the Standard Model prediction for $\Gamma(Z \to b\bar{b})$ will fade with time. Finally, it appears to be extremely difficult to find any unified MSSM model, regardless of the form of soft supersymmetry breaking, that can explain $\Gamma(Z \to b\bar{b})$ for large $\tan \beta$; in particular, no model with $t-b-\tau$ Yukawa coupling unification appears to be consistent with the experiments.

1. Introduction

Recent results from Fermilab [1] indicate that the top quark is rather heavy ($M_t = 174 \pm 17$ GeV), while recent results from LEP [2] indicate that $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ might be inconsistent (up to 2.5$\sigma$) with such a heavy top. In this letter we assume that both measurements are correct, and point out the rather powerful implications this has if nature is supersymmetric.

First, we give a brief discussion of the Standard Model (SM) prediction for $R_b$ which is approximately 2 to 2.5$\sigma$ away from the latest experimental measurement. We then consider the effect of supersymmetry (SUSY) on this process. In particular, we consider both the MSSM and the popular “super-unified” approach to supersymmetric model building. We demonstrate a No-Lose Theorem for discovery of superpartners at coming collider upgrades given 1$\sigma$ experimental bounds on $R_b$ and $M_t$, and a very minimal set of theoretical assumptions.

In considering the question of the $R_b$ discrepancy, one can take either of two attitudes. Perhaps the $R_b$ measurement is finally the one which directly demon-

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strates the existence of physics beyond the Standard Model; if so, its implications for the discovery of supersymmetry are dramatic (assuming SUSY is the origin of the deviation). Yet, as we will discuss in this paper, the "super-unified" SUSY models produce values for $R_b$ near those of the SM, not large enough to explain the $R_b$ discrepancy. Thus even if SUSY is the correct theory, it is not unlikely that the measurements of $R_b$ will approach the SM expectation as systematic effects are more fully understood.

2. The Standard Model prediction for $\Gamma(Z \rightarrow b\bar{b})$

The ratio $R_b$ is very sensitive to vertex corrections involving a heavy top quark. Within the Standard Model these corrections are negative and grow like $m_t^2$. (This can be seen best in the 't Hooft-Feynman gauge where the $\phi^+tb$ coupling is proportional to $m_t$.) On the other hand, the $m_t$-dependent oblique corrections are to a good approximation universal and therefore largely cancel out in the $R_b$ ratio. This effectively isolates the vertex corrections, thereby providing both an excellent test of the SM's self-consistency and a place to search for new physics beyond the reach of current experiments.

Using the program ZOPOLE [3] we have calculated $R_b$ in the on-shell scheme for $4.5 < M_b < 5.3$ GeV as a function of the top quark pole mass, $M_t$. In Fig. 1 we have plotted the experimental values (and their 1σ ranges) for $M_t$ and $R_b$. We also show the ZOPOLE calculation of the SM prediction as the shaded region in the figure. The SM prediction as quoted by LEP [2], which is calculated by ZFITTER in the on-shell scheme, falls on the right edge of the shaded region. To be conservative we utilize the rightmost edge of the SM prediction for $R_b$ in all our calculations and comparisons since it lies closest to the the experimental measurement.

With an SM prediction in hand we now can compare with LEP's measurement of $R_b$. We use $R_b = 0.2208 \pm 0.0024$ [2]. The uncertainty includes the quoted statistical and systematic errors added in quadrature. One sees from Fig. 1 that $R_{b}^{\text{theory}} = 0.2158$ for $M_t = 174$ GeV. This is approximately 2σ away from the above quoted experimental measurement. Disagreement of experiment and theory for other values of $M_t$ can likewise be read off from Fig. 1. Since a heavy top forces $R_b$ downward, a rather large positive contribution to $R_b$ from new physics clearly is required to lift $R_b$ to the reported central value, given CDF's measurement of a heavy top quark mass.

We understand of course that the measured $R_b$ may decrease as systematic effects are better understood. However, if $R_b$ is the long-awaited deviation from the SM that heralds the onset of new physics, its implications for SUSY are dramatic. More precisely, the question we consider here is the following: What implications are there for supersymmetry if the true value of $R_b$ is within one standard deviation of the current measurement at LEP and the true value of $M_t$ is within one standard deviation of the current measurement at FNAL? Henceforth we premise the true value of $R_b$ is within one standard deviation of the current measurement at LEP and the true value of $M_t$ is within one standard deviation of the current measurement at FNAL. Henceforth we premise the true value of $R_b$ and $M_t$ to be within $R_b = 0.2208 \pm 0.0024$ and $M_t = 174 \pm 17$ GeV respectively, and investigate the consequences of this statement. We find several surprising results.
3. Supersymmetry mass bounds from $\Gamma(Z \to b\bar{b})$

