The Effect of Applied Vertical Force on Static Coefficient of Friction Measurements for Industrial Floors

by

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ABSTRACT

Slips and falls continue to account for a significant number of industrial accidents. Efforts to eliminate such mishaps have been directed toward reducing the slipperiness of floors. The concept of coefficient of friction has often been used to quantify the traction expected between a shoe and a floor surface. The objective of this research was to investigate the effects of vertical force on static coefficient of friction (SCOF) measures for different types of industrial floors. Tests were conducted using two common shoe sole materials, under both wet and dry conditions, on four different industrial floor surfaces. It was found that the SCOF changed as a function of the applied vertical force. Generally, the SCOF increased as the vertical force increased. This was not true, however, for smooth tile floors. Significant first order effects on the SCOF were found between vertical weight and shoe material and between shoe and condition. Possible reasons for these findings and ramifications on slip testing were presented.

INTRODUCTION

Despite substantial improvements in floor surfaces and shoe sole materials, injuries resulting from slips and falls continue to constitute about 23% of all compensable injuries (Liberty Mutual,1984). The U.S. National Safety Council estimates that there are 250,000 to 300,000 injuries from occupational falls per year, including 1200 to 1600 fatalities (Pater,1985).

Research and experience have shown that environmental controls are required to prevent slipping and falling in the workplace. One approach in the study and prevention of slips and falls is to investigate the force interactions between the shoe and the floor. The basic force requirement is that the frictional force producing capabilities of the shoe/floor interface must be greater than imposed capabilities describe the "slip resistance" of a shoe/floor interface. In order to measure the slip resistance of a shoe/floor interface, the concept of Coefficient of Friction (COF) has been widely used, both statically and dynamically (Redfern and Chaffin, 1986; Perkins and Wilson, 1983; Andres and Chaffin, 1985). The static COF (SCOF) is the most widely used measure and is defined as the ratio of the force required to move one surface over another to the total vertical force applied to the two surfaces. The SCOF is therefore defined as:

$$\text{SCOF} = \frac{\text{HORIZONTAL FORCE}}{\text{VERTICAL FORCE}}$$

The concept that sliding force and normal force are proportional is the basis for nearly all COF research. It assumes that contacting surfaces are ideally smooth and hard, a situation seldom found in industrial settings. It is this fundamental assumption that this study seeks to explore, in particular, as it relates to the shoe/floor interface. This study addresses the question, "Does the static coefficient of friction for shoe and floor surfaces vary as a function of the applied normal force for shoe and floor surfaces?" If the answer to this question is negative, then past research on floor COF's is assumed to be valid. However, if static COF is found to vary as a function of applied normal force, then any standard SCOF test procedure should specify the normal force to be used.

METHODS

The concept behind this study was to vary the vertical force acting between two surfaces and then measure the horizontal force required to initiate sliding motion. SCOF's were then calculated and analyzed.

Apparatus

The test device used in this study is commonly called BIGFOOT. It consists of a rigid sled and a force measurement gauge. A 10 cm. by 11.5 cm. sample of shoe sole test material is attached to the flat bottom of the sled by double-stick carpet tape. Various weights are placed on the top of the sled to create the desired normal forces. Four weights were used, resulting in normal forces of 2.27, 4.54, 12.21, and 24.06 kg. The sled was designed to distribute weight evenly across the sole sample. A small tow wire was attached to one end of the sled to facilitate one-direction horizontal pull. The force measurement devices that were used were a Chatillon force gauge (R-Cat 719-10) and Chatillon model DPP-25KG force gauge. The former instrument measures forces up to 4.5 kg and the latter measures forces up to 25 kg. Both gauges retain the "peak force" reading mechanically. Gauge operation involves holding the gauge housing and manually pulling horizontally; a hook on the end of the instrument transmits the force to the sled tow wire.

Test Surfaces

Two common shoe sole materials were used: 9 1/2 - 10 1/2 iron, fine grade natural leather, and a rubber and clay compound hereafter referred to as simply rubber. Both sole materials were cut into quarter-inch thick rectangles, 10 cm x 11.5 cm. For dry condition tests, the leather samples were used in the new condition. Prior to wet condition tests, the leather samples were soaked under water for at least 30 minutes. Rubber samples were prepared for both the wet and dry test conditions by wiping the test surface with 5MG3 fine grit emery cloth four times in orthogonal directions. Sample surfaces were wiped clean with a soft cloth to remove contaminant particles.

