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STUDY AND INVESTIGATION OF A UHF-VHF ANTENNA

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ABSTRACT

Several theoretical and experimental problems were studied during this period. The study of diffraction of a plane wave by a ferrite sphere was extended to include a metal sphere enclosed within and concentric with the ferrite sphere. Computer results showed that resonant frequencies were changed slightly with the addition of the metal sphere. A study of plane-wave diffraction by a longitudinal magnetized ferrite cylinder was begun. A computer program is being prepared to evaluate the fields for various values of $\mu$ and $t$.

A shielded, balanced-loop antenna loaded with ferrite material was analyzed, showing that resonance of the loop could be maintained over a broad range of frequency. Radiation patterns from a ferrite-filled waveguide were evaluated on the computer, showing that with the use of ferrite loading the size of a waveguide radiator can be reduced by the factor $\sqrt{\mu_0\varepsilon_r}$ without substantial change in radiation pattern. The coaxial cavity equipment used to measure complex permeability and permittivity was improved to an accuracy of about 10% in $\varepsilon'$, $\mu'$ and about 20% in $\varepsilon''$, $\mu''$.

PURPOSE

This report summarizes the work done on Contract No. AF 33(616)-7180 during this period from April 1, 1961, to July 1, 1961.

The purpose of this task is to investigate the use of solid-state devices such as ferrites and dielectrics in their application to UHF-VHF antennas. More specifically, these materials are to be considered as loading devices or actual elements in the search for improvement of the following properties: (1) radiation resistance, (2) power gain and directivity, (3) broadbanding, (4) physical size, and (5) efficiency. Geometries now under consideration include dipoles, rods, slots, biconical dipoles, spirals, and yagis.
1. REPORTS, TRAVEL, AND VISITORS

During this period, no reports were issued, project personnel did not travel and no one visited the project.

2. FACTUAL DATA

2.1 THE PROBLEM OF A PLANE WAVE INCIDENT ON A MATERAL SPHERE

The problem of a plane wave incident on a sphere of arbitrary permeability and permittivity was formulated in Quarterly Report No. 4. The interior fields were formally solved, and numerical results obtained for the power density at the center. A factor $P$ was defined as the ratio of this power density to that of the incident wave. The factor $P$, when plotted against the radius of the sphere in material wavelengths, exhibits sharp resonant peaks where the concentration of power is very high.

This study has since been continued and extended to (1) evaluate the effects of higher modes throughout the volume, which gives information on the effect of the materials on the prominence of the various modes; (2) map the lines of power flow, which provides a basis for a physical explanation of the field distortion phenomena; and (3) treat the case of an enclosed, perfectly conducting sphere, which extends the problem to more antenna-like characteristics.

2.1.1 Higher Modes.—The basic formulas will be repeated for convenience. The incident plane wave can be expressed by:
\[ \bar{E}_b = E_0 e^{-j\omega t} \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} \left( \frac{\epsilon_1}{\mu_0} - j \frac{\epsilon_0}{\mu_1} \right) \]

\[ \bar{H}_b = -\sqrt{\mu_0 \epsilon_0} E_0 e^{-j\omega t} \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} \left( \frac{\epsilon_1}{\mu_1} + j \frac{\epsilon_0}{\mu_0} \right) \]

The interior fields are represented by a linear combination of the above functions, where the coefficients are \( a_n^t \) and \( b_n^t \):

\[ \bar{E}_t = E_0 e^{-j\omega t} \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} \left[ a_n \frac{\epsilon_1}{\mu_0} - j b_n \frac{\epsilon_0}{\mu_1} \right] \]

\[ \bar{H}_t = -\sqrt{\mu_1 \epsilon_1} E_0 e^{-j\omega t} \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} \left[ b_n \frac{\epsilon_1}{\mu_1} + j a_n \frac{\epsilon_0}{\mu_0} \right] \]

The coefficients \( a_n^t \) and \( b_n^t \) are expressed by:

\[ a_n^t = \frac{j\mu K}{K\epsilon'(k_1a)R_n^{(1)}(k_0a) - \mu\epsilon_n(k_1a)R_n^{(1)}(k_0a)} \]

\[ b_n^t = \frac{j\mu K}{\mu\epsilon'(k_1a)R_n^{(1)}(k_0a) - K\epsilon_n(k_1a)R_n^{(1)}(k_0a)} \]

Here, \( \mu = \mu_1 / \mu_0, \epsilon = \epsilon_1 / \epsilon_0 \), and \( K = k_1 / k_0 = \sqrt{\mu_0 \epsilon_0} \), where \( k_1 \) is the propagation constant in the material.

Plots of the coefficients vs. radius of the sphere in material wavelengths are shown in Figs. 1 and 2 for various values of \( \mu \) and \( \epsilon \). Inspection of Eqs. (1) and (2) shows that the plane-wave functions can be thought of as having the coefficients \( a_n^t = b_n^t = 1 \). The curves of Fig. 1 show that there are critical radii where the coefficients take on very high values. For example, for \( \mu = \epsilon = 10 \) at \( a / \lambda_m = .65 \), it can be seen that the first mode is resonant, while the other modes have considerably smaller coefficients.
Fig. 1. Coefficients for ferrite sphere for $\mu = \epsilon = 10$. 

$\mu = \epsilon = 10$

$\alpha_n = b_n$
Fig. 2. Coefficients for ferrite sphere for $\mu = \varepsilon = 3$. 
Similar resonances appear at larger radii for the higher modes, but the pre-
dominance of these higher modes at resonance is not so striking.

The resonant frequencies for a given radius can be found by minimizing the denominator. These are the frequencies that satisfy the transcendental equations:

\[
K_n(k_1a)C_n(k_0a) = \mu_n S_n(k_1a)C_n(k_0a)
\]

(1)

\[
\mu_n S_n(k_1a)C_n(k_0a) = K_n(k_1a)C_n(k_0a)
\]

These are plotted in Fig. 3 as a function of mode number.