In a supersymmetric theory there are additional corrections to the $Zbb$ vertex which scale like $m^2$ from the charged Higgs–top quark loops and the chargino–top squark loops. (It is the higgsino component of the chargino which couples like $m_t$; this will turn out to be crucial later in studying unified models of soft masses.) Furthermore, if $\tan \beta$ is very large then, for example, the pseudoscalar Higgs ($A^0$) coupling to $b\bar{b}$ is proportional to $m_b \tan \beta$ and the bottom squark–$b$–neutralino ($\tilde{b}_b \chi^0$) coupling similarly is proportional to $m_b / \cos \beta$. For $\tan \beta \gtrsim 40$ these contributions from “neutral exchanges” can be sizable. The supersymmetric electroweak vertex corrections to the $Zbb$ vertex at the Z pole have been analyzed by several authors [4–7]. We primarily draw from the formulas of Ref. [6] (see Appendix) in order to calculate the supersymmetric contributions to $R_b$.

We will now investigate the Minimal Supersymmetric Standard Model (MSSM), which we define as the minimal (but most general) SU(3) × SU(2) × U(1) gauge invariant Lagrangian containing the particle content of the SM, but which is supersymmetric (up to soft-breaking terms) and $R$-parity conserving.

Despite the large number of unknown soft-breaking parameters within the MSSM, any individual process usually depends only on a small subset. Thus it is with $R_b$, where only 12 unknown parameters of the MSSM enter. We will show that the experimental values for $R_b$ may be used to constrain certain of these parameters to ranges that can be easily probed in the near future. Since the SM prediction for $R_b$ is below the experimental one, the value which becomes most important to us for the present analysis is $R^\text{max}_b$, the maximum value which $R_b$ can take given any set of inputs and constraints.

The independent parameters which enter the calculation of $R_b$ are $\tan \beta$, $M_1$, $M_2$, and $\mu$ (from which one gets the chargino and neutralino masses and mixings), the physical top and bottom squark masses and their mixing angles ($m_{\tilde{t}_1}, m_{\tilde{b}_2}, \theta_t$, and $\theta_b$), the pseudoscalar Higgs mass, and $\tilde{V}_{tb}$ (the super-CKM angle between the bottom and top squarks). In theory, determining $R^\text{max}_b$ for any given set of assumptions is very difficult, for it will be a complicated function of all the free parameters which enter the process. Luckily, we can separate the dependences on many of the parameters and work with them independently. For example, $R_b(\tilde{V}_{tb})$ is always maximal when $|\tilde{V}_{tb}| = 1$, regardless of the values of the other parameters.

Some parameters are almost separable. The top squark mixing parameter, $\theta_t$, tends to maximize $R_b(\theta_t)$ for values which are small and negative. Yet the choice $\theta_t = 0$ always produces a near-maximal $R_b$, up to corrections of order 0.01$\sigma$, far too small for us to worry about here. Having now chosen $\theta_t = 0$ one then finds that $\tilde{t}_2$ (or $\tilde{t}_L$) decouples and its mass is no longer one of the parameters on which $R_b$ depends for low to intermediate $\tan \beta$. Also in this region of $\tan \beta$, the bottom squark–neutralino contributions decouple completely, leaving $R_b$ independent of $m_{\tilde{b}_1}$, $\theta_b$, and $M_1$.

Finally, $R_b$ is most strongly dependent on the masses of the light chargino(s), light top squark(s), and for large $\tan \beta$, neutral Higgs bosons. Lighter top squarks simply give larger contributions to $R_b$. The chargino contributions are much more complicated since they induce a local maximum of $R_b$ right at $m_{\chi^+_1} = \sqrt{s}/2 = m_Z/2$ where $R_b$ can be very large. Unfortunately, current bounds on the chargino mass fall right at $m_Z/2$; were these mass bounds a few GeV higher, much stricter bounds, for example on $m_{\chi^+_1}$ and $\tan \beta$, could be determined. This will be demonstrated explicitly later. Finally, the dependence on $m_{A^0}$ changes sign as $\tan \beta$ increases, weakly favoring large $m_{A^0}$ (actually, large $m_{H^\pm}$) for small to moderate $\tan \beta$, but strongly favoring small $m_{A^0}$ for large $\tan \beta$. One has in total twelve parameters, though only seven of them are relevant at low to intermediate values of $\tan \beta$; with any fewer we lose complete generality.

3.1. The No-Lose Theorem

One may summarize the nature of our No-Lose Theorem thusly: SUSY is a decoupling theory. That is, if SUSY is to make measurable contributions to the radiative corrections of SM processes, the mass scale of the SUSY partners must be near to the scale of the SM physics being studied. Only for masses of sparticles near to the current experimental limits can one expect to notice their effects through loops in SM diagrams. As these masses increase, the SM prediction for any given quantity is again realized. Thus, if we are to ex-
plain the discrepancy between the experimental and theoretical values for \( R_b \) using SUSY, the SUSY mass scale cannot be large.