Four industrial type floor surfaces were selected. These included a waxed (but traveled) tile floor, a steel slip-resistant safety plate with a grit type surface, unfinished concrete floor (normal sidewalk surface), and freshly sealed concrete floor. Each floor surface was brushed clean of contaminants prior to testing and care was taken to place the sled at the same location for each trial on a given floor.

Floor contaminants can significantly affect the coefficient of friction experienced by a shoe contacting a floor (Chaffin and Andres, 1982). Rather than deal with the vast number of contaminants present on most working surfaces, only water was included in this study. Thus two surface conditions were used: dry and wet. In each case the floor surface was kept as free as possible of other contaminants. For wet surface tests, water was poured onto the floor material until a small puddle formed which had an area larger than the sled. This assured that the entire "shoe" surface area was wet when placed on the test floor.

Procedure

The shoe sole materials and floor surfaces were prepared as described above. After a sole sample was attached to the sled, one of four weights was set on the sled. The sled tow wire was attached to the force gauge and the sled was placed on the test floor surface. A horizontal force was then slowly applied via the force measurement device until translation occurred. The force required to initiate motion was then recorded. When conducting wet condition trials, the front of the sled was held off the floor surface and a small horizontal force applied. The sled was then eased onto the water puddle as the applied force was increased. This effectively reduced adhesion effects resulting from intersurface vacuums and shortened the contact time before translation. In all tests, care was taken to maintain a direction of pull parallel to the floor surface.

All tests for a given floor were performed as a set, with all dry condition trials being completed prior to wet condition tests. 2.27 kg and 4.54 kg weights were alternated by sets of two, i.e., two 2.27 kg trials, followed by two 4.54 kg trials, then two 2.27 kg trials, etc. After performing the tests for the 2.27 kg and 4.54 kg weights using both leather and rubber soles, the 12.21 kg and 24.06 kg weight tests were similarly conducted using the alternating protocol. Ten trials were performed for each combination of floor, shoe, wet/dry condition, and weight, yielding 640 data points.

RESULTS

Beginning with the full model, an Analysis of Variance (ANOVA) of the SCOF measurements was performed within each floor condition. Those factors which were statistically insignificant at the 95 percent confidence level were eliminated from the model. The final model which resulted was: SCOF = constant + shoe + condition + weight + (shoe x condition) + (shoe x weight). The primary factors and first order interactions in this model were significant at a level of confidence of $p<.01$. Table 1 shows the coefficients of multiple determination (r^2) by floor type for this model.

Table 1: Coefficients of multiple determination (r^2).

FLOOR	(r^2)
tile	.78
steel	.88
rough concrete	.95
sealed concrete	.93

Given the high r^2 values for the model, the variance accounted for by each factor was then determined. Table 2 shows the sum of squares for each factor in the model, along with the percent of the total sum of squares attributed to that effect.

Table 2: Sum of Squares for factor effects, by floor, for the model SCOF = constant + shoe + condition + weight + (shoe x condition) + (shoe x weight).

	Tile		Steel		Rough Concrete		Sealed Concrete	
FACTOR	SS	%	SS	%	SS	%	SS	%
shoe	1.888	42	1.783	26	1.074	23	0.683	31
cond.	1.285	29	0.114	2	1.492	32	0.015	1
weight	0.079	2	0.369	5	0.485	10	0.280	13
s x cond	0.051	1	3.439	51	1.379	29	1.021	47
s x wt	0.170	4	0.292	4	0.027	1	0.022	1
Error	0.991	22	0.803	12	0.255	5	0.165	7
TOTAL	4.464		6.800		4.712		2.186	

The (shoe x condition) interaction effect accounted for much of the variance for all of the floors except tile. The shoe main effect was also important. The relative importance of the other factors was not consistent between floors.

Intuitively, two important factors affecting the coefficient of friction between the floor and a shoe are types of floor surface and sole material. The bar graph in Figure 1 shows the overall mean SCOF's broken down by weight and floor (shoe materials and conditions are pooled). A consistent SCOF rank by floor type is evident for each weight level. The steel safety plate provides the highest slip resistance, followed by rough concrete, tile and sealed concrete, respectively.

