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Report of Project **BAMIRAC**

THE EFFECT OF TRANSPORT PROPERTIES ON SUPERSONIC EXPANSION AROUND A CORNER

THOMAS C. ADAMSON, JR.

May 1966

INFRARED PHYSICS LABORATORY

Willow Run Laboratories

THE INSTITUTE OF SCIENCE AND TECHNOLOGY

UNIVERSITY OF MICHIGAN • 1817 •

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ABSTRACT

The compressible flow of a viscous heat-conducting gas around a corner is considered. The solutions are written in terms of asymptotic expansions, valid in the region far, compared to a viscous length, from the corner, so that the zeroth-order solutions are the classical Prandtl-Meyer solutions. The method of inner and outer expansions is used where the inner region encloses the first Mach line emanating from the corner. Boundary-layer effects are minimized to the extent that the initial flow is assumed to be uniform. The first corrections due to viscosity and heat conduction are calculated, and it is shown that the tangential velocity component can be calculated only to within an unknown function of the turning angle, which must be found by matching with a still-unknown boundary-layer solution. Although the text deals with the case where the initial flow is sonic, corresponding calculations for the case where the initial velocity is supersonic are given in an appendix.

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FIGURES

THE EFFECT OF TRANSPORT PROPERTIES ON SUPERSONIC EXPANSION AROUND A CORNER

1
INTRODUCTION

The Prandtl-Meyer solution [1] for the steady supersonic flow of a compressible inviscid gas around a corner is well known and has been used extensively. In this solution, viscous and heat-transfer effects are ignored; since there is then no physical characteristic length associated with the problem, the flow variables are independent of the radial length, depending only on the angle of turning. Although it is clear that such assumptions are justified for a large class of problems, the extent of the approximation involved by the use of the inviscid equation is not known. More precisely, in terms of an approximate solution the zeroth-order term of which is the Prandtl-Meyer solution, the magnitude and functional form of the first-order terms due to the inclusion of transport effects are unknown.

The general problem of the supersonic flow of a compressible viscous heat-conducting gas around a corner should include the effects of the wall before, at, and beyond the corner, unless the wall ends at the corner, in which case a mixing region exists downstream of the endpoint (nozzle problem). Since it is impossible to obtain a solution for the overall problem, and thus it is necessary to consider various regions of interest separately, a boundary-layer solution should be found which can be joined with the solution of the expanding flow outside the boundary layer. The solution for the compressible boundary layer under the conditions involved in turning a sharp corner is not known; the problem appears to be most difficult. However, it is possible to study the effects of the transport properties on the expansion wave alone, and it is this problem which is considered in this paper. Since the complete boundary layer solution is unknown, the effect of the boundary layer is minimized by assuming that the flow upstream of the expansion wave is uniform, although a brief discussion is given of the manner in which upstream boundary-layer terms could be included. Although it might be argued that the solutions obtained hold for the physically realizable problem of an upstream flow with boundary-layer suction, such arguments are specious; the solution which follows is simply one part of the complete solution to the flow of a gas around a corner. In order to emphasize and clarify the important aspects of the expanding flow, boundary-layer effects are minimized as much as is possible.

The solutions for the various flow variables are found in the form of expansions, with the zeroth-order term being the Prandtl-Meyer solution; the subsequent terms introduce the corrections due to viscosity and heat conduction as well as the radial distance. The problem is obviously a singular perturbation problem; the technique employed in the solution is the so-called method of inner and outer expansions developed by Kaplum, Lagerstrom, and Cole [2, 3, 4] and described in detail by Van Dyke [5]. Thus, the expansion wave is pictured as consisting of two separate regions; the first consists of a relatively thin region which contains the first Mach line and the second consists of the remainder of the expansion-wave region. Hereafter, the first will be referred to as the inner region and the second as the outer region. In the inner region the first effects of viscosity are felt in smoothing the discontinuity in second derivatives given by the Prandtl-Meyer solution. In the outer region, where the first-order effects of viscosity and heat conduction are less pronounced, the expansions are quite different. The expansions are matched asymptotically in the overlap region common to both the inner and outer regions.

It should be noted that there is a fundamental difference between the Prandtl-Meyer problem where the initial flow velocity is sonic and the problem where the initial flow is supersonic. In the former, which is the problem studied here, the above-mentioned discontinuities appear in the second derivatives; in the latter, they appear in the first derivatives (app. II).

2
FORMULATION OF THE PROBLEM

2.1. FLOW DESCRIPTION

Consider the steady flow of a compressible viscous heat-conducting gas around a corner. In direct analogy to the Prandtl-Meyer problem, the flow upstream of the expansion wave is assumed to be sonic; a discussion of the case where the initial flow Mach number is supersonic is given in appendix II. In addition, the flow upstream of the expansion wave is assumed to be uniform, with conditions denoted by the subscript 1 in the dimensional variables. In the absence of any boundary-layer effects, the characteristic length is the viscous length $\bar{\nu}_1/\bar{a}_1$, where ν and a are the kinematic viscosity and sound speed respectively, and the bar indicates a dimensional quantity. In this problem, then, a solution is sought for the flow far (compared to the viscous length) from the corner. Thus, $\bar{r}/(\bar{\nu}_1/\bar{a}_1) \gg 1$. Since it is somewhat simpler to work with parameter rather than coordinate expansions, and since this solution must ultimately be joined with a boundary-layer region, a length, L, is introduced to make the coordinates dimensionless. For the present problem, L is a fictitious length which scales the independent variables such that the dimensionless distance from the corner to a point in question is of order unity, and $L/(\bar{\nu}_1/\bar{a}_1) \gg 1$. All flow variables and fluid properties are made dimensionless with respect to their values in the original undisturbed flow, with the exception of the total enthalpy; thus,

$$P = \frac{\overline{P}}{\overline{P}_1}, \quad \rho = \frac{\overline{\rho}}{\overline{\rho}_1}, \quad T = \frac{\overline{T}}{\overline{T}_1}, \quad h_t = \frac{\overline{h}_t}{\overline{a}_1^2}, \quad \vec{q} = \frac{\vec{\overline{q}}}{\overline{a}_1}, \quad \vec{X} = \frac{\vec{\overline{X}}}{L} \tag{1}$$

where the symbols stand for pressure, density, temperature, total enthalpy, velocity vector, and position vector, in the order given. Figure 1 is a sketch of the flow field and the coordinate systems employed. Note that both a Cartesian and a polar coordinate system are indicated, with velocity components U, V, and u, v, respectively. The equations relating the velocity components of each coordinate system are simply

$$\begin{aligned} u &= U \sin \phi + V \cos \phi \\ v &= U \cos \phi - V \sin \phi \end{aligned} \tag{2}$$

In general, it is assumed that the specific heats and the Prandtl number are constant. Bulk viscosity is neglected, and the coefficients of viscosity and thermal conductivity are related to the temperature by a simple power law. Finally, the gas is assumed to follow the perfect gas law.

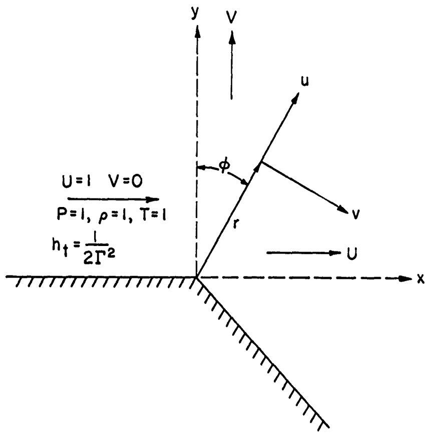

FIGURE 1. COORDINATE SYSTEMS AND BOUNDARY CONDITIONS

2.2. DIMENSIONLESS EQUATIONS

The equations expressing the conservation of mass, momentum, and energy are written below in polar coordinates. A similar set of equations written in Cartesian coordinates (app. I) is necessary for the inner region, but since they are relatively familiar they will not be reproduced here. In the following, subscripts r and ϕ indicate the partial derivative taken with respect to the indicated independent variable. The conservation equations, the equation of state, and the relation defining the total enthalpy are, then,

$$
\begin{aligned}
&\text{(a)} && (\rho u r)_r + (\rho v)_\phi = 0 \\
&\text{(b)} && \rho u u_r + \frac{\rho v}{r} u_\phi - \frac{\rho v^2}{r} + \frac{1}{\gamma} P_r = \frac{1}{Re} \mathbb{R}_u \\
&\text{(c)} && \rho u v_r + \frac{\rho v}{r} v_\phi + \frac{\rho u v}{r} + \frac{1}{\gamma r} P_\phi = \frac{1}{Re} \mathbb{R}_v \\
&\text{(d)} && \rho u (h_t)_r + \frac{\rho v}{r} (h_t)_\phi = \frac{1}{Re} \mathbb{R}_h \\
&\text{(e)} && P = \rho T \\
&\text{(f)} && h_t = \beta T + \frac{u^2}{2} + \frac{v^2}{2}
\end{aligned} \tag{3}
$$

where the various $\mathbb{R}$ functions, which represent the effects of viscosity and heat conduction are given by the following equations:

$$
\begin{aligned}
&\text{(a)} && \mathbb{R}_u = \left\{\mu\left[\frac{4}{3}u_r - \frac{2}{3r}(v_\phi + u)\right]\right\}_r + \frac{1}{r}\left\{\mu\left[r\left(\frac{v}{r}\right)_r + \frac{1}{r}u_\phi\right]\right\}_\phi + \frac{2\mu}{r}\left[r\left(\frac{u}{r}\right)_r - \frac{1}{r}v_\phi\right] \\
&\text{(b)} && \mathbb{R}_v = \left\{\mu\left[r\left(\frac{v}{r}\right)_r + \frac{1}{r}u_\phi\right]\right\}_r + \frac{1}{r}\left\{\mu\left[\frac{4}{3r}(v_\phi + u) - \frac{2}{3}u_r\right]\right\}_\phi + \frac{2}{r}\mu\left[r\left(\frac{v}{r}\right)_r + \frac{1}{r}u_\phi\right] \\
&\text{(c)} && \mathbb{R}_h = \frac{\beta}{Pr}\left[\frac{1}{r}(r\lambda T_r)_r + \frac{1}{r^2}(\lambda T_\phi)_\phi\right] + \frac{1}{r}\left(r\mu\left\{u\left[\frac{4}{3}u_r - \frac{2}{3r}(v_\phi + u)\right] + v\left[r\left(\frac{v}{r}\right)_r + \frac{1}{r}u_\phi\right]\right\}\right)_r \\
& && \qquad + \frac{1}{r}\left(\mu\left\{u\left[r\left(\frac{v}{r}\right)_r + \frac{1}{r}u_\phi\right] + v\left[\frac{4}{3r}(v_\phi + u) - \frac{2}{3}u_r\right]\right\}\right)_\phi
\end{aligned} \tag{4}
$$

In the above equations, μ and λ are the dimensionless coefficients of viscosity and thermal conductivity, respectively, $Pr = C_p\mu/\lambda$ is the Prandtl number, $Re = L\bar{a}_1/\bar{\nu}_1$ is the Reynolds number, and $\beta = (\gamma - 1)^{-1}$ where $\gamma = C_p/C_v$ is the ratio of specific heats.

2.3. STRETCHING FACTOR FOR INNER REGION

The inner region has been defined as that region where the effects of viscosity are first felt in smoothing the discontinuity in the second derivatives of the Prandtl-Meyer solution for the velocity. This region contains the first Mach line, and its thickness is small compared to the scale length, L. As is usual in this method of solution, the inner independent variable measuring the thickness is stretched in order to make it of order unity. It is not immediately apparent, in any given problem, as to what the stretching factor should be. In the present problem, it is possible to infer the proper form for this stretching factor from the equations and the zeroth-order matching condition.

First, the momentum equation in the x direction is written in Cartesian coordinates. Thus,

$$\rho U U_x + \rho V U_y + \frac{1}{\gamma} P_x = \frac{1}{Re}\left\{\left[\mu\left(\frac{4}{3}U_x - \frac{2}{3}V_y\right)\right]_x + \left[\mu\left(U_y + V_x\right)\right]_y\right\} \tag{5}$$

where subscripts x and y indicate partial differentiation. It is clear that the solutions in the inner region must join the values of the flow variables in the uniform stream to their corresponding outer values found from the Prandtl-Meyer solution for vanishing turning angle. Now, the derivatives in (5) may be ordered by considering the total change in the functions across the inner region, so that the equation effectively is used as a jump condition across the inner region. Then any excess momentum flux or pressure forces (terms on left-hand side of equation 5) must be balanced by viscous forces (right-hand side). That is, the existence of the differences found in evaluating the dependent variables on either side of the inner region is due to the effects of viscosity and heat conduction.

The Prandtl-Meyer solutions are as follows:

$$\begin{aligned} u_o &= \frac{\sin \Gamma\phi}{\Gamma} & v_o &= \cos \Gamma\phi \\ \rho_o &= v_o^{2\beta} & P_o &= v_o^{2\beta+2} \\ T &= v_o^2 & h_t &= \frac{1}{2\Gamma^2} \end{aligned} \tag{6}$$

where $\Gamma^2 = \left(\frac{\gamma - 1}{\gamma + 1}\right)$.