That said, it remains to be seen what kind of numerical bounds can be placed on the masses of the sparticles entering into the SUSY contributions to \( R_b \). The final result of such an analysis yields this: If the discrepancy in \( R_b \) is to be explained by the MSSM, then direct observation of superpartners at LEP II or an upgraded Tevatron will occur. Strict bounds may be placed on chargino and top squark masses if SUSY is to explain the value for \( R_b \): \( m_{\tilde{\chi}^\pm} < 85 \text{ GeV} \) and \( m_{\tilde{t}_1} < 165 \text{ GeV} \). Stronger bounds can also be found. For example, for \( \tan \beta \lesssim 30 \), \( \min(\frac{m_{\tilde{\chi}^\pm}, m_{\tilde{t}_1}}{m_{\tilde{t}_1}}) < 65 \text{ GeV} \). Under additional constraints, bounds exist also on \( m_{\tilde{\chi}^0}, m_{\tilde{t}_R} \) and \( \tan \beta \). This theorem holds under the following set of assumptions which we will examine below: (i) the true value for \( R_b \) is within 1\( \sigma \) of quoted LEP measurements, (ii) the true value for \( M_t \) is within 1\( \sigma \) of quoted CDF measurements, (iii) contributions from the MSSM are responsible for the difference between the actual and theoretical SM values, (iv) the Yukawa couplings of the MSSM remain perturbative up to scales \( \sim 10^{16} \text{ GeV} \) (a so-called perturbatively valid theory), and (v) we take various experimental lower bounds on sparticle masses from direct searches.

Let us examine these conditions. The very first condition is also the one least trusted by us. Current LEP bounds on \( R_b \) are 2 to 2.5\( \sigma \) from theoretical expectations; however, if the LEP measurement of \( R_b \) migrates over time to the SM value, our theorem will of course cease to be meaningful.

The second condition is important mostly for the 1\( \sigma \) lower bound on \( M_t \). In the SM as \( M_t \) decreases, the prediction for \( R_b \) increases such that the discrepancy between experiment and theory lessens. Also, when coupled with the requirement of perturbative validity (condition (iv) above), the lower bound on \( M_t \) places a lower bound on \( \tan \beta \). This lower bound on \( \tan \beta \) is necessary for the existence of upper mass bounds (see below).

The third condition listed above is self-explanatory.

The fourth condition of perturbative validity is necessary in order to place upper and lower bounds on \( \tan \beta \). The requirement that the top, bottom, and tau Yukawa couplings remain perturbative up to a scale \( \sim 10^{16} \text{ GeV} \) places limits on \( \tan \beta \) of

\[
\sin \beta > \frac{M_{t_{10}}}{M_t} \quad \text{and} \quad \tan \beta \lesssim 60
\]

where \( M_{t_{10}} \) is the perturbative validity bound on the top quark pole mass; one finds typically that \( 190 \lesssim M_{t_{10}} \lesssim 200 \text{ GeV} \). These limits are necessary in order to gain mass bounds on the chargino and top squark. This can be seen in Fig. 2, where the maximum value for \( R_b \) is plotted against \( \tan \beta \) for two choices of \( m_{\tilde{\chi}^\pm} \). Clearly as \( \tan \beta \rightarrow 0 \) and \( \tan \beta \rightarrow \infty \), \( R_b \) can become large even for heavy charginos and top squarks.

The fifth and final "condition" is the set of experimental lower bounds that we apply to the masses of the MSSM. We take \( m_{\tilde{\chi}^\pm} > 46 \text{ GeV} \) and the rather conservative \( m_{\tilde{t}_1} > 36 \text{ GeV} \) [8]. We take lower bounds on the pseudoscalar mass from the non-observation of \( Z \rightarrow h^0 A^0 \) (low \( \tan \beta \)) and \( Z^0 \rightarrow A^0 b \bar{b} \) (high \( \tan \beta \)) at LEP, with \( A^0 \) then decaying into \( \tau^+ \tau^- \). These bounds are \( \tan \beta \)-dependent. For \( 1 \lesssim \tan \beta \lesssim 3 < m_{A^0} > 20 \text{ GeV} \) [9]; for \( \tan \beta \gtrsim 30 \), \( m_{A^0} > 60 \text{ GeV} \) [10].
In the following sections we demonstrate how the bounds on chargino and top squarks are determined through the set of five conditions above.

One should note that the bounds given above can be easily tightened. In particular, if \( M_t \) is found to be larger than its 1sigma lower bound then our bounds will become stronger. For \( M_t = 174 \), the separate upper bounds on \( m_{\tilde{X}^\pm} \) and \( m_{\tilde{t}_1} \) decrease to 63 GeV and 77 GeV, with a corresponding decrease in the minimum of the two. Bounds on \( \tan \beta \) will also exist if values of \( m_{\tilde{X}^\pm} \) near to 46 GeV are ruled out experimentally, as shown by Fig. 2; there the lower line represents \( R_b^{\text{max}} \) for \( m_{\tilde{X}^\pm} > 60 \) GeV, in which case we see that the entire region of \( 2 \lesssim \tan \beta \lesssim 40 \) is ruled out by the 1sigma \( R_b \) bounds.