Figures 2, 3, 4, and 5 present the mean SCOF values for each shoe-condition combination on each floor type. Comparing Figures 4 and 5, rubber consistently offered a higher slip resistance than leather for all four dry floors that were tested. The difference between shoe soles was less marked for wet surfaces however, as indicated by Figures 2 and 3. Rubber appears to offer better slip resistance on wet tile, but leather had a higher SCOF value on wet steel, especially when heavier weights were applied. Although the reader might expect the condition main effect (i.e., wet vs. dry) to be important and consistent, the

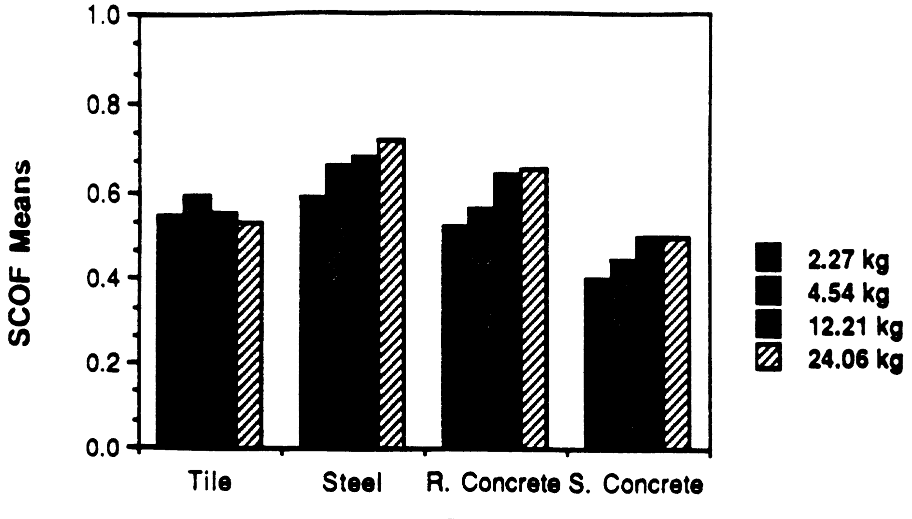

Figure 1: Pooled SCOF means for each floor by weight

relationship is not pure. A shoe x condition interaction effect accounted for more of the treatment mean differential than did the condition main effect for the steel and sealed concrete floors. (See Sum of Square values for (shoe x condition) and condition in Table 2).

As seen in Figures 2 - 5, rubber generally has a higher static coefficient of friction on dry surfaces than on wet, but that wet leather provides better traction than dry leather, except on smooth tile floors.

The results presented thus far are generally consistent with the findings of other researchers (Perkins & Wilson, 1983; Chaffin & Redfern, 1987; Redfern & Chaffin, 1986; James, 1983) The primary interest in this study is the significance of weight and (weight x shoe) factors, i.e. how the normal force interacts. Figures 2 - 5 show the mean effects of weight on static coefficient of friction for each shoe/condition combination. As the normal force is increased, the SCOF generally increases. The response on tile flooring is inconsistent with that on other floors; the SCOF either decreases or remains constant as weight is increased. To verify this result, Tukey pairwise comparisons were performed within each set of experimental parameters for each combination of weight treatment means. A family confidence level of 95 percent was used. The results are presented in Table 3.