2.1.2 Power Flow.—Near the first resonance one can say that the field energy is predominantly in the first mode; even the cross-terms between modes are small. Thus the power flow distribution can be satisfactorily described considering only the first mode. The P-factor defined earlier can be represented by:

\[
\bar{P} = \frac{\text{Re}(\overline{E} \overline{H}^*)}{\overline{E}_0^2} \frac{\epsilon_0}{\mu_0}
\]

\[
= 9/4 \text{ Re} \left[ a_{11}^t b_{11}^* \left[ M_{011} x M_{e11}^{(1)} \right] + a_{11}^t b_{11}^* \left[ M_{011} x M_{e11}^{(1)} \right] \right]
\]

\[
= 9/4 \text{ Re} \left( a_{11}^t b_{11}^* \right) \left[ M_{011} x \overline{M}_{e11}^{(1)} + \overline{M}_{011} x \overline{M}_{e11}^{(1)} \right]
\]

(5)

When broken into components, the expression in plane I is (where \( \phi = 0, \pi \))

\[
\bar{P} = 9/4 \text{ Re}(a_{11}^t b_{11}^*) \cdot \left[ \left( \frac{\overline{S}_1(k_1r)}{k_1r} \right)^2 + \left( \frac{\overline{S}_1(k_1r)}{k_1r} \right)^2 \cos \theta a_\theta^\wedge \right]
\]

\[
- \frac{2S_1(k_1r)S_{1}(k_1r)}{(k_1r)^3} \sin \theta a_\phi^\wedge \right]
\]

(6)
Fig. 3. Resonance plot for $\mu = \varepsilon = 10$. 
This is shown schematically in Fig. 4.

Several points are suggested by this equation. First, the shape of the field is not affected by changes in \( \mu_r \) and \( \varepsilon_r \) as long as \( \sqrt{\mu_r \varepsilon_r} = k_1/k_0 \) is left constant. Second, in the regions where the above assumptions hold (i.e., near the first resonance) the shape of the field depends on the value of \( k_1r \), regardless of the value of \( k_1a \), as long as \( k_1r \leq k_1a \). This implies that the streamlines are not always perpendicular at the surface. In fact, they are perpendicular only when the tangential term \( S_1(k_1r)S_1(k_1r) \) vanishes. Third, while the radial term must always have the sign of \( \cos \theta \), the tangential term alternates in sign as \( k_1r \) is increased. Thus at \( \theta = \pi/2 \), where the radial term vanishes, the streamlines will be in the positive or negative z-direction, which means that some streamlines circulate, much like the turbulence in a high-velocity stream of fluid.

2.2 THE PROBLEM OF A PLANE WAVE INCIDENT UPON A COMPOSITE STRUCTURE CONSISTING OF A METAL SPHERE INCLOSED WITHIN, AND CONCENTRIC WITH, A MATERIAL SPHERE

To resemble the problem of an antenna more closely, the problem of a plane wave scattered by a ferrite sphere can be modified by stipulating a spherical conducting boundary in the ferrite's interior (see Fig. 5). This allows one to evaluate the currents induced by the field on the surface.

The added conditions required by this modification are two:

(1) The tangential component of \( \mathbf{E} \) vanishes at the surface of the metallic sphere.

(2) Since the ferrite region does not now include the origin, the coefficient of the Neumann function is not zero. The most general formulation
Fig. 4. Power flow diagram for ferrite sphere.
Fig. 5. Conducting sphere problem.

includes both Bessel and Neumann functions, the latter of which becomes infinite at the origin. This is not physically permissible in the original problem, so the original coefficient must be zero. The expressions for the interior fields are now

\[
E_t = E_0 e^{-i \omega t} \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} a_n^t \overline{M}_n^{(1)} + c_n^t \overline{N}_n^{(2)} - j b_n^t \overline{N}_n^{(1)} + a_n^t \overline{N}_n^{(2)}
\]

\[
\overline{H}_t = -\frac{\epsilon_1}{\mu_1} E_0 e^{-i \omega t} \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} b_n^t \overline{M}_n^{(1)} + d_n^t \overline{N}_n^{(2)} + j a_n^t \overline{N}_n^{(1)} + c_n^t \overline{N}_n^{(2)}
\]

The boundary conditions are:

\[
a_r x (\overline{E}_i + \overline{E}_r) = a_r x \overline{E}_t \quad r = a
\]

\[
a_r x (\overline{H}_i + \overline{H}_r) = a_r x \overline{H}_t \quad r = a
\]

\[
a_r x \overline{E}_t = 0 \quad r = b
\]
These give rise to six simultaneous equations:

\[
\begin{align*}
    a_n^t S_n(k_1a) + c_n^t C_n(k_1a) - K_n R_n^-(1)(k_0a) &= K_S(nk_0a) \\
    c_n^t S_n'(k_1a) + c_n^t C_n'(k_1a) - \mu_R R_n^-(1)(k_0a) &= \mu R_n'(k_0a) \\
    a_n^t S_n(k_1b) + c_n^t C_n(k_1b) &= 0 \\
    b_n^t S_n'(k_1a) + a_n^t C_n'(k_1a) - K_R R_n^+(1)(k_0a) &= \mu R_n(k_0a) \\
    b_n^t S_n'(k_1b) + a_n^t C_n'(k_1b) &= 0
\end{align*}
\]

(9)

The coefficients can be solved explicitly. The interior coefficients are:

\[
\begin{align*}
    a_n^t &= \frac{\mu_R K}{K_R^+(k_1a) R_n^+(1)(k_0a) - \mu_R S_n(k_1a) R_n^-(1)(k_0a)} - \frac{S_n(k_1b)}{C_n(k_1b)} \frac{K_R^+(k_1a) R_n^+(1)(k_0a) - \mu_R C_n(k_1a) R_n^-(1)(k_0a)}{K_R^+(k_1a) R_n^+(1)(k_0a) - \mu_R C_n(k_1a) R_n^-(1)(k_0a)} \\
    b_n^t &= \frac{\mu_R K}{\mu_R S_n'(k_1a) R_n^+(1)(k_0a) - K_S(k_1a) R_n^+(1)(k_0a)} - \frac{S_n'(k_1b)}{C_n'(k_1b)} \frac{\mu_R S_n'(k_1a) R_n^+(1)(k_0a) - K_S(nk_1a) R_n^+(1)(k_0a)}{\mu_R S_n'(k_1a) R_n^+(1)(k_0a) - K_S(nk_1a) R_n^+(1)(k_0a)} \\
    c_n^t &= \frac{\mu_R K}{K_R^+(k_1a) R_n^+(1)(k_0a) - \mu_R S_n(k_1a) R_n^-(1)(k_0a)} - \frac{C_n(k_1b)}{S_n(k_1b)} \frac{K_R^+(k_1a) R_n^+(1)(k_0a) - \mu_R S_n(k_1a) R_n^-(1)(k_0a)}{K_R^+(k_1a) R_n^+(1)(k_0a) - \mu_R S_n(k_1a) R_n^-(1)(k_0a)} \\
    d_n^t &= \frac{\mu_R K}{\mu_R C_n'(k_1a) R_n^+(1)(k_0a) - K_S(k_1a) R_n^+(1)(k_0a)} - \frac{C_n'(k_1b)}{S_n'(k_1b)} \frac{\mu_R C_n'(k_1a) R_n^+(1)(k_0a) - K_S(nk_1a) R_n^+(1)(k_0a)}{\mu_R C_n'(k_1a) R_n^+(1)(k_0a) - K_S(nk_1a) R_n^+(1)(k_0a)}
\end{align*}
\]
It will be recalled that \( S_n, C_n, \) and \( R_n^{(1)} \) are adaptations of spherical Bessel, Neumann, and Hankel functions; that is, \( S_n(z) = z J_n(z) \), and similarly for the others. As \( z \) approaches zero, \( C_n(z) \) is proportional to \( z^{-1} \) so that for a small radius of the conducting sphere, the fractions, \( \frac{S_n(k_1 b)}{C_n(k_1 b)} \) and \( \frac{S_n'(k_1 b)}{C_n'(k_1 b)} \) are small.

Thus it is evident from the above equations that, as the conducting sphere vanishes, \( a_n^t \) and \( b_n^t \) approach the values which they had in the original problem, and \( c_n^t \) and \( d_n^t \) approach zero. This is intuitively reasonable, for as the metal sphere is shrunk, its effect on the fields vanishing in the limit should diminish.

Curves of the coefficients are shown in Figs. 6 and 7 for \( \mu_r = \varepsilon_r = 10 \) with an inner sphere of 0.1 ferrite wavelength. The resonant peaks are not at the same radius for the TM and TE coefficients. This separation between TM and TE coefficients decreases for the higher modes.

2.3 SCATTERING OF A NORMALLY INCIDENT PLANE WAVE BY A MAGNETIZED FERRITE CYLINDER

The solution of the boundary-value problem for a normally incident plane wave on a dielectric or ferrite cylinder, was described in Quarterly Progress Report No. 4, is now being explored thoroughly by computer programs. To investigate the various possible uses of a ferrite rod as a director of energy, the electromagnetic field distribution in and around an axially magnetized ferrite rod is now being determined.

It is expected that, with the biased ferrite rod, the radiating and impedance properties of solid-state cylindrical structures can be better con-
Fig. 6. Coefficients of ferrite sphere with conducting inner sphere. Inner radius = 0.1 $\lambda_m$. 
Fig. 7. Coefficients of ferrite sphere with conducting inner sphere. Inner radius = 0.5 $\lambda_m$. 
trolled. One of the principal advantages would be a broadbanding of the resonant-power-gathering ability of the ferrite cylinder.

The analysis shown below is an extension of the work of W. H. Eggimian, as presented in the July, 1960, issue of the Transactions of the PGMTT (pp. 440-445).

Only the TM case (incident field having an electric field in the z-direction only) will give rise to nonreciprocal effects. For TE (E_x, H_z only), the time-varying magnetic field is in the direction of the applied d-c magnetic field, and therefore no nonreciprocal interaction occurs. In the following, a time dependence, e^{-j\omega t}, is to be understood.

For the ferrite rod with d-c biasing, the permeability is a tensor quantity:

$$\mu = \begin{pmatrix} \mu & +jK \\ -jK & \mu \end{pmatrix}$$

with \mu and K given by

$$\mu = \mu_o \left[ 1 + \frac{\omega_o \omega_m}{\omega_o^2 - \omega^2} \right]$$

$$K = \mu_o \frac{\omega_o \omega_m}{\omega_o^2 - \omega^2}$$

with \omega_o \omega_m = \omega_M and \omega_o \omega_H = \omega_0. H_o is the effective internal magnetic field. For a thin rod, magnetized along its axis, H_o = H_{app}. For cylindrical rods with a length-to-width ratio not much over one, H_o = H_{app} - N_z M_z, where H_{app} = applied field in the z-direction, N_z = demagnetization factor in the z-direction, and M_z = demagnetization in the z-direction.

Since our problem is two-dimensional, we can write (for a time de-
pendence $e^{-j\omega t}$):

$$ B = \frac{1}{j\omega} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \nabla E_z = \frac{1}{j\omega} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} E_z $$

In air:

$$ H = \frac{1}{j\omega} \mu_0^{-1} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \nabla E_z $$

In ferrite:

$$ H = \frac{1}{j\omega} \mu^{-1} \cdot \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \nabla E_z $$

The equation for the $H$ field in ferrite becomes:

$$ H = \frac{1}{j\omega} \mu^{-1} \begin{pmatrix} +jK & \mu \\ -\mu & -jK \end{pmatrix} \cdot \nabla E_z $$

which gives in cylindrical coordinates:

$$ H_r = \frac{-1}{\omega(\mu^2-K^2)} \left( K \frac{\partial E_z}{\partial r} + j\mu \frac{1}{r} \frac{\partial E_z}{\partial \phi} \right) $$

$$ H_\phi = \frac{-1}{\omega(\mu^2-K^2)} \left( j\mu \frac{\partial E_z}{\partial r} - K \frac{1}{r} \frac{\partial E_z}{\partial \phi} \right) $$

(11)

To solve the problem completely, we need expressions for the field quantifiers inside and outside the cylinder. $H_r$ and $H_\phi$ can be derived from $E_z$ for a TM wave according to the equations given above. Inside the cylinder, $E_z$ is given as the solution of the homogeneous wave equation in cylindrical coordinates; i.e.,

$$ E_z = \sum_{n=-\infty}^{\infty} C_n J_n(\beta r)e^{-jn\phi}. $$

Outside the cylinder, $E_z$ is given by the sum of the homogeneous wave solution (representing a wave traveling outward) and of the expansion of the incident traveling plane wave in cylindrical wave form; i.e.,

$$ E_z = E_z^{\text{Scat}} + E_z^{\text{inc}} = $$
\[ \sum_{n=-\infty}^{\infty} a_n H_n^{(1)}(\beta_0 r) e^{-j\omega \phi} + \sum_{n=-\infty}^{\infty} J_n(\beta_0 r) e^{-j\omega \phi}. \]