If u_o and v_o are substituted in (2), U and V may be found in terms of ϕ. These two equations and the thermodynamic functions given in (6) may then be expanded for small ϕ. Finally, since for ϕ small compared to one, ϕ is approximately equal to x/y, the following expansions result for x/y << 1:

$$U \sim 1 + \left(\frac{1 - \Gamma^2}{2}\right)\left(\frac{x}{y}\right)^2 + \ldots$$

$$V \sim -\frac{1}{3}(1 - \Gamma^2)\left(\frac{x}{y}\right)^3 + \ldots$$

$$P \sim 1 - \left(\frac{1 + \Gamma^2}{2}\right)\left(\frac{x}{y}\right)^2 + \ldots \tag{7}$$

$$\rho \sim 1 - \left(\frac{1 - \Gamma^2}{2}\right)\left(\frac{x}{y}\right)^2 + \ldots$$

$$\frac{P}{\gamma} + \rho U^2 \sim \frac{\gamma + 1}{\gamma} + O\left[\left(\frac{x}{y}\right)^4\right]$$

Hence, if x is ordered by some function of the Reynolds number, say δ (Re), and since y is of order unity because the region considered is far from the boundary layer.

$$\frac{x}{y} = O(\delta)$$

Then, for example, the terms on the left-hand side of (5) may be ordered as follows, if $\Delta U = U(x, y) - U(-\infty, y)$, $\Delta x = O(\delta)$, $\Delta Y = O(1)$, etc.,

$$\rho U U_x = O(\delta)$$

$$\rho V U_y = O(\delta^5)$$

If the order of each term of (5) is found in the same manner, then it appears that $\delta = Re^{-1}$ is the proper order for x, if the largest viscous term is to balance the largest excess momentum flux term. However if the continuity equation is used to rearrange the left-hand side of (5), it may be rewritten in terms of the stream force in the x direction; i.e.,

$$\rho U U_x + \rho V U_y + \frac{1}{\gamma} P_x = \left(\frac{P}{\gamma} + \rho U^2\right)_x - U(\rho U)_x - \rho V U_y \tag{8}$$

This grouping is suggested by the small variation in stream force indicated by the last of (7). Thus

$$\left(\frac{P}{\gamma} + \rho U^2\right)_x = O(\delta^3)$$

When the grouping of terms indicated in (8) is employed, the largest terms are of order δ^3, and hence, from (5) again

$$\delta = Re^{-1/3} \tag{9}$$

That is, since $U(\rho U)_x = O(\delta\ )$, and $\rho V U_y = O(\delta^5)$, the momentum equation indicates that the viscous forces balance the first two terms on the right-hand side of (8). Since this combination of terms results in a change of smaller order, it is clear that the proper value for δ is that given in (9).

The fact that the thickness of the inner region is of order $Re^{-1/3}$ is notable in that the thickness of a weak discontinuity generally is considered to be of order $Re^{-1/2}$ (e.g., [6]). This difference arises because in the present problem the velocity of the gas entering the discontinuity is sonic. This is evidenced by the fact that with the δ given in (9), the relative orders of ΔU, ΔV, Δx, etc., are consistent with those found in investigations of transonic similarity [7]. When the initial velocity is supersonic, it can be shown that the thickness is of order $Re^{-1/2}$ as expected (6). For this problem, then, the stretched inner variables are defined as follows:

$$\begin{aligned} \tilde{x} &= Re^{1/3} x = x/\delta \\ \tilde{y} &= y \end{aligned} \tag{10}$$

so that in terms of the stretched coordinates, both x and y are of order unity in the inner region. Finally, the governing inner equations may be written in terms of the stretched variables, in Cartesian form:

$$\begin{aligned}
&\text{(a)} \quad (\rho U)_{\tilde{x}} + \delta(\rho V)_{\tilde{y}} = 0 \\
&\text{(b)} \quad \rho U U_{\tilde{x}} + \delta \rho V U_{\tilde{y}} + \frac{1}{\gamma} P_{\tilde{x}} = \frac{\delta}{Re} \mathbb{R}_U \\
&\text{(c)} \quad \frac{1}{\delta} \rho U V_{\tilde{x}} + \rho V V_{\tilde{y}} + \frac{1}{\gamma} P_{\tilde{y}} = \frac{1}{Re} \mathbb{R}_V \\
&\text{(d)} \quad \rho U (h_t)_{\tilde{x}} + \delta \rho V (h_t)_{\tilde{y}} = \frac{\delta}{Re} \mathbb{R}_H \\
&\text{(e)} \quad P = \rho T \\
&\text{(f)} \quad h_t = \beta T + \frac{U^2}{2} + \frac{V^2}{2}
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
&\text{(a)} \quad \mathbb{R}_U = \frac{1}{\delta}\left[\mu\left(\frac{4}{3\delta} U_{\tilde{x}} - \frac{2}{3} V_{\tilde{y}}\right)\right]_{\tilde{x}} + \left[\mu\left(U_{\tilde{y}} + \frac{1}{\delta} V_{\tilde{x}}\right)\right]_{\tilde{y}} \\
&\text{(b)} \quad \mathbb{R}_V = \frac{1}{\delta}\left[\mu\left(U_{\tilde{y}} + \frac{1}{\delta} V_{\tilde{x}}\right)\right]_{\tilde{x}} + \left[\mu\left(\frac{4}{3} V_{\tilde{y}} - \frac{2}{3\delta} U_{\tilde{x}}\right)\right]_{\tilde{y}} \\
&\text{(c)} \quad \mathbb{R}_H = \frac{\beta}{Pr}\left[\frac{1}{\delta^2} (\lambda T_{\tilde{x}})_{\tilde{x}} + (\lambda T_{\tilde{y}})_{\tilde{y}}\right] + \frac{1}{\delta}\left[\mu U\left(\frac{4}{3\delta} U_{\tilde{x}} - \frac{2}{3} V_{\tilde{y}}\right)\right. \\
&\qquad \left. + \mu V\left(U_{\tilde{y}} + \frac{1}{\delta} V_{\tilde{x}}\right)\right]_{\tilde{x}} + \left[\mu U\left(U_{\tilde{y}} + \frac{1}{\delta} V_{\tilde{x}}\right) + \mu V\left(\frac{4}{3} V_{\tilde{y}} - \frac{2}{3\delta} U_{\tilde{x}}\right)\right]_{\tilde{y}}
\end{aligned} \tag{12}$$

In order to simplify later calculations, it is necessary, at this point, to make use of a result which was found only after the approximate solutions had been found by straightforward methods. In order to find solutions to (11) and (12), the six unknown functions are expanded in appropriate forms, for Re >> 1. These expansions are substituted into the governing relations, and terms of like order are equated. In this manner, sets of equations for the zeroth order, first order, etc., approximation functions are found. Now, when the first-order equations are solved, it is found that, although there are six unknowns and six equations, one of the equations is redundant. Hence, it is necessary to go to the second-order equations, rearrange them, and find, finally, a relation between first-order functions only. This equation completes the set for the first-order equations, but now the third-order equations must be considered to find the missing second-order equation, etc. Such an occurrence is not uncommon; it is found in the weak, non-Hugoniot shock structure problem [8], for example. However, a considerable amount of work is involved, and it can be shown that, for this problem at least, a general equation can be derived from the governing conservation equations such that when the expansions are inserted and equations of various orders are found, the "missing" equation for each order of approximation results.

The redundancy arises, in the set of equations which holds for any order of approximation, because the equation of state gives the same relationships between the pressure, temperature, and density corrections as are found from the remaining equations. Hence, one can find the thermodynamic functions in terms of the velocity, but the velocity cannot be found as a function of the independent variables. It was found that, if the Eulerian derivative of the equation of state was taken, and the momentum, energy, and continuity equations were used to eliminate the resulting derivatives of pressure, temperature, and density, the resulting equation supplied the "missing" equation for the velocity to each order of approximation. Thus, in the following form, this equation removes the redundancy. It is written in terms of the stretched inner variables.

$$\frac{\delta}{\mathrm{Re}}\left(U\mathbb{R}_{U} + V\mathbb{R}_{U} - \frac{1}{\beta\gamma}\mathbb{R}_{H}\right) = \frac{1}{\gamma}\left\{\rho U_{\tilde{x}}\left[\left(\frac{\gamma+1}{2}\right)U^2 + \frac{V^2}{2\beta} - \frac{h_t}{\beta}\right]\right.$$
$$\left. + \rho UV\,(\delta\, U_{\tilde{y}} + V_{\tilde{x}}) + \delta\rho V_{\tilde{y}}\left[\left(\frac{\gamma+1}{2}\right)V^2 + \frac{U^2}{2\beta} - \frac{h_t}{\beta}\right]\right\} \tag{13}$$

2.4. BOUNDARY AND MATCHING CONDITIONS

A sketch of the various regions of flow discussed so far is shown in figure 2. As mentioned previously, boundary-layer effects are minimized to the point where the initial flow is considered to be uniform. Hence no detail of the interaction between the expansion wave and boundary layer

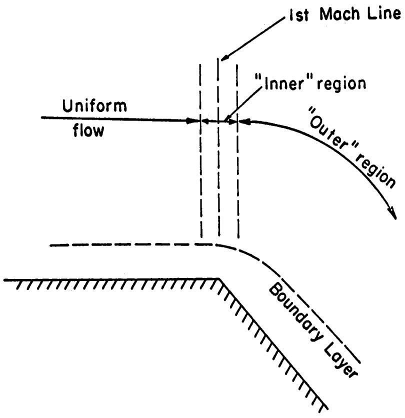

FIGURE 2. FLOW REGIONS

is shown. An additional point, which has not been discussed as yet, is that, for a given angle of deflection of the wall, a final Mach line would exist. Evidently another "inner" region would exist, about this final Mach line, bringing the flow to the final desired condition. This region is not considered in this report, the emphasis being on the relationship between the indicated uniform flow and the inner and outer regions.

In the inner region, the Navier-Stokes equations are to be satisfied. In general, because the unknown functions appear in the second-order derivatives which occur in the viscous terms, two boundary conditions are required. They are supplied by the uniform flow conditions as $\tilde{x} \to -\infty$, for all $\tilde{y} = O(1)$, and by the matching conditions between the inner and outer solutions as $\tilde{x} \to +\infty$, for all $\tilde{y} = O(1)$. In the outer region, because the solution is a perturbation from the inviscid solutions, the viscous terms are always calculated from known functions. Hence only first-order differential equations must be solved for the unknown functions. As a result only the initial conditions, supplied by the matching conditions, suffice for all first-order perturbation quantities and for two of the second-order perturbation quantities. For the remainder of the second-order perturbation quantities, additional information must be supplied by a matching condition between the boundary-layer region and the outer region. Since the solution for the boundary-layer flow is unknown, these quantities may be written only in terms of an unknown function of the turning angle, ϕ.

2.5. OUTER AND INNER EXPANSIONS

The outer solution is desired in the form which indicates the first-order effects of the transport properties for the condition $L\bar{a}_1/\bar{\nu}_1 = Re >> 1$. Thus, we wish to write the flow variables in the form of asymptotic expansions valid in the limit as $Re \to \infty$. Hence, for example, the expansion for the radial flow variable u is of the following form:

$$u \sim \sum_n \epsilon_n(\mathrm{Re})u_n(r, \phi) \tag{14}$$

where $\epsilon_o = 1$

$$u_o = u^{(0)}(\phi) = \text{Prandtl-Meyer solution}$$

and

$$\lim_{\mathrm{Re}\to\infty} \frac{\epsilon_{n+1}}{\epsilon_n} = 0$$

Upon examination of (3b, c, and d), it appears, at first glance, that since the viscous terms are of order Re^{-1}, $\epsilon_1 = \mathrm{Re}^{-1}$ should give the proper order to the first-order terms. Then, employing the principle of eliminability [9], which simply states that since L is a fictitious variable Re and r must appear in a form such that L may be eliminated from any term, the resulting expansion would be of the form

$$u \sim u^{(0)}(\phi) + \frac{1}{\mathrm{Re}} \frac{u^{(1)}(\phi)}{r} + \ldots \tag{15}$$

where in (15) and hereafter, the superscript notation for the outer flow variables (e.g., $u^{(0)}$, $u^{(1)}$) indicates that they are functions of ϕ alone. If expansions of the form shown in (15) are used in (3) and (4), solutions of the resulting first-order equations lead to conflicting results. Hence, it is clear that the expansion form given in (15) is not valid. Moreover, if we substitute such an expansion form for ρv in the continuity (3a), and integrate over r, it is seen that a $\ell n\, r/r$ term results. Hence, it appears that a proper form of expansion for the outer variables is

$$u \sim u^{(0)} + \frac{\ell n\, \mathrm{Re}}{\mathrm{Re}} \frac{u^{(1)}}{r} + \frac{1}{\mathrm{Re}} \left(\frac{\ell n\, r}{r} u^{(1)} + \frac{u^{(2)}}{r} \right) + \ldots \tag{16}$$

where, again, the principle of eliminability has been employed.

It should be noted that, if $r^* = \mathrm{Re}\, r = \overline{r}\overline{a}_1/\overline{\nu}_1$, equation (7) can be written as follows:

$$u \sim u^{(0)} + \frac{\ell n\, r^*}{r^*} u^{(1)} + \frac{1}{r^*} u^{(2)} + \ldots \tag{17}$$

i.e., a coordinate expansion in r^* appears to be of simpler form. However, in view of the complexities of the matching procedure which will become apparent later, the parameter expansion proves to be better suited to this problem. In addition, the final form of the solution is such that matching with a boundary-layer solution, when one is available, will be simpler.

It should be noted that, whereas $u^{(1)}$ and $u^{(2)}$, for example, are referred to as first- and second-order solutions for the radial velocity, they are in reality associated with terms of order ℓn Re/Re and Re^{-1}. Generally, such terms are considered to be essentially of the same order, since large differences between them become apparent only at the limit $Re \to \infty$. However, since different functions of ϕ are necessary, this simple notation and terminology is used throughout this paper.