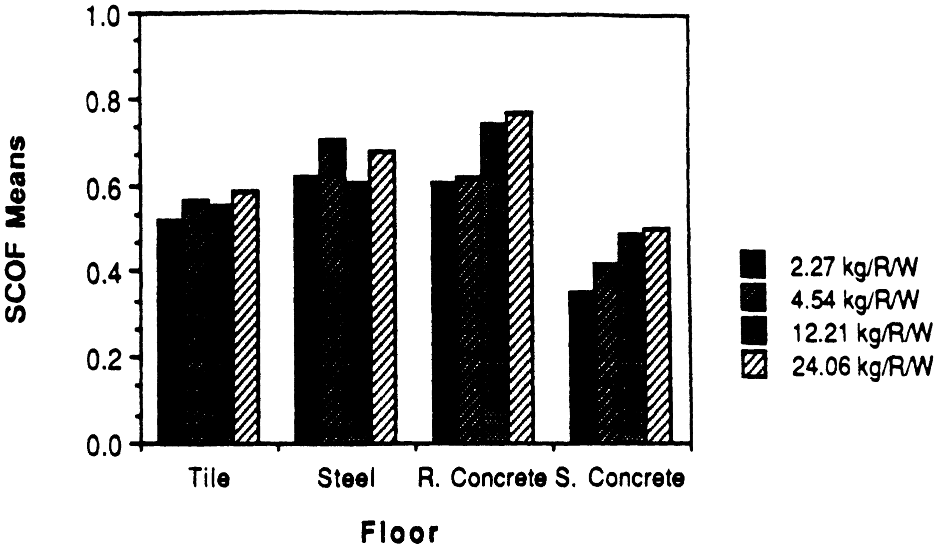

Figure 2: Mean SCOF values for Rubber/Wet

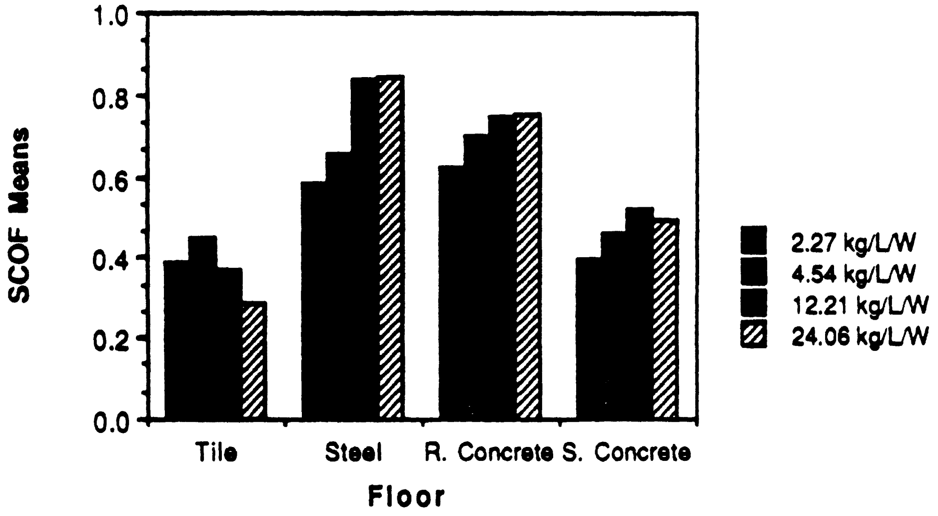

Figure 3: Mean SCOF values for Leather/Wet

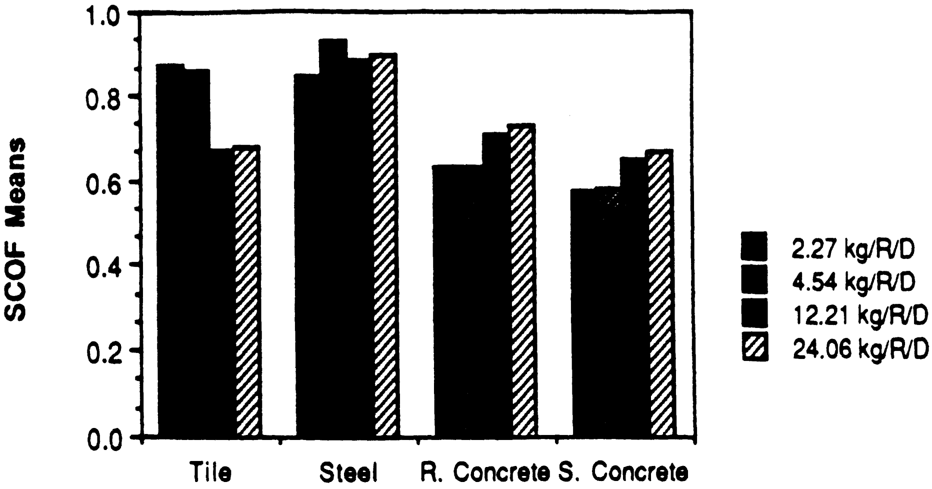

Figure 4: Mean SCOF values for Rubber/Dry

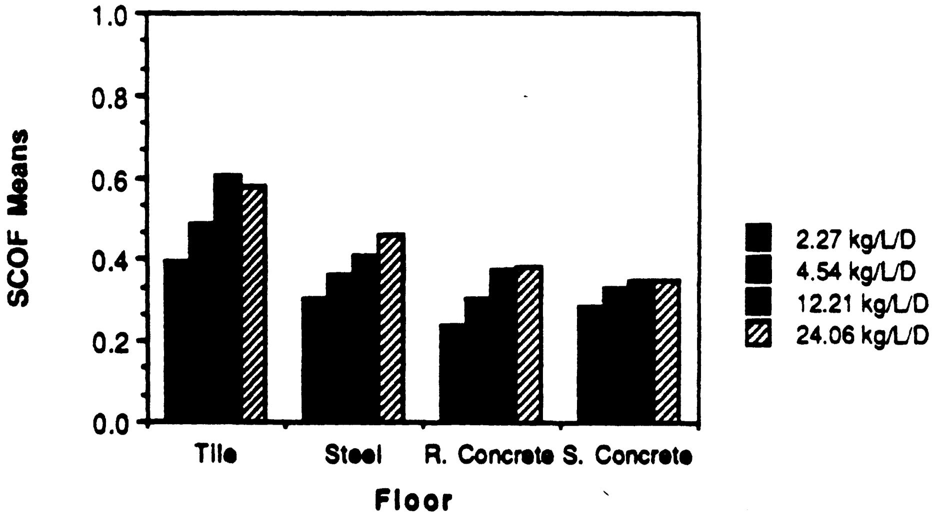

Figure 5: Mean SCOF values for Leather/Dry

Table 3: Significance of difference in SCOF weight treatment means as determined by Tukey pairwise comparisons with a 95 percent family confidence level.

Wt. Contrast (kg)	Tile	Steel	Rough Concrete	Sealed Concrete
4.54 - 2.27	S	S	S	S
12.21 - 2.27	NS	S	S	S
24.06 - 2.27	NS	S	S	S
12.21 - 4.54	S*	NS	S	S
24.06 - 4.54	S*	S	S	S
24.06 - 12.21	NS	S	NS	NS

S = significant NS = not significant

* = SCOF means decreased with increase in weight

Analysis of the Tukey contrast data indicates a significant difference in SCOF mean values as weight is increased for all floors except tile, where the results appear inconclusive. This difference in SCOF means was not significant at the heavier weights however, suggesting that the static coefficient of friction stabilizes at heavier weights (e.g. 12.21 kg of above) and may indeed be constant.

The standard deviation was calculated for the ten trial data points within each floor/shoe/condition/weight combination. These values are included in the Appendix, along with the ANOVA tables for the model and the SCOF means. Only one test set had a standard deviation greater than 0.100, with twenty-two of the sixty-four standard deviations being greater than 0.500. SCOF means and standard deviations were found to be consistent with those obtained in previous studies using similar floor surfaces and shoe sole materials (Chaffin and Andres,1982, Redfern and Bloswick,1987, and Miller, 1983).

DISCUSSION

This study found that applied normal force does affect static coefficient of friction measurements for different floors, shoes, and conditions. Specifically, as the applied weight increases the SCOF generally increases until a leveling off eventually occurs. This trend is not uniform however, since it was not observed on tile floor tests, suggesting that surface roughness may account, in part, for the phenomenon. A significant (weight x shoe) interaction effect was also found, suggesting that weight affects disparate shoes differently.

Consistent with previous research, the experimental data showed that SCOF was also a function of shoe sole material, surface conditions and contaminants, and the type of floor surface. A (shoe x condition) interaction effect also plays a role.