**Fields inside** \((r < a)\):

\[ E_z = \sum_{n=-\infty}^{\infty} b_n J_n(\beta r) e^{-jn\phi} \]

\[ H_r = \frac{-i}{\omega(\mu^2 - k^2)} \left[ K \beta \sum_{n=-\infty}^{\infty} b_n J_n(\beta r) e^{-jn\phi} - \frac{\mu}{r} \sum_{n=-\infty}^{\infty} b_n n J_n(\beta r) e^{-jn\phi} \right] \]

\[ H_\phi = \frac{-i}{\omega(\mu^2 - k^2)} \left[ -\mu \beta \sum_{n=-\infty}^{\infty} b_n J_n(\beta r) e^{-jn\phi} + K \frac{\mu}{r} \sum_{n=-\infty}^{\infty} b_n n J_n(\beta r) e^{-jn\phi} \right] \]

(12)

**Fields outside** \((r > a)\):

\[ E_z = \sum_{n=-\infty}^{\infty} a_n H_n(\beta_0 r) e^{-j\phi} + \sum_{n=-\infty}^{\infty} J_n(\beta_0 r) e^{-j\phi} \]

\[ H_r = \frac{1}{\omega \mu_0 r} \left[ \sum_{n=-\infty}^{\infty} a_n H_n^{(1)}(\beta_0 r) e^{-j\phi} + \sum_{n=-\infty}^{\infty} J_n(\beta_0 r) e^{-j\phi} \right] \]

\[ H_\phi = \frac{j \beta_0}{\omega \mu_0} \left[ \sum_{n=-\infty}^{\infty} a_n H_n^{(1)}(\beta_0 r) e^{-j\phi} + \sum_{n=-\infty}^{\infty} J_n(\beta_0 r) e^{-j\phi} \right] \]

(13)

\[ \beta^2 = \omega^2 \mu_0 \varepsilon_0 \]

\[ \beta^2 = \omega^2 \mu_{\text{eff}} \varepsilon = \omega^2 \frac{\mu^2 - k^2}{\mu} \varepsilon. \]

with:

The expansion coefficients \(a_n\) and \(b_n\) are obtained by matching the tangential \(H(= H_\phi)\), and the normal \(E(= E_z)\) at \(r = a\):
\[
a_n = -\left( \frac{j}{n} \right) \frac{J_n(\beta_0 a)D_n(\beta a) + J'_n(\beta_0 a)J_n(\beta a)}{J_n(\beta_0 a)D_n(\beta a) + J_n(\beta_0 a)J'_n(\beta a)}
\]

\[
b_n = \frac{j^n}{J_n(\beta_0 a)D_n(\beta a)H_n(\beta_0 a) + J_n(\beta a)H'_n(\beta_0 a)} \cdot \frac{1}{J_n(\beta a)}
\]

where the \( D_n \) is defined as

\[
D_n(\beta a) = \frac{\mu_0}{\mu_{\text{eff}}} \frac{\beta}{\beta_0} \left[ -J_n(\beta a) + \frac{K}{\mu} \frac{n}{\beta a} J_n(\beta a) \right]
\]

The \( a_n \)'s and \( b_n \)'s have to be evaluated for different \( \beta a, \mu, \) and \( K \) before any field quantities can be determined. A program now in preparation will provide the expansion coefficients for a range of different rod diameters and a range of different ferrite parameters \( \mu \) and \( K \). Then the amount of power flow through the cylinder can be evaluated, which will allow comparison of the power-gathering ability of a ferrite rod with that of a d-c biasing magnetic field.

A more detailed study of the fields inside and just outside the cylinder, involving some quite lengthy calculations, is planned to determine promising combinations of \( \beta a, \mu, \) and \( K \), i.e., for combinations where the energy densities inside the cylinder are considerably greater than what they would be in the absence of the cylinder.
2.4 THEORY OF LOOP ANTENNA

2.4.1 Analysis of the Shielded, Balanced-Loop Antenna.—The resonance condition for the shielded, balanced-loop antenna immersed in a ferrite medium has been analyzed. The method follows closely that given by Libby\(^3\) for a loop in air. The antenna, shown in Fig. 8, consists of a coaxial line bent into a loop. There is a small gap in the outer conductor at one point and a balanced feed directly across from the gap. As derived in Libby's paper, this configuration can be reduced to the equivalent circuit shown in Fig. 9. Basically, what has been done is that the outer shield has been transformed into the equivalent length of the two-wire transmission line shown in the left part of the figure. Assuming that the frequency is high enough, the fields inside the coaxial line are independent of the fields induced on the outer legs of the shield. This allows us to treat the inner coaxial line separately as an additional length of line, as shown at the center of the figure. The terms shown on the figure are:

\[
Z_{oo} = \text{characteristic impedance of the equivalent two-wire line replacing the outer shield}
\]

\[
Z_{oo} = \frac{\mu_{ro}}{\sqrt{\varepsilon_{ro}}} 276 \log \frac{D}{d}
\]

\(D = \text{diameter of the outer shield}
\]

\(d = \text{diameter of the outer shield wire}
\]

\[
Z_{oi} = \text{characteristic impedance of the inner coaxial line}
\]

\[
Z_{oi} = \frac{138}{\sqrt{\varepsilon_{ri}}} \log_{10} \frac{D'}{d'}
\]

\(D' = \text{inside diameter of the coaxial line: outer conductor}
\]

\(d' = \text{diameter of the coaxial line: inner conductor}
\]
Fig. 8. Shielded balanced-loop antenna.
\( \mu_{ro} = \) relative permeability of the medium surrounding the antenna

\( \varepsilon_{ro} = \) relative permeability of the medium surrounding the antenna

\( Z_{AH}, Z_{HD} = \) the terminating impedances

\( \varepsilon_{r1} = \) relative permittivity of the medium in the coaxial line.