3.2. Low and intermediate \( \tan \beta \) region

Consider first the case of low \( \tan \beta \) (i.e., \( \tan \beta \lesssim 5 \)). Here one finds that \( R_b \) increases monotonically as \( \tan \beta \) decreases (see Fig. 2). Therefore \( R_b^{\text{max}}(\tan \beta) \) is found at the boundary where \( \sin \beta = M_{\tilde{t}_1}/M_{10} \). For intermediate \( \tan \beta \) (i.e., \( 5 \lesssim \tan \beta \lesssim 30 \)), \( R_b^{\text{max}} \) increases with \( \tan \beta \) but remains below the values obtained at very small \( \tan \beta \). Therefore, one may simplify the analysis in this region by only considering the lowest possible value of \( \tan \beta \) for a given \( M_{10} \).

In Fig. 3 we have plotted contours of \( R_b^{\text{max}} = 0.2184 \) (the 1sigma lower experimental bound) against \( m_{\tilde{X}^\pm} \) and \( m_{\tilde{t}_1} \), for \( M_t = 157, 174, \) and 191 GeV. Recall that \( R_b^{\text{max}} \) is the largest value that the theory can produce, regardless of the values of the other parameters not shown in the plot. To be conservative, we take \( M_{10} = 205 \) GeV, thereby allowing smaller \( \tan \beta \) (and thus larger \( R_b^{\text{max}} \)) for a given top quark mass. We emphasize that what is shown are the maximum values of \( m_{\tilde{t}_1} \) and \( m_{\tilde{X}^\pm} \) that can give \( R_b \) within 1sigma of its reported value. In the figure, the regions to the left and below each line are compatible with the 1sigma bounds on \( R_b \); the regions above and to the right are excluded.

One can read the central result of the No-Lose Theorem from this graph: by combining the 1sigma bounds on the top quark mass from CDF and on \( R_b \) from LEP, one can place upper bounds of 85 GeV on the mass of the lighter chargino and 100 GeV on the mass of the lighter top squark in the low to intermediate \( \tan \beta \) region. However, one or the other must always be lighter than about 65 GeV. For \( M_t \) above 157 GeV, these limits become stronger. These bounds are to our knowledge the strongest experimental bounds on the MSSM to date. They do not depend on any scheme for soft-breaking parameters, on constraints from dark matter or proton decay, or any assumptions other than those few listed. They imply that with an upgrade in energy to \( \sqrt{s} = 130 \) GeV at LEP a chargino or a top squark must be found, and with an upgrade in luminosity at the Tevatron both the chargino and the top squark should be found. Therefore, the "1sigma" ranges of chargino and top squark masses as shown in Fig. 3 could be completely probed if \( \tan \beta \lesssim 30 \).

Further, if \( m_{\tilde{X}^\pm} \gtrsim 60 \) GeV and \( \tan \beta \lesssim 40 \), then \( \tan \beta \) is restricted to be close to 1. Therefore, the mass of the lightest Higgs approaches zero at tree level, most of its mass then being due to radiative corrections. Since we bound the lighter top squark mass, the leading term in the corrections to the Higgs mass (which goes as \( m_{\tilde{t}_1}^2 \)) is determined up to the value of
the heavier top squark. Ignoring contributions due to
top squark mixing, one finds that for $m_{t_2} < 1 \text{ TeV}$,
$m_{h^0} < 75 \text{ GeV}$; likewise for $m_{t_2} < 2 \text{ TeV}$, $m_{h^0} <
80 \text{ GeV}$. Thus in the low to intermediate $\tan \beta$ regime,
h$^0$ may also be accessible at LEP or FNAL.

3.3. The high $\tan \beta$ region

Large values of $R_b$ can also be attained for high
$\tan \beta$. Once again, it is at the perturbative edge
($\tan \beta \simeq 60$) that we obtain the largest $R_b^{\text{max}}$. Inter-
estingly, it is also this region of very high $\tan \beta$ that is motivated by $t \rightarrow b \tau$ Yukawa unification within
minimal SO(10) models. In this region interactions
proportional to $m_b \tan \beta$ become comparable to and
possibly more significant than the $m_t$ dependent in-
teractions. Therefore, neutralino-bottom squark loops
and neutral Higgs-bottom quark loops must be con-
sidered.