The observation that SCOF increases with applied normal force leads the researchers to believe that surface roughness and shoe material properties are the primary cause. It is suggested that a rough floor surface grabs the shoe at the shoe/floor interface, interfering with sliding action. A tearing action may be occurring, with the opposing forces no longer being comprised totally of shearing forces, but including tensile forces within the shoe material itself. Since shoe material penetration is a function of floor roughness and sole hardness, it is reasonable to assume that greater tearing forces are generated in the soft wet leather than in the hard dry rubber. The significance of the (shoe x weight) interaction supports this hypothesis. Heavier weights effect greater penetration for softer materials. The apparent inconsistent response on tile floor further supports this hypothesis; the smooth tile does not penetrate any of the shoe surfaces, consequently very little (shoe x condition) interaction effect occurs. The finding that SCOF either remains constant or decreases for tile as weight increases also supports this contention.

There may be other factors which also contribute to or account for the results of this study. Vacuum between the shoe/floor interface, deformation of the shoe material, and viscoelastic properties of shoe soles may impose some effect, but the tearing/internal tensile force hypothesis seems to offer the most likely explanation. Whatever the correct cause for the weight effect on SCOF measurements, the fact remains that SCOF and applied normal force are not independent for the shoe/floor interface.

This study suggests that care must be taken in selecting vertical force levels to use when conducting floor SCOF tests. These levels should be based on a biomechanical analysis of foot forces applied during activity. Since slips and subsequent falls often occur at heel strike, we suggest that the amount of applied forces occurring at this aspect of gait be used. Additional research is currently being directed toward determining such appropriate force levels for testing of most industrial activities. Once these force levels are determined, then meaningful SCOF testing procedures can be standardized for field use.

CONCLUSION

This study found that SCOF between shoe soles and floors does vary with vertical force, especially when those applied forces are relatively small. Generally, as weight increased, SCOF increased. In addition, an important weight/shoe interaction was found which depended on the floor condition. This indicates the need both for exercising care in selecting weights for SCOF testing and for additional research prior to standardization of test procedures for determining slip resistance of floor surfaces.