---

**Fig. 9.** Equivalent circuit of shielded loop antenna.

The shielded loop receives energy through the induction, by the propagated field, of electromotive forces on the outside surface of the shield, along its legs, causing current to flow and thus producing voltage \( V_{EG} \) across shield gap.

Letting \( \theta_o \) be the electrical length of the equivalent two-wire line and \( \theta_1 \) be the electrical length of the coaxial line, the total length (\( \theta_t \)) of the transmission line can be found. \( Z_{AH} \) and \( Z_{HD} \) will be assumed open circuits for the following derivation.

Were \( Z_{oo} \) is equal to twice \( Z_{o1} \), \( \theta_t \) would be the sum of \( \theta_o \) and \( \theta_1 \). This is not true, so an equivalent electrical length \( \theta_{eq} \) of the outside transmission line with target to the inner line must be found. This is done by equating the impedance to the left of BC (Fig. 9) in terms of \( Z_{oo} \) to the same impedance in terms of twice \( Z_{o1} \). Then
\[ j Z_{oo} \tan \theta_o = 2j Z_{o1} \tan \theta_{eq} \]  

(15)

Solving for \( \theta_{eq} \),

\[ \theta_{eq} = \arctan \left( \frac{Z_{oo} \tan \theta_o}{2 Z_{o1}} \right) \]  

(16)

Then

\[ \theta_t = \theta_1 + \theta_{eq} = \theta_1 + \arctan \left( \frac{Z_{oo} \tan \theta_o}{2 Z_{o1}} \right) \]  

(17)

The frequency at which this transmission line network goes through resonance is found by setting \( \theta_t \) equal to 90° and solving for the wavelength, \( \lambda \).

From Eq. (3),

\[ \tan \theta_t = \tan \left[ \theta_1 + \arctan \left( \frac{Z_{oo} \tan \theta_o}{2 Z_{o1}} \right) \right] \]  

(18)

Using the identity

\[ \tan (A + B) = \frac{\tan A \tan B}{1 - \tan A \tan B} \]  

(19)

then

\[ \tan \theta_t = \frac{\tan \theta_1 + \frac{Z_{oo}}{2 Z_{o1}} \tan \theta_o}{1 - \frac{Z_{oo}}{2 Z_{o1}} \tan \theta_o \tan \theta_1} \]  

(20)

\[ \tan \theta_t - \tan \theta_t \left( \frac{Z_{oo}}{2 Z_{o1}} \tan \theta_o \tan \theta_1 \right) = \frac{Z_{oo}}{2 Z_{o1}} \tan \theta_o + \tan \theta_1 \]  

(21)

Dividing through by \( \tan \theta_t \) and letting \( \theta_t \) be 90°,

\[ 1 - \frac{Z_{oo}}{2 Z_{o1}} \tan \theta_o \tan \theta_1 = 0 \]  

(22)
\[
\tan \theta_0 \tan \theta_1 = \frac{2 Z_{oi}}{Z_{oo}}
\]  
(23)

where

\[\theta_1 = \beta_1 l\]

\[\beta_1 = \frac{360}{\lambda_1} = \frac{360f}{c} \sqrt{\mu_{r1}\varepsilon_{r1}} = \frac{360}{\lambda_0} \sqrt{\mu_{r1}\varepsilon_{r1}}\]

\[l = \frac{P}{2}, \text{ the length of coaxial line, the length of the two-wire line}\]

\[\Theta = \beta_0 l\]

\[\beta_0 = \frac{360}{\lambda_0} = \frac{360f}{c} \sqrt{\mu_{ro}\varepsilon_{ro}} = \frac{360}{\lambda} \sqrt{\mu_{ro}\varepsilon_{ro}}\]

\[\mu_{r1} = \text{relative permeability of the medium in the coaxial line}\]

\[\lambda = \text{free space wavelength of antenna}\]

\[\lambda_1 = \text{wavelength in coaxial line}\]

\[\lambda_0 = \text{wavelength in the two-wire line}\]

Equation (23) becomes

\[
\tan \left(\frac{360}{\lambda} \sqrt{\mu_{ro}\varepsilon_{ro}} \frac{P}{2}\right) \tan \left(\frac{360}{\lambda} \sqrt{\mu_{r1}\varepsilon_{r1}} \frac{P}{2}\right) = \frac{2 Z_{oi}}{Z_{oo}}
\]  
(24)

Equation (24) is the design equation for resonance. An illustrative design has been calculated as follows:

Assume:

\[Z_{oi} = 50 \text{ ohms}\]

\[\varepsilon_{ro} = 10\]

\[D = 4 \text{ in.}\]
\[ d = 0.142 \text{ in.} \]
\[ \frac{p}{2} = 6.28 \text{ in.} \]
\[ \varepsilon_{ri} = 2.1 \]
\[ \mu_{ri} = 1 \]
\[ \mu_{ro} = \text{frequency-dependent (to be determined)} \]

then

\[ \theta_1 = \frac{3275}{\lambda} \]
\[ \theta_0 = \frac{7150}{\lambda_0} \sqrt{\mu_{ro}} \]
\[ Z_{oo} = \sqrt{\mu_{ro}} \frac{276}{\sqrt{10}} \log \frac{1}{0.142} = \sqrt{\mu_{ro}} 26.5 \]

Equation (24) then becomes

\[ \tan \left( \frac{7150}{\lambda_0} \sqrt{\mu_{ro}} \right) \tan \frac{3275}{\lambda_0} = \frac{1790}{\sqrt{\mu_{ro}}} \quad (25) \]

As can be seen from the equation, the resonant wavelength depends on the value of \( \mu_{ro} \). This transcendental equation was solved graphically for \( \mu_{ro} \) vs. \( \lambda \). The wavelength is converted to frequency and plotted in Fig. 10, which shows, for a particular loop antenna, the value of \( \mu_{ro} \) needed to resonate at any frequency in the range 1 Mc to 200 Mc. It also reveals that a broadband antenna can be realized if the permeability varies with frequency according to the curve. In the given frequency range, the ferrite materials exhibit a dispersion characteristic similar to the curve. The dotted line in the figure exemplifies the frequency characteristic for an experimental ferrite. The major problem in constructing a model of this antenna is the high loss presently associated
Fig. 10. $\mu_r$ as a function of frequency needed to resonate the shielded-loop antenna of Fig. 8.
with ferrites of this character. Derivations of this type indicate the need for improved materials. For the design given above, a reasonable bandwidth would be from 5 Mc to 100 Mc (20 to 1 bandwidth), since values of $\mu_{ro}$ greater than 500 and less than .9 are unlikely in this frequency range.