Four of the independent parameters of our model
which could be ignored in the low $\tan \beta$ region, namely
the bottom squark masses and mixing angle, and $M_1$,
must now be included when we maximize $R_b$. This
significantly complicates the process and the demon-
stration of our theorem, so we again separate those
variables that can be taken independent from the oth-
ers. One simplification would be to set $M_1 \simeq \frac{1}{2} M_2$, as
indicated by wide classes of supergravity and super-
string scenarios. We have taken this simplification as
well-motivated and base most of our numerical results
in the high $\tan \beta$ limit on it; however, we have checked
that perturbing the ratio of $M_1$ to $M_2$ by an additional
factor of two in either direction only changes our cal-
culations of $R_b^{\text{max}}$ by less than 0.1$\sigma$.

The bottom squark masses and mixing provide an-
other complication. Our calculations indicate that $R_b$
is maximized when both bottom squarks are light, near
to their lower bound which we take to be 45 GeV. The
squarks are then nearly degenerate in mass and the
mixing angle becomes arbitrary.

The Higgs sector behaves in the high $\tan \beta$ limit
very differently than in the opposite limit. Unlike in
the low $\tan \beta$ case, a light pseudoscalar Higgs yields
an overall large positive contribution to $R_b$. The con-
tribution of the light charged Higgs is still negative, but
is overwhelmed by a large contribution from neutral
Higgs-bottom quark loops which increases with $\tan \beta$.
Therefore, we must reintroduce the pseudoscalar mass
as a variable when working in this region. However
the experimental constraint that $m_{A^0} > 60 \text{ GeV}$ for
$\tan \beta > 30$ [10] means that large chargino contribu-
tions are still needed, leading to chargino bounds at
large $\tan \beta$ which are tighter than those at small $\tan \beta$.

How do discovery limits compare in the high $\tan \beta$
region? Most significantly, one finds that $m_{A^0} < 70 \text{ GeV}$,
a tighter bound than existed in the low $\tan \beta$
limit. However, it is not solely the chargino-top squark
loops that are responsible for this bound, but also
the neutralino-bottom squark loops. As the chargino
mass becomes larger and its contributions decou-
ples, $R_b^{\text{max}}$ plummets towards the SM value, thereby
placing a bound on $m_{A^0}$. Yet as $\tan \beta$ increases,
the neutralino contributions can be sizable, bringing $R_b$
back into agreement with experiment. Thus there is
really a bound on a combination of the neutralino and
chargino masses. But the appearance of the same $\mu$ in
both sectors means that bounds can be placed on each
individually, leading to the 70 GeV for the chargino
given above. The comparable neutralino bound is then
approximately 67 GeV. If the 1$\sigma$ lower bound of $R_b$
shifted to 0.2172, then the bounds on the chargino and
neutralino would relax to 98 and 95 GeV respectively.

The same interplay we found between chargino
and neutralino masses is of course also found in
the top and bottom squark sectors. Under the as-
sumption of nearly degenerate bottom squarks, one
finds that either a bottom or a top squark must be
lighter than 165 GeV (85 GeV) for $\tan \beta < 60$ and
$m_{A^0} > 45 \text{ GeV}$ ($m_{A^0} > 60 \text{ GeV}$), a weaker bound
than was found in the low $\tan \beta$ limit. But unlike the
case for the charginos and neutralinos, no individual
bounds exist on the top and bottom squarks alone.

Finally, there are also the contributions due to $A^0$-
bottom quark loops. Once again an upper bound can be
placed on $m_{A^0}$ consistent with $R_b$ if $m_{A^0} > 60 \text{ GeV}$.
Here we find that $m_{A^0} < 95 \text{ GeV}$ is necessary. Thus,
little pseudoscalars are implied by $R_b$ for high $\tan \beta$,
one once constrains charginos to be slightly heavier
than the current bound of 46 GeV.

4. Implications for the super-unified MSSM

Much work has been completed recently by a
number of groups on the phenomenology of "super-
unified” minimal SUSY (see Ref. [11] and references therein). These models are constructed under the assumption of not only gauge coupling unification at some high scale, but also unification of various soft mass parameters in the MSSM Lagrangian (a common gaugino and a common scalar mass). One then connects these high scale assumptions to low-energy phenomenology through the renormalization group equations (RGEs) of the parameters, under the condition that electroweak symmetry-breaking occurs radiatively at scales \( \sim m_Z \).

In two previous works [11,12], the super-unified MSSM was assumed and a number of constraints stemming from direct experimental searches for SUSY, CLEO bounds on BR(\( b \to s \gamma \)), relic abundances of the lightest SUSY particle, etc. were assumed. What remained of the original parameter space of the super-unified models was called the Constrained Minimal Supersymmetric Standard Model (CMSSM). The natural question is, then, what ranges of values for \( R_b \) are predicted for solutions consistent with the CMSSM? In fact, one finds that the constraints of the CMSSM force \( R_b \) to lie below approximately 0.2166, some 1.5\( \sigma \) below the reported \( R_b \) for \( M_t = 174 \) GeV. Fig. 1 shows the CMSSM’s “line of closest approach” to the experimentally measured value of \( R_b \). This closest approach line does not enter the 1\( \sigma \) area, and therefore CMSSM cannot bring LEP’s measurement of \( R_b \) into agreement with FNAL’s measurement of \( M_t \); conversely, one could say that the CMSSM predicts \( R_b \) below the line given in Fig. 1.