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APPENDIX

ANOVA Tables for SCOF

Model: SCOF = constant + shoe + condition + weight + (shoe x condition) + (weight x shoe) (N=160 for each floor)

Tile   Analysis of Variance $r=.882$ $r^2=.778$

SOURCE	SS	DF	MEAN-SQUARE	F-RATIO	P
shoe	1.888	1	1.888	285.837	0.000
condition	1.285	1	1.285	194.480	0.000
weight	0.079	3	0.026	4.007	0.009
shoe x cond	0.051	1	0.051	7.762	0.006
wt x shoe	0.170	3	0.057	8.588	0.000
ERROR	0.991	150	0.007		

Steel   Analysis of Variance $r=.939$ $r^2=.882$

SOURCE	SS	DF	MEAN-SQUARE	F-RATIO	P
shoe	1.783	1	1.783	333.125	0.000
condition	0.114	1	0.114	21.316	0.000
weight	0.369	3	0.123	22.998	0.000
shoe x cond	3.439	1	3.439	642.627	0.000
wt x shoe	0.292	3	0.097	18.176	0.000
ERROR	0.803	150	0.005		

Rough Concrete   Analysis of Variance $r=.973$ $r^2=.946$

SOURCE	SS	DF	MEAN-SQUARE	F-RATIO	P
shoe	1.074	1	1.074	630.333	0.000
condition	1.492	1	1.492	875.965	0.000
weight	0.485	3	0.162	95.017	0.000
shoe x cond	1.379	1	1.379	809.686	0.000
wt x shoe	0.027	3	0.009	5.283	0.002
ERROR	0.255	150	0.002		

Sealed Concrete   Analysis of Variance $r=.962$ $r^2=.925$

SOURCE	SS	DF	MEAN-SQUARE	F-RATIO	P
shoe	0.683	1	0.683	622.048	0.000
condition	0.015	1	0.015	13.314	0.000
weight	0.280	3	0.093	85.108	0.000
shoe x cond	1.021	1	1.021	930.040	0.000
wt x shoe	0.022	3	0.007	6.645	0.000
ERROR	0.165	150	0.001		

STATIC COEFFICIENT OF FRICTION DATA SUMMARY

			Normal forces (kg)					
Floor	Shoe	Dry/Wet	2.27	4.54	12.21	24.06	AVE.	STD.DEV.
Tile	Leather	Dry	0.399	0.487	0.606	0.575	0.517	0.081
Tile	Leather	Wet	0.388	0.451	0.370	0.283	0.373	0.060
Tile	Rubber	Dry	0.871	0.860	0.669	0.678	0.769	0.096
Tile	Rubber	Wet	0.518	0.562	0.551	0.586	0.554	0.025
Steel	Leather	Dry	0.305	0.363	0.838	0.460	0.492	0.207
Steel	Leather	Wet	0.587	0.655	0.838	0.847	0.732	0.114
Steel	Rubber	Dry	0.846	0.931	0.882	0.899	0.889	0.031
Steel	Rubber	Wet	0.617	0.700	0.607	0.673	0.649	0.039
R Con	Leather	Dry	0.243	0.304	0.375	0.380	0.326	0.056
R Con	Leather	Wet	0.622	0.700	0.745	0.751	0.704	0.052
R Con	Rubber	Dry	0.633	0.628	0.710	0.729	0.675	0.045
R Con	Rubber	Wet	0.601	0.618	0.742	0.769	0.682	0.074
Seal Con	Leather	Dry	0.283	0.332	0.348	0.349	0.328	0.027
Seal Con	Leather	Wet	0.398	0.463	0.520	0.491	0.468	0.045
Seal Con	Rubber	Dry	0.574	0.579	0.651	0.667	0.618	0.042
Seal Con	Rubber	Wet	0.352	0.416	0.485	0.503	0.439	0.060

Name: ____________________
S.N. #: ________-__-________

PROBLEM V - THE MONEY MARKETS

Congratulations! You just won $10,000 in the lottery. Instead of spending the money, you plan to invest it in one or more of the following three alternatives. These alternatives are independent. However, they are not necessarily mutually exclusive. The investment life of each alternative is 4 years.