The inner expansions have the same general construction as the outer. However, in this case, the zeroth-order functions correspond to the uniform flow. Thus, if we choose the velocity component in the x direction, for example,

$$U \sim \sum_n \tilde{\epsilon}_n(Re)\tilde{U}_n(\tilde{x}, \tilde{y}) \tag{18}$$

where $\tilde{\epsilon}_o = 1$ and $\tilde{U}_o = 1$, and

$$\lim_{Re \to \infty} \frac{\tilde{\epsilon}_{n+1}}{\tilde{\epsilon}_n} = 0$$

In problems involving inner and outer expansions, the forms of the various functions of the parameters (e.g., $\tilde{\epsilon}_n$ and ϵ_n) are found for each order by careful examination of the differential equations, boundary conditions, and matching conditions for each order. For example, in this case, the zeroth-order outer solutions (Prandtl-Meyer solution) are matched by the zeroth- and first-order inner solutions, the second-order inner by the first-order outer, etc. Although this construction could be demonstrated step by step, and the functions ϵ_n and $\tilde{\epsilon}_n$ derived, considerable algebraic manipulation is involved both because of the number of functions involved and by the fact that two different coordinate systems are employed. Hence, the final forms of the various expansions will be shown immediately. Their validity will be demonstrated in the matching conditions. In the following equations, both the inner and outer expansions are shown for the thermodynamic functions. Since different coordinate systems are involved, separate expansions are given for the inner and outer velocity components. The zeroth-order outer functions have been given in (6).

$$
\begin{aligned}
u &\sim u^{(0)} + \frac{\ln \mathrm{Re}}{\mathrm{Re}}\frac{u^{(1)}}{r} + \frac{1}{\mathrm{Re}}\left(\frac{\ln r}{r}u^{(1)} + \frac{u^{(2)}}{r}\right) + \ldots \\
v &\sim v^{(0)} + \frac{\ln \mathrm{Re}}{\mathrm{Re}}\frac{v^{(1)}}{r} + \frac{1}{\mathrm{Re}}\left(\frac{\ln r}{r}v^{(1)} + \frac{v^{(2)}}{r}\right) + \ldots \\
U &\sim 1 + \frac{1}{\mathrm{Re}^{2/3}}\frac{\tilde{U}^{(1)}}{\tilde{y}^{2/3}} + \frac{\ln \mathrm{Re}}{\mathrm{Re}^{4/3}}\frac{\tilde{U}^{(2)}}{\tilde{y}^{4/3}} + \frac{1}{\mathrm{Re}^{4/3}}\left(\frac{\ln \tilde{y}}{\tilde{y}^{4/3}}\tilde{U}^{(2)} + \frac{\tilde{U}^{(3)}}{\tilde{y}^{4/3}}\right) + \ldots \\
V &\sim \frac{1}{\mathrm{Re}}\frac{\tilde{V}^{(1)}}{\tilde{y}} + \frac{\ln \mathrm{Re}}{\mathrm{Re}^{5/3}}\frac{\tilde{V}^{(2)}}{\tilde{y}^{5/3}} + \frac{1}{\mathrm{Re}^{5/3}}\left(\frac{\ln \tilde{y}}{\tilde{y}^{5/3}}V^{(2)} + \frac{\tilde{V}^{(3)}}{\tilde{y}^{5/3}}\right) + \ldots \\
h_t &\sim h_t^{(0)} + \frac{\ln \mathrm{Re}}{\mathrm{Re}}\frac{h_t^{(1)}}{r} + \frac{1}{\mathrm{Re}}\left(\frac{\ln r}{r}h_t^{(1)} + \frac{h_t^{(2)}}{r}\right) + \ldots \\
&\sim \frac{1}{2\Gamma^2} + \frac{1}{\mathrm{Re}^{2/3}}\frac{\tilde{h}_t^{(1)}}{\tilde{y}^{2/3}} + \frac{\ln \mathrm{Re}}{\mathrm{Re}^{4/3}}\frac{\tilde{h}_t^{(2)}}{\tilde{y}^{4/3}} + \frac{1}{\mathrm{Re}^{4/3}}\left(\frac{\ln \tilde{y}}{\tilde{y}^{4/3}}\tilde{h}_t^{(2)} + \frac{\tilde{h}_t^{(3)}}{\tilde{y}^{4/3}}\right) + \ldots \\
A &\sim A^{(0)} + \frac{\ln \mathrm{Re}}{\mathrm{Re}}\frac{A^{(1)}}{r} + \frac{1}{\mathrm{Re}}\left(\frac{\ln r}{r}A^{(1)} + \frac{A^{(2)}}{r}\right) + \ldots \\
&\sim 1 + \frac{1}{\mathrm{Re}^{2/3}}\frac{\tilde{A}^{(1)}}{\tilde{y}^{2/3}} + \frac{\ln \mathrm{Re}}{\mathrm{Re}^{4/3}}\frac{\tilde{A}^{(2)}}{\tilde{y}^{4/3}} + \frac{1}{\mathrm{Re}^{4/3}}\left(\frac{\ln \tilde{y}}{\tilde{y}^{4/3}}\tilde{A}^{(2)} + \frac{\tilde{A}^{(3)}}{\tilde{y}^{4/3}}\right) + \ldots
\end{aligned}
\tag{19}
$$

where A represents either P, ρ, or T.

It should be noted that, in the inner expansions above, the principle of eliminability has been invoked in each term. Thus, since Re $\propto$ L and $\tilde{y} \propto L^{-1}$, the product Re $\tilde{y}$ is independent of L. As a result, when Re appears with a negative fractional exponent in any given term, $\tilde{y}$ must appear in this term with the same negative fractional exponent. By the same token the various order functions must also be independent of L. Hence, for example,

$$\tilde{U}^{(n)} = \tilde{U}^{(n)}(t)$$

where $t = t(\tilde{x}, \tilde{y})$ is a similarity variable independent of L. In general, then, each of the various-order inner functions, identified by a tilde, is a function of t. The superscript notation used for these inner functions in (19) and hereafter indicates that they are functions of t alone.

In view of the fact that t must be independent of L, and is a function of both $\tilde{x}$ and $\tilde{y}$, it must be of the form $\tilde{x}\,\tilde{y}^{-m}$. Now

$$\tilde{x}\tilde{y}^{-m} = Re^{1/3}\, xy^{-m} \propto L^{\frac{1}{3}-1+m}$$

Hence

$$m = \frac{2}{3}$$

and

$$t \propto \frac{\tilde{x}}{\tilde{y}^{2/3}} \tag{20}$$

It will be shown later that, when $\tilde{U}^{(n)}$, $\tilde{V}^{(n)}$, etc., are assumed to be functions of the variable t as defined in (20), the governing partial differential equations are reduced to ordinary differential equations, so t is a similarity variable and the expansions and arguments presented above are consistent.

3
SOLUTIONS

If the outer expansions given in (20) are substituted in (3) and (4) and terms of like order in Re are gathered, the governing equations for the various functions of each order approximation are obtained. The zeroth-order outer equations are the inviscid-conservation equations with no r dependence, and the solutions are the well known Prandtl-Meyer solutions.

3.1. INNER SOLUTIONS

The inner expansions given in (20) are substituted into (11), (12), and (13) in order to find the necessary equation for each order. The zeroth-order functions, being constant, satisfy their equations identically. The first-order function must satisfy the following set of equations:

$$\left.\begin{aligned}
&\text{(a)} && (\tilde{\rho}_1 + \tilde{U}_1)_{\tilde{x}} = 0\\
&\text{(b)} && (\tilde{U}_1)_{\tilde{x}} + \frac{1}{\gamma}(\tilde{P}_1)_{\tilde{x}} = 0\\
&\text{(c)} && (\tilde{V}_1)_{\tilde{x}} + \frac{1}{\gamma}(\tilde{P}_1)_{\tilde{y}} = 0\\
&\text{(d)} && \left(\tilde{h}_{t_1}\right)_{\tilde{x}} = 0\\
&\text{(e)} && \tilde{P}_1 = \tilde{\rho}_1 + \tilde{T}_1\\
&\text{(f)} && \tilde{h}_{t_1} = \beta\tilde{T}_1 + \tilde{U}_1
\end{aligned}\right\} \tag{21}$$

$$\text{(g)}\quad \frac{4}{3}\left[\tilde{\mu}_o(\tilde{U}_1)_{\tilde{x}}\right]_{\tilde{x}} - \frac{1}{\gamma\beta}\left\{\frac{\beta}{Pr}\left[\lambda_o(\tilde{T}_1)_{\tilde{x}}\right]_{\tilde{x}} + \frac{4}{3}\left[\mu_o(\tilde{U}_1)_{\tilde{x}}\right]_{\tilde{x}}\right\} = \frac{1}{\gamma}\left\{\left[(\gamma+1)\tilde{U}_1 - \frac{h_{t_1}}{\beta}\right](\tilde{U}_1)_{\tilde{x}} - (\tilde{V}_1)_{\tilde{y}}\right\}$$

where the number subscript notation is used to indicate all variable parts of a given order function. For example, for this and following orders,

$$\tilde{U}_1 = \frac{\tilde{U}^{(1)}}{\tilde{y}^{2/3}}$$

$$\tilde{U}_2 = \frac{\tilde{U}^{(2)}}{\tilde{y}^{4/3}} \tag{22}$$

$$\tilde{U}_3 = \frac{\ell n\, \tilde{y}}{\tilde{y}^{4/3}} \tilde{U}^{(2)} + \frac{\tilde{U}^{(3)}}{\tilde{y}^{4/3}}$$

Since the zeroth-order functions satisfy the uniform flow conditions exactly, the boundary conditions for (20) are simply that all first-order functions tend to zero as $\tilde{x} \to -\infty$, for all $\tilde{y} = O(1)$. Hence the solutions to (21a, b, d, and e) and the resulting form of (21c) are

$$\begin{array}{ll} \text{(a)} & \tilde{\rho}_1 = -\tilde{U}_1 \\ \text{(b)} & \tilde{P}_1 = -\gamma \tilde{U}_1 \\ \text{(c)} & (\tilde{V}_1)_{\tilde{x}} - (\tilde{U}_1)_{\tilde{y}} = 0 \\ \text{(d)} & \tilde{h}_{t_1} = 0 \\ \text{(e)} & \tilde{T}_1 = -U_1/\beta \end{array} \tag{23}$$

Since $\tilde{h}_{t_1} = 0$, it is obvious that (21f) simply repeats the solution for $\tilde{T}_1$, equation 23e. Thus, as mentioned previously, a redundancy occurs when the original governing equations are used, and so the new equation, (21g), must be employed. However, before it can be solved, the viscosity and thermal conductivity must be considered.

In view of the assumptions concerning constant Prandtl number and constant specific heats, it is clear that

$$\lambda = \mu \tag{24}$$

Furthermore, if a power-law dependence on temperature is assumed, the viscosity may be written in terms of inner variables as follows:

$$\mu = T^{\omega} \sim \left(1 + \frac{\omega}{Re^{2/3}} \tilde{T}_1 + \frac{\omega\, \ell n\, Re}{Re^{4/3}} \tilde{T}_2 + \ldots \right) \tag{25}$$

Hence,

$$\lambda_o = \mu_o = 1$$
$$\lambda_1 = \mu_1 = \omega\tilde{T}_1 \qquad (26)$$
$$\lambda_2 = \mu_2 = \omega\tilde{T}_2$$

When the above results for $\tilde{T}_1$, $\tilde{h}_{t_1}$, $\tilde{\mu}_o$, and $\tilde{\lambda}_o$ are substituted into (21g), the result is as follows:

$$b(\tilde{U}_1)_{\tilde{x}\tilde{x}} = \left(\frac{2}{1 - \Gamma^2}\right)\tilde{U}_1(\tilde{U}_1)_{\tilde{x}} - (\tilde{V}_1)_{\tilde{y}} \qquad (27)$$

where

$$b = \frac{4}{3}\gamma\left[1 - \frac{1}{\gamma\beta}\left(1 - \frac{3}{4\,\mathrm{Pr}}\right)\right] \qquad (28)$$

Equation 27 is to be solved simultaneously with (23c), which is an irrotationality condition. Hence, a velocity potential function Φ_1 may be defined such that

$$\tilde{U}_1 = \frac{b(1 - \Gamma^2)}{2}(\Phi_1)_{\tilde{x}}$$
$$\tilde{V}_1 = \frac{b(1 - \Gamma^2)}{2}(\Phi_1)_{\tilde{y}} \qquad (29)$$

and (23c) is satisfied identically. Finally, if new variables X and Y are defined as follows:

$$X = \tilde{x}b^{-1/3}$$
$$Y = \tilde{y} \qquad (30)$$

then equations 29 become

$$\tilde{U}_1 = b^{2/3}\frac{(1 - \Gamma^2)}{2}(\Phi_1)_X$$
$$\tilde{V}_1 = b\,\frac{(1 - \Gamma^2)}{2}(\Phi_1)_Y \qquad (31)$$

and (27) may be written in terms of the potential function and the new variables X and Y as follows:

$$(\Phi_1)_{XXX} = -(\Phi_1)_{YY} + (\Phi_1)_X(\Phi_1)_{XX} \tag{32}$$

Equation 32 is the so-called viscous-transonic equation. It has been studied recently by Sichel [10] in connection with his work on shock-wave structure and nozzle flow, and by n-Szaniawski [11], with reference, again, to nozzle flow.

If a new dimensionless potential function, F_1, is defined as follows:

$$\Phi_1(X, Y) = F_1(t) \tag{33}$$

where t is defined as

$$t = \frac{X}{Y^{2/3}} \tag{34}$$

then (32) may be written in terms of F_1. Thus,

$$F_1''' - F_1'F_1'' + \frac{4}{9}t^2F_1'' + \frac{10}{9}tF_1' = 0 \tag{35}$$

where the primes indicate differention with respect to t. Hence, similarity solutions of (32) in terms of t, the similarity variable which holds for the present problem, evidently do exist. Finally, if the obvious substitution

$$g_1(t) = F_1'(t) \tag{36}$$

is made in (35), the resulting equation is

$$g_1'' - g_1g_1' + \frac{4}{9}t^2g_1' + \frac{10}{9}tg_1 = 0 \tag{37}$$

and

$$\left.\begin{aligned} \tilde{U}_1 &= b^{2/3}\frac{(1 - \Gamma^2)}{2}\frac{g_1(t)}{Y^{2/3}} \\ \tilde{V}_1 &= -b\frac{(1 - \Gamma^2)}{3}\frac{tg_1(t)}{Y} \end{aligned}\right\} \tag{38}$$

Equation 37 is in exactly the same form as that studied by Sichel [10]. However, he was interested in solutions which are bounded as $t \to \pm\infty$. In the present problem, interest lies in solutions which are bounded and in fact tend to zero as $t \to -\infty$. However, as $t \to +\infty$, the solutions must match with the outer solution, and it will be shown that the desired solution is unbounded as $t \to +\infty$.