A shielded loop similar to the one described above has been constructed. It will be tested, insofar as possible with available material, and the results will be compared with the theory.

2.4.2 Broadbanding Discussion.—The method of broadbanding described in Section 2.4.2 can also be applied to other antenna structures. The basis of the theory is that the wavelength in the medium either (1) remains constant over a given frequency range, or (2) changes more slowly than the frequency variation according to some calculated curve.

For (1), the wavelength is given by

$$\lambda = \frac{v}{f} = \frac{c}{f \sqrt{\mu_r \varepsilon_r}} \quad (26)$$

where:

- $v$ = velocity of propagation in the medium
- $f$ = frequency of operation
- $c$ = velocity of light in free space
- $\mu_r$ = relative permeability, frequency-dependent
- $\varepsilon_r$ = relative permittivity

For constant wavelength, we then need

$$f \sqrt{\mu_r \varepsilon_r} = \text{constant} \quad (27)$$
Among the antennas that could use this type of broadbanding are horns, slots, dipoles, and simple arrays.

An example of an antenna utilizing the second type of permeability characteristic is given in Section 2.4.1 on the shielded loop.

Although materials are not now available to test the theory properly, we will continue our efforts along this line to increase the bandwidth of antennas.

2.5 WAVEGUIDE RADIATORS

2.5.1 General Discussion.—Derivations of the characteristics of material-filled waveguide radiators have been undertaken. The same type of antenna filled with air, has high efficiency, is easy to construct, and has been built in a number of forms, some of which might benefit by using solid-state materials in their construction. It is expected that a practical antenna will evolve from these derivations.

One disadvantage of waveguide radiators at the frequencies of interest (around 150 Mc) is their large size. Filling the waveguide with a high \( \mu, \epsilon \) material will reduce the wavelength considerably and thus reduce the cutoff frequency and over-all size of the waveguide. As an example, consider the rectangular waveguide shown in Fig. 11. For the dominant TE\(_{10}\) mode, the cutoff wavelength is given by \( \lambda_c = 2a \). For a cutoff frequency of 100 Mc, the dimension "a" must be 59 in. If, however, the waveguide is filled with material of \( \mu = \epsilon = 10 \), the wavelength is reduced by a factor of 10 and thus "a" is reduced to 5.9 in.
Fig. 11. (a) Open-end rectangular waveguide radiator.
(b) Coordinate system for the representation of the far fields.
While the size advantage is readily apparent, changes in antenna characteristic such as gain, efficiency, bandwidth, etc., must be investigated using a detailed analysis. Of several derivations now in progress, one has been completed, and is presented in the next section.

2.5.2 Theoretical Results.—The antenna to be discussed in this section is the open-ended waveguide radiator shown in Fig. 11. The waveguide is assumed filled with a material of relative permeability $\mu_r$ and relative permittivity $\varepsilon_r$. The derivation for the radiated fields is similar to that of Silver for an air-filled waveguide. Huygens principle is used to replace the source by the fields in the aperture. To simplify the solution, higher-order modes and the current distribution over the exterior surface of the waveguide are neglected.

As shown by Silver, the radiated fields from an aperture are given by:

\[
\begin{align*}
E_R &= 0 \\
E_\Theta &= \frac{\jmath k e^{-\jmath kR}}{4\pi R} \left[ 1 + \frac{\beta mn}{\alpha n} \frac{(1-\Gamma)}{(1+\Gamma)} \sqrt{\frac{\mu_r}{\varepsilon_r}} \cos \Theta \right] (N_x \cos \phi + N_y \sin \phi) \\
E_\phi &= \frac{\jmath k e^{-\jmath kR}}{4\pi R} \left[ \cos \Theta + \frac{\beta mn (1-\Gamma)}{\alpha n (1+\Gamma)} \sqrt{\frac{\mu_r}{\varepsilon_r}} \right] (N_x \sin \phi - N_y \cos \phi)
\end{align*}
\]  

(28)

where

\[
N = \int_{\text{aperture}} E_t e^{\jmath k(x \cos \Theta \cos \phi + y \sin \Theta \sin \phi)} ds
\]

(29)

\[
= (1 + \Gamma) \int_{\text{aperture}} (E_t) e^{\jmath k(x \cos \Theta \cos \phi + y \sin \Theta \sin \phi)} ds
\]

and

\[
E_t = \text{resultant electric field of the dominant mode over the aperture}
\]
\((E_t)_i \Delta y = - \frac{j \omega \mu_0 \gamma}{K^2 a} \sin \frac{ma}{a} \cos \frac{ny}{b}\)

\((E_t)_i \Delta x = \frac{j \omega \mu_0 \gamma}{K^2 b} \cos \frac{ma}{a} \sin \frac{ny}{b}\)

where

\[K^2 = (\frac{ma}{a})^2 + (\frac{nb}{b})^2\]

From Eq. (29), \(N_x\) and \(N_y\) are computed

\[N_x = \frac{j n \mu_0 u (1 + \Gamma)}{K^2 b} \int_0^a \cos \frac{ma}{a} e^{(j k x \sin \theta \cos \phi)} dx \int_0^b \sin \frac{ny}{b} e^{(j k y \sin \theta \sin \phi)} dy\]

\[N_y = - \frac{j n \mu_0 u (1 + \Gamma)}{K^2 a} \int_0^a \sin \frac{ma}{a} e^{(j k x \sin \theta \cos \phi)} dx \int_0^b \cos \frac{ny}{b} e^{(j k y \sin \theta \sin \phi)} dy\]