Why is the CMSSM incapable of producing larger values for \( R_b \)? Naively, one would expect contributions from supersymmetric masses \( \sim m_W \) to have a large contribution to the \( Z \to b \bar{b} \) partial width just as the \( W \) boson itself has. As described earlier, there exist interactions of the charginos with top squarks and quarks which are large, proportional to the top Yukawa coupling (\( h_t \)), and enhance \( R_b \). Similar contributions exist at large tan \( \beta \) for the neutral Higgs bosons with bottom squarks and squarks, proportional to the tan \( \beta \)-enhanced bottom Yukawa coupling. These interactions could lead to \( R_b \) consistent with experiment. We now summarize the reasons that they do not.

The coupling of the top quarks to charginos and top squarks, proportional to the large top Yukawa coupling, is given by \( \mathcal{L} = h_t \bar{t} H^\pm \bar{t}_R \). In order to maximize the impact of this coupling, we need a light top squark with a significant right-handed component and a light chargino with a significant higgsino component. These are both difficult requirements for the CMSSM, and they are quite impossible to satisfy simultaneously.

Within CMSSM, the choice of common scalar masses means that \( m_{\tilde{t}_R} \) is invariably smaller than \( m_{\tilde{t}_L} \) due to the running of the RGEs from the GUT scale down to the weak scale. Yet the resulting \( m_{\tilde{t}_L} \) is rarely smaller than \( m_W \) unless we impose large mixing between the \( \tilde{t}_L \) and \( \tilde{R} \) eigenstates. This in turn means that \( \tilde{t}_L \) will have a significant \( \tilde{t}_L \) component which does not couple to the charginos with \( h_t \).

The lightest chargino is even more troublesome. The chargino mass matrix depends on \( M_2, \mu \) and tan \( \beta \). With radiative breaking \( \mu \) scales roughly as \( \max\{M_{1/2}, m_0\} \) and thus generally dominates the chargino matrix (for a discussion, see Ref. [11]). This means that the charginos with significant higgsino components are heavy, generally well above \( m_W \), and even though they couple with a factor of \( h_t \), these contributions will be kinematically suppressed. Solutions with a chargino as light as 46 GeV can be obtained, but since they will be almost all \( \tilde{t}_L \) they do not couple as \( h_t \).

When we try to simultaneously satisfy the two requirements of a light \( \tilde{t}_L \) \( \simeq \tilde{R} \) and a light \( \chi^\pm \simeq H^\pm \), we find that the CMSSM is incapable of providing a solution. The two requirements actually push solutions in different, incompatible directions. Large mixing in the top squark sector generally requires \( \mu \) (or \( m_0 \)) to be large, while light higgsino-like charginos require \( \mu \) (thus \( m_0 \) also) to be small.

In the high tan \( \beta \) region, the additional contributions coming from the pseudoscalar–bottom squark loops are likewise suppressed. The CMSSM generally produces heavy bottom squarks and also heavy pseudoscalars, so that their contributions will be small and cannot bring the CMSSM into agreement with experiment.

For these reasons, and after much numerical work, we can conclude that the CMSSM is incompatible (to 1.5\( \sigma \)) with the experimental measurements of \( R_b \) and \( M_t \). Those who wonder how easy it is to vary SUSY parameters in order to fit data should note this result. Clearly, the CMSSM’s ability to fit experiment for this particular observable is inextricably tied to the Standard Model’s since the predictions are essentially
the same.

What of models which unify the gauge couplings but not the soft-breaking parameters? One may use a "bottom-up" approach in building models at the GUT scale, starting from the constraints of the low-energy theory. Of particular interest is the case motivated by $t - b - \tau$ Yukawa coupling unification in SO(10). Here one finds that large $\tan\beta$ is necessary, which could be consistent with $R_b$ given the increase in $R_b^{\text{max}}$ that occurs at very large $\tan\beta$ (see Fig. 2). However, the RGEs for the soft masses can be analytically solved in this (pseudo-fixed point) limit \[13\], and are consistent with a tree-level $m_{\chi^0}^2 > 0$ only if $\mu^2 \gtrsim 3M^2_{1/2}$.

In this case, the light charginos/neutralinos are predominantly wino/bino, not higgsino, and $R_b$ will approach the SM value. Therefore, these SO(10)-type models appear to not be able to yield $R_b$ within 1$\sigma$ of experiment, regardless of the GUT-scale structure of the soft-breaking terms.