In (7), the Prandtl-Meyer solutions for U and V are shown for $x/y \ll 1$, and y fixed. It is these forms of U and V which must be matched by the inner solutions for $t \gg 1$. Now, U and V may be written in terms of t by using (10), (30), (34), and (7). Thus,

$$
\begin{aligned}
U &\sim 1 + b^{2/3}\,\mathrm{Re}^{-2/3}\frac{(1-\Gamma^2)}{2}\frac{t^2}{Y^{2/3}} + \ldots \\
V &\sim b\,\mathrm{Re}^{-1}\frac{(1-\Gamma^2)}{3}\frac{t^3}{Y} + \ldots
\end{aligned}
\tag{39}
$$

A comparison of (39) with the inner velocity expansions, (19), and the forms of U_1 and V_1 shown in (38) indicate that the matching condition on $g_1(t)$ is

$$g_1(t) \sim t^2 + \ldots \tag{40}$$

That is, it is desired to find an asymptotic solution to (37) for $t \gg 1$, where the first term is t^2. Hence, the solution is of the form

$$g_1(t) \sim t^2 + h(t) \tag{41}$$

where $h \ll t^2$ for $t \gg 1$.

If (41) is substituted in (37) under the above conditions, the resulting equation for h is

$$h'' - hh' - \frac{5}{9}t^2h' - \frac{8}{9}th + 2 = 0 \tag{42}$$

It is clear that, if h is to be small compared to t^2, the last three terms in (42) are dominant and hence

$$h \sim \frac{6}{t} + \ldots \tag{43}$$

It can be shown that, if the next term in h is desired, the nonlinear term in (42) must be taken into account, and this term is $-36t^{-4}$. Thus, the desired asymptotic solution for $g_1(t)$ is

$$g_1(t) \sim t^2 + \frac{6}{t} - \frac{36}{t^4} + \ldots \tag{44}$$

The asymptotic form of all the first-order inner variables may then be found by substituting (44) into (38), and the resulting equation for U_1 into (23). They match with the corresponding zeroth-order terms at least in the first term of each expansion, of course, since this matching condition was used to find the first term of $g_1(t)$. The next terms will be matched with higher order outer solutions later.

The boundary conditions on the first-order inner functions are simply that they all must tend to zero as $t \to -\infty$. This condition will be satisfied by the condition $tg_1(t) \to 0$ as $t \to -\infty$. Hence it is necessary to examine the behavior of g for large negative values of t. It has been shown by Sichel [10] that the asymptotic solution of (37) may have one of the two forms below:

$$\underset{t \to -\infty}{g_1} \sim (-t)^{-5/2}$$

$$\underset{t \to -\infty}{g_1} \sim (-t)^{1/2} \exp\left[(4/27)(-t)^3\right] \tag{45}$$

i.e., as indicated by the first of the above equations, a solution for which tg_1 tends to zero as $t \to -\infty$ does exist.

The last inner terms to be considered are those of order $Re^{-4/3}\ \ell n\ Re$ and $Re^{-4/3}$, designated here as the second- and third-order terms.

The equations which hold for the second-order terms are

(a) $$(\tilde{\rho}_2 + \tilde{U}_2)_{\tilde{x}} = 0$$

(b) $$(\tilde{U}_2)_{\tilde{x}} + \frac{1}{\gamma}(\tilde{P}_2)_{\tilde{x}} = 0$$

(c) $$(\tilde{V}_2)_{\tilde{x}} + \frac{1}{\gamma}(\tilde{P}_2)_{\tilde{y}} = 0$$

(d) $$\left(\tilde{h}_{t_2}\right)_{\tilde{x}} = 0 \tag{46}$$

(e) $$\tilde{P} = \tilde{\rho}_2 + \tilde{T}_2$$

(f) $$\tilde{h}_{t_2} = \beta\tilde{T}_2 + \tilde{U}_2$$

(g) $$\frac{4}{3}\gamma[\tilde{\mu}_o(\tilde{U}_2)_{\tilde{x}}]_{\tilde{x}} - \frac{1}{\beta}\left\{\frac{\beta}{Pr}[\tilde{\lambda}_o(\tilde{T}_2)_{\tilde{x}}]_{\tilde{x}} + \frac{4}{3}[\tilde{\mu}_o(\tilde{U}_2)_{\tilde{x}}]_{\tilde{x}}\right\}$$
$$= (\tilde{U}_1)_{\tilde{x}}\left[(\gamma+1)\tilde{U}_2 - \frac{\tilde{h}_{t_2}}{\beta}\right] + (\tilde{U}_2)_{\tilde{x}}\left[(\gamma+1)\tilde{U}_1 - \frac{\tilde{h}_{t_1}}{\beta}\right] - (\tilde{V}_2)_{\tilde{y}}$$

The boundary conditions for (46) state that all second-order functions tend to zero as $x \to -\infty$ for all $\tilde{y} = O(1)$. Hence,

$$
\begin{aligned}
&\text{(a)} && \tilde{\rho}_2 = -\tilde{U}_2 \\
&\text{(b)} && \tilde{P}_2 = -\gamma\tilde{U}_2 \\
&\text{(c)} && (\tilde{V}_2)_{\tilde{x}} - (\tilde{U}_2)_{\tilde{y}} = 0 \\
&\text{(d)} && \tilde{h}_{t_2} = 0 \\
&\text{(e)} && \tilde{T}_2 = -\tilde{U}_2/\beta
\end{aligned}
\tag{47}
$$

Again, a velocity potential function, Φ_2, may be defined, satisfying (47c) identically. If this potential function is introduced into (46g), and the independent variables are transformed according to (30), the resulting equation for Φ_2 is

$$(\Phi_2)_{XXX} = (\Phi_2)_X(\Phi_1)_{XX} + (\Phi_1)_X(\Phi_2)_{XX} - (\Phi_2)_{YY} \tag{48}$$

where

$$
\begin{aligned}
\tilde{U}_2 &= b^{-1/3}(\Phi_2)_X \\
\tilde{V}_2 &= (\Phi_2)_Y
\end{aligned}
\tag{49}
$$

Just as in the first-order calculations, it can be shown that a similarity solution exists if in this case

$$\Phi_2(X, Y) = Y^{-2/3}F_2(t) \tag{50}$$

Then (48) becomes,

$$F_2''' + \left(\frac{4}{9}t^2 - g_1\right)F_2'' + (2t - g_1')F_2' + \frac{10}{9}F_2 = 0 \tag{51}$$

and

$$
\begin{aligned}
\tilde{U}_2 &= b^{-1/3}Y^{-4/3}F_2' \\
\tilde{V}_2 &= -\frac{2}{3}(F_2 + tF_2')Y^{-5/3}
\end{aligned}
\tag{52}
$$

It can be shown readily that the solution to (51) which exhibits the highest order allowable terms (i.e., nonexponential) for $t \gg 1$, is

$$F_2(t) = A_1g_1(t) \tag{53}$$

where A_1 is a constant.

Hence, the asymptotic expansions ($t >> 1$) for each of the second-order terms may be found to within an unknown constant, A_1, by using (53) and (49) in (52) and (47). Standard methods may be used to show that, in the limit as $t \to -\infty$, the three asymptotic solutions for F_2 are:

$$(F_2)_1 \sim (-t)^{-1} + \ldots$$
$$(F_2)_2 \sim (-t)^{-5/2} + \ldots \tag{54}$$
$$(F_2)_3 \sim (-t)^{-5/2} \exp [(4/27)(-t)^3]$$

Hence, solutions do exist [i.e., $(F_2)_1$, $(F_2)_2$] such that F_2', F_2, and tF_2' all tend to zero as $t \to -\infty$, thus satisfying the boundary conditions that both $\tilde{U}_2$ and $\tilde{V}_2$ vanish in this limit.

The only third-order inner term which must be calculated is $\tilde{h}_{t_3}$, if matching with the outer expansions is to be made only up to terms of order $(r\ Re)^{-1}$. The reasoning behind this statement is as follows.

Consider the expansion for the inner velocity component, U, up to third-order terms. The only orders which could lead to terms of order $(Re\ r)^{-1}$ in terms of the outer variables are the first- and third-order terms. Thus, since $g_1(t)$ has a term in it which varies as t^{-1}, a term of order $(Re^{-1}\ r)$ arises from the term $Re^{-2/3}\tilde{y}^{-2/3}\tilde{U}^{(1)}(t)$, i.e.,

$$Re^{-2/3}\tilde{y}^{-2/3}t^{-1} = \text{Const. } Re^{-2/3}\tilde{X}^{-1} = \text{Const. } Re^{-1}(r\phi)^{-1}$$

since $\tilde{x} = Re^{1/3}x$, and $x = r\phi$ for $\phi << 1$. By the same token, if a term of the asymptotic expansion for $\tilde{U}^{(3)}(t)$ is to result in a term of order $(Re^{-1}\ r)$, when written in the outer variables, then $U^{(3)}(t)$ must contain a term which varies as t. Thus,

$$Re^{-4/3}\tilde{y}^{-4/3}\ t = \text{Const. } Re^{-4/3}\tilde{y}^{-2}\ \tilde{x} = \text{Const. } (Re\ r)^{-1}\phi$$

However, since matching is to be accomplished for $\phi << 1$, it is apparent that the dominant term should arise from the first-order functions. This is indeed the case for all second-order functions except for h_t. Since $\tilde{h}_{t_1} = 0$, $\tilde{h}_{t_3}$ must be calculated. From the above discussion, it is evident that, if matching is to occur, the first term of the asymptotic expansion for $\tilde{h}_t^{(3)}$ will be proportional to t.

The equation for $\tilde{h}_{t_3}$ is

$$\left(\tilde{h}_{t_3}\right)_{\tilde{x}} + (\tilde{\rho}_1 + \tilde{U}_1)\left(\tilde{h}_{t_1}\right)_{\tilde{x}} = \frac{\beta}{Pr}\left[\tilde{\lambda}_o(\tilde{T}_1)_{\tilde{x}}\right]_{\tilde{x}} + \frac{4}{3}\left[\tilde{\mu}_o(\tilde{U}_1)_{\tilde{x}}\right]_{\tilde{x}} \tag{55}$$

with boundary condition

$$\tilde{h}_{t_3} \to 0 \qquad \text{as } \tilde{x} \to -\infty,\ \tilde{y} = O(1)$$

The solution, written now for $h_t^{(3)}$, is simply

$$\tilde{h}_t^{(3)}(t) = b^{-1/3}\left(\frac{4}{3} - \frac{1}{Pr}\right)(U^{(1)})' \tag{56}$$

Before we proceed to the outer solutions, we will write the inner solutions in a form which facilitates the matching procedure. Of course, one matching condition has been used already, in choosing the proper form of the asymptotic solution for $g_1(t)$. This matching condition may be written, formally, as follows, where the x velocity component has been chosen for an example:

$$1 + \frac{1}{Re^{2/3}}\tilde{U}_1 + \ldots \sim (\sin\phi)u_o + (\cos\phi)v_o + \ldots \tag{57}$$

$$t \gg 1,\quad Y \text{ fixed}$$

That is, the outer solutions are written in terms of the inner variables since information concerning the inner solutions is desired. Then for the next-order terms, the matching condition is written in terms of the outer variables. Thus, if the radial velocity component, u, is chosen for example,

$$u_o + \frac{\ln Re}{Re}u_1 + \frac{1}{Re}u_2 + \ldots \sim (\sin\phi)\left(1 + \frac{1}{Re^{2/3}}\tilde{U}_1 + \frac{\ln Re}{Re^{4/3}}\tilde{U}_2 + \frac{1}{Re^{4/3}}\tilde{U}_3 + \ldots\right)$$
$$+ (\cos\phi)\left(\frac{1}{Re}\tilde{V}_1 + \frac{\ln Re}{Re^{5/3}}\tilde{V}_2 + \frac{1}{Re^{5/3}}\tilde{V}_3 + \ldots\right) \tag{58}$$

$$\phi \ll 1,\quad r \text{ fixed}$$

Similar expressions may be written for the remaining variables. A summary of the most important asymptotic inner solutions follows. The remaining functions may be constructed easily from them. They are written in terms of the outer variables, using the transformation given previously.

$$\text{(a)}\qquad u \sim \phi - \frac{\Gamma^2\phi^3}{3!} + \frac{1}{Re}b\frac{(1 - \Gamma^2)}{r} + \ldots$$

$$\text{(b)}\qquad v \sim 1 - \frac{\Gamma^2\phi^2}{2!} + \frac{\ln Re}{Re}\frac{2A_1}{b^{2/3}}\frac{\phi}{r} + \frac{1}{Re}\left[\frac{2A_1}{b^{2/3}}\frac{\phi \ln r}{r} + \frac{3b(1 - \Gamma^2)}{r\phi}\right] + \ldots \tag{59}$$

$$\text{(c)}\qquad h_t \sim \frac{1}{2\Gamma^2} + \frac{1}{Re}\frac{4}{3}\left(1 - \frac{3}{4Pr}\right)(1 - \Gamma^2)\frac{\phi}{r} + \ldots$$

$$\phi \ll 1,\quad r \text{ fixed}$$

Thus, (59a) is the right-hand side of (58), having been formed from the various inner solutions mentioned previously. Equations 59b and c are the corresponding equations for v and h_t, to which the outer solutions must be matched. Terms of order Re^{-1} ℓn Re have been omitted from the expansion for u, since the largest of these terms is proportional to ϕ^2. As we will show later, it follows that u_1 is zero.

3.2. OUTER SOLUTIONS

The terms to be considered here are those in the outer expansions which are of order ℓn Re/Re and Re^{-1}. In the given notation, they are the first- and second-order outer terms. The equations which result for the first-order outer variables are as follows:

$$
\begin{aligned}
&(a) \quad (\eta^{(0)})' = 0 \\
&(b) \quad (\eta^{(0)}u^{(1)})' - \eta^{(0)}v^{(1)} - \frac{P^{(1)}}{\gamma} = 0 \\
&(c) \quad (\eta^{(0)}v^{(1)})' + \eta^{(1)}((v^{(0)})' + u^{(0)}) + \eta^{(0)}u^{(1)} + \frac{1}{\gamma}(P^{(1)})' = 0 \\
&(d) \quad \left(\eta^{(0)}h_t^{(1)}\right)' = 0 \\
&(e) \quad P^{(1)} = \rho^{(0)}T^{(1)} + \rho^{(1)}T^{(0)} \\
&(f) \quad h_t^{(1)} = \beta T^{(1)} + u^{(0)}u^{(1)} + v^{(0)}v^{(1)}
\end{aligned}
\tag{60}
$$

where the prime denotes differentiation with respect to ϕ and where

$$\eta = \rho v \tag{61}$$

so that $\eta^{(0)} = \rho^{(0)}v^{(0)}$, $\eta^{(1)} = \rho^{(0)}v^{(1)} + \rho^{(1)}v^{(0)}$, etc.