Performing the integration, we obtain

\[N_x = \frac{n^2 \pi^2 \mu_0 (1 + \Gamma)}{K^2 b} k \sin \theta \cos \phi \left[ \frac{1 - e^{j(k x \sin \theta \cos \phi + ma)}}{K^2 \sin^2 \theta \cos^2 \phi - (\frac{ma}{a})^2} \right]\]

\[N_y = - \frac{n^2 \pi^2 \mu_0 (1 + \Gamma)}{K^2 a} k \sin \theta \sin \phi \left[ \frac{1 - e^{j(k x \sin \theta \cos \phi + ma)}}{K^2 \sin^2 \theta \sin^2 \phi - (\frac{ma}{a})^2} \right] - \frac{1 - e^{j(k x \sin \theta \sin \phi + nx)}}{K^2 \sin^2 \theta \sin^2 \phi - (\frac{mn}{b})^2}\]
By substituting into Eq. (29), we get for the far fields

\[ E_\Theta = \mu_r \sqrt{\frac{\mu_0}{\varepsilon_0}} \left( \frac{\pi ab}{2} \right) \sin \theta \left\{ 1 + \frac{\beta_{mn}}{\omega \mu} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos \theta + \Gamma \left( 1 - \frac{\beta_{mn}}{\omega \mu} \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos \theta \right) \right\} \]

\[ \left[ \left( \frac{m}{a} \right)^2 \sin^2 \phi - \left( \frac{n}{b} \right)^2 \cos^2 \phi \psi_{mn}(\theta, \phi) \right] \]

\[ E_\phi = \mu_r \sqrt{\frac{\mu_0}{\varepsilon_0}} \left( \frac{\pi ab}{2} \right) \frac{\sin \theta \sin \phi \cos \phi}{\lambda_0^2 R} \]

\[ \left[ \cos \theta + \frac{\beta_{mn}}{\omega \mu} \sqrt{\frac{\mu_0}{\varepsilon_0}} + \Gamma \left( \cos \theta - \frac{\beta_{mn}}{\omega \mu} \sqrt{\frac{\mu_0}{\varepsilon_0}} \right) \psi_{mn}(\theta, \phi) \right] \]

where

\[ \psi_{mn}(\theta, \phi) = \left[ \frac{\sin \left( \frac{\pi a}{\lambda_0} \sin \theta \cos \phi + \frac{\pi n}{2} \right)}{\left( \frac{\pi a}{\lambda_0} \sin \theta \cos \phi \right)^2 - \left( \frac{\pi n}{2} \right)^2} \right] \left[ \frac{\sin \left( \frac{\pi b}{\lambda_0} \sin \theta \sin \phi + \frac{\pi m}{2} \right)}{\left( \frac{\pi b}{\lambda_0} \sin \theta \sin \phi \right)^2 - \left( \frac{\pi m}{2} \right)^2} \right] 

\[ e^{-j[kR - \frac{\pi}{\lambda_0} \sin \theta (a \cos \phi + b \sin \phi) - (m + n + 1)\frac{\pi}{2}]} \]

Equations (33), (34), and (35) can be simplified when only the TE_{10} mode propagating in the waveguide. Letting \( m = 1, n = 0 \), we obtain

\[ E_\Theta = -\mu_r \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\pi ab}{2} \sin \phi \left\{ 1 + \frac{\beta_{10}}{\omega \mu} \sqrt{\mu_0 \varepsilon_0} \Gamma \left( 1 - \frac{\beta_{10}}{\omega \mu} \sqrt{\mu_0 \varepsilon_0} \right) \right\} \]

\[ \left[ \frac{\cos \left( \frac{\pi a}{\lambda_0} \sin \theta \cos \phi \right)}{\left( \frac{\pi a}{\lambda_0} \sin \theta \cos \phi \right)^2 - \left( \frac{\pi}{2} \right)^2} \right] \left[ \frac{\sin \left( \frac{\pi b}{\lambda_0} \sin \theta \sin \phi \right)}{\left( \frac{\pi b}{\lambda_0} \sin \theta \sin \phi \right)^2} \right] 

\[ e^{-j[kR - \frac{\pi}{\lambda_0} \sin \theta (a \cos \phi + b \sin \phi)]} \]
\[ E_\phi = -\mu_r \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{2a^2 b}{2\lambda_0^2 R} \cos \phi \left[ \cos \theta + \frac{\beta_{10}}{\omega \mu_r \sqrt{\mu_0 \varepsilon_0}} + \Gamma \left( \cos \theta - \frac{\beta_{10}}{\omega \mu_r \sqrt{\mu_0 \varepsilon_0}} \right) \right] \]

\[
\begin{bmatrix}
\cos \left( \frac{\pi a}{\lambda_0} \sin \theta \cos \phi \right) \\
\left( \frac{\pi a}{\lambda_0} \sin \theta \cos \phi \right)^2 - \left( \frac{\pi}{2} \right)^2 \\
\frac{\pi b}{\lambda_0} \sin \theta \sin \phi
\end{bmatrix}
\begin{bmatrix}
\sin \left( \frac{\pi b}{\lambda_0} \sin \theta \sin \phi \right) \\
\frac{\pi b}{\lambda_0} \sin \theta \sin \phi
\end{bmatrix}
\]

(37)

\[ e^{-j[kR - \frac{\pi}{\lambda_0} \sin \theta (a \cos \phi + b \sin \phi)]} \]

The phase factor, \( kR - \frac{\pi}{\lambda_0} \sin \theta (a \cos \phi + b \sin \phi) \), can be simplified for the far field by shifting the origin to the center of the aperture. The phase factor then becomes \( kR \) where \( R \) is now measured from the center of the aperture.

Since the electric field is polarized in the y-direction in the waveguide, the yz-plane is the E-plane of the system and the xz-plane is the H-plane.

The patterns are:

A. E-plane, \( \phi = \pi/2 \)

\[ E_\phi = 0 \]

\[ E_\theta = \mu_r \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{2a^2 b}{\pi \lambda_0^2 R} \left[ 1 + \frac{\beta_{10}}{\omega \mu_r \sqrt{\mu_0 \varepsilon_0}} \cos \theta + \Gamma \left( 1 - \frac{\beta_{10}}{\omega \mu_r \sqrt{\mu_0 \varepsilon_0}} \cos \theta \right) \right] \]

\[
\begin{bmatrix}
\sin \left( \frac{\pi b}{\lambda_0} \sin \theta \right) \\
\frac{\pi b}{\lambda_0} \sin \theta
\end{bmatrix}
\]

\[ e^{-j kR} \]
B. H-plane, $\phi = 0$

\[
E_\theta = 0
\]

\[
E_\phi = -\mu_r \frac{2 \mu_0 \mu \lambda_0^2 R}{\epsilon_0 2 \lambda_0^2 R} \left[ \cos \theta + \frac{\beta_{10}}{\omega \mu_r \sqrt{\mu_0 \epsilon_0}} + \Gamma \left( \cos \theta - \frac{\beta_{10}}{\omega \mu_r \sqrt{\mu_0 \epsilon_0}} \right) \right] e^{-jkR}
\]

\[
\frac{\cos \left( \frac{\lambda}{\lambda_0} \sin \theta \right)}{\left( \frac{\lambda}{\lambda_0} \sin \theta \right)^2 - \left( \frac{\beta_{10}}{\Gamma} \right)^2}
\]