5. Conclusions

We have explicitly demonstrated a type of No-Lose Theorem for direct observation of SUSY partners at either LEP II or upgraded Tevatron. The central assumption for this theorem is that the actual values of $R_b$ and $M_t$ lie at or above their 1$\sigma$ lower bounds.

By coupling these measurements to the requirement of perturbative validity of the Yukawa couplings up to a GUT-like scale, we derived bounds on charginos and top squarks leading one to conclude that one or both should be directly observed at LEP II with $\sqrt{s} > 140\,\text{GeV}$, and at an upgraded (in luminosity) Tevatron, for any SUSY breaking and any (perturbative) $\tan\beta$. We also pointed out the existence of interesting bounds on $\tan\beta$ if the lower bound on charginos could be pushed up a little from its current value. Of course, these bounds may never be saturated. Other observables, such as the $\rho$-parameter, the forward-backward $b$-asymmetry ($A_{FB}^{b}$), or $B^0 - \bar{B}^0$ mixing ($\chi_B$) could significantly tighten our upper bounds; we are currently examining this issue. However, we have checked that full spectra can be found which are consistent with both the experimental value for $R_b$ and a cosmological relic density of $\Omega \simeq 1$.

Finally, we explained the inability of the simplest class of super-unified models to explain the $R_b$ discrepancy. We found that these models are strongly decoupled from the decay of $Z \rightarrow b\bar{b}$ despite their often low mass scales, so that their prediction for $R_b$ bears little difference from that of the SM. We are currently exploring the question of what kinds of GUT models can be found which are consistent with $R_b$.

In particular, one wishes to know what hierarchies of soft-breaking masses are required at the GUT scale to replace the assumption of common masses. Surprisingly, we have found that it is extremely difficult to construct a unified theory for any set of soft-breaking parameters if $\tan\beta$ is in the range required by unification of all third generation Yukawa couplings, as in SO(10) models. The solution to this problem may lead to progress in understanding the mechanism of soft SUSY breaking and to the breaking of gauge symmetries at the GUT scale, assuming the experimental $R_b$ discrepancy persists over time.

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Appendix

Supersymmetric corrections to the $Zb\bar{b}$ vertex are $m_t$ and $m_b\tan\beta$ dependent, and can be quite large. In this appendix we present the calculation of the shift in $R_b$ due to supersymmetric contributions from charged Higgs-top quark loops, chargino-top squark loops, and neutralino-bottom squark loops. Our calculations are in agreement with those of Ref. \[6\], and in this appendix we follow the notation of this reference as closely as possible. However, we attempt to clear up possible confusion by presenting the equations strictly in terms of ordinary Passarino-Veltman functions with full arguments.
\[ \nabla_{b}^{\text{susy}} = 2 \, \text{Re} \left( \frac{v_{b}^{*} \cdot \nabla_{b}^{*}}{2} \right) \]

Fig. 4. Feynman diagram representation of \( \nabla_{b}^{\text{susy}} \).

The expansion of \( R_{b} \) can be separated into SM contributions and supersymmetric contributions [6]:

\[
R_{b} = \frac{\Gamma(Z \rightarrow b \bar{b})}{\Gamma(Z \rightarrow \text{hadrons})} = R_{b}^{\text{SM}}(m_{t}, m_{b}) + R_{b}^{\text{SM}}(0, 0) \left[ 1 - R_{b}^{\text{SM}}(0, 0) \right] \\
\times \left[ \nabla_{b}^{\text{susy}}(m_{t}, m_{b}) - \nabla_{b}^{\text{susy}}(0, 0) \right].
\]

\( R_{b}^{\text{SM}}(0, 0) \) is the SM prediction of \( R_{b} \) for massless top and bottom quarks which we take to be 0.220. The value for \( \nabla_{b}^{\text{susy}}(m_{t}, m_{b}) \) is the sum of one-loop interferences with the tree graph divided by the squared amplitude of the tree graph as shown in Fig. 4.

A convenient parameterization of the supersymmetric vertex corrections \( \nabla_{b}^{\text{susy}}(m_{t}, m_{b}) \) is

\[
\nabla_{b}^{\text{susy}}(m_{t}, m_{b}) = \frac{\alpha}{2\pi} \frac{v_{L}F_{L} + v_{R}F_{R}}{\sin^{2}\theta_{W}(u_{L})^{2} + (v_{R})^{2}},
\]

where

\[
u_{L} = -\frac{1}{2} + \frac{1}{3} \sin^{2}\theta_{W}, \quad v_{R} = \frac{1}{3} \sin^{2}\theta_{W}.
\]