Alternative A - Unites States Money Market

Interest Rates: Market research indicates that **nominal** annual interest rates during the next 4 years can be expressed in decimal form by the following function:

$$R(n) = \mathrm{Ln}(1.24 - 0.39\,n + 0.21\,n^2 - 0.03\,n^3), \quad n=1, 2, 3, \text{and } 4$$

Restrictions:

- You may invest any portion of your money in this alternative at the beginning of any year.
- You may withdraw any portion of your money from this alternative at the end of any year.

Alternative B - Japanese Money Market

Interest Rates: 10% effective annual interest rate during each of the next 4 years.

Restrictions:

- You may invest a maximum of $5,000 in this alternative at the beginning of both the first and second years.
- No investment is allowed in this alternative during the third or fourth year.
- Once money is invested in this alternative, it must remain invested until the end of year 4.

Alternative C - German Money Market

Interest Rates: 7% effective annual interest rate during each of the next 4 years.

Restrictions:

- You may invest in this alternative only one time.
- You may invest any portion of your money in this alternative at the beginning of any year.
- You may withdraw any portion of your money from this alternative at the end of any year.

Name: ________________
S.N. #: ______-___-___

20 POINTS) A. What is your investment strategy to maximize your future worth at the end of 4 years? In other words, in which alternative(s) should you invest? When? Why?

Ⓐ CONVERT NOMINAL INTEREST RATES TO EFFECTIVE INTEREST RATES.

$$i_e = e^R - 1 = e^{LN(1.24 - 0.39N + 0.21N^2 - 0.03N^3)} - 1$$

$$= 0.24 - 0.39N + 0.21N^2 - 0.03N^3$$

$$\Rightarrow i_{e_1} = \underline{3\%},\ i_{e_2} = \underline{6\%},\ i_{e_3} = \underline{15\%},\ i_{e_4} = \underline{12\%}$$

PROBLEM SUMMARY

	Ⓐ		Ⓑ		Ⓒ	
YR	MAX $ INVEST	i_e	MAX $ INVEST	i_e	MAX $ INVEST	i_e
1	10,000	3	5,000	10	10,000	7
2	10,000	6	5,000	10	10,000	7
3	10,000	15	10,000	10	10,000	7
4	10,000	12	10,000	10	10,000	7

NOTE:
THIS TABLE SHOWS THE MAXIMUM AMOUNT OF MONEY THAT CAN BE INVESTED IN THE ALTERNATIVES ON A YEAR BY YEAR BASIS. IT ALSO SHOWS THE ASSOCIATED EFFECTIVE INTEREST RATES. COMBINATIONS OF ALTERNATIVES ARE SUBJECT TO RESTRICTIONS

TWO ALTERNATIVE(S) DOMINATE:

- Ⓐ & Ⓒ
  - Ⓒ FOR YEARS 1 & 2
  - Ⓐ FOR YEARS 3 & 4

$$i_1 = \underline{7\%},\ i_2 = \underline{7\%},\ i_3 = \underline{15\%},\ i_4 = \underline{12\%}$$

$$F = P(1.07)(1.07)(1.15)(1.12) = \underline{1.4746P}$$

- Ⓑ & Ⓒ
  - 5,000 IN Ⓑ AND 5,000 IN Ⓒ FOR YEAR 1 (5,000 IN Ⓑ MUST REMAIN IN Ⓑ THROUGH YEAR 4)
  - REINVEST 5,000 (FROM Ⓒ) IN Ⓑ FOR YEAR ② (5,000 IN Ⓑ MUST REMAIN IN Ⓑ THROUGH YEAR 4)

$$F = P\left(\tfrac{1}{2}(1.07) + \tfrac{1}{2}(1.10)\right)(1.10)^3 = \underline{1.4441P}$$

$\underline{1.4746P > 1.4441P}$ CHOOSE Ⓐ & Ⓒ COMBINATION

INVEST IN GERMAN MONEY MARKET Ⓒ FOR YEARS 1 & 2, THEN INVEST IN U.S. MONEY MARKET Ⓐ FOR YEARS 3 & 4.

Name: ____________________
S.N. #: ______-___-____

Points) B. Using your investment strategy, how much money will you have at the end of 4 years?

$F = 1.4746\,P = 1.4746\,(10{,}000) = \$\underline{14{,}746}$

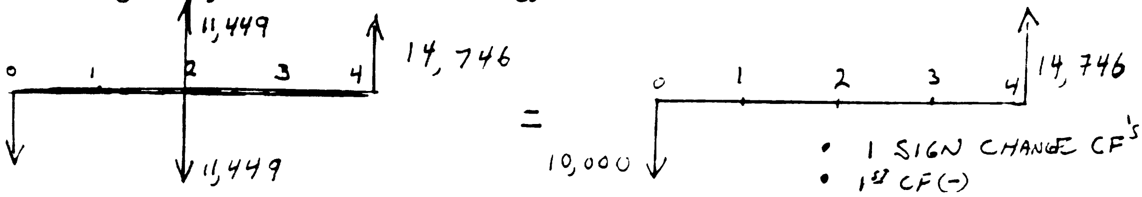
Answer: $ 14,746

Points) C. What is the internal rate of return of your investment strategy?