The solutions to equations 60 are:

$$
\begin{aligned}
&(a) \quad \rho^{(1)} = -\rho^{(0)}v^{(1)}/v^{(0)} + \eta^{(1)}/v^{(0)} \\
&(b) \quad u^{(1)} = -\eta^{(1)}u^{(0)}/\eta^{(0)} \\
&(c) \quad P^{(1)} = -\gamma(\eta^{(0)}v^{(1)} + \eta^{(1)}v^{(0)}) \\
&(d) \quad h_t^{(1)} = H_1\big/\eta^{(0)}, \qquad H_1 = \text{constant} \\
&(e) \quad \eta^{(1)} = -\Gamma^2 H_1 \\
&(f) \quad T^{(1)} = -v^{(0)}v^{(1)}\big/\beta + H_1\big/\beta\eta^{(0)} + \eta^{(1)}u^{(0)^2}/\beta\eta^{(0)}
\end{aligned}
\tag{62}
$$

Thus, again a redundancy exists in that (59e) gives only a relationship between two constants which already appear in other equations. As a result, the remainder of the first-order functions are given in terms of $v^{(1)}$, and it is necessary to find $v^{(1)}$ from solutions of the second-order equations. Note that it is not possible to remove this redundancy, in general, as was done for the inner equations by deriving a new governing relation, since in this case it can be demonstrated that the redundancy exists only in the first- and second-order equations.

Before we proceed to the second-order equations, we may evaluate the constants appearing in (62) by employing the matching condition (58) and the inner form (59a). Equation 59a shows that no term proportional to $\phi \ln \mathrm{Re}/\mathrm{Re}$ appears in the expansion for u. Hence, $u_1 = u^{(1)}/r$ must be identically zero; i.e., (eq. 62b, e, and d),

$$\begin{aligned} \eta^{(1)} &= 0 \\ H_1 &= 0 \\ h_t^{(1)} &= u^{(1)} = 0 \end{aligned} \tag{63}$$

As a result, the first-order solutions given in (62) and the second-order equations are simplified considerably.

The following equations hold for the so-called second-order outer variables. The known first-order solutions have been employed so that the only first-order variable which appears is the unknown, $v^{(1)}$.

$$\begin{aligned}
&\text{(a)} \quad (\eta^{(2)})' - \rho^{(0)}u^{(0)}v^{(1)}/v^{(0)} = 0 \\
&\text{(b)} \quad (\eta^{(0)}u^{(2)})' - \eta^{(0)}v^{(2)} - \eta^{(0)}v^{(1)} - P^{(2)}/\gamma = -f^{(0)} \\
&\text{(c)} \quad (\eta^{(0)}v^{(2)} + P^{(2)}/\gamma)' + \rho^{(0)}u^{(0)}v^{(1)} + \eta^{(2)}(v^{(0)})' + u^{(0)} + \eta^{(0)}u^{(2)} = (f^{(0)})' \\
&\text{(d)} \quad \left(\eta^{(0)}h_t^{(2)}\right)' = \left(1 - \frac{3}{4\mathrm{Pr}}\right)(v^{(0)}f^{(0)})' \\
&\text{(e)} \quad P^{(2)} = \rho^{(0)}T^{(2)} + \rho^{(2)}T^{(0)} \\
&\text{(f)} \quad h_t^{(2)} = \beta T^{(2)} + u^{(0)}u^{(2)} + v^{(0)}v^{(2)}
\end{aligned} \tag{64}$$

where

$$f^{(0)} = \frac{4}{3}\mu^{(0)}[(v^{(0)})' + u^{(0)}]$$

and η has been defined by (61).

It is possible to integrate the differential equations in (64). However, because of the redundancy and because $v^{(1)}$ is an unknown, all second-order terms cannot be found. From the six governing equations, solutions can be found for $v^{(1)}$, $u^{(2)}$, $h_t^{(2)}$, $(P^{(2)}/\gamma + \eta^{(0)}v^{(2)})$, and $\eta^{(2)}$, in terms of ϕ. Thus, $P^{(2)}$, $\rho^{(2)}$, and $T^{(2)}$ are given only in terms of $v^{(2)}$, which is unknown. A consideration of higher order terms (up to order Re^{-2}) leads to the conclusion that the redundancy disappears, so that if $v^{(2)}$ were known these higher order solutions could be found. It seems clear that the unknown $v^{(2)}$, (and hence $P^{(2)}$, $\rho^{(2)}$, and $T^{(2)}$) must be found from a matching with a boundary-layer solution near the wall, as discussed previously. For this calculation then, the solutions are given only up to an unknown function of ϕ. It is interesting to note that, although we might expect the correction due to a boundary layer to be evident first in the displacement velocity component (e.g., the radial velocity component, u, in this case) it is in the streamwise velocity component, v, that this correction appears.

The solutions to (64) may be written in terms of the known zeroth-order functions.

$$\text{(a)} \qquad h_t^{(2)} = \frac{4}{3}\left(1 - \frac{3}{4Pr}\right)(1 - \Gamma^2)\mu^{(0)}u^{(0)}/\rho^{(0)} + C_o/\eta^{(0)}$$

$$\text{(b)} \qquad u^{(2)} = [\gamma G + (3/4)bf^{(0)} - N]/\rho^{(0)}u^{(0)}$$

$$\text{(c)} \qquad P^{(2)}/\gamma + \eta^{(0)}v^{(2)} = G - \rho^{(0)}u^{(0)}u^{(2)} + f^{(0)} \qquad (65)$$

$$\text{(d)} \qquad \rho^{(0)}v^{(2)} + \rho^{(2)}v^{(0)} = N/v^{(0)}$$

$$\text{(e)} \qquad v^{(1)} = (N'/\rho^{(0)}u^{(0)} + \Gamma^2 N/\eta^{(0)})$$

where

$$N = (\rho^{(0)}u^{(0)}v^{(0)})^{1/2}\int^{\phi}(\rho^{(0)}u^{(0)}v^{(0)})^{-1/2}\Big((\gamma/2)\Big\{(G + 3bf^{(0)}/4\gamma)'$$

$$- [v^{(0)}/u^{(0)} - (1 - \Gamma^2)u^{(0)}/v^{(0)}](G + 3bf^{(0)}/4\gamma)\Big\} - u^{(0)}G/2v^{(0)}\Big)d\phi + C_2\Big] \qquad (66)$$

$$G = -(3b/4)(1 - \Gamma^2)v^{(0)}\int^{\phi}(u^{(0)}f^{(0)}/v^{(0)2})\,d\phi + C_1 v^{(0)}$$
$$= v^{(0)}\left[C_1 + \beta b(1 - \Gamma^2)(u^{(0)}v^{(0)} - \phi)\right] \tag{67}$$

for a linear viscosity-temperature relationship.

Again, the prime indicates differentiation with respect to ϕ, and C_o, C_1, and C_2 are constants of integration. $T^{(2)}$ can be found in terms of $v^{(2)}$ from (64e).

In order to evaluate the constants, the matching conditions (e.g., eq. 58) must be employed. That is, if the outer solutions are written for $\phi << 1$, r fixed, and substituted in the outer expansions, the results must agree with the corresponding equations, (59). If the expansions of the zeroth-order functions, for $\phi << 1$, are substituted into (65a), (66), and (67), and the resulting expansions for N and G substituted into (65b and e), it can be shown that, for a linear μ-T relationship,

$$h_t^{(2)} \sim C_o + \frac{4}{3}\left(1 - \frac{3}{4}Pr\right)(1 - \Gamma^2)\phi + \dots$$
$$u^{(2)} \sim -C_2\phi^{-1/2} + b(1 - \Gamma^2) + O(\phi) + \dots$$
$$v^{(1)} \sim C_2\phi^{-3/2}/2 - C_1(2\Gamma^2/3\beta)(1 + \gamma\beta(1 - 2\Gamma^2)/\Gamma^2)$$
$$+ C_2\Gamma^2(1 + (6\beta + 1)/12)\phi^{1/2} - (2/5)\Gamma^2 b(1 - \Gamma^2)(6 - \gamma\beta)\phi + \dots \tag{68}$$

Now when the above expansions for $h_t^{(2)}$ and $u^{(2)}$ are multiplied by $(r\ Re)^{-1}$ and then matched with the corresponding terms of (59), it can be seen that

$$C_o = C_2 = 0 \tag{69}$$

Likewise, if $v^{(1)}$ is multiplied by $(r\ Re)^{-1}\ \ell n\ Re$ and matched with the corresponding term in the v expansion in (59), then,

$$C_1 = 0$$
$$A_1 = -b^{5/3}\Gamma^2(1 - \Gamma^2)(6 - \gamma\beta)/5 \tag{70}$$

Although matching has been demonstrated only for the velocity components and total enthalpy, it is easily shown from (62) (outer) and (23) and (47) (inner) that all known functions match properly with no other constants arising. It is of interest to note that, as $\phi \rightarrow 0$, $u^{(2)} \rightarrow$ constant $\neq 0$.

That is, in the matching region, the perturbations do not necessarily tend toward zero as the turning angle tends toward zero. In fact, while the ϕ variation of the outer velocity component, $v^{(2)}$, is unknown, it must behave as ϕ^{-1} for $\phi << 1$, as indicated in (59). Thus, in the outer coordinate system $v^{(2)}$ has a singularity at $\phi = 0$.

4
DISCUSSION OF RESULTS

The outer solutions for the velocities, pressure, temperature, density, and total enthalpy may be written up to order Re^{-1}, within an unknown function of ϕ, by substituting the solutions found above (eqs. 62, 63, 64, 69, and 70) into the outer expansions (eq. 19). One of the more interesting aspects of the solution is the fact that logarithmic terms occur immediately, rather than in higher order terms as is generally the case. Thus, the first-order terms, those which indicate the order of magnitude of the error involved where viscous terms are ignored, are of order $Re^{-1} \ell n\, Re$. Further calculations (app. II) for the case where the initial uniform stream Mach number is arbitrary indicate that, although the stretching factor for the inner region changes from $Re^{1/3}$ to $Re^{1/2}$, the same form of expansion for the outer flow exists, i.e., the first viscous terms are of order $Re^{-1} \ell n\, Re$.

The change in entropy due to the viscous and heat-conduction effects may be calculated since it involves only the known functions. Since the entropy of the zeroth-order inviscid expansion is a constant, the change in entropy can be written with respect to $S^{(0)}$. Thus,

$$\Delta S = S - S^{(0)} = \ell n\,(T/T^{(0)}) - [(\gamma - 1)/\gamma]\,\ell n(P/P^{(0)})$$

$$\sim (Re\ r)^{-1}\left[\left(h_t^{(2)} - u^{(0)}u^{(2)}\right)\Big/T^{(0)} - \eta^{(2)}/\eta^{(0)}\right] + \ldots \qquad (71)$$

where, in the second equation, the asymptotic expansions have been employed and simple substitutions made in order to include only known terms. The fact that there is no entropy change in the terms of order $Re^{-1} \ell n\, Re$ is consistent with the result that in the inner expansion, up to terms of this order, the flow was irrotational.

A physical picture of the changes in the flow caused by the inclusion of viscosity and heat transfer can be gained by a consideration of the signs of the various corrections. These are indicated for the velocities and total enthalpy for $\phi << 1$, for example by the expansions given in (59). Since $A_1 < 0$, it is seen that the radial velocity component, u, is increased, while the angular velocity component, v, is decreased. The magnitude of the velocity is decreased, as expected, since the highest order correction is $u^{(0)}v^{(1)}(\mathrm{Re}\ r)^{-1}\ \ell n\ \mathrm{Re}$, and $v^{(1)}$ is negative. In figure 3 a sketch of the differences in velocity and streamline pattern between the viscous and inviscid flows is shown. As indicated, the result of the given changes in velocity is that at a given turning angle, ϕ, and radius, r, the viscous heat-conducting flow has turned less than the corresponding inviscid flow.

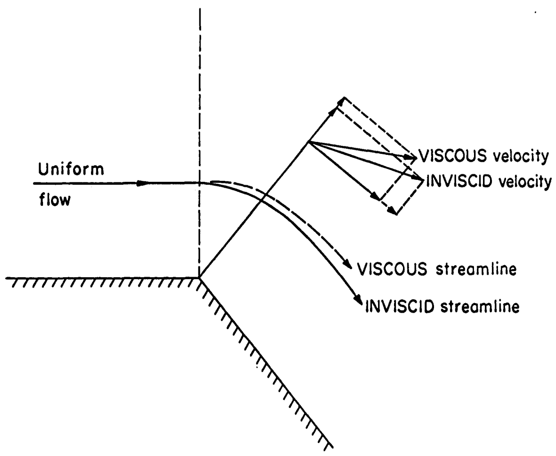

FIGURE 3. PRANDTL-MEYER FLOW VARIATIONS DUE TO EFFECTS OF VISCOSITY AND THERMAL CONDUCTIVITY

The total enthalpy corrections may be positive or negative, depending on the Prandtl number. From (59) or (65), it can be seen that for Pr < 3/4 the total enthalpy is decreased, and for Pr > 3/4 it is increased. Further, in view of the fact that $h_t^{(1)} = 0$, the first correction is of order Re^{-1}. Since the uniform flow and the expansive flow for $r \rightarrow \infty$ are at the same total enthalpy, the change in total enthalpy must be balanced by an energy exchange with the boundary layer. As mentioned previously, this means that the asymptotic expansion for the total enthalpy (outer) must be matched by a corresponding expansion from the boundary-layer solution.