Loops involving the charged Higgs with the top quark yield the following contributions to \( F_{L} \) and \( F_{R} \):

\[
F_{L,R}^{a} = S_{1}(m_{t}, m_{H^{\pm}}) u_{L,R} \Lambda_{L,R}^{2}
\]

\[
F_{L,R}^{b} = \frac{1}{2} S_{2}(m_{t}, m_{H^{\pm}}, m_{t}) v_{L,R}^{(1)}
\]

\[
F_{L,R}^{c} = m_{t}^{2} S_{3}(m_{t}, m_{H^{\pm}}, m_{t}) v_{L,R}^{(1)} \Lambda_{L,R}^{2}
\]

\[
F_{L,R}^{d} = S_{4}(m_{H^{\pm}}, m_{t}, m_{H^{\pm}}) \left( \frac{1}{2} - \sin^{2}\theta_{W} \right) \Lambda_{L,R}^{2}
\]

where

\[
u_{L}^{(1)} = \frac{1}{2} - \frac{5}{3} \sin^{2}\theta_{W}, \quad v_{R}^{(1)} = -\frac{2}{3} \sin^{2}\theta_{W}
\]

\[
\lambda_{L} = \frac{m_{t} \cot \beta}{\sqrt{2} m_{W}}, \quad \lambda_{R} = \frac{m_{t} \tan \beta}{\sqrt{2} m_{W}}.
\]

Loops involving charginos and top squarks yield the following contributions to \( F_{L} \) and \( F_{R} \):

\[
F_{L,R}^{e} = \sum_{i,j} S_{1}(m_{t}, m_{H^{\pm}}, m_{t}) u_{L,R} \Lambda_{L,R}^{2}
\]

\[
\times \left( \frac{3}{2} \sin^{2}\theta_{W} \delta_{ij} - \frac{1}{2} T_{L} T_{R} \right) \Lambda_{L,R}^{2} \Lambda_{L,R}^{2}
\]

\[
F_{L,R}^{f} = \sum_{i,j,k} \left[ S_{2}(m_{t}, m_{t}, m_{t}, m_{t}) O_{ij} \Lambda_{L,R}^{2} \right.
\]

\[
+ m_{t}^{2} m_{t}^{2} S_{3}(m_{t}, m_{t}, m_{t}, m_{t}) O_{ij} \Lambda_{L,R}^{2} \Lambda_{L,R}^{2}
\]

\[
+ m_{t}^{2} m_{t}^{2} S_{3}(m_{t}, m_{t}, m_{t}, m_{t}) O_{ij} \Lambda_{L,R}^{2} \Lambda_{L,R}^{2}
\]

where

\[
\Lambda_{L,R}^{ij} = \frac{1}{2} N_{j}^{*} \tan \theta_{W} - N_{j}^{*} B_{i}
\]

\[
- \frac{m_{b}}{\sqrt{2} m_{W} \cos \beta} N_{j}^{*} B_{i}
\]

\[
\Lambda_{L,R}^{ij} = \frac{1}{2} N_{j}^{*} \tan \theta_{W} - N_{j}^{*} B_{i}
\]

\[
O_{L,R}^{ij} = \frac{1}{2} N_{j}^{*} N_{j} - \frac{1}{2} N_{j}^{*} N_{j}
\]

\[
O_{L,R}^{ij} = -O_{L,R}^{ij}
\]

Again, we follow the conventions of Ref. [15], and define the neutralino mixing matrix \( N \) in the
Note that this convention is different from that of Ref. [6] which chose to diagonalize the neutralinos in the \( \{ \tilde{B}, \tilde{W}^3, \tilde{H}_d, \tilde{H}_u \} \) basis, thereby interchanging \( N_{13} \) and \( N_{14} \) in the above equations.

The \( S_n \) functions are defined in terms of the non-divergent parts of Passarino-Veltman scalar integral functions [16]:

\[
S_1(m_1, m_2) = B_1(-m_1^2; m_1^2, m_2^2) \\
S_2(m_1, m_2, m_3) = -\frac{1}{2} \left[ 2C_{24} - m_2^2C_{12} - m_3^2C_{23} \right] \\
\times (-m_1^2, -m_1^2, -m_2^2; m_1^2, m_2^2, m_3^2) \\
S_3(m_1, m_2, m_3) = C_0(-m_1^2, -m_2^2, -m_3^2; m_1^2, m_2^2, m_3^2) \\
S_4(m_1, m_2, m_3) \\
= -2C_{24}(-m_1^2, -m_1^2, -m_2^2; m_1^2, m_2^2, m_3^2).
\]

All logs encountered in the scalar integrals are made dimensionless by inserting the 't Hooft mass \( \mu_R \).

Note that our \( S_2 \) function contains \( C_{12} \). In Ref. [6], the calculation of \( R_R \) is presented in terms of reduced Passarino-Veltman functions [17]. However, the translation there of the \( \epsilon_6 \) reduced Passarino-Veltman function leaves one to believe that \( S_2 \) contains \( C_{11} \) rather than the correct \( C_{12} \).

### References