(39)

The radiation patterns were calculated from Eqs. (38) and (39) using the IBM 704 computer. For this calculation $\Gamma$ was assumed to be zero. Referring to Fig. 11, the plot was made for the following conditions:

\[
a/b = 2
\]

\[
a/\lambda_0 = 0.625 / \sqrt{\mu_r \epsilon_r}
\]

\[
b/\lambda_0 = 0.312 / \sqrt{\mu_r \epsilon_r}
\]

\[
f/f_c = 1.25
\]

where:

- $f_c$ is the cutoff frequency in the waveguide,
- $f$ is the frequency of operation, and
- $\lambda_0$ is the free-space wavelength of operation.

The pattern is shown in Fig. 12 for several combinations of $\mu_r$, $\epsilon_r$ values. As can be seen from the graph, there is a small broadening of the pattern compared to the same antenna filled with air for $\mu_r$ equal to 3, $\epsilon_r$ equal to 3, and for $\mu_r = 10$, $\epsilon_r = 10$. The patterns are changed more radically for $\mu_r$ different from $\epsilon_r$. For $\mu_r \gg \epsilon_r$, the H-plane pattern is similar to the air case for $0 < \phi < 90^\circ$ and exhibits radiation in the reverse direction. For the E-
Fig. 12. Theoretical patterns for material-filled rectangular waveguide radiator.
plane, the pattern is practically omni-directional. For the case of $\varepsilon_r >> \mu_r$, the situation is reversed with a narrow beam in the E-plane and a broad beam in the H-plane.

An open-ended waveguide antenna is being constructed and will soon be tested.

2.6 MATERIALS

2.6.1 Powdered Ferrite. — Date were received for the powdered ferrite characteristics from the manufacturer. The measurements were taken for a sample in the solid form. A large difference characteristics exists between the material ordered and that received. Reproduction of the original material is a problem requiring additional time and funds. Data on the original sample and the new sample as measured by the manufacturer are given in Table I.

<table>
<thead>
<tr>
<th></th>
<th>$\mu'$</th>
<th>$\mu''$</th>
<th>$\mu'$</th>
<th>$\mu''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Sample</td>
<td>7.1</td>
<td>0.028</td>
<td>6.1</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>New Sample</td>
<td>10.5</td>
<td>2.1</td>
<td>8.0</td>
<td>3.6</td>
</tr>
</tbody>
</table>

The high losses associated with this material make use in experimental testing of the antennas of doubtful value.

Tests have been made in our laboratory of the properties of this ferrite in the powdered and in the solid form. Results are shown in Table II.
### TABLE II

RESULTS OF LABORATORY TESTS

<table>
<thead>
<tr>
<th>Sample</th>
<th>Measurement Method</th>
<th>Frequency</th>
<th>(\mu')</th>
<th>(\mu'')</th>
<th>(\varepsilon')</th>
<th>(\varepsilon'')</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>Resonant Cavity</td>
<td>260Mc</td>
<td>8.06</td>
<td>3.34</td>
<td>8.2</td>
<td>0.53</td>
</tr>
<tr>
<td>Solid</td>
<td>VSWR</td>
<td>260Mc</td>
<td>7.6</td>
<td>3.4</td>
<td>7.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Powder</td>
<td>Resonant Cavity</td>
<td>260Mc</td>
<td>3.27</td>
<td>.55</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

As was expected, the permeability \((\mu')\) of the powder is roughly half of the permeability of the solid, while the magnetic \(Q(\mu'/\mu'')\) of the powder is greater than the \(Q\) of the solid ferrite.

2.6.2 **Experimental Results on Material Measurements.**—Measurements of permeability and permittivity have been made using the Perturbation method described in the second Quarterly Progress Report. Results of measurements made on several materials are given in Table III.
### TABLE III
MEASURED VALUES OF PERMEABILITY AND PERMITTIVITY

<table>
<thead>
<tr>
<th>Material</th>
<th>$\mu'$</th>
<th>$\sigma_m$</th>
<th>$\varepsilon'$</th>
<th>$\sigma_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceramag 22 5A</td>
<td>14.45</td>
<td>2.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ceramag 22 5A</td>
<td>12.83</td>
<td>2.16</td>
<td></td>
<td></td>
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3. **ACTIVITIES FOR THE NEXT PERIOD**

(1) Experimental work on the ferrite-loaded biconical antenna, the shielded-loop antenna, and the waveguide radiator will be completed.

(2) The theoretical studies of plane-wave diffraction by a ferrite sphere and a ferrite cylinder will be extended to analyze the power flow near resonance.

(3) A theoretical study of plane-wave diffraction by a ferrite spheroid will be initiated.

(4) Theoretical results of radiation from sectoral horns will be analyzed on the computer.

(5) Experimental work on the ferrite-loaded spiral will be initiated.
4. SUMMARY

The theoretical study of plane-wave diffraction by a ferrite sphere has been extended to (1) evaluate the effect of higher modes, (2) map the lines of power flow, and (3) treat the case of an enclosed, perfectly conducting sphere.

The theoretical study of plane-wave diffraction by a ferrite cylinder has been extended to treat the longitudinally magnetized case.

An analysis of the resonance condition for the shielded, balanced-loop antenna immersed in a ferrite medium has been made. Conditions for maintenance of resonance over a broad frequency range have been obtained and found to be fairly consistent with the published properties of known ferrite materials. The analysis suggests the possibility of tailoring ferrite properties for broadband use in microwave components and antennas.

Radiation from a ferrite-filled rectangular waveguide has been analyzed. The results show that, for $\mu = \varepsilon$, the radiator size can be decreased with very little change in beam pattern.

Experimental results of the measurement of ferrite materials are given.
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