Examination of the highest order correction terms (eq. 62) in the solution for the pressure, density, and temperature, indicates that they all increase over their zeroth order (Prandtl-Meyer) solutions. This means that the effect of the transport properties is to decrease the pressure drop at a given point, for example, so that for a given pressure drop, the flow must turn through a greater angle.

The inclusion of boundary-layer effects in the initial flow would result in additional terms of order $Re^{-1/2}$ in both the inner and outer expansions. That is, the initial flow would not be uniform, but instead have terms of order $Re^{-1/2}$. In order to match this condition, the inner solution would include terms of the same order which would be carried through to the outer expansions. These terms would also have to match with corresponding terms in the downstream boundary-layer solution. Finally, it should be noted that the existence of terms of order $Re^{-1/2}$ means that there would be new additions to the existing terms of order Re^{-1}.

Appendix I
GOVERNING EQUATIONS IN CARTESIAN FORM ($M_1 = 1$)

The conservation equations are written below in terms of Cartesian coordinates. The independent and dependent variables are nondimensionalized as described in the text. Subscripts x and y indicate partial differentiation.

Continuity:

$$(\rho U)_x + (\rho V)_y = 0$$

Momentum, x direction:

$$\rho U U_x + \rho V U_y + \frac{1}{\gamma} P_x = \frac{1}{Re} \mathbb{R}_U \tag{I-1}$$

Momentum, y direction:

$$\rho U V_x + \rho V V_y + \frac{1}{\gamma} P_y = \frac{1}{Re} \mathbb{R}_V$$

Energy:

$$\rho U (h_t)_x + \rho V (h_t)_y = \frac{1}{Re} \mathbb{R}_H$$

The $\mathbb{R}$ functions are as follows:

$$\mathbb{R}_U = \left[\mu\left(\frac{4}{3} U_x - \frac{2}{3} V_y\right)\right]_x + \left[\mu(U_y + V_x)\right]_y$$

$$\mathbb{R}_V = \left[\mu(U_y + V_x)\right]_x + \left[\mu\left(\frac{4}{3} V_y - \frac{2}{3} U_x\right)\right]_y \tag{I-2}$$

$$\mathbb{R}_H = \frac{\beta}{Pr}\left[(\lambda T_x)_x + (\lambda T_y)_y\right] + \left\{\mu\left[U\left(\frac{4}{3} U_x - \frac{2}{3} V_y\right) + V(U_y + V_x)\right]\right\}_x + \left\{\mu\left[U(U_y + V_x) + V\left(\frac{4}{3} V_y - \frac{2}{3} U_x\right)\right]\right\}_y$$

The state equation and the equation defining total enthalpy are, again,

$$P = \rho T$$

$$h_t = \beta T + \frac{U^2}{2} + \frac{V^2}{2} \tag{I-3}$$

and the additional equation, necessary to remove the redundancy discussed in the text, is

$$U\mathbb{R}_U + V\mathbb{R}_V - \frac{1}{\beta\gamma}\mathbb{R}_H = \frac{1}{\gamma}\left\{\rho U_x\left[\frac{(\gamma+1)}{2}U^2 + \frac{V^2}{2\beta} - \frac{h_t}{\beta}\right]\right.$$

$$\left. + \rho UV(U_y + V_x) + \rho V_y\left[\frac{(\gamma+1)}{2}V^2 + \frac{U^2}{2\beta} - \frac{h_t}{\beta}\right]\right\} \qquad \text{(I-4)}$$

Appendix II
EFFECTS OF TRANSPORT PROPERTIES ON SUPERSONIC EXPANSION AROUND A CORNER WHEN THE INITIAL FLOW IS SUPERSONIC

The problem when the initial uniform flow is supersonic differs in several important aspects from the case where it is sonic, as was mentioned in the text. In the following, the problem is formulated briefly, and the inner and outer solutions for the velocity components and total enthalpy are found and matched.

The initial stream is again assumed to be uniform, with a velocity $\bar{q}$, and a Mach number M_1, and with properties $\bar{\rho}_1$, $\bar{P}_1$, $\bar{T}_1$, and $\bar{h}_{t_1}$, where the notation is that used in the text, and the bar indicates a dimensional quantity. The variables are nondimensionalized as follows.

$$U = \frac{\bar{U}}{\bar{q}_1}, \quad V = \frac{\bar{V}}{\bar{q}_1}, \quad P = \frac{\bar{P}}{\bar{P}_1}, \quad \rho = \frac{\bar{\rho}}{\bar{\rho}_1}, \quad T = \frac{\bar{T}}{\bar{T}_1},$$

$$u = \frac{\bar{u}}{\bar{q}_1}, \quad v = \frac{\bar{v}}{\bar{q}_1}, \quad h_t = \frac{\bar{h}_t}{\bar{q}_1^2}, \quad \vec{X} = \frac{\vec{\bar{X}}}{L}, \quad Re = \frac{\bar{\rho}_1 \bar{q}_1 L}{\bar{\mu}_1} \qquad \text{(II-1)}$$

i.e., the nondimensionalization is carried out using $\bar{q}_1$ for the velocities and energy. This is, of course, exactly the same procedure used in the text, where $\bar{q}_1 = \bar{a}_1$.

A sketch of the coordinate systems employed and the boundary conditions is shown in figure 4. Just as in the problem considered in the text, Cartesian and polar coordinate systems are constructed relative to the first Mach line of the expansion. That is, the y coordinate, which corresponds to the r coordinate for $\phi = 0$, lies along the first Mach line of the expansion.

In terms of the above variables, and from figure 4, it is clear that, in the uniform stream,

$$U_1 = \sin \mu_1 = \frac{1}{M_1}$$

$$V_1 = \cos \mu_1 = \frac{\sqrt{M_1^2 - 1}}{M_1} \qquad \text{(II-2)}$$

where $\mu_1 = \sin^{-1}\left(M_1^{-1}\right)$ is the Mach angle associated with M_1. The values attained by the rest of the variables, in the uniform stream, are shown in figure 4.

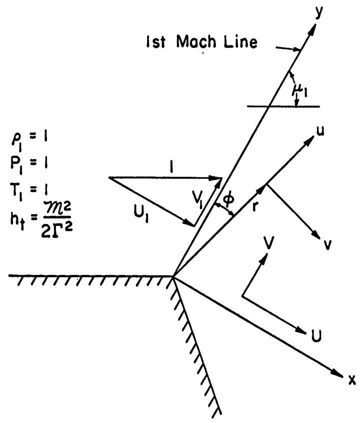

FIGURE 4. COORDINATE SYSTEMS AND BOUNDARY CONDITIONS FOR THE CASE $M_1 > 1$

The zeroth-order outer solutions (Prandtl-Meyer) can be written as follows:

$$
\begin{aligned}
u_o &= \frac{\mathbb{M}}{\Gamma} \sin \Gamma(\phi + \theta_1) \\
v_o &= \mathbb{M} \cos \Gamma(\phi + \theta_1) \\
T_o &= M_1^2 v_o^2 = v_o^2 / \sin^2 \mu_1 \\
P_o &= T_o^{\gamma\beta} \\
\rho_o &= T_o^{\beta} \\
h_{t_o} &= \frac{\mathbb{M}^2}{2\Gamma^2}
\end{aligned}
\qquad \text{(II-3)}
$$

where

$$
\mathbb{M}^2 = \frac{1 + \Gamma^2 (M_1^2 - 1)}{M_1^2} \qquad \text{(II-4)}
$$

and where θ_1 is the Prandtl-Meyer turning angle associated with M_1. Thus,

$$\cos \Gamma\theta_1 = \frac{1}{M_1 \mathbb{M}} = \frac{\sin \mu_1}{\mathbb{M}}$$

$$\sin \Gamma\theta_1 = \frac{\Gamma}{\mathbb{M}} \frac{\sqrt{M_1^2 - 1}}{M_1} = \frac{\Gamma}{\mathbb{M}} \cos \mu_1 \tag{II-5}$$

Hence, as $M_1 \to 1$, $\mathbb{M}^2 \to 1$, and $\theta_1 \to 0$, so that all the zeroth-order functions given in (II-3) reduce to the forms given in the text, where the $M_1 = 1$ case was considered.

The relationships between the Cartesian and polar coordinate velocity components is, again,

$$u = U \sin\phi + V \cos\phi$$

$$v = U \cos\phi - V \sin\phi \tag{II-6}$$

The general considerations mentioned in the introduction hold for this case. That is, the expansion flow field is divided into two regions: the inner region, which contains the first Mach line, and the outer region, which includes the main part of the expansion.

If we calculate the derivatives of the velocity components given in (II-3) (Prandtl-Meyer solution), evaluate these derivatives at $\phi = 0$, and compare them with the corresponding values of the derivatives found from the uniform flow relations [(II-6) with U_1 and V_1 from (II-2)] we see that discontinuities exist in the first derivatives. This differs from the $M_1 = 1$ case considered in the text, where discontinuities do not appear until the second derivatives. Hence, in this case, the inner region is a thin region containing the first Mach line, in which the effects of viscosity and heat conduction are to smooth discontinuities which arise in the first derivatives of the isentropic Prandtl-Meyer solution, at $\phi = 0$.

II.1. ORDER OF THE THICKNESS OF THE INNER REGION

The method by which the order of the thickness of the inner region is calculated is that used previously in the $M_1 = 1$ case. Briefly: the various zeroth-order outer functions necessary to evaluate the terms in the x-direction momentum equation used as a jump condition across the inner region are expanded for $\phi << 1$. Since for $\phi << 1$, $\phi \cong x/y$, and since in the inner region, $y = O(1)$ and $x = O(\delta)$, where δ is the order of the thickness, the derivatives in the momentum equation may be ordered in terms of δ. Finally, by balancing the changes in the momentum flux and pressure forces by the viscous forces, δ may be found in terms of the Reynolds number.

First, the zeroth-order outer functions are expanded for $\phi << 1$. The expansions are found from the zeroth order solutions given in (II-3). The Cartesian velocity components are found from (II-6), written for $\phi << 1$. The resulting equations are:

$$U \sim \sin\mu_1 + (\cos\mu_1)(1 - \Gamma^2)\phi + (\sin\mu_1)\left(\frac{1 - \Gamma^2}{2}\right)\phi^2 - (\cos\mu_1)\left(\frac{1 - \Gamma^4}{6}\right)\phi^3$$

$$- (\sin\mu_1)\left(\frac{1 - \Gamma^2}{24}\right)(3 + \Gamma^2)\phi^4 + \ldots$$

$$V \sim \cos\mu_1 - (\cos\mu_1)\left(\frac{1 - \Gamma^2}{2}\right)\phi^2 - (\sin\mu_1)\left(\frac{1 - \Gamma^2}{3}\right)\phi^3$$

$$+ (\cos\mu_1)\left(\frac{1 - \Gamma^2}{24}\right)(1 + 3\Gamma^2)\phi^4 \ldots$$

$$T \sim 1 - 2(\cot\mu_1)\Gamma^2\phi - \left(1 - \Gamma^2\cot^2\mu_1\right)\Gamma^2\phi^2 + \frac{4}{3}(\cot\mu_1)\Gamma^4\phi^3 + \left(1 - \Gamma^2\cot^2\mu_1\right)\frac{\Gamma^4}{3}\phi^4 + \ldots$$

$$P \sim 1 - (\cot\mu_1)(1 + \Gamma^2)\phi - \left(1 - \cot^2\mu_1\right)\left(\frac{1 + \Gamma^2}{2}\right)\phi^2 + (\cot\mu_1)\Big[3 + \Gamma^2$$

$$- (1 - \Gamma^2)\cot^2\mu_1\Big]\left(\frac{1 + \Gamma^2}{6}\right)\phi^3 + \Big[3 + \Gamma^2 - 2(3 - \Gamma^2)\cot^2\mu_1$$

$$+ (1 - \Gamma^2)(1 - 2\Gamma^2)\cot^4\mu_1\Big]\left(\frac{1 + \Gamma^2}{24}\right)\phi^4 + \ldots \qquad \text{(II-7)}$$

$$\rho \sim 1 - (\cot\mu_1)(1 - \Gamma^2)\phi - \left[1 - (1 - 2\Gamma^2)\cot^2\mu_1\right]\left(\frac{1 - \Gamma^2}{2}\right)\phi^2$$

$$+ (\cot\mu_1)\left[3 - 5\Gamma^2 - (1 - 2\Gamma^2)(1 - 3\Gamma^2)\cot^2\mu_1\right]\left(\frac{1 - \Gamma^2}{6}\right)\phi^3$$

$$+ \left[3 - 5\Gamma^2 - (3 - 5\Gamma^2)(3 - 7\Gamma^2)\cot^2\mu_1 + (1 - 2\Gamma^2)(1 - 3\Gamma^2)(1 - 4\Gamma^2)\cot^4\mu_1\right]\frac{(1 - \Gamma^2)}{24}\phi^4 + \ldots$$

$$\frac{\sin^2\mu_1 P}{\gamma} + \rho U^2 \sim \left(\frac{\gamma + 1}{\gamma}\right)\sin^2\mu_1 - (\sin\mu_1)(\cos\mu_1)\left(1 + 5\Gamma^2 + \cot^3\mu_1\right)\left(\frac{1 - \Gamma^2}{3}\right)\phi^3$$

$$- \left(\sin^2\mu_1\right)\Big\{6 - [6 - 8\Gamma^2 - (3 + 5\Gamma^2)(7 - 11\Gamma^2)]\cot^2\mu_1$$

$$- 6(1 - 2\Gamma^2)\cot^4\mu_1\Big\}\left(\frac{1 - \Gamma^2}{24}\right)\phi^4 + \ldots$$

In the above equations the functions have been expanded up to terms of order ϕ^4 so that they may be used for the case $M_1 = 1$, $\mu_1 = \frac{\pi}{2}$ as well, thus illustrating the change in order which occurs as $M_1 \to 1$. This change in order is, of course, the reason for the change in δ as $M_1 \to 1$.