$(1+i^*)^4 = 1.4746 \Rightarrow i^* = \sqrt[4]{1.4746} - 1 = \underline{10.20\%}$

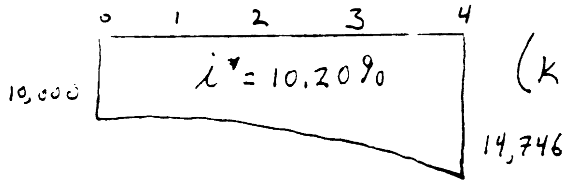
Answer: 10.20 %

Points) D. Categorize your investment strategy:



- 1 SIGN CHANGE CF's ⇒ SIMPLE
- 1st CF (−) THEREFORE PURE
- ΣCF (+)
- K DOES NOT CHANGR SIGN ⇒ [illegible]

Answer: Simple ✓ Non Simple ______
Pure ✓ Mixed/Pseudo ______ Complex ______ UN[illegible]T



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II (C) ii) ALTERNATE SOLUTION

NOTE: SINCE THE LAST PAYMENT PAYS OFF THE LOAN, THE FW = 0.

$$FW = 0 = \left(\text{PRINCIPLE} + \text{PRINCIPLE}(\overset{0.0153}{i})\right) - 1548.^{\underline{25}}$$

$$\Rightarrow \text{PRINCIPLE} = \frac{1548.^{\underline{25}}}{1.0153} = \$1524.^{\underline{90}}$$

$$\Rightarrow \text{INTEREST} = 1548.^{\underline{25}} - 1524.^{\underline{90}} = \underline{\$23.^{\underline{35}}}$$

III (A) ALTERNATE SOLUTION

NOTE: TEST ACCEPTABILITY OF EACH ALTERNATIVE $\Rightarrow i^* > \text{MARR}$

FIND PW @ $i^* = 0$ (ΣCF's)

$$PW_A = -1000 + 500 + 100 + 300 + 200 + 100 = \underline{\$\ 200}$$

$$PW_B = -4000 + 1150 + 750 + 950 + 850 + 1750 = \underline{\$\ 1450}$$

$$PW_C = -3000 + 1090 + 690 + 890 + 790 + 690 = \underline{\$\ 1150}$$

$$PW_D = -2000 + 500 + 100 + 300 + 200 + 1750 = \underline{\$\ 850}$$

FIND PW @ MARR = 12%

$$PW_A = -1000 + 500(P/F,12,1) + 100(P/F,12,2) + 300(P/F,12,3) + 200(P/F,12,4) + 100(P/F,12,5) = \underline{\$-76.4}$$

$$PW_B = -4000 + 1150(\text{"}) + 750(\text{"}) + 950(\text{"}) + 850(\text{"}) + 1750(\text{"}) = \underline{\$-165.}$$

$$PW_C = -3000 + 1090(\text{"}) + 690(\text{"}) + 890(\text{"}) + 790(\text{"}) + 690(\text{"}) = \underline{\$+50.}$$

$$PW_D = -2000 + 500(\text{"}) + 100(\text{"}) + 300(\text{"}) + 200(\text{"}) + 1750(\text{"}) = \underline{\$-140.}$$

$\Rightarrow$ FOR Ⓐ, Ⓑ, & Ⓓ $0 < i^* < 12$ (PW CHANGES SIGN BETWEEN $i^* = 0$, $i^* = 12$)

TRY PW @ $i^* = 15\%$ FOR Ⓒ

$$\Rightarrow PW_C = -3000 + 1090(P/F,15,1) + 690(P/F,15,2) + 890(P/F,15,3) + 790(P/F,15,4) + 690(P/F,15,5) = \underline{\$-150.^{\underline{51}}}$$

FOR Ⓒ, $12 < i^* < 15$

FOR Ⓐ, Ⓑ, Ⓓ : $i^* < \text{MARR}$ NOT ACCEPTABLE

FOR Ⓒ : $i^* > \text{MARR}$ ACCEPTABLE

PICK Ⓒ, ONLY ACCEPTABLE ALTERNATIVE