Equations II-7 represent the various functions on the downstream side of the inner region. On the upstream side they are given by (II-2) and the boundary conditions shown in figure 4. Hence, the differences, which are used to calculate the orders of the derivatives, may be found easily. Thus, since for $\phi << 1$,

$$\phi \cong \frac{x}{y} = O(\delta) \qquad \text{(II-8)}$$

then

$$U_x = O(1), \qquad U_y = O(\delta)$$

$$\left[\left(\sin^2 \mu_1\right)\frac{P}{\gamma} + \rho U^2\right]_x = O(\delta^2), \qquad (\rho U)_x = O(\delta) \qquad \text{(II-9)}$$

$$P_x = O(1)$$

The momentum equation, written in Cartesian coordinates, for the x direction, may be used to illustrate the calculation of δ. It should be noted that just as in the case $M_1 = 1$, the inviscid terms in the equation must be rearranged so as to emphasize the group of terms $P/\gamma M_1^2 + \rho U^2$. The x-direction momentum equation is, then, in dimensionless form,

$$\begin{array}{ccccc} \left[\left(\sin^2 \mu_1\right)\frac{P}{\gamma} + \rho U^2\right]_x & - U(\rho U)_x & + \rho V U_y & = \frac{1}{Re}\left\{\left[\mu\left(\frac{4}{3}U_x - \frac{2}{3}V_y\right)\right]_x \right. & \left. + (U_y + V_x)_y\right\} \\ \delta^2 & \delta & \delta & \frac{1}{Re}\left\{\quad \frac{1}{\delta} \qquad \delta \qquad \right. & \left. \delta \quad \delta \right\} \end{array} \qquad \text{(II-10)}$$

The order of each term appears beneath it. Although the terms involving μ_1 do not appear in the order estimates, it is clear that μ_1 cannot approach too closely to $\pi/2$, if the given orders are to remain valid. If the various trigonometric functions of μ_1 are included in the order estimates, the omitted terms are small compared with terms retained provided that

$$\cos \mu_1 >> \frac{1}{Re} \qquad \text{(II-11)}$$

With this condition, then, it can be seen from (II-10) that

$$\delta = Re^{-1/2} \qquad \text{(II-12)}$$

gives the proper order to the thickness of the inner region. Hence the inner variables are stretched according to the following transformation:

$$\tilde{x} = Re^{1/2}x = \frac{x}{\delta} \qquad \text{(II-13)}$$

$$\tilde{y} = y$$

II.2. GOVERNING EQUATIONS

The dimensionless governing equations are written in Cartesian form for the inner region. In the following equations, the stretched coordinates are employed. Thus,

(a) $$(\rho U)_{\tilde{x}} + \delta(\rho V)_{\tilde{y}} = 0$$

(b) $$\rho U U_{\tilde{x}} + \delta\rho V U_{\tilde{y}} + \left(\frac{\sin^2\mu_1}{\gamma}\right) P_{\tilde{x}} = \frac{\delta}{Re}\mathbb{R}_U$$

(c) $$\frac{\rho U}{\delta} V_{\tilde{x}} + \rho V V_{\tilde{y}} + \left(\frac{\sin^2\mu_1}{\gamma}\right) P_{\tilde{y}} = \frac{1}{Re}\mathbb{R}_V$$

(d) $$\rho U(h_t)_{\tilde{x}} + \delta\rho V(h_t)_{\tilde{y}} = \frac{\delta}{Re}\mathbb{R}_H$$

(e) $$P = \rho T \qquad \text{(II-14)}$$

(f) $$h_t = \left(\sin^2\mu_1\right)\beta T + \frac{U^2}{2} + \frac{V^2}{2}$$

(g) $$\delta\left(U\mathbb{R}_U + V\mathbb{R}_V - \frac{1}{\beta\gamma}\mathbb{R}_H\right) = \frac{1}{\gamma}\left\{\rho U_x\left[\left(\frac{\gamma+1}{2}\right)U^2 + \frac{V^2}{2\beta} - \frac{h_t}{\beta}\right]\right.$$
$$\left. + \delta\rho V_{\tilde{y}}\left[\left(\frac{\gamma+1}{2}\right)V^2 + \frac{U^2}{2\beta} - \frac{h_t}{\beta}\right] + \rho UV\left(\delta V_{\tilde{y}} + V_{\tilde{x}}\right)\right\}$$

where $\mathbb{R}_U$ and $\mathbb{R}_V$ are given in (12) of the text, and

$$\mathbb{R}_H = \left(\sin^2\mu_1\right)\frac{\beta}{\gamma}\left[\frac{1}{\delta^2}(\lambda T_{\tilde{x}})_{\tilde{x}} + (\lambda T_{\tilde{y}})_{\tilde{y}}\right] + \frac{1}{\delta}\left\{\mu\left[U\left(\frac{4}{3\delta}U_{\tilde{x}} - \frac{2}{3}V_{\tilde{y}}\right)\right.\right.$$
$$\left.\left. + V\left(U_{\tilde{y}} + \frac{1}{\delta}V_{\tilde{x}}\right)\right]\right\}_{\tilde{x}} + \left\{\mu\left[U\left(U_{\tilde{y}} + \frac{1}{\delta}V_{\tilde{x}}\right) + V\left(\frac{4}{3}V_{\tilde{y}} - \frac{2}{3\delta}U_{\tilde{x}}\right)\right]\right\}_{\tilde{y}} \qquad \text{(II-15)}$$

Again, (II-14g) is a relation derived from the other governing equations. In the form given, this equation removes the redundancy which arises when the six original equations are employed.

The dimensionless equations which hold in the outer region are similar to those employed in the $M_1 = 1$ case.

$$\text{(a)} \qquad (\rho u r)_r + (\rho v)_\phi = 0$$

$$\text{(b)} \qquad \rho u u_r + \frac{\rho v}{r} u_\phi - \frac{\rho v^2}{r} + \frac{\sin^2 \mu_1}{\gamma} P_r = \frac{\mathbb{R}_u}{Re}$$

$$\text{(c)} \qquad \rho u v_r + \frac{\rho v}{r} v_\phi + \frac{\rho u v}{r} + \frac{\sin^2 \mu_1}{\gamma r} P_\phi = \frac{\mathbb{R}_v}{Re} \qquad \text{(II-16)}$$

$$\text{(d)} \qquad \rho u (h_t)_r + \frac{\rho v}{r} (h_t)_\phi = \frac{\mathbb{R}_h}{Re}$$

$$\text{(e)} \qquad P = \rho T$$

$$\text{(f)} \qquad h_t = \left(\sin^2 \mu_1\right) \beta T + \frac{u^2}{2} + \frac{v^2}{2}$$

where $\mathbb{R}_u$ and $\mathbb{R}_v$ are given by (4a and b) and

$$\mathbb{R}_h = \frac{\beta \sin^2 \mu_1}{Pr} \left[\frac{1}{r}(r\lambda T_r)_r + \frac{1}{r^2}(\lambda T_\phi)_\phi\right]$$
$$+ \frac{1}{r}\left(r\mu \left\{u\left[\frac{4}{3}u_r - \frac{2}{3r}(v_\phi + u)\right] + v\left[v_r + \frac{1}{r}(u_\phi - v)\right]\right\}\right)_r \qquad \text{(II-17)}$$
$$+ \frac{1}{r}\left(\mu \left\{u\left[v_r + \frac{1}{r}(u_\phi - v)\right] + v\left[\frac{4}{3r}(v_\phi + u) - \frac{2}{3}u_r\right]\right\}\right)_\phi$$

II.3. INNER AND OUTER EXPANSIONS

The inner asymptotic expansions for the six unknown functions are somewhat different in form, from the corresponding expansions for the $M_1 = 1$ case. This difference arises as a result of the fact that, in the present case, $\delta = O(Re^{-1/2})$, whereas for $M_1 = 1$, $\delta = O(Re^{-1/3})$. Thus,

$$U \sim \sin \mu_1 + \frac{1}{Re^{1/2}} \frac{\tilde{U}^{(1)}}{\tilde{y}^{1/2}} + \frac{\ln Re}{Re} \frac{\tilde{U}^{(2)}}{\tilde{y}} + \frac{1}{Re}\left(\frac{\ln \tilde{y}}{\tilde{y}} U^{(2)} + \frac{\tilde{U}^{(3)}}{\tilde{y}}\right) + \ldots$$

$$V \sim \cos \mu_1 + \frac{1}{Re} \frac{\tilde{V}^{(1)}}{\tilde{y}} + \frac{\ln Re}{Re^{3/2}} \frac{\tilde{V}^{(2)}}{\tilde{y}^{3/2}} + \frac{1}{Re^{3/2}}\left(\frac{\ln \tilde{y}}{\tilde{y}^{3/2}} V^{(2)} + \frac{1}{\tilde{y}^{3/2}}\tilde{V}^{(3)}\right) + \ldots \qquad \text{(II-18)}$$

$$h_t \sim \frac{\mathbb{M}^2}{2\Gamma^2} + \frac{1}{Re^{1/2}} \frac{\tilde{h}_t^{(1)}}{\tilde{y}^{1/2}} + \frac{\ln Re}{Re} \frac{\tilde{h}_t^{(2)}}{\tilde{y}} + \frac{1}{Re}\left(\frac{\ln \tilde{y}}{\tilde{y}}\tilde{h}_t^{(2)} + \frac{1}{\tilde{y}}\tilde{h}_t^{(3)}\right) + \ldots$$

$$A = 1 + \frac{1}{Re^{1/2}} \frac{\tilde{A}^{(1)}}{\tilde{y}^{1/2}} + \frac{\ln Re}{Re} \frac{\tilde{A}^{(2)}}{\tilde{y}} + \frac{1}{Re}\left(\frac{\ln \tilde{y}}{\tilde{y}}\tilde{A}^{(2)} + \frac{1}{\tilde{y}}\tilde{A}^{(3)}\right) + \ldots$$

where A represents P, ρ, or T. In (II-18), the superscript notation is used to indicate that these functions depend on t alone, where t is a similarity variable. From the principle of eliminability, it is seen that in this case

$$t \propto \frac{\tilde{x}}{\tilde{y}^{1/2}} \tag{II-19}$$

if t is to be independent of L.

The outer expansions are of the same form as those used in the $M_1 = 1$ case and hence are given in (19).

II.4. SOLUTIONS

If the inner expansions (II-18) are substituted in the inner equations, (II-14), and terms of like order in Reynolds number are equated, the governing equations for each order of approximation are obtained. The first-order equations are similar to (21), and if the boundary conditions (that all first-order functions tend to zero as $\tilde{x} \to -\infty$ for $\tilde{y}$ fixed) are applied to those equations which are immediately integrable, the resulting solutions are:

$$\begin{aligned} \tilde{\rho}_1 &= -\frac{\tilde{U}_1}{\sin\mu_1} \\ \tilde{P}_1 &= -\frac{\gamma\tilde{U}_1}{\sin\mu_1} \\ \tilde{T}_1 &= -\frac{\tilde{U}_1}{\beta\sin\mu_1} \\ \tilde{h}_{t_1} &= 0 \end{aligned} \tag{II-20}$$

where the number subscript notation again indicates all variable parts of a given order function; e.g., $\tilde{U}_1 = \tilde{y}^{-1/2}\,\tilde{U}^{(1)}(t)$, $\tilde{V}^{(1)} = \tilde{y}^{-1}\,\tilde{V}^{(1)}(t)$, etc. In addition, the following equations are found:

$$(\tilde{V}_1)_{\tilde{x}} - (\tilde{U}_1)_{\tilde{y}} = 0$$

$$\begin{aligned} &\frac{1}{\gamma}\left[(\tilde{U}_1)_{\tilde{x}}(\gamma+1)\tilde{U}_1 \sin\mu_1 + (\sin\mu_1)(\cos\mu_1)\left((\tilde{U}_1)_{\tilde{y}} + (\tilde{V}_1)_{\tilde{x}}\right)\right] \\ &= \frac{4}{3}(\sin\mu_1)(\tilde{U}_1)_{\tilde{x}\tilde{x}} - \frac{\sin\mu_1}{\beta\gamma}\left[\frac{\beta}{P_r}(\tilde{T}_1)_{\tilde{x}\tilde{x}}\sin\mu_1 + \frac{4}{3}(\tilde{U}_1)_{\tilde{x}\tilde{x}}\right] \end{aligned} \tag{II-21}$$

Hence, a potential function, $\Phi_1(X, Y)$, may be defined such that

$$
\begin{aligned}
U_1 &= \left(\frac{1 - \Gamma^2}{2}\right)\sqrt{b \cos \mu_1}\,(\Phi_1)_X \\
V_1 &= \left(\frac{1 - \Gamma^2}{2}\right) b (\Phi_1)_Y
\end{aligned}
\tag{II-22}
$$

where

$$
\begin{aligned}
X &= \sqrt{\frac{\cos \mu_1}{b}}\,\tilde{x}, \qquad Y = \tilde{y} \\
b &= \frac{4}{3}\gamma\left[1 - \frac{1}{\gamma\beta}\left(1 - \frac{3}{4\,Pr}\right)\right]
\end{aligned}
\tag{II-23}
$$

and where the X and Y subscripts indicate partial differentiation. Then, the second of equations II-21 may be written in terms of Φ_1 as follows:

$$
(\Phi_1)_{XXX} = (\Phi_1)_X(\Phi_1)_{XX} + 2(\Phi_1)_{XY} \tag{II-24}
$$

Just as in the $M_1 = 1$ case, $\Phi_1(X, Y)$ may be written in terms of a similarity variable, and (II-24) is reduced to an ordinary differential equation. Thus, in this case, if

$$
\Phi(X, Y) = F_1(t) \tag{II-25}
$$

and

$$
f_1(t) = \frac{dF_1}{dt} = F_1'
$$

then (II-24) becomes

$$
f_1'' - (f_1 - t)f_1' + f_1 = 0 \tag{II-26}
$$

and

$$
\begin{aligned}
\tilde{U}_1 &= \left(\frac{1 - \Gamma^2}{2}\right)\sqrt{b \cos \mu_1}\,\frac{f_1}{Y^{1/2}} \\
\tilde{V}_1 &= -\left(\frac{1 - \Gamma^2}{4}\right) b \frac{tf_1}{Y}
\end{aligned}
\tag{II-27}
$$

As $t \to -\infty$, it can be shown that the two solutions for f_1 are

$$f_1 \sim \text{Const. } t^{-1}$$

$$\sim \text{Const. } t\, e^{-t^2/2} \tag{II-28}$$

Thus, the second solution is the one with which both boundary conditions, $f_1 \to 0$ and $tf_1 \to 0$, may be satisfied.

In order that the velocity components will match with their counterparts in the outer solution, it is clear that $\tilde{U}_1$ and $\tilde{V}_1$ must be proportional to ϕ and ϕ^2, respectively (see eqs. II-7). Hence, f must be proportional to t, as $t \to \infty$. Then, the next term must be small compared to t. In fact, it can be shown that the asymptotic form of the solution for f_1 is

$$f_1 \sim 2t + \frac{D_1}{t} + \ldots, \qquad D_1 = \text{Const.} \tag{II-29}$$

The first term, when substituted in (II-27), results in perfect matching with the corresponding terms in the zeroth-order outer expansion (II-7). However, there is a significant difference between the above solution and the corresponding one for $M_1 = 1$. In the $M_1 = 1$ case, there was no undetermined constant in the first-order inner solution. In fact, the inner solution was employed through the matching conditions, to find the unknown constants in the outer solution. This will not be possible in the present case, because D_1 is at present, unknown. The difference between the two cases arises as a result of the order of the first terms in the inner expansions in each case. In the present case, where M_1 is arbitrary, the first terms are of order $Re^{-1/2}$. Now, the first-order terms in a boundary-layer solution are also of order $Re^{-1/2}$, so it seems clear that the unknown constant in (II-29) must be found by matching with the boundary-layer solution. This situation does not arise in the $M_1 = 1$ case, where the first-order inner terms are of order $Re^{-2/3}$ and no matching with the boundary layer is necessary. It is seen that, in the $M_1 = 1$ case, the initial effects of the boundary layer may be eliminated by ignoring the $O(Re^{-1/2})$ terms that would be necessary, were a boundary layer considered. In the $M_1 \neq 1$ case, it is impossible to ignore the initial boundary-layer effects since the first-order terms are, themselves, of order $Re^{-1/2}$.

The second-order inner solutions are found in a similar fashion. Thus,

$$\tilde{\rho}_2 = -\frac{\tilde{U}_2}{\sin\mu_1}$$

$$\tilde{P}_2 = -\frac{\gamma}{\sin\mu_1}\tilde{U}_2$$

$$\tilde{T}_2 = -\frac{\tilde{U}_2}{\beta\sin\mu_1} \qquad \text{(II-30)}$$

$$\tilde{h}_{t_2} = 0$$

$$(\tilde{V}_2)_{\tilde{x}} - (\tilde{U}_2)_{\tilde{y}} = 0$$

$$(\tilde{U}_2)_{\tilde{x}}(\gamma + 1)\tilde{U}_1 \sin\mu_1 + (\tilde{U}_1)_{\tilde{x}}(\gamma + 1)\tilde{U}_2 \sin\mu_1 + 2(\sin\mu_1)(\cos\mu_1)(\tilde{U}_2)_{\tilde{y}} = b(\sin\mu_1)(\tilde{U}_2)_{\tilde{x}\tilde{x}}$$

Again, a potential function $\Phi_2(X, Y)$, is defined such that

$$\tilde{U}_2 = \sqrt{\frac{\cos\mu_1}{b}}(\Phi_2)_X \qquad \text{(II-31)}$$

$$\tilde{V}_2 = (\Phi_2)_Y$$

and the last of equations II-30 becomes

$$(\Phi_2)_{XXX} = (\Phi_1)_X(\Phi_2)_{XX} + (\Phi_2)_X(\Phi_1)_{XX} + 2(\Phi)_{XY} \qquad \text{(II-32)}$$

Finally, if

$$\Phi_2(X, Y) = Y^{-1/2}F_2(t)$$

and (II-33)

$$f_2 = \frac{dF_2}{dt} = F_2'$$

equations II-32 and II-31 reduce to the following:

$$f_2'' + f_2'(t - f_1) + f_2(2 - f_1') = 0$$

$$\tilde{U}_2 = \sqrt{\frac{\cos\mu_1}{b}}\frac{f_2}{Y} \qquad \text{(II-34)}$$

$$\tilde{V}_2 = -\frac{1}{2Y^{3/2}}(F_2 + tF_2')$$

It can be shown that as $t \to -\infty$ a solution exists such that f_2, F_2, and tF_2' all tend to zero. Also, as $t \to +\infty$, the proper asymptotic solution is

$$f_2 \sim b_o\left(1 - \frac{D_1}{2t^2} + \ldots\right) \tag{II-35}$$

where b_o is an unknown constant.

Again, there is a difference between the $M_1 = 1$ case and the present case in that the unknown b_o appears in the latter. Although the first $\ln$ Re term which appears in the semi-infinite flat-plate boundary layer is the term of order $Re^{-3/2}$ $\ln$ Re (e.g., ref. [5]) it may very well be that in the present case, where the boundary layer turns a corner, the first term involving $\ln$ Re is of order Re^{-1} $\ln$ Re. However, this is only conjecture and cannot be proved, since the boundary-layer solution is not known. In any event, the above solution shows that another unknown constant arises in the inner solutions.

The only third-order term that is necessary is $\tilde{h}_{t_3}$, for the same reasons given in the text for the $M_1 = 1$ case. The equation is

$$(\tilde{h}_t)_{3\tilde{x}} = \left(\frac{4}{3} - \frac{1}{Pr}\right)(U_1)_{\tilde{x}\tilde{x}} \tag{II-36}$$

Hence, integrating, and applying the boundary condition that $\tilde{h}_{t_3} \to 0$ as $\tilde{x} \to -\infty$, for $\tilde{y}$ fixed,

$$\tilde{h}_{t_3} = \left(\frac{4}{3} - \frac{1}{Pr}\right)(\tilde{U}_1)_{\tilde{x}} \tag{II-37}$$

If the inner solutions are written in the outer variables, the resulting expansions for u, v, and h_t are:

$$\begin{aligned}
u &\sim \cos\mu_1 + (\sin\mu_1)\phi - \frac{\Gamma^2}{2}(\cos\mu_1)\phi^2 + \frac{1}{Re}D_1\left(\frac{1-\Gamma^2}{4}\right)\frac{b}{r} + \ldots \\
v &\sim \sin\mu_1 + \Gamma^2(\cos\mu_1)\phi - \frac{1}{2}(\sin\mu_1)\phi^2 + \frac{\ln Re}{Re}\sqrt{\frac{\cos\mu_1}{b}}\,\frac{b_o}{r} \\
&\quad + \frac{1}{Re}\left[\frac{\ln r}{r}\sqrt{\frac{\cos\mu_1}{b}}\,b_o + D_1\frac{(1-\Gamma^2)b}{2r\phi}\right] + \ldots \\
h_t &\sim \frac{\mathbb{M}^2}{2\Gamma^2} + \frac{1}{Re}(\cos\mu_1)\left(\frac{4}{3} - \frac{1}{Pr}\right)(1-\Gamma^2)\frac{1}{r} + \ldots \\
&\qquad \phi \ll 1 \qquad r = O(1)
\end{aligned} \tag{II-38}$$

Just as in the $M_1 = 1$ case, the fact that there is no term of order Re^{-1} $\ln$ Re in the expansion for U indicates that, within this term, terms of order ϕ canceled, so that at least to order ϕ^2

this term is zero. As it turns out, the corresponding term in the outer expansion has a constant as its first term; so this constant is zero. Equations II-38, then, are the expansions to which the outer solutions must be matched term by term.

The outer solutions may be found more simply than the inner solutions. If we compare (II-16) with the corresponding $M_1 = 1$ equations, (3), it is clear that, if P and T in the $M_1 = 1$ equations are replaced by $P \sin^2 \mu_1$, and $T \sin^2 \mu_1$, the general $M_1 \neq 1$ equations result. Hence, since the expansions have exactly the same form in each case, the general solutions for each order approximation must be the same, if the above substitutions are made for P and T.

The zeroth-order solutions are the Prandtl-Meyer solutions and have been given in (II-3). The first-order equations may be written immediately, in view of the above argument, and are as follows:

$$\rho^{(1)} = -\frac{\rho^{(0)}}{v^{(0)}}v^{(1)} + \frac{\eta^{(1)}}{v^{(0)}}$$

$$u^{(1)} = -\frac{\eta^{(1)}u^{(0)}}{\eta^{(0)}}$$

$$\left(\sin^2 \mu_1\right)\frac{P^{(1)}}{\gamma} = -\eta^{(0)}v^{(1)} - \eta^{(1)}v^{(0)} \tag{II-39}$$

$$h_t^{(1)} = \frac{H_1}{\eta^{(0)}}$$

$$\left(\sin^2 \mu_1\right)T^{(1)} = \frac{1}{\beta\eta^{(0)}}\left(H_1 + \eta^{(1)}u^{(0)2} - \eta^{(0)}v^{(0)}v^{(1)}\right)$$

where

$$\eta^{(1)} = \text{constant}$$

$$H_1 = \text{constant} \tag{II-40}$$

$$\eta = \rho v$$

If $u^{(0)}$, $\rho^{(0)}$, and $v^{(0)}$ (see eqs. II-3), are expanded for $\phi << 1$ and the results substituted in (II-39), the first-order terms, of order Re^{-1} ℓn Re, may be compared with the corresponding terms from the inner solutions, equations II-38. From this matching, it is seen that

$$\eta^{(1)} = 0, \qquad H_1 = 0$$

so that

$$u^{(1)} \equiv 0$$
$$h_t^{(1)} \equiv 0 \tag{II-41}$$

Again, the first-order solutions contain a redundant equation, so that, although $u^{(1)}$ and $h_t^{(1)}$ may be found, $P^{(1)}$, $\rho^{(1)}$, and $T^{(1)}$ may be found only in terms of $v^{(1)}$. In order to obtain $v^{(1)}$, the second-order equations must be solved. The solutions yield $h_t^{(2)}$, $u^{(2)}$, and $v^{(1)}$, but $P^{(2)}$, $\rho^{(2)}$, and $T^{(2)}$ may be found only in terms of $v^{(2)}$, which must be found from matching with a boundary-layer solution. The second-order solutions are, then,

$$h_t^{(2)} = \left(\frac{4}{3} - \frac{1}{Pr}\right)(1 - \Gamma^2)\frac{\mu^{(0)}u^{(0)}}{\rho^{(0)}} + \frac{H_2}{\eta^{(0)}}$$

$$u^{(2)} = [\gamma G + (3/4)bf^{(0)} - N]/\rho^{(0)}u^{(0)}$$

$$\left(\sin^2 \mu_1\right)\frac{P^{(2)}}{\gamma} + \eta^{(0)}v^{(2)} = G - \rho^{(0)}u^{(0)}u^{(2)} + f^{(0)} \tag{II-42}$$

$$\rho^{(0)}v^{(2)} + \rho^{(2)}v^{(0)} = N/v^{(0)}$$

$$v^{(1)} = N'/\rho^{(0)}u^{(0)} + \Gamma^2 N/\eta^{(0)}$$

where H_2 is a constant, N is given in (66), G is given in (67), and $f^{(0)}$ is given following (64).

If the solutions for $h_t^{(2)}$, $u^{(2)}$, and $v^{(1)}$ are expanded for $\phi \ll 1$, the following expressions result:

$$u^{(2)} \sim b(1 - \Gamma^2) - B_2 \tan^{1/2}\mu_1 + \gamma B_1 \tan \mu_1 + \ldots$$

$$v^{(1)} \sim -2b(1 - \Gamma^2)\Gamma^2 \cot \mu_1 + \frac{B_2}{2}\left(\tan^{1/2}\mu_1\right)\left(\tan \mu_1 - (1 - 2\Gamma^2)\cot \mu_1\right) - \gamma\frac{B_1}{2}\tan \mu_1 \tag{II-43}$$

$$h_t^{(2)} \sim \left(\frac{4}{3} - \frac{1}{Pr}\right)(1 - \Gamma^2)\cos \mu_1 + \frac{H_2}{\sin \mu_1}$$

In the above equations, B_1 and B_2 are constants, replacing C_1 and C_2 respectively in (66) and (67) for this case, where definite integral between 0 and ϕ were written for G and N. Equations II-43 are to be compared with the terms multiplied by Re^{-1} in (II-38). Thus it is seen that

$$H_2 = 0$$

$$D_1 \frac{(1 - \Gamma^2)b}{4} = b(1 - \Gamma^2) - B_2 \tan^{1/2}\mu_1 + \gamma B_1 \tan\mu_1 \qquad \text{(II-44)}$$

$$\sqrt{\frac{\cos\mu_1}{b}}\, b_o = -2b(1 - \Gamma^2)\Gamma^2 \cot\mu_1 + \frac{B_2}{2}\left(\tan^{1/2}\mu_1\right)\left[\tan\mu_1 - (1 - 2\Gamma^2)\cot\mu_1\right] - \frac{\gamma B_1}{2}\tan\mu_1$$

As mentioned previously, there are four constants and two equations. However, the inner solutions are to be matched with boundary-layer solution as yet unknown, which will set D_1 and b_o. Then B_1 and B_2, the outer constants, may be found from (II-44). In any event, there is a functional matching which does exist, validating the general problem formulation and expansion forms.

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The compressible flow of a viscous heat-conducting gas around a corner is considered. The solutions are written in terms of asymptotic expansions, valid in the region far, compared to a viscous length, from the corner, so that the zeroth-order solutions are the classical Prandtl-Meyer solutions. The method of inner and outer expansions is used where the inner region encloses the first Mach line emanating from the corner. Boundary-layer effects are minimized to the extent that the initial flow is assumed to be uniform. The first corrections due to viscosity and heat conduction are calculated, and it is shown that the tangential velocity component can be calculated only to within an unknown function of the turning angle, which must be found by matching with a still-unknown boundary-layer solution. Although the text deals with the case where the initial flow is sonic, corresponding calculations for the case where the initial velocity is supersonic are given in appendix.

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